Understanding and Optimizing the SMX Static Mixer

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Using the Mapping Method different designs of SMX motionless mixers are analyzed and optimized. The three design parameters that constitute a specific SMX design are: The number of cross-bars over the width of channel, \( N_x \), the number of parallel cross-bars per element, \( N_p \), and the angle between opposite cross-bars \( \theta \). Optimizing \( N_x \) somewhat surprisingly reveals that in the standard design with \( N_p = 3, N_x = 6 \) is the optimum using both energy efficiency as well as compactness as criteria. Increasing \( N_x \) results in under-stretching and decreasing \( N_x \) leads to over-stretching of the interface. Increasing \( N_p \) makes interfacial stretching more effective by co-operating vortices. Comparing realized to optimal stretching, we find the optimum series for all possible SMX\((n)\) designs to obey the universal design rule \( N_p = (2/3) N_x - 1 \), for \( N_x = 3, 6, 9, 12, \ldots \).

Introducing the SMX

Static Mixers

Motionless mixers are widely used in a range of applications such as in continuous mixing of viscous liquids, blending, chemical reactions, and heat and mass transfer. Most designs of static mixers are geometrically very different, but operationally very similar. Their invention dates back to the middle of the previous century. One of the most used motionless mixers in industry is the Sulzer SMX static mixer.\(^{[2]}\) It is generally believed to provide the most compact mixing device as compared to the other static mixers like the Kenics\(^{[2,3]}\), the Ross LPD\(^{[4–6]}\), the Ross ISG\(^{[4]}\), the Multi-Flux mixer\(^{[7,8]}\), the Pulsating Mixing Reactor (PMR), the PSM, the Erestat Mixer, N-shaped pipe mixer, and the Hi-mixer (for details of these designs see Pahl and Muschelknautz)\(^{[9]}\).

As all continuous static mixers, the SMX is composed of two elements, periodically repeated in an axial direction and placed in a circular tube. The second element is an identical copy of the first element with 90° rotation in tangential direction. Each static element consists of multiple X-shaped cross-bars and the angle between these opposite cross-bars, \( \theta \), is 90°. The standard Sulzer SMX element consists of eight cross-bars (four X-shaped pairs of crossed plates over the width of the channel). Flow is induced by applying a pressure difference. If we move, in a Lagrangian way, with the fluid through the tube, we experience the crossed bars acting as intermingled combs moving in opposite direction from one wall to another. The interface between two fluids, each occupying half the channel, is touched by these eight combs that move in two pairs of four combs perpendicular to its orientation, causing the interface to be stretched and folded eight times while moving through the first element. When the fluid moves through the second 90° rotated element, stretching and folding of all interfaces formed in the first element is
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performed in a perpendicular direction. Repeating the procedure eventually leads to exponential growth in the interface lengths, and in the standard design in interface grows at a rate of (8 – 1)Nx, with Nxelem the number of successive elements.

In this paper we apply one of the most advanced and efficient modern tools available to analyze complex processes like mixing based on chaotic advection, known as the Mapping Method, to analyze mixing in the one of the most established efficient mixers, the Sulzer SMX, in an attempt to investigate whether we can understand its working principle, whether we can quantify its performance, and most challenging, whether still innovations in its design are possible and realizable. The paper is organized as follows. First, we start with introducing the design parameters of the SMX. Second, we briefly review prior studies on the SMX as reported in literature. Third, we describe the problem at hand, and the flow field calculations. Fourth, in short the Mapping Method used to analyze mixing is introduced. Finally, the most relevant outcomes of the mixing analyses and optimization results of the SMX are given followed by the ultimate design: the SMX’.

