Effects of confinement on the motion of a single sphere in a sheared viscoelastic liquid

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Abstract
The motion of a single, inertialess, non-Brownian sphere immersed in a suspending fluid is studied in a confined geometry. A simple shear flow is imposed as external flow field and the impact of the gap between the particle and the wall on the flow fields is investigated.

The suspending fluid is considered both Newtonian and viscoelastic, using for the latter case two different constitutive equations in order to separately highlight the influence of typical non-linear phenomena of non-Newtonian fluids. The rotation rate of the sphere is investigated for different Deborah numbers and gap sizes, with the sphere always at the cell center.

The study is carried out through full 3D numerical simulations. We solve the balance equations by means of a finite element method. The particle rigid-body motion is imposed through constraints on the sphere surface. Therefore, the unknown particle rotation is recovered by solving the full system of equations.

Simulations for a Newtonian suspending liquid show a slowing down of the particle if the gap decreases, in quantitative agreement with other numerical results in literature. For a viscoelastic matrix, this effect is even more pronounced since the nature of the fluid leads itself to a slower rotation, even in unconfined geometries.

Finally, the streamlines around the particle show the existence of a recirculation zone even in Newtonian suspending fluids. Such a zone is larger and closer to the sphere as the viscoelasticity of the suspending fluid increases.

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1. Introduction

Recently, interest in the use of microdevices for chemical and biochemical processes has increased. A microfluidic device is identified by the fact that it has one or more channels with at least one dimension less than 1 mm.

Quite often, suspensions are processed in these devices, with characteristic dimensions of the particles comparable to the channel width. In such a system, due to the increased surface to volume ratio, interfacial effects become more and more important, leading to a completely different particle motion and/or interactions with respect to macrosuspensions. It is quite obvious that the understanding of complex fluids in a confined geometry is necessary in order to optimize the setup of such systems.

Common fluids used in microfluidic devices include whole blood samples, bacterial cell suspensions [1–3], protein [4,5] or antibody solutions [6] and various buffers. These fluids often show a non-Newtonian behavior.

The effect of a confinement on a single particle motion has been analyzed by Bikard et al. [7] by numerical simulations. The results show a slowing down effect in the particle rotation rate in shear flow, due to a finite wall-particle distance. The authors impose the rigid-body motion on the particle surface by considering the sphere as a fluid with a much higher viscosity than the external suspending liquid. In such a way, the interior of the particle needs to be discretized as well and a mesh refinement close to the liquid–particle interface is required in order to achieve a sufficient accuracy. However, the work is limited to the Newtonian case only.

In unbounded geometries, the effect of a viscoelastic suspending fluid on a single particle rotation has been studied by D’Avino [8] and D’Avino et al. [9], through numerical simulations. In agreement with experimental data [10,11], the authors found a slowing down of the particle rotation rate in shear if a viscoelastic suspending liquid is considered. Aim of this paper is to analyze the...
effect of a confined geometry on the motion of a single, rigid, non-Brownian, inertialess sphere immersed in a viscoelastic suspending liquid, in simple shear flow. The effect of the confinement as well as the viscoelasticity of the suspending fluid are investigated through 3D numerical simulations.

A proper numerical scheme is used and stabilization techniques are implemented in order to improve the convergence at finite Deborah numbers.

The rotation rate of the particle is systematically investigated with varying the particle–wall distance, for both Newtonian and viscoelastic fluids. In the latter case, two different constitutive equations are used in order to emphasize the effect of the shear thinning. Streamlines around the particle are also presented in order to show the effect of the confinement on the fluid flow field.

2. Governing equations

The problem of a single sphere in an externally imposed flow field, under isothermal conditions, consists of the continuity (mass balance) and momentum balance equations, plus a constitutive equation depending on the nature of the suspending liquid. Assuming also incompressibility, negligible inertia, and buoyancy free conditions, the governing equations then read:

\[ \nabla \cdot \mathbf{v} = 0 \]  

\[ -\nabla p + \nabla \cdot \mathbf{\tau} = 0 \]  

(1)\hspace{1cm}(2)

where \( p \) is the pressure, \( \mathbf{v} \) is the velocity and \( \mathbf{\tau} \) is the constitutive extra stress.

