On the consequences of non linear constitutive modelling of brain tissue for injury prediction with numerical head models

M. Hrapko, J. A. W. van Dommelen, G. W. M. Peters and J. S. H. M. Wismans


(Received 27 March 2008; final version received 11 July 2008)

The objective of this work was to investigate the influences of constitutive non linearities of brain tissue in numerical head model simulations by comparing the performance of a recently developed non linear constitutive model [10, 11] with a simplified version, based on neo-Hookean elastic behaviour, and with a previously developed constitutive model [6]. Numerical simulation results from an existing 3D head model in the explicit Finite Element code MADYMO were compared. A head model containing a sliding interface between the brain and the skull was used and results were compared with the results obtained with a previously validated version possessing a tied skull-brain interface. For these head models, the effects of different constitutive models were systematically investigated for different loading directions and varying loading amplitudes in both translation and rotation. In the case of the simplified and fully non linear version of the model of Hrapko et al. [10, 11], the response predicted with a head model for varying conditions (i.e. severity and type of loading) varies consistently with the constitutive behaviour. Consequently, when used in a finite element head model, the response can be scaled according to the constitutive model used. However, the differences found when using the non linear model of Brands et al. [5] were dependent on the loading conditions. Hence this model is less suitable for use in a numerical head model.

Keywords: brain tissue; constitutive model; mechanical properties; finite element method

1. Introduction

More than one third of all injuries are traumatic brain injuries (TBI), which also represent one of the major causes of death resulting from traffic accidents [15]. Despite the major advances in prevention and treatment, head injury remains a major health and social problem. TBI can be caused when the head is suddenly struck by an object with or without the object penetrating the skull and the brain. These injuries can be divided into primary injuries, which occur at the moment of impact, and secondary injuries, which develop at a later stage. The majority of brain injuries are caused by diffuse axonal injury (DAI) characterised by microscopic damage of axons. DAI can occur without any direct impact on the head, as it can be the result of rapid acceleration/deceleration. DAI is thought to be the most common and important pathology in mild, moderate and severe traumatic brain injury [13]. It may develop over a period of hours or even days after an accident.

To develop protective measures, an accurate assessment of the risk of injury is required. In the early sixties, the currently used Head Injury Criterion was developed [25] on the basis of the Wayne State Tolerance Curve [9]. However, it is based on translational head acceleration only and it does not allow for a distinction between different injury mechanisms. By using a detailed Finite Element (FE) model of the head, the behaviour of the brain can be predicted for any loading condition and such models can serve to provide improved injury criteria which can be implemented into safety standards. These FE models often contain a detailed geometrical description of the anatomical components but lack accurate descriptions of the mechanical behaviour of the brain tissue.

A number of constitutive models have been developed to describe the mechanical behaviour of brain tissue. Some authors propose integral models [16, 17, 20, 21, 23, 24] often in combination with Ogden hyper-elasticity, whereas others propose differential models [2, 6, 8, 10, 23] which are more suitable for implementation in numerical codes. Important for the use of these models to predict injury may be the ability to correctly describe the non linear behaviour for complex loading histories and large deformations in different deformation modes.

To be able to use uniform tissue level injury criteria, i.e. injury criteria that can be transferred between different head models, accurate stress and strain levels should be predicted, for which the correct description of the constitutive response of brain tissue is required. Therefore, the objective of this study is to investigate the consequences of using different constitutive descriptions of the mechanical response of brain tissue in FE head models for injury...
Table 1. Material parameters of the various anatomical components of the FE head model.