**Design Parameters**

A typical standard SMX (Sulzer Chemtech) geometry with four elements is shown in Figure 1. Each element consists of eight crossing bars over the width, Nw, forming four X-shaped pairs of cross-bars, each at ±45° with the axis of the pipe. The central crosses are once copied both in positive and negative axial direction, resulting in a total of three sets of parallel cross-bars, Np, in an element. Every second element is rotated by 90° with respect to the previous one. The aspect ratio, length to diameter l/D, is equal to one, within the standard mixer D = 52 mm, and a cross-bar thickness of t = 2 mm. The width of the cross-bars next to the tube walls (wwall) is 8 mm whereas all others are 6 mm wide.

Clearly, three design parameters determine the final fate of mixing in the SMX, see Figure 2:

- Nw, the number of cross-bars over the width of the channel (compare Figure 2(a) and 2(b)).
- Np, the number of parallel cross-bars (compare Figure 2(a) and 2(c)).
- θ, the angle between opposite cross-bars (compare Figure 2(a) and 2(d)).

**Prior Studies**

Numerous experimental and computational studies have been reported in the literature regarding the performance of SMX mixers [2,5,10–17] Tanguy and coworkers (see[15] and references therein) performed computational studies on the SMX mixer, characterizing its performance and comparing it with different alternative motionless mixers. Zalc et al[2] computed mixing patterns in the Koch-Glitsch SMX static mixer and validated the computed relative standard deviation as a mixing measure with an experimental relative standard deviation; a good agreement is reported. Mickaily-Huber et al[14] modified the design of
a SMRX mixer by changing the crossing angles between two opposite cross-bars. They found the standard 90° crossing angle to be the optimum for mixing. Note that in a SMRX, \( R \) for reactive, the internal elements are composed of circular pipes. Hrymak and coworkers (see\[16,17\] and references therein) performed numerical as well as experimental studies to characterize mixing for Newtonian as well as non-Newtonian fluids, and analyzed the effect of the number of cross-bars over width \( N \), ranging from 4 to 18. They found that a SMX design with 10 cross-bars provides the best mixing.

Most of the numerical mixing analyses reported in literature start with forward particle tracking and evaluate performance by defining statistical measures like the coefficient of variation (COV) or the standard deviation (see\[5,2,15\]). However, mixing analysis using forward particle tracking brings several inherent drawbacks. First, it requires the tracking of a huge number of particles to generate high-resolution images at far downstream locations. Second, there is no guarantee that all the space of interest at the desired location will be completely occupied by particles, due to the fact that any ordered array of particles at the inlet becomes disordered at downstream positions. This leads to a loss of accuracy in quantification of mixing. In addition, proper care must be taken while computing a statistical measure like the COV: the measure must be independent of the initial number of particles for the given grid size on which the COV is computed. Also, the grid used to compute the COV must be sufficiently fine to capture a reasonable level of mixing characteristics. Some of the studies used a small injection area of tracers (10 or 20 percent of the inlet cross-section) and then follow them in the flow field. The better the distribution of tracers at a required downstream position, the better is the mixer (see\[5,15,17\]). Here, we will show that mixing analyses using a small injection area can sometimes lead to erroneous conclusions regarding the performance of different layouts. This is explained in Section 3.1 where it is shown that the optimum found can be dependent on the injection location. One of the important suggestions mentioned by Liu et al.\[17\] indeed reflected the requirement of a larger number of passive particles to characterize all the designs in an accurate way.

Summarizing all the above aspects, we can conclude that a more advanced method is required to overcome the above mentioned disadvantages of forward particle tracking approaches. In this respect, we will show that the Mapping Method, which is based on backward particle tracking,\[18,19\] can indeed be a useful tool.