In this work, in addition to the classical Newtonian constitutive equation, \( \mathbf{\tau} = 2\eta_0 \mathbf{D} \), with \( \mathbf{D} = (\nabla \mathbf{v} + (\nabla \mathbf{v})^T)/2 \) the rate-of-deformation tensor and \( \eta_0 \) the solvent viscosity, a viscoelastic matrix is also considered. Specifically, two different constitutive viscoelastic equations are used:

\[ \lambda \nabla \mathbf{\tau} + \mathbf{\tau} = 2\eta_0 \mathbf{D} \quad (\lambda \mathbf{\tau} + \mathbf{c} - \mathbf{I} = 0) \]  

\[ \lambda \nabla \mathbf{\tau} + \frac{\alpha\lambda}{\eta_0} \mathbf{\tau} + \mathbf{\tau} = 2\eta_0 \mathbf{D} \quad (\lambda \nabla \mathbf{\tau} + \mathbf{c} - \mathbf{I} + \alpha(\mathbf{c} - \mathbf{I})2 = 0) \]  

(3)\hspace{1cm}(4)

where \( \nabla \) indicates the upper convected (UC) time derivative of \( \mathbf{\tau} \):

\[ \frac{\partial \mathbf{\tau}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{\tau} - (\nabla \mathbf{v})^T \cdot \mathbf{\tau} - \mathbf{\tau} \cdot \nabla \mathbf{v} \]  

\[ \mathbf{v} = \frac{\partial \mathbf{r}}{\partial t} \]  

\[ \mathbf{r} = G(\mathbf{c} - \mathbf{I}) \]  

where \( G = \eta_0/\lambda \).

The Giesekus model contains an additional parameter, \( \alpha \). We recall that, under steady state simple shear flow, the Maxwell model predicts a constant viscosity, a first normal stress difference \( (N_1) \) quadratic in the shear rate, and no second normal stress difference \( (N_2) \). In the Giesekus model, both normal stress differences do exist. The viscosity and both normal stress coefficients \( (\Psi_1 = N_1/\gamma^2 \) and \( \Psi_2 = -N_2/\gamma^2) \) are all shear thinning for this model. At any given shear rate, \( \gamma \), the parameter \( \alpha \) modulates the extent of the thinning.

The analyzed system is schematized in Fig. 1. A sphere of radius \( R \) is located between two plates at a distance \( W \). The plates move along the \( x \)-direction in different sense with an imposed shear rate \( \dot{\gamma} \). The cell dimensions along the \( x \)- and \( z \)-directions were chosen large enough in order to have deviations in the local fields within a tolerance (about 0.5%) if the cell size is further increased. Shear flow conditions are imposed on the external fluid boundaries.

The sphere is located at the center of the gap and symmetry implies that it can only rotate. We assume that the sphere is torque-free, or “freely rotating” [12,13], i.e. its rotation is only due to the motion of the surrounding fluid. Thus, the torque-free boundary condition at \( r = R \) is:

\[ \int_{\partial S_p} r \times T \cdot n \, dA = 0 \]  

(7)

where \( \partial S_p \) is the sphere surface, \( n \) is the normal at the sphere surface, \( r \) the position vector from the sphere center, the tensor \( T \) is \( T = -pI + \mathbf{\tau} \), and the integral of the local torque \( r \times T \cdot n \, dA \) spans the sphere surface. Due to the symmetry of the imposed shearing flow, only the vorticity component \( z \) of Eq. (7) is relevant, the other two components being identically zero. In fact, the velocity at the sphere surface is known, and can be written as:

\[ \mathbf{v}(R, \tau) = \omega(\tau) \times R = \omega(\tau)\mathbf{e}_z \times R \]  

(8)

\( \mathbf{e}_z \) being the unit vector along \( z \). The particle angular velocity, \( \omega(\tau) \), is, however, unknown and it depends on the nature of the fluid as well as on the gap/particle radius ratio. The analysis is carried out by making the above equations dimensionless, using \( \dot{\gamma} \) as the characteristic time scale, and \( \eta_0\dot{\gamma} \) as the scale for the stress. Then, the Deborah number, \( De = \lambda / \dot{\gamma} \), appears in all the equations.

In this work, we study the effect of the confinement \( (W/2R) \) and of the matrix nature \( (De) \) on the particle rotation rate.