<table>
<thead>
<tr>
<th></th>
<th>Young’s modulus $E$ (Pa)</th>
<th>Poisson’s ratio $\nu$ (-)</th>
<th>Mass density $\rho$ (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skull</td>
<td>—</td>
<td>—</td>
<td>2070</td>
</tr>
<tr>
<td>Dura mater</td>
<td>$3.15 \times 10^7$</td>
<td>0.45</td>
<td>1130</td>
</tr>
<tr>
<td>Pia mater</td>
<td>$3.15 \times 10^7$</td>
<td>0.45</td>
<td>1130</td>
</tr>
</tbody>
</table>

Elastic and viscous parameters

Viscoelastic parameters

<table>
<thead>
<tr>
<th>Brain (NLVE-A)</th>
<th>$G_0 = 182.9$ Pa</th>
<th>$G_1 = 9884$ Pa</th>
<th>$\lambda_1 = 0.00013$ s</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A = 0.73$</td>
<td>$G_2 = 835.5$ Pa</td>
<td>$\lambda_2 = 0.012$ s</td>
<td></td>
</tr>
<tr>
<td>$C = 15.6$</td>
<td>$G_3 = 231.2$ Pa</td>
<td>$\lambda_3 = 0.35$ s</td>
<td></td>
</tr>
<tr>
<td>$n = 1.65$</td>
<td>$G_4 = 67.1$ Pa</td>
<td>$\lambda_4 = 4.62$ s</td>
<td></td>
</tr>
<tr>
<td>$\tau_0 = 9.7$ Pa</td>
<td>$G_5 = 2.79$ Pa</td>
<td>$\lambda_5 = 12.1$ s</td>
<td></td>
</tr>
<tr>
<td>$k = 0.39$</td>
<td>$G_6 = 2.79$ Pa</td>
<td>$\lambda_6 = 54.3$ s</td>
<td></td>
</tr>
<tr>
<td>$K = 2.5$ GPa</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Prediction. For this purpose, a non linear viscoelastic constitutive model for brain tissue is used [10]. This model has shown to provide a good prediction of the response to both shear and compression [11]. Numerical simulations using the constitutive model in a 3D head model were compared with predictions using a simplified version of this model and the constitutive model developed by Brands et al. [6]. These constitutive models are used within a 3D head model with a sliding interface between the brain and the skull. However first, this sliding interface model is also compared to a previously validated 3D head model with a tied interface [5, 7]. The effect of the size of a subdural space in FE models has been investigated by Kleiven and Von Holst [14], showing an increase in relative motion between the brain and the skull with an increasing size of the subdural space. In a previous study, Al-Bsharat et al. [1] have compared predictions of models with tied and sliding interfaces and found the results of a sliding interface model to compare better with experimental pressure data of Nahum et al. [19]. However, Bradshaw and Morfey [3] have concluded that varying the shear modulus by an order of magnitude produces little change in pressure response in an FE model of TBI, while the peak maximum principal strain is very sensitive to changes in the shear modulus. Therefore, the head model with the sliding interface will be used to assess the consequences of the different constitutive models for brain tissue for equivalent stress and strain predictions.

2.1. Effect of the skull-brain interface conditions

To investigate the consequences of using different skull-brain interfaces in a finite element head model, two head models are compared.

First, a model developed by Claessens et al. [7] and Brands et al. [5] is used. In this model the brain, dura mater and skull are tied, not allowing sliding and separation of each other. This model will be referred to as the tied interface model and was validated [7] using the experimental data of Nahum et al. [19]. All anatomical components are described by solid, reduced integration, eight node brick elements. The skull consists of 3212 elements, the dura mater consists of 3178 elements, and the brain is modelled by 7478 elements. The skull is assumed to be rigid and the dura mater is modelled as a linear elastic material. Material parameters of these components are summarised in Table 1.

For a comparison of these models, the brain tissue was assumed to be isotropic, homogeneous and it was modelled with the non linear viscoelastic model developed in Hrapko et al. [10]. This model was validated for both shear and compression [10]. The model, which is summarised in Appendix A, is of a multi-mode Maxwell type and consists of a non linear elastic mode in combination with a number of viscoelastic modes. These viscoelastic modes consist of an elastic Mooney-Rivlin model and a viscous Ellis model. The non linear equilibrium mode is described.

