The objective of this paper is to present a method to arrive at a linear parametrically varying (LPV) model for vehicle occupants. These LPV models are especially useful in the development of controlled restraint systems. In controlled restraint systems, the restraint elements, such as the seat belt force or airbag outflow, are controlled during the crash to minimize the risk of injuries to the occupant. These control algorithms contain low-order LTI models of the occupant and restraint system. Since the occupant may vary largely in size and mass, LPV models are well suited account for these variations. It has been shown that LPV models well describe the occupant dynamics, and are useful in the design and development of controlled restraint systems.

Passive Safety, Driver Model, Modeling and Simulation Technology

1. INTRODUCTION

Today’s restraint systems typically include a number of airbags, and a three-point seat belt with load limiter and pretensioner. The design and development of these passive safety devices is largely oriented towards car occupants of average height and weight and for a set of standardized, high speed crashes. In general, the systems are not able to adjust their characteristics during a crash event, with the consequence that an occupant that deviates from the average will not be optimally protected in every crash.

Occupant safety can be significantly improved when the restraint systems are continuously manipulated as a function of measured signals [1]. In that case they are referred to as real-time controlled restraint systems. The continuous manipulation can be performed through a control algorithm that aims at minimizing one or more occupant injury criteria. A schematic representation of the components of a controlled restraint system is shown in Fig. 1. These systems do not yet exist in today’s passenger vehicles, but numerical simulations with a controlled seat-belt and/or airbag [2], [3], [4] showed that a significant injury reduction can indeed be achieved. Therefore, this class of systems will be a main focus of future restraint system development.

1.1 Problem definition

Vehicle occupants largely differ in mass and length, and the controlled restraint system has to cope with these occupant variations. In a previous simulation study [2], it has been showed that the chest motion of different percentile dummies cannot be robustly controlled by a single controller. Hence, control parameters have to be selected based on the occupant properties.

Secondly, low order occupant models are part of the controlled restraint system. These occupant models are employed for real-time applications such as state estimators [4] and optimization algorithms [5]. The former is useful in the estimation of occupant responses that can not be measured by sensors during a crash event, so the responses have to be obtained in a model-based manner. The optimization algorithm determines an optimal (future) occupant trajectory in terms of injury reduction, which serves as an input to a restraint controller. The models that are required in the aforementioned algorithms are linear and time-invariant (LTI). For this type of applications, it is important that the model closely corresponds to the actual occupant, and the selection of the correct model parameters is therefore an important step.
1.2 Approach

It is proposed in this paper to describe the occupant variations by a linear parameter varying (LPV) modeling approach, in which a scheduling variable $\delta$ is introduced. This variable represent the occupant type and can be determined a priori, i.e. before the crash. So instead of generating a large database of LTI models for every possible type of occupant, we calculate a single model that is a function of the $\delta$. Since the models have to be stored in the vehicle, this is far more attractive.

From a control design perspective, LPV models are not required. Individual controllers could be designed for members of the model database, and the control parameters could subsequently be interpolated as a function of $\delta$. However, since LPV models are available anyhow, well-known techniques can be used to design a single gain-scheduled controller directly from the LPV model, see [6]. For example, in the special case where the model can be written in an LFT representation, standard $\mathcal{H}_\infty$ control design can be used. Obviously, this has an advantage over designing a individual controllers.

The approach that is followed is as follows: a) a number of nonlinear models are used that represent different occupant sizes, b) the parameters of these models are interpolated as a function of an appropriate scheduling variable, c) the nonlinear models are linearized and reduced to manageable dimensions d) the family of LTI models are interpolated using the same scheduling variable to obtain an LPV system. Finally, as a cross-validation, the LPV model is evaluated on a grid of $\delta$ values, and compared with original LTI models on the same grid.