**Fig. 1.** Geometrical domain used in the simulation (a) and the projection on the \( xy \)-plane (b).
3. Numerical procedure

3.1. Weak form

The governing equations Eqs. (1) and (2) (plus (3) or (4)) are solved by the finite element method on a cell containing a single sphere at the center of the cell (see Fig. 1). The lengths of the cell along the x- and z-directions are chosen 20 times the particle radius in order to assure unperturbed conditions. Due to the symmetry of the problem, only one-quarter \((y > 0 \text{ and } z > 0)\) of the full domain can be considered in order to optimize the computational effort. Indeed the following conditions hold in the xy-plane:

\[
\begin{align*}
\nu_x(x, y, 0) &= 0 \\
\tau_{xy}(x, y, 0) &= 0 \\
\tau_{yz}(x, y, 0) &= 0
\end{align*}
\]  

(9)

and in the xz-plane:

\[
\begin{align*}
\nu_x(x, 0, z) &= -\nu_x(-x, 0, z) \\
\nu_y(x, 0, z) &= -\nu_y(-x, 0, z) \\
\nu_z(x, 0, z) &= \nu_z(-x, 0, z) \\
\tau_{ij}(x, 0, z) &= \tau_{ij}(-x, 0, z) \quad \text{for } ij = xx, xy, yy, zz \\
\tau_{ij}(x, 0, z) &= -\tau_{ij}(-x, 0, z) \quad \text{for } ij = xz, yz
\end{align*}
\]  

(10)

On the other three faces (that have no intersection with the sphere), shear flow boundary conditions are imposed. It is important to point out that the boundary conditions imposed in such a way lead to the existence of one inflow section on the domain boundaries (the yz-side for negative x values). As discussed below, the conformation tensor needs to be imposed on these sections.

A mesh with tetrahedral elements was chosen, with a higher density of elements close to the sphere, where larger gradients are expected. In particular, a finer mesh close to the sphere is required as expected, the most critical situation occurs when \(D\varepsilon\) is high and the gap size is small. Convergence results showed that a good accuracy is achieved with considering about 9000 elements. An example of the mesh used is depicted in Fig. 2.

In order to deal with the viscoelastic models, the continuity and momentum equations (Eqs. (1) and (2)) need to be solved together with the constitutive one (Eqs. (3) or (4)). However, in our numerical method [14], the momentum and continuity equations are decoupled from the constitutive equation, and, as discussed later, an implicit stress formulation is used. In this formulation the time-discretized constitutive equation is substituted into the momentum balance in order to obtain a Stokes-like system (the stress tensor computed in the previous time step is used).

It is well known that the resultant system leads to convergence problems at relatively low Deborah number, if standard finite element discretization for the unknowns \((\nu, p, \tau)\) is implemented. In order to improve the convergence at high \(De\), stabilization techniques are required. In this regard, we discretized the momentum equation with the DEVSS formulation [15] by using in the constitutive equation the projected velocity gradient \((G)\) instead of \((\nabla \nu)^T\) (DEVSS-G formulation) [16]. The scheme is combined with the SUPG method [17] for the constitutive equation. In addition, we use the recently proposed log-conformation representation, which leads to a significantly improvement of numerical stability [18,19]. According to this formulation, the original equation for the conformation tensor \(c\) is transformed to an equivalent equation for \(s = \log(c)\):

\[
\dot{s} = \frac{\partial s}{\partial t} + \nu : \nabla s = g(\nabla \nu^T, s)
\]  

(11)

An expression for the function \(g\) can be found in [19]. Solving the equation for \(s\) instead of the equation for \(c\) leads to a substantial improvement of stability for high Deborah numbers.

Finally, the torque-free condition (Eq. (7)) is imposed through constraints on the sphere surface, by means of Lagrange multipliers. Therefore, by solving the full system of equations, we directly get the particle angular velocity without the necessity to consider a particle made by a highly viscous fluid and to mesh the interior of the sphere (as in [7]).