### Modeling Aspects

#### Flow Field Calculations

The question is posed whether a change in one of the three design parameters of the SMX (as mentioned in Section 1.2) can yield better mixing or worse, and the next question is how these parameters can be tuned to achieve an optimum in mixing. For all cases we need an accurate three-dimensional velocity field, and Fluent 5 is used to solve the Navier-Stokes equations. The inlet and outlet sections are composed of two empty circular tubes, each with a length of two times the diameter of the cylinder to avoid the effect of a developing flow. The mesh is generated using Gambit and contains 421,408 nodal points and 2,134,186 first-order tetrahedral elements for the standard SMX design. In most of the cases investigated here, the geometry becomes more complex and, hence, mesh sizes of typically around 6 million elements and 1 million nodal points are used. At the inlet a fully developed velocity profile is taken, and a no-slip boundary condition is applied at the tube walls and surfaces of the static elements. The fluid is assumed to be Newtonian with density and viscosity equal to 846 kg·m\(^{-3}\) and 1 Pa·s, respectively. The average inlet velocity is 0.01 m·s\(^{-1}\), yielding a Reynolds number of 0.44. Hence, the flow is clearly in the Stokes regime.

Optimization of the SMX mixer is cumbersome, since any change in the design requires re-computation of velocity field as well as of the mapping matrices. This in contrast with the optimization of lid-driven cavity flow\[20\] and the Kenics mixer,\[9\] where a few mapping computations were sufficient to analyze various designs.
Defining the Mapping Matrices

The new computationally simple-to-implement approach to obtain the mapping matrix based on backward particle tracking (see\(^\text{[18,19,21]}\)) is used. For the calculation of a mapping matrix, the cross-sectional area is divided into a grid consisting of 200 \(\times\) 200 cells, and the number of particles per cell (NPPC) used is 100 (applying a 10 \(\times\) 10 array) and, therefore, in total 4 \(\times\) 10\(^6\) particles are tracked in the flow field. Note that the NPPC should be sufficient to obtain a converged quantitative mixing measure, the flux-weighted intensity of segregation \(I_d\)\(^\text{[18]}\). To do a full analysis of mixing, we compute two separate mapping matrices \(F_i\) \((i = 1, 2)\) representative for two types of elements of a typical SMX design as shown in Figure 1 with its (1, 2, 1, 2, \ldots, etc.) sequence of elements from left to right. The A–B region is denoted as matrix \(F_2\) and the B–C region as matrix \(F_1\). Only the two middle elements are used in computing mapping, assuming that the flow field is developed, such that the periodic structure indeed is represented by these two elements. The matrices are used to obtain the concentration evolution after a number of elements \(N_{\text{elem}}\) via a computationally very fast matrix-vector multiplication:

\[
\begin{align*}
C^0 & = \Phi_1 C^0, \\
C^1 & = \Phi_2 C^0, \\
C^2 & = \Phi_2 (\Phi_2 (\ldots (\Phi_1 (\Phi_2 C^0))))
\end{align*}
\]

where \(C^0\) is initial concentration distribution.

Results

Effect of the Number of Cross-Bars over the Width, \(N_x\)

Mixing profiles for designs (fixed \(N_p = 3\) and \(\theta = 90^\text{o}\)) with a different number of cross-bars \(N_x\) are shown in Figure 3. We start the analyses of these results with the standard design with eight cross-bars. As evident from the mixing profile after 1 element of mixing \(C^1\), 4 black and 4 white striations with 7 \((8 - 1)\) interfaces in between are found, and hence this mixer results in an interface increase with a factor \((8 - 1)^{N_{\text{elem}}}\) after having passed a \(N_{\text{elem}}\) number of elements. But, of course, this is an idealized situation and some deficiencies are readily observed by examining \(C^1\) and the mixing profile after the next element \(C^2\). For example, \(C^1\) reveals that interfaces are not covering the whole cross-section and the thickness of striations is not uniform in the cross-section. Finally, we observe more “white” in top-right part of the cross-section and more “black” in the bottom-left part. This situation rotates by 90\(^\text{o}\) in each element. This leads to a non-uniform distribution in striations, which is evident even after eight elements of mixing, \(C^8\) showing the same pattern, of course with a change in intensity. All this becomes much more clear by investigating the mixing profiles for 10, 12, and 16 cross-bars, where more inhomogeneity results and mixing by interface stretching is more and more restricted to the center part of the tube. For example, the SMX with \(N_x = 16\) cross-bars shows the initial segregation of white and black material in left and right parts even after eight elements of mixing (see \(C^8\)).