2. Nonlinear, Low-order Dummy Models

Nonlinear, low-order design models are developed in [7] that represent the 5, 50 and the 95 %-ile seated Hybrid III dummies with a belt restraint system. Fig. 2. The model has 14 degrees of freedom, 10 bodies and contains a force actuated belt at the load limiter. All parameters, dimensions and constitutive equations are derived from the numerical Hybrid III dummies in the MADYMO database[8]. It has been constructed such that mechanical responses of the chest region are biofidelic. Responses of interest, $y$, are chest acceleration $a_{\text{chest}}$, belt displacement $x_{\text{belt}}$, chest compression $\Delta x_{\text{chest}}$ and its time derivative $\Delta v_{\text{chest}}$. Inputs to the model are exogenous signals $w$, here the vehicle acceleration pulse $a_{\text{veh}}$ and control inputs $u$, here the belt force $F_{\text{belt}}$. For a EuroNCAP frontal impact with a small family car, the responses of both the 50th percentile MADYMO dummy and the corresponding design model are shown in Fig. 3. It shows good resemblance for all 4 responses, and also in other crash scenarios or for other dummy types, the design model has a good fidelity, see [7].

As mentioned, three design models have been constructed, and they can be represented by the following nonlinear, continuous time, state and output equation

\[
\dot{x}(t) = f_i(x(t), w(t), u(t)), \quad i = 1, 2, 3 \quad (1)
\]

\[
y(t) = g_i(x(t)) \quad (2)
\]

in which $x \in \mathbb{R}^{28}, y \in \mathbb{R}^4, u, w \in \mathbb{R}$. The subscript $i$ denotes the dummy type, i.e. the 5, 50 and 95 %-ile dummy respectively.

3. Selection of Scheduling Variable

The nonlinear occupant model consists of many parameters to either represent a 5, 50 or 95 %-ile percentile dummy. The parameters of each of the three dummy types are time-invariant, and consist of (i) mass parameters, (ii) dimensions and (iii) parameters from the constitutive equations. For convenience of notation, they are stacked in the parameter vector $p_i \in \mathbb{R}^8$, where the subscript $i = 1, 2, 3$ again denotes dummy type. Our model from (1) can now be written as the family of plants

\[
\dot{x} = f(x, u, w, p_i), \quad i = 1, 2, 3 \quad (3)
\]

\[
y = g(x, p_i)
\]
with \( p \), the vector of time-invariant parameters.

A Linear Parameter Varying (LPV) system is a linear system whose describing matrices depend on a real-valued scheduling variable \( \delta \), that can either be exogenous or endogenous signals with respect to the plant [6, 9]. The first step to describe our family of plants as an LPV system, is to have a dependency on a scheduling variable \( \delta \). Therefore, each parameter vector \( p_i \) is replaced by a vector function \( \alpha (\delta) : \mathbb{R} \rightarrow \mathbb{R}^v \), where \( \delta \in (-1, 1) \).

This yields the three parameter vectors in the family of plants

\[
p_i = \alpha (\delta_i)
\]

We associate each \( \delta_i \) with a dummy percentile, referred to as parameter \( d \). Since \( d \) has a maximum of \( d = 100 \) and a minimum of \( d = 0 \), appropriate scaling leads

\[
\delta = \frac{2d - (d + d)}{d - 0} \frac{2d - 100}{100},
\]

and hence \( \delta_0 = -0.9, \delta_1 = 0 \) and \( \delta_2 = 0.9 \).

Next, we have find a function \( \alpha \) that satisfies (4). With three parameter vectors, \( p_1, p_2 \) and \( p_3 \), a cubic interpolation between the parameter values is most straightforward. Therefore, a second order polynomial function in \( \delta \) is chosen for the vector function \( \alpha \), so we can write

\[
\alpha(\delta) = P_0 + P_1 \delta + P_2 \delta^2 = P \begin{pmatrix} 1 & \delta & \delta^2 \end{pmatrix}^T
\]

with \( P \in \mathbb{R}^{v \times 3} \) and \( P_j \) the \( j^{th} \) column of \( P \). Now (5) can be plugged in (4) to yield

\[
p_i = P \begin{pmatrix} 1 & \delta_i & \delta_i^2 \end{pmatrix}^T
\]

Above equation can be solved for \( P \) as follows

\[
P = \begin{pmatrix} p_1 & p_2 & p_3 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ -0.9 & 0 & 0.9 \\ 0.81 & 0 & 0.81 \end{pmatrix}^{-1}
\]

An example of the interpolation is shown in Fig. 4 for a small number of mass and dimensional parameters. It shows that the vector function \( \alpha \) allows interpolation of the parameters by choosing an intermediate value of \( \delta \).