The weak form of the system of Eq. (1)–(2) and the constitutive Eq. (3) or (4) then reads: For \(t > 0\), find \(\nu \in U, p \in P, s \in S, G \in H, \omega \in \Omega, \lambda \in L^2(\partial S_p)\) such that:

\[
- \int_V \nabla \cdot \nu p \, dV + \int_V \mathbf{a}(\nabla \nu)^T : \nabla \nu \, dV - \int_V \mathbf{a}(\nabla \nu)^T : G^T \, dV + \int_V (\mathbf{u} - (\chi \times \mathbf{r}) : \lambda)_{\partial S_p} = - \int_V \mathbf{D}(\mathbf{u}) : \mathbf{r} \, dV
\]  

(12)

\[
\int_V q \cdot \nabla \cdot \nu \, dV = 0
\]  

(13)

\[
\int_V H : G \, dV - \int_V H : (\nabla \nu)^T \, dV = 0
\]  

(14)

\[
\int_V (\mathbf{S} + \mathbf{r} \nu \cdot \nabla \mathbf{S}) : \left( \frac{\partial s}{\partial t} + \nu : \nabla s - g(\mathbf{G}, s) \right) \, dV = 0,
\]  

(15)

\[
(\mathbf{u}, \nu - (\chi \mathbf{r})_{\partial S_p})_{\partial S_p} = 0
\]  

(16)

\[
\mathbf{s} = \mathbf{s}_0 \quad \text{at } t = 0, \quad \text{in } V
\]  

(17)

\[
\mathbf{s} = \mathbf{s}_{\phi = 0} \text{ on inflow section}
\]  

(18)

for all \(\mathbf{u} \in U, q \in P, S \in S, H \in H, \chi \in \Omega, \mathbf{G} \in L^2(\partial S_p)\), where \(U, P, S, H\) are suitable functional spaces and:

\[
\langle \mathbf{a}, \mathbf{b} \rangle_{\partial S_p} = \int_{\partial S_p} \mathbf{a} \cdot \mathbf{b} \, dA
\]

The \(t\) parameter in Eq. (15) is given by \(t = \beta h^2/2U_e\), where \(\beta\) is a dimensionless constant, \(h\) is a typical size of the element and \(U_e\) is a characteristic velocity for the element. In our simulations, we have chosen \(\beta = 1\) and for \(U_e\) we take the average of the magnitude of the velocities in all integration points. In addition, \(a\) in Eq. (12) is chosen equal to the viscosity, \(a = \eta_0\). We take the initial value of \(\mathbf{s}_0 = 0\), corresponding to zero initial stress. Finally, \(\mathbf{s}_{\phi = 0}\) is the
conformation tensor for an unfilled fluid in the same conditions as the system and generally is a function of time. In this way, we impose an unperturbed flow condition on the inflow section of the domain (being sufficiently far from the sphere).

Notice that the angular velocity, \( \omega \), is treated as an additional unknown and is included in the weak form of momentum equation. Only the \( z \)-component of \( \omega \) is set different to zero since, for the symmetry of the problem, the sphere can rotate around the vorticity axis only. The torque-free condition is imposed through the Lagrange multipliers, \( \lambda \), in each node of the sphere surface. Only the \( x \)- and \( y \)-component of \( \lambda \) are set different to zero since the Lagrange multipliers act as constraints on the \( x \)- and \( y \)-component of the sphere velocity, the \( z \)-component being always nil.

### 3.2. Discretization

For the discretization of the weak form, we use tetrahedral elements with continuous quadratic interpolation \( (P_2) \) for the velocity \( \mathbf{v} \), linear continuous interpolation \( (P_1) \) for the pressure \( p \), linear continuous interpolation \( (P_1) \) for the velocity gradient \( \mathbf{G} \) and linear continuous interpolation \( (P_1) \) for the log-conformation tensor \( \mathbf{s} \).

