Design and realization of a mechanism with negative stiffness

4GC10 Mechanical design project, November 2019
Hans Vermeulen, Pascal Etman, Piet Schreurs, Sander Hermanussen
Goal

Design and realize an adjustable negative stiffness mechanism

• Design and realize a mechanism with negative stiffness, which is used to reduce the positive stiffness of an existing straight guide elastic mechanism

• The negative stiffness has to be adjustable such that a total net stiffness is created, which is as low as possible

Elastic straight guide mechanism based on six folded leaf springs [Rosielle, 2017].
Limitations in using elastic elements are the limitation of stroke defined by the elastic limit (incl. safety factor), and the increase of lateral force as a result of the positive spring stiffness. The latter effect, however, can be overcome by using an additional spring element to provide negative stiffness. Negative spring stiffness implies a co-operating (helping) force when the spring is displaced.
Introduction negative stiffness (2/4)

Negative stiffness is delivering a ‘helping’ force delivering energy

Positive stiffness: Counteracting force that is aligned with displacement storing energy

Negative stiffness: Helping force that is opposite to displacement delivering energy

Positive / negative spring stiffness implies a counteracting / ‘helping’ force when the spring is displaced

Positive stiffness $c_p$ compensated by negative stiffness $c_n$ consisting of preloaded springs with positive stiffness $c_{p'}$
Introduction negative stiffness (3/4)
A buckled leaf spring may provide a force opposite to displacement

Central part of buckled leaf spring $M$ brought back to the central position experiences a negative stiffness

Bucking force:

$$F_b = \frac{4\pi^2 EI}{l^2}$$

Leaf spring (left) brought from first buckling mode (middle) to second bucking mode (right)
Introduction negative stiffness (4/4)

Negative stiffness adjustable e.g. by variation of mounting angle $\varphi$

Second buckling mode ($\Delta l/l < 0.25$):

$$c_n = -210 \frac{EI}{l^3}$$

$$\sigma_{b,\text{max}} = 8.9Eh \sqrt{\frac{\Delta l}{l^3}}$$

Third buckling mode:

$$c_n = -64\pi^2 \frac{EI}{l^3} = -631 \frac{EI}{l^3}$$

$$\sigma_{b,\text{max}} = 4\pi Eh \sqrt{\frac{\Delta l}{l^3}} = 12.6Eh \sqrt{\frac{\Delta l}{l^3}}$$
Negative stiffness vibration isolator (1/3)

Adding negative stiffness allows for independent optimization of isolation frequency and load capacity.

Pneumatic isolator stiffness:

\[ c_p = \gamma \Delta p \frac{A^2}{V} \]

Isolation frequency:

\[ f_e = \frac{1}{2\pi} \sqrt{\frac{c_p}{m}} \]

Load capacity:

\[ F_p = \Delta p A \]

Note:

\( \Delta p \) = pressure difference [bar]

\( A \) = surface area of piston [m²]

\( V \) = gas volume [m³]

\( \gamma \) = polytrope exponent (1.4 for air) [-]

\( m \) = supported mass [kg]
Negative stiffness vibration isolator (2/3)

Third order buckling for high negative stiffness, hence low isolation frequency

Schematic representation of an unloaded leaf spring (top), and buckled leaf spring (bottom) that is brought from its first- into its third-order buckling mode.

Force-displacement diagram of a buckled leaf spring in the third-order buckling mode.
Negative stiffness vibration isolator (3/3)
Various analysis results match rather closely

Third buckling mode according to:

- [Hoek, 1985]: \( c_n = -567 \frac{EI}{l^3} \)
- [Dijksman, 1979]: \( c_n = -631 \frac{B}{l^3} \)

, where \( EI \leq B \leq \frac{EI}{(1 - \nu^2)} \)

\( B = EI \) if \( w \approx t \) or \( w^2/Rt \ll 1 \)

\( B = \frac{EI}{(1 - \nu^2)} \) elsewise

Ref:


Note:

- \( l \) = unbuckled length of the leaf spring [m]
- \( B \) = bending stiffness [Nm/rad]
- \( \nu \) = Poisson ratio [-]
- \( w \) = width of the leaf spring [m]
- \( t \) = thickness of the leaf spring [m]
- \( R \) = radius of curvature the leaf spring [m]
Negative stiffness mechanism design:

- Based on one or more buckled leaf springs made out of spring steel with thickness $t = 0.3$ mm
- Leaf springs fixed such that a reproducible mechanism is created
- Mounted via a specific interface to elastic straight guide mechanism with a positive stiffness $c_p$ between $2.0e3$ and $4.0e3$ N/m (see next slide)
- Include a tuning mechanism with a resolution such that the negative stiffness matched the positive straight guide stiffness as close as possible (verified experimentally)

See assignment description for further details
Negative stiffness mechanism realization:

- Collective outsourcing of laser machining to Feijen Metaaltechniek during Christmas break
- Detailed design drawings provided in DXF files sent to Sander Hermanussen (s.j.hermanussen@tue.nl)
- Machine shop of the Dynamics & Control group of the department of Mechanical Engineering via Pieter van Hoof (p.w.c.v.hoof@tue.nl)
- Interaction on manufacturing is possible (see Project information), maximum half an hour of manufacturing time available, limited to drilling, milling and turning

See assignment description for further details, e.g. on tolerances and timing
Project assignment (3/3)
Test setup for experimental verification prior to oral defense

Schematic representation the test setup for the final experimental verification.

Front view of the straight guide mechanism incl. interface for mounting the negative stiffness mechanism.
References

- Soemers, H.M.J.R., Design principles for precision mechanisms, 2010