With the obtained matrix \( P \), the vector of parameter values in (3) can now be replaced by the vector function \( \alpha (\delta) \). This leads to the following nonlinear plant model

\[
\begin{align*}
\dot{x} &= \tilde{f}(x, u, w, \delta), & \delta \in (-1, 1) \\
y &= \tilde{g}(x, \delta)
\end{align*}
\]

The last step consist of finding a value for \( \delta \) for every occupant. It is proposed to use the mass of the occupant, \( m_{\text{total}} \), to obtain this value. Many dummy scaling procedures also use the length of the occupant [10], but this parameter is difficult to obtain from today’s in-vehicle sensors. The mass can easily be measured, for example by load cells in the seat, see [11].

Now let the masses of the \( N \) bodies be collected in the vector \( m \in \mathbb{R}^N \), which is a subset of parameter vector \( p \). Therefore

\[
m = \tilde{P} \begin{pmatrix} 1 & \delta & \delta^2 \end{pmatrix}^T
\]

with \( \tilde{P} \in \mathbb{R}^{N \times 3} \). Let \( \tilde{P}_{(i,j)} \) denote the \((i,j)\) element of \( \tilde{P} \). Then the mass of the occupant is given by

\[
m_{\text{total}} = \sum_{j=1}^N \tilde{P}_{(1,j)} + \delta \sum_{j=1}^N \tilde{P}_{(2,j)} + \delta^2 \sum_{j=1}^N \tilde{P}_{(3,j)}
\]

So, for every occupant that enters the vehicle, the total mass is measured or estimated by sensors and equation (7) is used to obtain a corresponding scheduling variable \( \delta \). In addition, the angle and position of the seat can be used to approximate the initial conditions of the model (6).

4. FAMILY OF LTI MODELS

The next step is to obtain a linear description of the nonlinear plant, as a function of the scheduling variable. An LPV plant could directly be obtained by a jacobian linearization of the parameter dependent plant model in (6). However, this leads to an LPV model with a very high order in \( \delta \), which is not desirable. Secondly, the mass matrix is dependent on \( \delta \), and has many off-diagonal terms. An explicit inversion due to its size could not be realized, and the model in (6) could not be obtained as an explicit function of \( \delta \).

4.1 Linearization

Therefore, a jacobian linearization of the models in (5) is performed, along the operating points \( \bar{x}, \bar{u} \) and \( \bar{w} \). This leads to the following family of LTI models

\[
\begin{pmatrix} \dot{x}_i \\ y_i \end{pmatrix} = \begin{pmatrix} A_i & B_i \\ C_i & 0 \end{pmatrix} \begin{pmatrix} x_i \\ u_i \end{pmatrix}, \quad i = 1, 2, 3
\]

with state space matrices defined as

\[
A_i = \frac{\partial f(x, u, w, p_i)}{\partial x} (\bar{x}, \bar{u}, \bar{w})
\]

The deviation variables are defined in a straightforward manner, for example

\[
x_i = x(t) - \bar{x}, \quad u_i(t) = u(t) - \bar{u}
\]
The linearization is performed at multiple operating points along the trajectory, namely $\bar{x} = x(\tau)$ for $\tau = [0, 5, 10, \ldots, 100]$ ms. Simulations are performed with each LTI model for the given inputs $u(t)$ and $w(t)$, and based on output responses of these LTI models and the nonlinear model, it is concluded that the LTI models obtained at $\tau = [5, 10, \ldots, 50]$ ms mimic the behavior of the nonlinear model very well. For ease of notation, the subscript $i$ is omitted from the system descriptions in the remainder of this paper.