Regarding the time-discretization, since we do not consider any solvent, an implicit stress formulation is required. Indeed, let us consider the DEVSS-G explicit stress formulation of the momentum solvent, an implicit stress formulation is required. Indeed, let us consider the DEVSS-G explicit stress formulation of the momentum equation at iteration \( n + 1 \) if the solvent is taken into account:

\[
- \nabla \cdot (2\eta_s \mathbf{D}(e^{n+1})) + \nabla p^{n+1} - \alpha \nabla \cdot (\nabla v^{n+1} - G^{n+1} T) = \nabla \cdot \mathbf{r}(e^{n+1})
\]  
(19)

\[
\nabla \cdot \mathbf{v}^{n+1} = 0
\]  
(20)

\[
- \nabla v^{n+1} + G^{n+1} T = \mathbf{0}
\]  
(21)

where \( \eta_s \) is the solvent viscosity and \( e^{n+1} \) has already been computed from a previous time step. If \( \eta_s = 0 \), remembering that \( G = \nabla \mathbf{v} \), the system becomes singular and no update of the velocity field is possible. Instead of fully couple the system unknowns \( (v, p, \mathbf{G}, \mathbf{s}) \), we can find an expression for \( \mathbf{r}(e^{n+1}) \) which involves still unknown terms for the velocity \( v^{n+1} \). Let us recall, then, the explicit-Euler formulation of a general viscoelastic constitutive equation:

\[
\frac{\mathbf{e}^{n+1}}{\Delta t} = \frac{\mathbf{e}^n}{\Delta t} - \mu \mathbf{v}^n \cdot \nabla \mathbf{e}^n + \mu \mathbf{v}^n \cdot \nabla \mathbf{v}^n + \mathbf{e}^n \cdot \nabla \mathbf{v}^n + \mathbf{f}(\mathbf{e}^n)
\]  
(22)

where \( \mathbf{f}(\mathbf{e}) \) is a function depending on the model. A dependence on \( \mathbf{v}^{n+1} \) can be achieved in the following way:

\[
\frac{\mathbf{c}^{n+1}}{\Delta t} = \frac{\mathbf{c}^n}{\Delta t} - \mu \mathbf{c}^n \cdot \nabla \mathbf{c}^n + \mu \mathbf{v}^{n+1} \cdot \mathbf{c}^n + \mathbf{c}^n \cdot \nabla \mathbf{c}^n + \mathbf{f}(\mathbf{c}^n)
\]  
(23)

In the models investigated in this work, a linear relationship between \( \mathbf{c} \) and \( \mathbf{e} \) holds (Eq. (6)) so the stress term in Eq. (19) can be written as:

\[
\mathbf{c}^{n+1} = G \Delta t (-\mathbf{v}^{n+1} \cdot \nabla \mathbf{c}^n + \mathbf{v}^{n+1} \cdot \mathbf{c}^n + \mathbf{c}^n \cdot \nabla \mathbf{v}^n + \mathbf{f}(\mathbf{c}^n))
\]  
(24)

Substituting this expression into the momentum equation (without solvent) leads to the implicit stress formulation:

\[
\nabla \mathbf{v}^{n+1} - \alpha \nabla \cdot (\nabla \mathbf{v}^{n+1} - G^{n+1} T) - \nabla \cdot (G \Delta t (-\mathbf{v}^{n+1} \cdot \nabla \mathbf{c}^n + \mathbf{v}^{n+1} \cdot \mathbf{c}^n \cdot \nabla \mathbf{c}^n + \nabla \mathbf{c}^{n+1} \cdot \mathbf{c}^n \cdot \nabla \mathbf{c}^n)) = \nabla \cdot (G \mathbf{c}^n + \Delta t \mathbf{f}(\mathbf{c}^n) - \mathbf{I})
\]  
(25)

\[
\nabla \cdot \mathbf{v}^{n+1} = 0
\]  
(26)

\[
- \nabla \mathbf{v}^{n+1} + G^{n+1} T = \mathbf{0}
\]  
(27)

Finally, the time-stepping procedure can be stated as follows.

### Initialization

At \( t = 0 \), the log-conformation tensor \( \mathbf{s} \) is set to \( \mathbf{0} \), representing a zero initial stress condition. No initial condition for the velocity is required since we neglect the inertia.