Next, we investigate mixing in designs with less cross-bars than in the standard SMX, \(N_x = 4\). \(C^2\) and \(C^4\) now reveal that the interface stretching per element is that high that it...
leads to overstretching, and more white material is transported from the right into the left part, while the opposite is true for the black material. Over-stretching for $N_x = 4$ and under-stretching for $N_x \geq 8$, suggest that a design with six cross-bars could be superior and indeed the $C^0$ of the design with $N_x = 6$ cross-bars shows interfaces covering almost the total cross-section and, therefore, a more uniform distribution of striations is found as compared to any of the other designs in Figure 3. Since an increase in $N_x$ increases the pressure drop per element,[16] a quantitative comparison must be made using the flux-weighted intensity of segregation; results are shown in Figure 4(a). Indeed, also this quantitative plot reveals what it was observed qualitatively: the SMX mixer with $N_x = 6$ is superior to all others, although also the mixer with four cross-bars performs almost as good. Sometimes, not energy efficiency, as measured as the dimensionless pressure drop, is relevant to compare mixer designs but instead compactness, aiming at a minimum length (see Figure 4(b)). Surprisingly, also according to this criterion of compactness, $N_x = 6$ is the best. These findings are different from the results of Liu et al.[17] who reported that ten cross-bars was optimum. To investigate the reason for the differences found, we repeat their computations using two designs of the SMX: $N_x = 6$ and 10. Like Liu et al.,[17] we injected 40 000 particles uniformly distributed in a circle of radius 1 mm placed at the origin $(0, 0, 0)$. Next, we shifted the position of injection to the left side $(-0.01, 0, 0)$ (note that $D = 0.026$ m), see Figure 5. The top part of Figure 5 reproduces the results reported in Liu et al.[17] and, indeed, comparing mixing profiles here reveals that $N_x = 10$ performs better than $N_x = 6$. In contrast, the bottom part of Figure 5 leads to a different conclusion regarding the optimum and $N_x = 6$ is better. Apparently, two contradictory conclusions can be drawn and, therefore, using a mixing quality criterion that is based on the injection of tracers in a small area does not give reliable results. Therefore, we will keep using interfacial stretching in our investigations of finding the optimum designs in the different SMX series.

**Effect of the Number of Parallel Cross-Bars, $N_p$**

From the previous section, it becomes obvious that SMX designs with $N_p = 3$ and $N_x > 6$ under-stretch the interface and as the number of cross-bars increases, mixing rapidly deteriorates. We now consider the worst design with $N_x = 16$ in an attempt to learn how to improve mixing in this extreme case. Transverse interfacial stretching is clearly not sufficient and, therefore, ways should be found to increase the efficiency of the transverse components of the velocity. The number of parallel cross-bars, $N_p$, decisively influences this aspect, see Figure 6 where results for the SMX design with $N_x = 16$, and $N_p = 3$ are compared to the design with $N_x = 16$ and $N_p = 9$. Evidently, the design with $N_p = 9$ is able to effectively stretch all interfaces to cover the total cross-section and the mixing profiles are much more close to a perfect bakers
transformation. Of course any increase in the number of parallel cross-bars leads to an increase in pressure drop. Therefore, we will have to analyze mixing using either energy efficiency or compactness as a criterion. This will be done in Section 4.

If we now compare the results of the two designs that this far showed almost ideal interfacial stretching (no under-stretching, neither over-stretching) and thus the best mixing; \( N_x = 6, N_p = 3 \) (see Figure 3) and \( N_x = 16, N_p = 9 \) (see Figure 6) and realizing that \( N_p \) because of symmetry reasons should be odd, see Figure 2, the relation \( N_p = (2/3) N_x - 1 \) is suggested. This is checked in Figure 7 by comparing the standard SMX \( N_x = 8, N_p = 3 \) with \( N_x = 9, N_p = 3 \) and with the modified versions \( N_x = 8, N_p = 5 \) and \( N_x = 9, N_p = 5 \) that roughly obey this relation. Indeed, compared to the standard SMX both designs with \( N_p = 5 \) give superior interface stretching and, as a result, superior mixing.