Now the transfer functions are defined by

$$y(s) = H(s)u(s)$$

and $H_{(j,k)}$ denotes the transfer from the $k$th input to the $j$th output. An example is given in figure 5 where the 50th %-ile system from first input to first output, i.e. $H_{11}(s)$, is plotted in a bode diagram for $\tau = [5, 10, \ldots, 50]$. It shows that the FRF vectors are very similar, and it is chosen to use the LTI models obtained at $\tau = 10$ ms in the remainder of the paper.

Fig. 5. Bode plot of the frequency response function $H_{11}(j\omega)$ for the 50th percentile dummy, obtained from a linearization around a point of the trajectory at time instances $\tau = [5, 10, \ldots, 50]$ ms.

### 4.2 Model reduction

Model reduction is part of dynamic analysis, control design, system identification, etc. Structural models based on first principles are often oversized, and may contain modes with high natural frequencies that are not relevant for control design. Additionally, it may contain weakly controllable and observable parts that could be removed. Finally, modern control design methods such as $H_\infty$ lead to controllers that have an order that is at least the plant order, and these controllers are often too complex to practically implement. Model reduction is therefore applied to our system, and two methods of truncation are discussed.

1) Modal truncation: The first method we apply is called modal truncation, and consist of removing the high-frequency modes of a state-space realization. Our plant model in (8) is transformed to a Jordan form, i.e. a realization $H(s) = (A, B, C, 0)$ such that

$$A = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n)$$

with $|\lambda_1| < |\lambda_2| < \cdots < |\lambda_n|$. The order of the plant model in (8) is $n = 28$. Reduction is achieved by simple truncation of the states that correspond to the fastest modes. In all three LTI models, two states had natural eigenfrequencies $\omega_n = |\lambda| > 1$ kHz, and these were removed. After truncation, the realization reads $H_r(s) = (A_r, B_r, C_r, 0)$, with $A_r \in \mathbb{R}^{n_r \times n_r}$ and $n_r = 26$.

2) Balanced truncation: A second method is balanced truncation, and is based on a balanced realization. In this realization, every state is equally observable as controllable, which is quantified by the Hankel singular values $\sigma_i$. The balanced realization is obtained when the controllability and observability grammians $P$ and $Q$ are made equal. In that case they have the Hankel singular values on their diagonals, i.e. $P = Q = \text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_n)$.

The truncated LTI models $H_r(s)$ are balanced, and their Hankel singular values are obtained. In Fig. 6 they are plotted for the 50th percentile dummy. It can be seen that the Hankel singular values of the states 21 – 26 are very low, indicating that these states are weakly observed by the outputs and weakly controlled by the input. Therefore, these states are removed, leading to a realization $H_b(s) = (A_b, B_b, C_b, 0)$, with $A_b \in \mathbb{R}^{n_b \times n_b}$ and $n_b = 20$.

Fig. 6. Hankel singular values of the modal truncated system $H_r(s)$ representing the 50th percentile dummy.\[0x0] Note that removing the states 15 – 26 resulted in an error between reduced and original model that was considered to be too large. The error is namely given by

$$\|H - H_b\|_\infty \leq 2(\sigma_{n_b+1} + \cdots + \sigma_{n_r})$$

so removing more states increases the error by twice the sum the their Hankel values.

In Fig. 7, the frequency response functions of the original model $H_{11}(s)$ and the balanced, truncated model $H_{b,11}(s)$ are shown. It can be seen that the truncated system matches well at the frequencies of interest.
4.3 Results

The obtained LTI models are now given by

$$\begin{pmatrix} \dot{x}_b^\delta \\ y_s \end{pmatrix} = \begin{pmatrix} A_b & B_b \\ C_b & 0 \end{pmatrix} \begin{pmatrix} x_b^\delta \\ u_s \end{pmatrix}$$

with $x_b^\delta \in \mathbb{R}^{n_b}$. These models are validated in the time domain, and the responses are compared with the responses from the nonlinear design mode. An example is plotted in Fig. 8, which shows the responses of the 50th percentile dummy model. It can be seen that the linearization and model reduction has a minor effect on the responses, at least in the considered crash scenario. Error responses with the 5th and 95th percentile dummy have similar trends.