**Step 1.** The unknowns \(( \mathbf{G}, \mathbf{v}, \mathbf{p}, \omega)\) as well as the Lagrange multipliers \(( \lambda)\) are found by solving the following system according to the implicit stress formulation:

\[
- \int_\Omega \nabla \cdot \mathbf{u} p^{n+1} \, dV + \int_\Omega (\nabla \mathbf{u})^T : \nabla p^{n+1} \, dV
\]

\[
- \int_\Omega (\nabla \mathbf{u})^T : G^{n+1} T \, dV + G \Delta t \int_\Omega (\nabla \mathbf{u})^T : (-\mathbf{v}^{n+1} \cdot \nabla \mathbf{c}^n + \nabla \mathbf{v}^{n+1} \cdot \mathbf{c}^n + \mathbf{c}^n \cdot \nabla \mathbf{v}^{n+1} + (\mathbf{u} - (\mathbf{\omega} \times \mathbf{r}) \cdot \mathbf{I}) s_p) = - \int_\Omega \mathbf{D}(\mathbf{u}) : (\mathbf{c}^n + \Delta t \mathbf{f}(\mathbf{c}^n) - \mathbf{I}) \, dV,
\]  
(28)

\[
\int_\Omega q \nabla \cdot \mathbf{v}^{n+1} \, dV = 0,
\]  
(29)

\[
\int_\Omega \mathbf{H} : \mathbf{c}^{n+1} \, dV - \int_\Omega \mathbf{H} : (-\nabla v^{n+1})^T \, dV = \mathbf{0},
\]  
(30)

\((\mathbf{\mu}, \mathbf{v}^{n+1} - (\mathbf{\omega}^{n+1} \times \mathbf{r}) \cdot \mathbf{I}) s_p = \mathbf{0}.
\]  
(31)

Notice that, in Eq. (28), \( \mathbf{c}^n \) is the conformation tensor evaluated in the previous time step.

**Step 2.** The log-conformation tensor at the next time step, \( \mathbf{s}^{n+1} \), is evaluated by integrating the constitutive equation (15). A combined first-order Euler forward/backward scheme is used and Eq. (15) is replaced by the following time-discretized form:

\[
\int_\Omega (\mathbf{S} + \tau v^{n+1} \cdot \nabla \mathbf{S}) : \left( \frac{\mathbf{s}^{n+1} - \mathbf{s}^n}{\Delta t} + \tau v^{n+1} \cdot \nabla \mathbf{s}^{n+1} - \mathbf{g}(\mathbf{c}^{n+1}, \mathbf{s}^n) \right) \, dV = \mathbf{0},
\]  
(32)

In both Steps 1 and 2, an unsymmetric sparse linear system needs to be solved. We use the parallel direct solver PARDISO [20].

As remarked in previous studies [8,9], the time step of the simulations should be carefully chosen. The time-convergence is checked by decreasing the step size. In Fig. 3, the start-up of the rotation rate is reported for \( De = 1.0 \) and \( W/2R = 1.3 \) and different \( \Delta t \). As expected, for sufficiently small \( \Delta t \), the curves tend to overlap. It is important to notice that, for the parameters chosen, even when the
time-convergence is satisfied (compare $\Delta t = 0.005$ and 0.0025), a maximum in the start-up is still present. Such a feature is different from an unbounded and “almost unbounded” suspension, as will be discussed in next section.

4. Simulation results

In microfluidic devices the influence of the walls has a strong impact on the particle motion. On the other hand, the viscoelastic behavior of the suspending fluid leads by itself to different dynamics with respect to the Newtonian case. As experimentally reported by Astruc et al. [10] and Snijkers et al. [11], and confirmed by the simulations of D’Avino [8] and D’Avino et al. [9], the rotation rate of an isolated particle immersed in a sheared viscoelastic liquid is slower than in a Newtonian suspending fluid. As shown below, the confinement leads to a further slowing down.

The calculations are performed by taking into account a Newtonian fluid as well in order to test the code by directly comparing our results with Bikard et al. [7].

In Fig. 4, the steady state rotation rate of the particle as a function of the gap size/particle diameter ratio, $W/2R$, is reported. The line with open circles is the Newtonian prediction ($\text{De} = 0$), whereas the lines with full symbols are for different Deborah numbers, using for the fluid a Giesekus model with $\alpha = 0.2$. We recall that $\alpha$ parameter should range between 0 and 0.5 to be realistic. Our choice is made just to show the effect of the shear thinning on the particle rotation. The dashed lines on the right are the results for an unbounded system for Newtonian as well as viscoelastic fluid (the corresponding data are adapted from [8] and [9]). The data from [7] are also reported with stars.