2. Methods

An existing 3D numerical head model with different skull-brain interfaces is used to investigate the consequences of constitutive non linearities. First, the effect of the different interface conditions will be examined. Thereafter, influences of constitutive non linearities of brain tissue in numerical head model simulations will be investigated by comparing the performance of different constitutive models. For these simulations the explicit FE code MADYMO, version 6.3.2 is used.
by a Mooney-Rivlin-type model, modified with a non linear prefactor. The material parameters of the model (see Table 1) were determined in Hrapko et al. [10] and extended with an extra viscoelastic mode for higher frequencies based on experimental data by Shen et al. [23] and Hrapko et al. [12] who applied time-temperature superposition, see Table 1.

In addition, a more realistic skull-brain interface is used for which the original model is extended with a sliding interface [4, 18] (see Figure 1). This model will be referred to as the sliding interface model. The interface between the skull and the brain was modelled by a 0.5 mm gap between the dura mater and the pia mater. In addition to the previous model, the pia mater is composed of 3210 shell elements with 0.5 mm thickness which completely envelope the brain. Sliding and separation is enabled between the dura mater and pia mater by a frictionless contact condition. The skull is assumed to be rigid, whereas the dura mater and the pia mater are modelled as linear elastic materials, see Table 1 for material parameters.

For a comparison of the two head models with different interface conditions, the skull was subjected to sinusoidal acceleration pulses for anterior-posterior (AP) translation and posterior-anterior (PA) rotation with six different acceleration pulse amplitudes (see Figure 2). The translational acceleration amplitudes were chosen according to HIC values of 10, 100, 500, 1000, 1500 and 2000. These values are chosen in order to study the model response in a range of loading conditions covering the non-injurious to the injurious regime. For rotational acceleration, the axis of rotation was positioned in the anatomical origin of the model corresponding to the ear hole projected to the sagittal plane, see Figure 3. The acceleration amplitudes for rotation were chosen to produce translational acceleration in the upper brain-skull interface approximately corresponding to the previously chosen translational acceleration levels.

2.2. Effect of different constitutive models
To investigate the consequences of using different constitutive models for brain tissue in an FE head model, the head model developed by Claessens et al. [7] and Brands et al. [5], extended with a sliding interface [18] was used. The brain tissue was assumed to be homogeneous, isotropic and was described by three different constitutive models.
• NHVE - a simplified version of the constitutive model developed in Hrapko et al. [10], obtained with $k = 1$, $A = 1$. The resulting model is a visco-hyperelastic model in which the elastic behaviour is neo-Hookean. The model is also known as the Upper Convected Maxwell (UCM) model.

• NLVE-A - the non linear viscoelastic model developed in Hrapko et al. [10] which is an extension of the UCM and was described in the previous section. This model is summarised in Appendix A.

• NLVE-B - the non linear constitutive model developed in Brands et al. [5], using the material parameters obtained from the same study. The model is a non linear extension of a multi-mode UCM model. The elastic behaviour is modelled by a hyper-elastic, higher order Mooney–Rivlin formulation. The inelastic, time dependent behaviour is modelled using a Newtonian law, acting on the deviatoric part of the stress only.

For each constitutive model, the skull was subjected to translational and rotational accelerations in three directions each, to study the model response in a total of six different loading directions (see Figure 3). In addition to the previously described AP translation and PA rotation, translation in lateral and superior-inferior direction and rotation in lateral and axial direction were applied. In all rotational cases, the axis of rotation was chosen to coincide with the anatomical origin of the model, corresponding to the ear hole projected to the sagittal plane. The translational and rotational acceleration pulses are similar as described in the previous section and are depicted in Figure 2. Results will be presented in terms of equivalent V on Mises stress and strain levels which are defined as: $\tilde{\sigma} = \sqrt{\frac{3}{2} \sigma^d : \sigma^d}$ and $\tilde{\varepsilon} = \sqrt{\frac{3}{2} \varepsilon^d : \varepsilon^d}$, respectively, with $\sigma$ the Cauchy stress tensor and $\varepsilon$ the Left Green-Lagrange strain tensor.

3. Results

3.1. Effect of the skull-brain interface conditions

In the case of the tied interface model, the largest deformations were produced mainly in the interior regions of the brain. For the sliding interface model, the highest deformations were exclusively found in the outer regions of the brain. This was mainly caused by the brain being obstructed by the dura mater, the falx cerebri or the tentorium cerebelli. In the sliding interface model, the brain is lagging behind the motion of the skull during a few initial milliseconds, caused by the gap between the brain and other parts.