**Effect of the Crossing Angle between Opposite Plates, \( \theta \)**

As known from our Ross LPD analyses,\(^6\) we can also change interface stretching by changing the angle between opposite cross-bars. Increasing the angle increases the axial length \( (l/D > 1) \) and, therefore, the transverse components act for a longer axial length, and therefore longer time, before re-orientation of flow occurs. To investigate this design aspect also for the SMX, we again take the extreme design with \( N_x = 16, N_p = 3 \) where mixing was worst. Figure 8 compares mixing profiles for \( \theta = 90^\circ \) and \( 120^\circ \) revealing that little influence is found since increased interface stretching is limited to regions close to the tube walls only, basically making the concentration distribution less homogeneous. The only result is a decrease in pressure drop per element of a factor of 1.3 at cost of axial length of course, that increases with a factor of 1.7. However, simulations with simpler mixer designs with less than this high number of 16 cross-bars (not shown here) revealed that slightly more stretching in the interface folding results (longer “hairs” at the interface), yielding slightly better mixing for 70% more length.

**Influence of an Odd or Even Number of Cross-Bars**

All existing SMX designs have a \( N_x \) which is even e.g., \( N_x = 4, 6, 8, 10, \) etc., and a \( N_p \) that is odd, usually \( N_p = 3 \). The drawback of an even \( N_x \) is as already illustrated in the \( C^1 \) and \( C^2 \) mixing profiles in Figure 3: almost never a equal distribution of black and white striations over the cross-section is realized, indicating that the ideal bakers transformation is not yet approached. An odd number of cross-bars \( N_x \) provides better mixing, see Figure 7, while Figure 9(a) illustrates that in spite of a higher pressure drop per element for the \( N_x = 9 \) design, its efficiency is still better. Besides that, Figure 9(b) shows the
superiority of the $N_p = 5$ in this $N_x = 8–9$ layout in terms of compactness.

**Understanding the SMX**

Now, we are ready to identify the optimum SMX design. As before in Figure 4 and 9, we will use both criteria energy (measured with the pressure drop needed) and compactness (measured with dimensionless mixer length). First let us consider the number of parallel cross-bars within one element, $N_p$, which, given the central cross and the symmetry around that one, per definition is odd: $N_p = 1, 3, 5, 7, 9$, etc, see Figure 10. It is clear that the parallel cross-bars split the channel in $n$ parts according to:

$$N_p = 2n - 1$$

(2)

with $n = 1, 2, 3, 4, 5,$ etc. Now we check the influence of the number of cross-bars over the width of the element, $N_x$. In Section 3.2 we concluded that the best SMX designs suggest the relation:

$$N_p = \left(\frac{2}{3}\right)N_x - 1$$

(3)

which using Equation (2) yields:

$$N_x = 3n$$

(4)

The stunning conclusion is that the basic unit of the SMX, the working horse so to say, which is the most left mixer depicted in Figure 10, and which is found for $n = 1$, is the $(n, N_p, N_x) = (1, 1, 3)$ configuration, see Figure 11(a).