The Newtonian curve, in quantitative agreement with [7], clearly shows that the particle slows down when the gap/particle diameter, $W/2R$, decreases. The viscoelasticity of the suspending fluid works in the same direction leading to further slowdown the higher is $\text{De}$. In conclusion, both confinement effects and high viscoelasticity slow down the particle rotation rate.

A comparison between one of the depicted curves and the corresponding dashed curves for an unbounded domain suggests that the sphere still feels the presence of the wall up to distances equal to 3–4 particle radii (in the range depicted in Fig. 4, the solid black lines are always below the dashed ones). This confirms what is found in [8] and [9]: a distance equal to at least 10 particle radii is necessary in order to assure unperturbed conditions on the external boundaries of the domain.

Our numerical method does not allow to investigate particle–wall distances less than $W/2R = 1.2$ (the smallest one in Fig. 4), because a high mesh resolution in the gap is required to have sufficient elements between the particle and the wall.

The Giesekus model represents a shear thinning viscoelastic fluid with both first and second normal stresses. In [8] and [9], the authors found that a shear thinning fluid without any normal stress (pure viscous fluid) does not lead to a slowing down of the particle. On the other hand, a Maxwell fluid (no shear thinning and only first normal stress difference) as suspending matrix leads to the slowest rotation rate.

In Fig. 5, the steady state particle rotation rate is reported as a function of $W/2R$ for a single sphere in a Maxwell fluid (grey lines). For the sake of comparison, the corresponding Giesekus curves (black lines and full circles) and the Newtonian one (line with open circles) are plotted as well.

The same features as for a Giesekus fluid are found: the viscoelasticity as well as a closer particle–wall distance lead to slow down the particle rotation. Regarding a comparison with the shear thinning fluid, for large gap sizes, our results agree with previous works for unbounded domain [8,9]: a suspending Maxwell fluid predicts a slower rotation rate. However, by reducing the gap, the trends invert and the Maxwell model predicts a faster rotation than Giesekus one when the wall is close to the particle.

The results are shown up to $\text{De} = 1.0$ because the numerical procedure gives less reliable results for larger $\text{De}$. Indeed, strong elongational components around the particle arise and a pure Maxwell fluid predicts a diverging viscosity for $\text{De} = 0.5$ in pure elongational flow.

Let us consider now the analysis of the transients. In Fig. 6, start-up trends for the same system as in Fig. 4 with $\text{De} = 1.0$ are reported, at different gap size/particle diameter ratios.

For large gap sizes, a monotonic decreasing trend is observed, in agreement with previous works [8,9]. On the contrary, if $W/2R$ decreases, a maximum in the rotation rate in the start-up phase occurs. The existence of such an overshoot is related to the gap size as well as the viscoelasticity of the material, i.e. high viscoelastic suspending fluids and strong confinements enhance the maximum value in the start-up.

![Fig. 5. Steady state rotation rate of the particle as a function of gap size/particle diameter ratio. A Newtonian fluid (grey lines) is considered at different $\text{De}$. For comparison, a Giesekus fluid (black lines and full symbols) and Newtonian one (line with open circles) are reported as well.](image-url)
Fig. 6. Rotation rate of the particle as a function of time for different gap size/particle diameter ratio. A Giesekus fluid \((\alpha = 0.2)\) is considered with \(De = 1.0\).

A transient analysis has been carried out with considering a Maxwell suspending fluid also. The results (non-reported) show the same features observed for a Giesekus fluid, with the presence of an overshoot for a small gap and at finite Deborah numbers.

Finally, the effect of the confinement on the flow field is investigated through the streamlines around the particle. In Fig. 7, the streamlines on the \(xy\)-plane \((z = 0)\) are reported. Since the computational domain is one-quarter of the cell \((y > 0, z > 0)\), the velocity field for \(y < 0\) is recovered by using the antisymmetry with respect to the sphere center:

\[
\begin{align*}
  v_x(x, y, 0) &= -v_x(-x, -y, 0), \\
  v_y(x, y, 0) &= -v_y(-x, -y, 0)
\end{align*}
\]  

(33)

To show the impact of the confinement, the unbounded case is depicted as well (left column). In the confined case (right column) the gap is chosen such that \(W/2R = 1.5\) (strong confinement) and the figures refer to a Newtonian suspending fluid (row (a)), a Giesekus fluid with \(\alpha = 0.2\) and \(De = 1.0\) (row (b)) and a Giesekus fluid with \(\alpha = 0.2\) and \(De = 2.0\) (row (c)).