Figure 4. FE model predictions of Von Mises stress in parasagittal (5 mm off-centre) cross-section for (a)–(b) AP translation at 4.5 ms, (c) PA rotation at 9 ms, and (d) PA rotation at 12.5 ms. (a), (c) tied interface model, (b), (d) sliding interface model.
of the model. During simulations with AP translation, the tied interface model showed the highest deformations in the anterior part of the corpus callosum at approximately 4 ms, indicated by A in Figure 4a. They originated from the brain being obstructed by the anterior part of the falx cerebri. The sliding interface model showed the maximum deformations in the posterior and inferior side of the region corresponding to the occipital lobe of the brain at approximately 5 ms, indicated by B in Figure 4b. During simulations with PA rotation, the tied interface model showed the maximum deformations in thecentre of the thalamus and midbrain regions at 12.5 ms, indicated by C in Figure 4c. The second major deformation area was observed to be in the superior side of the regions corresponding to the frontal and parietal lobe at 5.5 ms, indicated by D in Figure 4c. The sliding interface model showed the highest deformations in the anterior and superior side of the region corresponding to the frontal lobe and anterior side of the region corresponding to the temporal lobe of the brain at approximately 9 ms, indicated by D and E in Figure 4d.

For the two head models with different interface conditions, the Von Mises stress or Von Mises strain levels that are exceeded by a certain percentage of elements are displayed in Figure 5. In the case of the tied interface model, the predictions of Von Mises stress and strain are linearly increasing with the increasing translational acceleration levels, while they are increasing non-linearly for the sliding interface model. The predictions of the Von Mises stress during rotational acceleration are increasing non-linearly for both models, while the predictions of Von Mises strains are increasing almost linearly. The relative differences between the Von Mises stress and strain predictions of the tied and sliding interface models are non-linearly decreasing with applied translational acceleration from approximately 9 in the case of 290 ms$^{-2}$ to 3 in the case of 2400 ms$^{-2}$, see Figure 6. The differences between the Von Mises stress and strain predictions of the tied and sliding interface models are non-linearly decreasing with applied translational acceleration from approximately 9 in the case of 290 ms$^{-2}$ to 3 in the case of 2400 ms$^{-2}$. See Figure 6. The differences between the Von Mises stress and strain predictions of the tied and sliding interface models are non-linearly decreasing with applied translational acceleration from approximately 9 in the case of 290 ms$^{-2}$ to 3 in the case of 2400 ms$^{-2}$.
strain predictions of the tied and sliding interface models are mostly constant (approximately 1.15) with the level of applied rotational acceleration in all cases.

In addition it is remarked that the mesh of the model with a sliding interface was found to distort more at the interface than the mesh of the model with the tied interface during some stages of loading, as illustrated in Figure 7.

3.2. Effect of different constitutive models

The patterns of Von Mises stress and Von Mises strain predictions with the FE head model using different constitutive models for brain tissue are compared in Figure 8. Stress and strain predictions obtained with the different constitutive models show similar patterns. However, the magnitudes of stress and strain concentrations differ. During simulations with translational acceleration, the highest deformations were in all cases caused by the brain being obstructed by relatively stiff parts of the dura mater, the falx cerebri or the tentorium cerebelli. For all models during AP translation, the maximum deformations were observed in the posterior and inferior side of the region corresponding to the occipital lobe of the brain at approximately 5 ms, indicated by A in Figure 8.
In the case of lateral translation, the maximum deformations were observed in the lateral side of the region corresponding to the occipital lobe and the cerebellum of the brain at approximately 5 ms for the NHVE and NLVE-A models and at approximately 8 ms for the NLVE-B model. For SI translation, the maximum deformations were observed in the anterior and posterior side of the thalamus and midbrain regions of the brain at approximately 10 ms.