Analyzing this conclusion is done by computing mixing in the -for simplicity chosen square- channels of the $(n, N_p, N_x) = (n, 2n - 1, 3n)$ series for $n = 1, 2, 3, \ldots$, see Figure 12 and 13. The basic unit $(n, N_p, N_x) = (1, 1, 3)$ design represents the mixer that gives best mixing for the lowest overall pressure drop (but the longest length), see Figure 13(a) and the highest order $(n, N_p, N_x) = (4, 7, 12)$ design represents the most compact mixer investigated here (while of course higher values of $n$ give even more compact mixers at costs of higher pressure drops), see Figure 13(b). Considering that the pressure drop in a rectangular channel with square cross-section $D$, $\Delta P_3$, when splitting the one channel into $n$ channels with edges $D/n$ increases to scale with

$$\Delta P_n = \Delta P_3 \sim n^2$$

(5)

and that mixing $M$ is proportional to total interface stretching:

$$M \sim (3n - 1)^{N_{\text{elem}}}$$

(6)

with $N_{\text{elem}}$ the number of successive elements positioned within the mixer in axial direction, we can rationalize these results, see Figure 13(c) and 13(d).
The Influence of the Transverse Velocities

The Importance of Counter-Rotating Vortices

Our understanding of the SMX essentially relates to the conclusion that a basic unit exists, based on which all different designs were intuitively developed without having this notion. Rather interesting further is that not one design realized in the SMX practice over all its years of use has one of the optimal structures, \( N_p = 2n - 1 \) and \( N_x = 3n \), except for two schematic, cut-off drawings (numbers 5 and 6) in a recent patent.\(^{[22]}\) Reason is that the basic unit, our “working horse”, was never recognized. Why is the basic unit so beautiful?

The \((n, N_p, N_x) = (1, 1, 3)\) design combines three cross-bars, two going up, one in the middle going down (or the other way around). It shows symmetry in itself, being mirrored around the middle of the square cross-section. And mirroring is important in chaotic advection. Two examples:

- The mirrored Kenics design RL-180 (right-left, co- and counter-rotating) performs in all designs with different blade twist (see\(^{[3]}\)) much better than its unmirrored counterpart RR-180, see Figure A1 in the appendix.
- Similarly gives the mirrored Ross RL-90 LPD with its co- and counter-rotating vortices, much better results in all designs (for different crossing angles) than its unmirrored RR-90 counterpart, see Figure A2 in the Appendix.

Co- and counter-rotating (clock and anti-clock wise) vortices are known to be a prerequisite for global chaotic advection throughout the whole mixing domain of interest, while in only one way rotating vortices almost always give rise to the presence of KAM boundaries separating unmixed regions in the flow, forming three dimensional islands, from the well mixed regions. The basic element \((n, N_p, N_x) = (1, 1, 3)\) integrates the two counter-rotating vortices within one element. It creates two interfaces each with length \( D \) from the one with length \( D \) that is present at its entrance. Its stretch is basically 2, see Figure 11(a) and 12(a). Of course we can integrate more than one basic element in one mixing element by putting them parallel and in series, see e.g. Figure 11(b) and Figure 12(b–d).

The Importance of Co-Operating Vortices

The basic element of a SMX with three vertically placed cross-bars and with edges \( D/n \), only functions properly if a horizontal interface, e.g. with length \( D/n \), is present at its entrance. Upon passing the element, the interface is split into two parts of 0.5 \( D/n \) each by the counter rotating vortices of the secondary flow and stretched into two vertical interfaces of length \( D/n \). Obviously if no interface...
is present at the entrance (but only black or only white material) the element does not function and only unnecessarily contributes to the pressure drop. If a vertical interface is entering a vertical basic element, it only rotates back, see Figure 14(a). The first element stretches the interface with roughly a factor 2, changing its orientation from horizontal to vertical; but subsequent rotations in the following cubes (one quarter each) transforms the pattern formed (which does not contribute to interfacial stretching) and basically after passing four cubes, we arrive at more or less the same pattern that was found at the entrance of the mixer (a little bit more hairy interfaces result). Apparently only the first element functions in interface stretching and the other \((n/C_0)\) elements just contribute to space and pressure consumption. This situation completely changes if material exchange with upper and lower mixing cubes is possible, see Figure 14(b). The explanation is given in Figure 15 that shows that cooperative vortices of the secondary flow result in effective stretching of an originally horizontal interface entering the mixer in the middle. Sufficient stretch is only found when the cross-section of the basic unit is square and either under-stretching results, see e.g. in the \((n, N_p, N_x) = (2, 3, Nx)\) mixers \(N_{smx} > 6\), in Figure 3, and the \((n, 2n-1, 9)\) mixers in Figure 15 with \(n = 1, 2\), or in the opposite, which is overstretching for \(N_{smx} < 6\) in Figure 3 and \(n = 4\) in Figure 15.