As it is well known, for an unbounded domain \((W \to \infty)\) the Einstein’s solution [21] for an isolated sphere in a Newtonian fluid under shear flow predicts the existence of a zone surrounding the particle where the streamlines are closed curves, i.e. a fluid particle in this zone will rotate indefinitely around the sphere (Fig. 7(a1)). Theoretically, such a zone has an infinite extension even if, above a distance from the sphere of few radii, it becomes extremely thin. Outside this region, every fluid particle coming from infinity approaches the sphere and then moves far from it to the opposite side.

The presence of the confinement strongly modifies the flow field leading to the appearance of a recirculation zone, enclosed in the grey bold line (see Fig. 7(a2)). A fluid particle in such a zone never crosses the \(yz\)-plane \((x = 0)\) but inverts its motion on the \(x\)-axis, coming back to the infinity. However, closed streamlines around the particle still exist but, in this case, a finite amount of fluid is involved. The extension of the recirculation zone increases as the confinement is stronger and it is in quantitative agreement with the predictions of Bikard et al. [7]. Finally, notice that, due to the symmetry of the system, the streamlines are symmetric as well along the \(x\)- and \(y\)-direction (and in general with respect to the \(xz\)-and \(yz\)-plane).

Let us consider now a viscoelastic suspending liquid (second and third rows in Fig. 7). D’Avino et al. [9] reported the streamlines for different \(De\) values in the unbounded case. They found the opening-up of the closed orbits around the sphere for very low \(De\), with respect to the Newtonian case. By increasing the Deborah number, an attractive closed orbit surrounding the sphere appears. A fluid particle starting close to the sphere or far from it, but still within a zone enclosed by two separatrices, moves towards such an orbit. By further increasing \(De\), the attractive orbit approaches the particle.

Fig. 7. Streamlines on the \(xy\)-plane \((z = 0)\) with \(W/2R = 1.5\). The different plots refer to: (a) Newtonian suspending fluid (b) Giesekus fluid with \(\alpha = 0.2\) and \(De = 1.0\) (c) Giesekus fluid with \(\alpha = 0.2\) and \(De = 2.0\).
and two recirculation zones arise, as reported in Fig. 7(b1) and (c1) (see [9] for further details).

The confined system shows, similarly to the Newtonian case, large recirculation regions that approach the sphere with respect to the unconfined case, as depicted in Fig. 7(b2) and (c2). Such an effect is more pronounced as the viscoelasticity of the suspending fluid is higher. A finite region filled of a fluid moving around the particle still exists. Finally, a symmetry breaking is also evident and the extreme limit of the recirculation zone moves up the x-axis, according to the particle rotation sense.

5. Conclusions

In this work, a suspended sphere in shear flow is investigated when the gap size between the walls is comparable with the particle radius. Both a Newtonian and viscoelastic suspending liquid is considered, neglecting inertia effects. The analysis is carried out through numerical simulations, using the finite element method for the discretization of governing equations with proper stabilization techniques to improve the convergence at high Deborah numbers.

Our results can be summarized as follows: (i) the presence of walls slows down the particle rotation, (ii) such a slowing down is more and more pronounced as the gap gets smaller, (iii) the viscoelasticity of the fluid enhances the slowing down phenomenon, and (iv) the start-up is characterized by a small overshoot as the particle–wall distance is sufficiently small and De high. Moreover, we studied the effect of the shear thinning. We found that the absence of any thinning leads to a faster rotation compared to a shear thinning suspending fluid. Such an effect goes in the opposite direction with respect to an unbounded domain.

Finally, the streamlines evidenced the existence of a recirculation zone both in Newtonian and viscoelastic fluid. Such a region is larger as the confinement is stronger and the viscoelasticity of the fluid is higher.

In this study the particle is always located in the center of the channel. Therefore, due to the symmetry imposed on the flow it cannot translate. However, a small perturbation will cause the arising of a lift directed towards the walls. This is a different problem that must be approached by a completely different numerical scheme that leaves the sphere free to migrate. Part of future work will then be focused on removing this constraint and studying the lift experienced by the particle.

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References