Figure 9. FE model predictions of Von Mises strain during lateral rotation in (a)–(c) a coronal cross-section (24 mm off-centre) with an acceleration of 7.3 krad\(^{-2}\) and (d)–(f) a coronal cross-section (−14 mm off-centre) with an acceleration of 14 krad\(^{-2}\) at 9 ms. (a), (d) NHVE model, (b), (e) NLVE-A model, (c), (f) NLVE-B model.

Figure 10. FE model predictions of Von Mises strains and Von Mises stress in (a) anterior-posterior translation, (b) lateral translation, and in (c) superior-inferior translation. ◦ = NHVE; • = NLVE-A; × = NLVE-B.
for the NHVE and NLVE-A models and at approximately 5 ms for the NLVE-B model.

For all models during PA rotation, the maximum deformations were observed in the anterior, superior and inferior side of the region corresponding to the frontal lobe and anterior side of the region corresponding to the temporal lobe of the brain at approximately 9 ms. A comparison of the strain fields obtained with the different constitutive models for lateral rotation is shown in Figure 9. The maximum deformations were observed in the posterior side of the corpus callosum of the brain which is caused by the falx cerebri at 10–13 ms, indicated by A in Figures 9 a–c. The second major deformation area was found to be the inferior side of the region corresponding to the frontal lobe at the same time, indicated by B in Figure 9 d–f. For axial rotation, the maximum deformations were observed in the anterior side of the region corresponding to the frontal lobe and the corpus callosum of the brain at approximately 8 ms for all constitutive models. The second major deformation areas were observed in the anterior and the posterior side of the corpus callosum at the same time.

Von Mises stress and Von Mises strain values that are exceeded by various amounts of elements in the predictions with the sliding interface model, using different constitutive

![Graphs showing PA, lateral, and axial rotation predictions](image)

Figure 11. FE model predictions of Von Mises strains and Von Mises stress in (a) posterior-anterior rotation, in (b) lateral rotation, and in (c) axial rotation. \( \circ = \) NHVE; \( \bullet = \) NLVE-A; \( \times = \) NLVE-B.

![Graphs showing strain and stress levels](image)

Figure 12. Difference in the Von Mises strain and Von Mises stress levels that are exceeded by 10% of elements of results plotted in Figures 10 and 11. \( \circ = \) PA rotation; \( \bullet = \) AP translation; \( \times = \) lateral rotation; translation; \( \circ = \) axial rotation; SI translation.
models for brain tissue, are shown in Figure 10 and Figure 11. For simplicity, only maximum values obtained during the first 15 ms of the simulation are shown in these figures. The entire time dependent response can be found in Figures 13 and 14 in Appendix B.

The differences found between the NLVE-A and NHVE models are slightly dependent on the applied acceleration amplitude in both translation and rotation. Also the differences found between predictions with the NLVE-A and NLVE-B model for translation are slightly dependent on the applied acceleration amplitude. However, the differences found between the NLVE-A and NLVE-B model predictions in rotation are strongly dependent on the applied acceleration amplitude. In the case of translational acceleration, the relative differences between predictions with different constitutive models are up to 55%. When assuming the NLVE-A model to be the reference model, the NHVE constitutive model predicts up to 10% smaller strains and up to 40% larger stresses, depending on the acceleration amplitude. The strain predictions from the NLVE-B constitutive model were found to be up to 15% larger than the strains obtained with the NLVE-A constitutive model, and the stresses predicted were found to be up to 55% smaller than the stresses predicted when using the NLVE-A constitutive model. In the case of rotational acceleration, the relative differences between predictions of different constitutive models are found to be up to 630% in the case of 24 krad s−2. The NHVE constitutive model predicts up to 17% smaller strains and up to 27% larger stresses than the NLVE-A model. The strain predictions from the NLVE-B constitutive model were found to be up to 450% smaller than the stress obtained with the NLVE-A constitutive model, and the strain predictions were found to be up to 630% larger than the strains predicted when using the NLVE-A constitutive model.