**Optimization**

**First thoughts, the SMX**

The optimum design of any motionless SMX mixer should contain only elements if interfaces with proper position and orientation are present. The first thought yields a design in which all elements with increasing order, \(n = 1, 2, 3, 4\), etc., are put in a row within one mixer, see Figure 16. Interestingly, the pressure drop in this hierarchical design of the SMX, the SMX, that is focused on the
Figure 14. The importance of coordinating vortices from the transverse flow. (a) Flow through 5 successive identical \((n, N_p, N_x) = (1, 1, 3)\) mixing cubes on a row. (b) Flow through 3 successive identical mixing cubes on a row, now forming the middle part of a \((n, N_p, N_x) = (3, 5, 9)\) mixer, illustrating that black material extends to cross the total mixer.

Figure 15. Effect of the number of parallel cross-bars \(N_p\) on interface stretching in \(N_x = 9\) mixers. Transverse velocity vectors are shown in three different cross-sections in the first element on locations 0.5 \(l\), 0.75 \(l\), and \(l\), for four mixers with increasing complexity given by \(n = 1, 2, 3, 4\). Under-stretching is found for \(n = 1, 2\) for the \((n, N_p, N_x) = (1, 1, 9)\) and \((2, 3, 9)\) mixers; correct stretching for \(n = 3\), the \((n, N_p, N_x) = (3, 5, 9)\) mixer; and over-stretching for \(n = 4\), \((n, N_p, N_x) = (4, 7, 9)\) mixer.
mixing performance of the element with the highest value of $n$, but that tries to reach that with a pressure drop related to that of the lowest value of $n$, scales as:

$$\frac{\Delta P_n}{\Delta P_1} \sim n \log(n)$$

(7)

which is substantially lower than the $\Delta P_n/\Delta P_1 \sim n^2$ found earlier in Equation (5) for using just higher ($n$) order elements.\[23\]

Last Thoughts, the SMX$^n$

However, we even can do better. Since in a mixer with order $n$, where at the end of the first element $3n-1$ interfaces with proper orientation are entering the second (90° rotated) element, only the first $n^2$ cubes, forming the basic elements with edges $D/n$ in the first plane, function by rotating all interfaces, while the following $(n-1) \times n^2$ cubes do nothing else than consuming space and pressure. Consequently, the optimum design of every SMX mixer starts with deciding what $n$ should be, given the maximum pressure drop available or given manufacturing limits or given stiffness requirements of the cross-bars themselves. Subsequently we need a first full element obeying $(n, N_\nu, N_x) = (n, 2n-1, 3n)$ that acts to extend the interface entering (horizontally) in the middle of the (vertically oriented) element, into $3n-1$ vertical interfaces. (This first element could if wanted partly be cut-off at the entrance on places where no interface is present or will appear and only either white or black material enters). The second element (of course 90° rotated with respect to the orientation of the first element) consist of only the first layer of $n^2$ cubes and the next $(n-1) \times n^2$ cubes are removed. The same holds for the third, fourth, fifth, etc. elements, see Figure 17(b) and 17(c). Neglecting pressure consumption in the first element, which is allowed for sufficiently large $N_{\text{elem}}$, the length $L_n$ and thus also pressure drop $\Delta P_n$ are, in this optimized device of the SMX$^n$, a factor $1/n$ lower than in all $(n, N_\nu, N_x) = (n, 2n-1, 3n)$ designs and, therefore, compare Equation (5), only scales linearly with $n$, while the mixing efficiency, Equation (6), remains unaltered:

$$\frac{L_n}{L_1} \sim n^{-1}$$

(8)

$$\frac{\Delta P_n}{\Delta P_1} \sim n$$

(9)
Mixing efficiency in the SMX\(^n\) design is demonstrated in Figure 18 for \(n = 3\). The influence of the injection location is demonstrated in Figure 19 and 20, illustrating that injection somewhere in the middle gives the best results.