Finally, the family of FRFs $H_b(j\omega)$ is plotted in figure 9, which clearly shows that the gain nicely (inversely) scales with occupant mass for outputs 1, 3 and 4. The occupant type seems to have less effect on the magnitude of $H_{b,21}(j\omega)$. These results gave rise to the idea of interpolating the models as a function of $\delta$, which will be presented in the next section.

5. LPV Dummy Model

In the previous section, a family of three LTI models have been obtained as a function of a scheduling variable $\delta$. In this section, the obtained models are interpolated to form a single model, which can then be marked as an LPV model.

An appropriate way of doing this, see [14] is as follows: a) write the models in an convenient state-space notation, b) fit the elements of the matrices by a polynomial function, c) put the polynomial function in a single model and d) verify the LPV model at intermediate values of $\delta$.

First, the models are transformed to a controller canonical form [15] in order to reduce the number of coefficients $H_b(s) \doteq (A_c, B_c, C_c, 0)$

$$A_c = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ a_1 & a_2 & a_3 & \cdots & a_{n_b} \end{pmatrix}, \quad B_c = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix},$$

$$C_c = \begin{pmatrix} c_{1,1} & c_{1,2} & \cdots & c_{1,n_b} \\ \vdots & \vdots & & \vdots \\ c_{4,1} & c_{4,2} & \cdots & c_{4,n_b} \end{pmatrix}$$

(11)

The $n_b$ elements of the $A_c$ matrix are shown in figure [14] for different values of $\delta$. It shows that the majority of the elements $a_j$ of $A_c$ vary as a function of $\delta$, which also applies to the elements of $C_c$. All the
individual elements, i.e. the $n_k$ elements of $A_c$ and $4n_k$ of $C_c$, are now fitted by a second order polynomial in $\delta$. This results in two polynomial functions

$$
A(\delta) = A_0 + A_1 \delta + A_2 \delta^2
$$

$$
C(\delta) = C_0 + C_1 \delta + C_2 \delta^2
$$

The $B_c$ matrix is independent of the $\delta$ and can directly put is the LPV model. Our LPV model in state space notation now reads

$$
\dot{x}(t) = A(\delta)x(t) + B_c u(t), \quad \delta \in (-1, 1)
$$

$$
y(t) = C(\delta)x(t)
$$

The last step consists of validating the LPV model with an LTI model at intermediate values of $\delta$, and the following grid is used: $\delta = [-0.9, -0.7, -0.5, \ldots, +0.9]$. Equation 6 is used to obtain intermediate nonlinear models, which are linearized, and the same model reduction techniques are applied as discussed earlier. LPV models are calculated on the same $\delta$-grid, and are compared with corresponding LTI models. Fig. 11 shows the LTI and LPV model for $\delta = 0.5$, which corresponds to a total occupant mass of 90.6 kg, see [7]. Both models match quite well, which also applies to different values of $\delta$.

So it is concluded that a second order polynomial LPV description in the mass parameter $\delta$ is a good choice to describe the family of LTI models. It should be noted that this conclusion is based on comparison of frequency responses functions.

6. CONCLUSIONS

In this paper, nonlinear models of Hybrid III dummies are written as a single LPV model with polynomial dependence on a scheduling variable, that represents the mass of the occupant. It has been shown that LPV models well describe the occupant dynamics, and are useful in the development design for controlled restraint systems. The next step consists of control and observer design for the LPV models, and to test these algorithms on existing MADYMO models.

![Fig. 10. Elements $a_{ij}$ of the $A$ matrix in controller canonical form for different values of scheduling variable $\delta$.](image)

![Fig. 11. Magnitude plot of the FRF of the LTI model $H(s)$ (black) and the LPV model $H_\delta(s)$ (gray) for an intermediate value $\delta = 0.5$.](image)

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