4. Discussion and conclusions

To obtain a reliable prediction of a mechanical response from any FE model, the constitutive models used for the materials involved have to be chosen carefully. The aim of the current study was to analyse the consequences of using different material models for brain tissue in 3D FE head models for injury prediction. For this purpose, a non linear viscoelastic constitutive model for brain tissue developed in Hrapko et al. [10], a simplified version of this model, and a previously developed non linear model [6] were compared.

Before analysing the consequences of different constitutive models, two 3D head models with different skull-brain interface conditions were compared. A more realistic, sliding interface model showed higher deformations for both translational and rotational acceleration than the model containing a tied interface. The differences were most pronounced in the case of translational acceleration and they were dependent on the applied acceleration level. For rotational acceleration the differences were not dependent on the applied acceleration level and were approximately 15%. For the sliding interface model, the higher relative motion is causing the brain to be obstructed by the relatively stiff dura mater, falx cerebri or tentorium cerebelli, therefore the higher deformations are found in the outer regions of the brain. Kleiven and Von Holst [14] showed that by increasing the thickness of the subdural space (1−7.3 mm), the relative motion between the brain and the skull is also increasing. Al-Bsharat et al. [1] have found a relative motion of about 5 mm during impact studies on animal and human cadaver heads. They have also compared FE head models with a tied and sliding interface with experimental pressure data of Nahum et al. [19] and concluded the sliding interface to be more realistic. Both of these studies have used solid elements with a low shear modulus representing the CSF layer instead of a frictionless contact between the dura mater and the pia mater, and found similar results. Based on these findings, the head model with a sliding interface was chosen for studying the influences of constitutive non linearity of brain tissue in FE simulations. In addition to being more realistic, a sliding interface was found to lead to more severe mesh distortions, compared to a fully tied interface model. To prevent these mesh distortions, an Arbitrary Lagrange Eulerian (ALE) formulation may be used, as was for example done by Willinger et al. [26].

Simulations using a non linear constitutive model developed in Hrapko et al. [10] in a 3D head model extended with a sliding interface for the skull-brain interaction were compared with predictions obtained from its simplified viscoelastic version and the constitutive model developed by Brands et al. [6]. Different constitutive models show similar patterns for the stress and strain predictions; however, the magnitudes differ. Regions exhibiting maximum deformations were similar for different constitutive models. Comparing predictions of the two non linear constitutive models, relative differences of 4.5 for Von Mises strain and 6.3 for Von Mises stress levels that are exceeded by 10% of elements were found in the case of rotational acceleration with the highest amplitude. Differences found in translational acceleration were more than a decade smaller. An important observation is that these differences were non linearly increasing with an increasing amplitude of the acceleration applied. This is due to the non linearity of the stress response of the NLVE-B constitutive model above shear strains of 0.25, see Appendix C. This non linearity is considered to be non-physical. Contrary, the variation in the Von Mises stress and Von Mises strain response between the simplified and the non linear version of the constitutive model developed in Hrapko et al. [10] which was found to accurately describe the non linear response of brain tissue in both shear and compression, was found to be only up to 17% and 40%, respectively. These differences were not dependent on the applied acceleration level in both
translation and rotation. Therefore, the simplified version of the recently developed model could be used instead of the non linear model to obtain reliable injury predictions with FE simulations, since the response can easily be scaled according to the constitutive model used.

In the current study it was shown that choosing a different constitutive model for brain tissue to be used in an FE model can have large consequences, depending on the presence of non linearities in the model. However, in the case of a simplified and non linear version of a model that has been shown to match the non linear response of brain tissue, the response predicted with a numerical head model for different conditions (i.e. severity and type of loading) varies consistently with the constitutive behaviour used. Therefore, still a reliable assessment of injury can be made with the less accurate simplified constitutive model by using a model-specific injury criterion that then is not a true threshold for injury of brain tissue.

Acknowledgements

This work was conducted within the APROSYS Integrated Project supported by the European Commission. The authors would like to thank TNO Defence, Security and Safety, Department Explosions, Ballistics and Protection for providing the numerical head model with the sliding interface. The authors would also like to thank Marike van der Horst for her support.

References

Appendix A: Compressible version of the NLVE-A constitutive model

A compressible version of the differential constitutive model by Hrapko et al. [10] has been implemented in the explicit FE code MADYMO. Material parameters for brain tissue can be found in Table 1.