Although the SMX\(^n\) is indeed the most compact but effective SMX mixer possessing the lowest possible pressure drop, see also ref.\(^{[24]}\), a closer examination of its performance reveals its limitations, see Figure 21.

**Discussion**

Figure 21(a) demonstrates that from the optimal design series of the SMX\((n)\) elements of the SMX\(^n\) mixer \((n, N_p, N_x) = (n, 2n - 1, 3n)\) the most simple mixer, our basic unit and working horse \((1, 1, 3)\), provides the best mixing at the lowest pressure drop. However, given its long length, see Figure 21(b), it should not so much be compared with higher complexity SMX\((n)\) mixers, \((n > 1)\), but more with Kenics or Ross LPD mixers.\(^{[25]}\) Now we compare the SMX\((n)\) with the SMX\(^n\) design. Both are indeed very compact mixers, see Figure 21(b), and the higher the value of \(n\) the more compact the mixer is. Also recognizable is that, certainly in initial stages of mixing, the SMX\(^n\) outperforms the SMX\((n)\), both in compactness and energy consumption. However, after a while, sooner for higher values of \(n\), a change in slope is found, disappointingly almost approaching that of the \((1, 1, 3)\) mixer line, see Figure 21(b). This can be understood since a change from global mixing (first \(n\) elements of the SMX\(^n\) mixer) to local mixing (following rows of elements) could ideally only be done if the average concentration entering each individual cell of the local

\[
\frac{M_n}{M_1} \sim \left( \frac{3n - 1}{2} \right)^{N_{\text{el}}} \tag{10}
\]
mixers would be equal to the average concentration in the total domain. And that is apparently not yet reached after one first full element.

**Conclusion**

Quantitative mixing analyses based on the Mapping Method applied on motionless SMX mixers teach that...
optimal interface stretching only happens within an element that is build from basic units with a square cross-section. Deviations from this local square cross-section yield either under-stretching or over-stretching of the interface within one element, both of which are bad for optimal mixing. The most simple example is the \( n = 1 \) design, thus \( N_\theta = 1 \) with three crossing bars (\( N_x = 3 \)). When the crossing angle is chosen 90° then the basic unit is even a simple cube. When the angle is chosen larger, for slightly better interfacial stretching and lower pressure drop per element, the basic unit extends to a cuboid. Based on this basic unit, more complex optimal mixers can be build, just by changing the value of \( n \) to \( n = 2, 3, 4, \ldots \) in the \( (n, N_\theta, N_x) = (n, 2n-1, 3n) \) series. It proves that \( n = 1 \) gives the best mixing for the lowest energy consumption. Higher values of \( n \) yield more compact mixers. Finally, both the hierarchical SMX\(^8\) and the extreme compact SMX\(^5\) initially yield better results in terms of mixing quality reached within short lengths, but when extremely high mixing qualities are required, thus rather low values of \( \log(I) \) (the discrete, cross-section averaged, flux weighted intensity of segregation), then the SMX\( (n) \) proves to remain superior.

### Appendix

- **Figure A1.** Mixing profiles for RL-180 and RR-180 design of Kenics mixer.
- **Figure A2.** Mixing profiles for RL-90 and RR-90 design of LPD mixer.

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[23] In retrospective this proves to be the idea behind a relatively recent patent: US 5605399, 1995.

[24] In retrospective this proves to be the idea behind a relatively recent patent, US 7438464, 2004 and EP 1510247, 2004, that was just a reaction on US 5605399, 1995.