Case description 4GC10 Mechanical Design Project (2019-2020)
(Also read the descriptions for 4GC10 in Canvas, and the information posted on the 4GC10 website http://www.mate.tue.nl/~piet/)

Design and realization of a mechanism with negative stiffness

Project coordinators:

Dr. Ir. L.F.P. Etman, GEM-Z 0.123, tel. 040-2472879, l.f.p.etman@tue.nl
Dr. Ir. P.J.G. Schreurs, GEM-Z 4.129, tel. 040-2472778, p.j.g.schreurs@tue.nl
Prof. Dr. Ir. J.P.M.B. Vermeulen, GEM-Z 0.115/GEM-N 0.68, tel. 06-12591693, j.p.m.b.vermeulen@tue.nl
Ir. S.J. Hermanussen, GEM-N 0.68, tel. 040-2474580, s.j.hermanussen@tue.nl

Education and research technician:

P.W.C. van Hoof, GEM-Z -1.13, tel. 040-2472824, P.W.C.v.Hoof@tue.nl

Goal

The goal for this project is to design and realize a mechanism with negative stiffness, which is used to reduce the positive stiffness of an existing elastic straight guide mechanism (see example in Figure 1). The negative stiffness has to be adjustable such that a total net stiffness is created, which is as low as possible.

Negative stiffness

Elastically deforming elements, such as flexures, are highly useful to create mechanisms or rotational joints that are free of backlash, friction and hysteresis, which is essential for minimal positioning errors. However, elastic elements, require a force \( F \) or moment \( M \) when brought outside the undeformed position, which is proportional to the displacement \( u \) or rotation \( \varphi \) according to:

\[
F = cu \quad (1)
\]

\[
M = k\varphi \quad (2)
\]

The constant of proportionality is called ‘stiffness’ \( c \) in [N/m] or \( k \) in [Nm/rad].

In addition to the required force, the positive stiffness of elastic mechanisms or pneumatic springs as applied in vibration isolation systems, implies a limitation in isolation (or resonance) frequency \( f_e \) [Hz], defined by the (positive) stiffness \( c_p \) [N/m] and the supported mass \( m \) [kg] according to:

\[
f_e = \frac{1}{2\pi} \sqrt{\frac{c_p}{m}} \quad (3)
\]

Negative spring stiffness implies a co-operating (helping) force that is delivering energy when the spring is displaced, as opposed to a positive spring stiffness that is providing a counteracting force and is storing energy (see Figure 2 and Figure 3). In addition to using opposing magnetic fields from permanent magnets, negative stiffness can be created mechanically through springs under pretension at an angle approximately orthogonal to the direction of movement (see Figure 4) or buckled leaf spring that is
brought from a first buckling mode into a higher order buckling mode (such as the lid of jam jar), see [Eijk, 1979] and [Hoek, 1985].

In Figure 5, both an unloaded leaf spring with length \( l \) (top), and a buckled leaf spring (bottom) are shown. Through a horizontal relative displacement \( \delta l \) of the points of fixation of the leaf spring, the leaf spring buckles into its first buckling mode, thereby moving the center point C to position \( h_1 \) or \( h_2 \). Moving point C from \( h_1 \) over some distance in the direction of \( h_2 \) (or vice versa), will bring the buckled leaf spring into the third order buckling mode (polynomial with three zero slope sections). Figure 6 shows the resulting force-displacement diagram. The solid line shows a nearly constant negative stiffness over a large stroke from \( h_{min} \) to \( h_{max} \).

The negative stiffness of a buckled leaf spring depends on multiple parameters, such as leaf spring dimensions, material properties, and displacement \( \delta l \) (see [Dijksman, 1979] and [Hoek, 1985]). To cope with tolerances in the manufacturing that result in a deviation of the negative stiffness from its targeted value, it is desired to use a tuning knob or mechanism, i.e. make the negative stiffness adjustable in a specified range.
Assignment

The assignment is to design a negative stiffness mechanism based on one or more buckled leaf springs including a tuning mechanism as mentioned, which allows to adjust the negative stiffness to a desired value that matches the value of a positive stiffness as close as possible.

The design of the adjustable negative stiffness mechanism is subject to the following requirements:

- The negative stiffness mechanism will be based on leaf springs made out of steel with a thickness $t$ of 0.3 mm. Negative stiffness is required over a range of ± 5 mm (10 mm total range).
- The leaf springs should be fixed such that a reproducible mechanism is created, which is stable over a large number of deflections.
- The negative stiffness mechanism shall be mounted via a specified interface to the elastic straight guide mechanism (see Figure 7), which has a positive stiffness $c_p$ between $2.0e3$ and $4.0e3$ N/m. The interface consist of a flat plate with a diameter of $\varnothing 180$ mm and a thickness of 15 mm with an array of $3x3$ M4x0.7 through holes at a pitch of 25 mm (see Figure 8).
- The resolution of the tuning mechanism should be such that it allows for an adjustment of the negative stiffness that matches the value of a positive stiffness as close as possible, thereby creating a resonance frequency of the mass-spring system, which is as low as possible. This is one of the criteria for the evaluation of the assignment.
- Available materials:
  - Steel plate of 200x300 mm with a thickness of 0.3 mm.
  - Aluminum plate of 300x400 with a thickness of 10 mm.
  - M4 nuts, washers and bolts in length 20 mm to 50mm, strength 8.8.
  - Tension springs 2.8 N/mm (Tevema T31361).
- Manufacturing of the designed components can be done via:
  - Collective outsourcing of laser machining to Feijen Metaaltechniek. Outsourcing will be done during the Christmas break. Design drawings have to be provided in DXF files, and have to be sent to Sander Hermanussen via e-mail (s.j.hermanussen@tue.nl). Additional information, e.g. on manufacturing tolerances and color coding for detailed drawings will be provided separately. Timing will be provided in the Project information 4GC10.
  - The machine shop of the Dynamics & Control group of the department of Mechanical Engineering via Pieter van Hoof. In view of the large amount of groups, interaction on manufacturing (interaction on design via the design team in GEM-Z 3A13) is limited as specified in the Project Information 4GC10. Please be aware, that maximum half an hour of manufacturing time is available and machining operations are limited to drilling, milling and turning.
- Focus of the assignment is on the concept phase incl. analyses (phase 2), selection phase (phase 3), detailing phase (phase 4), and test phase (phase 6) in accordance with the 7-phases plan used in the Design Based Learning projects and the 4WBB0 Engineering design course.

![Figure 7](image1.png)  
*Figure 7 Schematic representation of the test setup for the final experimental verification.*

![Figure 8](image2.png)  
*Figure 8 Front view of the straight guide mechanism incl. interface for mounting the negative stiffness mechanism.*
Additional information regarding the non-linear FEM (buckled leaf spring), CAD-to-DXF generation, and technical drawings incl. manufacturing tolerances will be given during the project supporting workshops (for details see Project information).

**Test setup**

A test setup will be used for experimental verification. Prior to the final oral defense, the negative stiffness mechanism shall be mounted at a table (specification will follow) and to the straight guide mechanism and tuned to a minimum resonance frequency. Please be aware, that maximum 15 minutes are available for install and adjustment.

**References**