The constitutive model consists of an elastic part, denoted by the subscript ‘e’ and a (deviatoric) viscoelastic part, denoted by the subscript ‘ve’, with $N$ viscoelastic modes. The total Cauchy stress tensor $\sigma$ is written as

$$\sigma = \sigma^h + \sigma^d + \sum_{i=1}^{N} \sigma_{ve_i}$$

in which superscripts ‘h’ and ‘d’ denote the hydrostatic and the deviatoric part, respectively. The hydrostatic part is defined as

$$\sigma^h = K(J - 1)I$$

Figure 13. FE model predictions of the Von Mises stress or Von Mises strain levels that are exceeded by a certain percentage of elements for translation in different directions.
where $K$ is the bulk modulus, $J = \sqrt{\det(F)}$ is the change in volume and $I_3$ is the third invariant of the Finger tensor $B$. The non linear deviatoric elastic mode is given by:

$$
\sigma^d = \frac{G_\infty}{\sqrt{I_3}} \left[ (1 - A) \exp \left( -C \sqrt{\tilde{I}_1 + (1 - b) \tilde{I}_2} - 3 \right) + A \right] \times \left[ b \tilde{B}^d - (1 - b) (\tilde{B}^{-1})^d \right],
$$

where $G_\infty$ is the elastic shear modulus, $I_3$ is the third invariant of the Finger tensor $B$, $\tilde{B} = J^{-\frac{1}{2}} B$ is the isochoric part of the Finger tensor $B$, and $I_1$ and $I_2$ are the first and second invariant of the isochoric Finger tensor $\tilde{B}$, respectively. The third term on the right hand side of Equation (1) consists of the summation of the viscoelastic modes, which are based on a decomposition of the deformation gradient tensor $F$ into an elastic deformation gradient tensor $F_e$ and a viscous deformation gradient tensor $F_v$,

$$
F = F_e \cdot F_v. \quad (4)
$$

The viscoelastic contribution to the stress is given by:

$$
\sigma^d_{ve} = \frac{G_\infty}{\sqrt{I_3}} \left[ a \tilde{B}^d - (1 - a) (\tilde{B}^{-1})^d \right]. \quad (5)
$$
with $G_i$ the shear modulus, $I_3$ the third invariant of the Finger tensor $\mathbf{B}$, $\mathbf{B}_e$ the isochoric part of the elastic Finger tensor $\mathbf{B}_e$. The viscous deformation $\mathbf{F}_v$ is assumed to be volume-invariant. The viscous rate of deformation tensor, is calculated from the flow rule, assuming incompressibility, as

$$D_v = \frac{\sigma_{\nu_0}}{2\eta_i(\tau)}$$

(6)

where the dynamic viscosity $\eta_i$ is a function of the scalar equivalent stress measure $\tau = \sqrt{\frac{1}{2} \sigma^2}$, It is described by the Ellis model, which states

$$\eta_i(\tau) = \eta_{\infty} + \frac{\eta_0 - \eta_{\infty}}{1 + \left(\frac{\tau}{\tau_0}\right)^n}$$

(7)

with subscripts $0$ and $\infty$ denoting the initial and infinite values, respectively. The initial value for viscosity is defined as $\eta_0 = G_i \lambda$, whereas the infinite viscosity is defined as $\eta_{\infty} = k \eta_0$.

Appendix B: Influence of constitutive models for varying loading conditions

The Von Mises stress and Von Mises strain levels that are exceeded by a certain percentage of elements, obtained with FE simulations with different constitutive models are shown in Figures 13 and 14.

Appendix C: Model preformance

Figure 15 shows the response of the NHVE, NLVE-A, and NLVE-B models for pure shear deformation with $\dot{\gamma} = 1 \text{ s}^{-1}$ and unconfined compression with $\dot{\varepsilon} = 0.05 \text{ s}^{-1}$, where the compressive strain is defined as $\varepsilon = \lambda - 1$ and $\lambda$ is the stretch ratio.