Development and validation of a vibration model for a complete vehicle

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Abstract

This report focuses on the development and validation of a multi-body model of a complete car. The purpose of this model is to help to predict the behaviour of the vehicle with respect to vibrations early in the development process of a car. The model is kept as simple as possible to be able to make adjustments easily to both the geometry and parameters of the model. The outcome of this study is a multi-body model featuring nine masses which include four tyres, three vehicle body parts, an engine and a windscreen. The model is validated using measurements on a real vehicle in the modal analysis laboratory of the BMW group. From this validation, the main discrepancies can be found in the connection between engine and vehicle body as well as the damping behaviour in the entire model. However, the resonance frequencies between model and real vehicle are comparable.
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<th>Description</th>
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</thead>
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<td>m/s$^2$</td>
<td>Acceleration</td>
</tr>
<tr>
<td>$A$</td>
<td>-</td>
<td>Constant</td>
</tr>
<tr>
<td>$I$</td>
<td>kgm$^2$</td>
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<tr>
<td>$z$</td>
<td>m</td>
<td>Length</td>
</tr>
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<td>$\omega_n$</td>
<td>rad/s</td>
<td>Natural undamped frequency</td>
</tr>
<tr>
<td>$\omega_d$</td>
<td>rad/s</td>
<td>Natural damped frequency</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>-</td>
<td>Damping ratio</td>
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</table>

<table>
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<th>Index</th>
<th>Unit</th>
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<td>$(..)_x$</td>
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<td>In the x-direction</td>
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<td>$(..)_{xx}$</td>
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<td>Around the x-axis</td>
</tr>
<tr>
<td>$(..)_y$</td>
<td>-</td>
<td>In the y-direction</td>
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<td>$(..)_{yy}$</td>
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<td>Front body</td>
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<td>$(..)_2$</td>
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<td>Middle body</td>
</tr>
<tr>
<td>$(..)_3$</td>
<td>-</td>
<td>Rear body</td>
</tr>
</tbody>
</table>
1. Introduction

The first chapter gives an introduction concerning the work described in this report. The introduction consists of a motivation, background, aim and the scope of this project. Finally, a brief outline of the contents of the report is given.

1.1 Motivation and background

In the development of a car, there are many different aspects that have to be taken into account. One of these aspects are the vibrations caused by driving a car over roads. The excitation of the wheels results in vibrations in the entire vehicle. Vibrations do not play an important role in the performance of the vehicle. However, it does influence the comfort a driver experiences during driving. These vibrations cause unwanted accelerations and noise. Therefore, car manufacturers put large efforts in the analysis and reduction of these vibrations.

The analysis of vibrations can be done by modal analysis. Modal analysis focuses on the vehicle response to a range of input frequencies typically between 0 and 100 Hz. At BMW group, there is a special laboratory where a modal analysis can be done on both Bodies in White and complete vehicles. Body in White refers to the stage in automobile manufacturing in which the car body sheet metal has been assembled but before the components and trim have been added. In this laboratory, the input on a vehicle consists of a white noise force signal with a frequency range from 0 to 100 Hz and an amplitude of 49 N. This force signal is applied to the vehicle body on the front right-hand side in the z-direction. The acceleration on different parts of the vehicle is then measured and a transfer function between in- and output is made.

Modal analysis of a car is a costly and time consuming process. In order to reduce costs and to be able to make quick predictions about the behaviour of a vehicle with respect to vibrations, a model can be made. This can be done by using for example finite elements (finite element model) or rigid masses (multi-body model). Finite element models have the advantage over multi-body models that they can be more accurate with extensive tuning. However, the complexity of a finite element model makes adjusting the model harder.

1.2 Aim and scope

The focus in this report is on the development of a simple multi-body model of a complete vehicle in Matlab/Simulink. This model can be used to help to predict the behaviour of a vehicle to a range of input frequencies. In this study, the choice has been made to develop a multi-body model because of its relative simplicity compared to a finite element model. The advantage of a multi-body model is that changes can easily be made to the geometry of the vehicle as well as different parameters such as stiffness, damping, inertia or geometry. Furthermore, the SimMechanics toolbox of Matlab/Simulink is used for the development of the model because of the possibility to process the obtained signals easily using Matlab itself. This is useful for the conversion from time to frequency domain in a modal analysis.

Building a model for a complete vehicle requires obtaining parameters from the vehicle. These parameters include stiffness coefficients, damping coefficients, masses, moments of inertia, coordinates and lengths. These parameters are obtained by using (static) measurements and estimations. For some of the parameters, a finite element model that is built by BMW engineers can be used. This finite element model is made for the Body in White and can be used to obtain mass and moment of inertia parameters of parts of the vehicle body. The finite element model can also be used to obtain stiffness properties of the vehicle body. There is no finite element model available for the complete vehicle.

Finally, the model is validated using the measurements from the modal analysis laboratory as described earlier. Another measurement that is used for validation is from the ‘hydropuls’ laboratory. In this measurement, a sine input is applied to either the front or rear tyres with the right and left side out of
phase. The acceleration at different places on the vehicle is then measured and a transfer function is derived between in- and output. The focus in these validations is on vibrations in the z-direction.

1.3 Contents of the report

The outline of this report is as follows. In chapter 2 a model is built for the Body in White. This model consists of three rigid bodies which represent the vehicle body. The model is validated using measurements from the modal analysis laboratory. The same is done for the complete vehicle in chapter 3. Components such as the tyres, suspension, engine and windscreen are added to the model. The model is again validated using measurements from the modal analysis laboratory in chapter 3 and measurements from the hydropuls laboratory in chapter 4. Finally, conclusions and recommendations for future research are given in chapter 5.
2. Modal analysis of the Body in White

Because of its relative simplicity, a good way to start the modelling and validation process is to use a Body in White. In spite of the simplicity of the Body in White, it can be used to validate the torsional and bending properties of the multi-body model that is modelled in this chapter. The advantage of this model is that a part of the system is tested without having problems caused by components such as tyres, suspension and engine.

2.1 The model

A schematic representation of the multi-body model for the Body in White as well as the coordinate system is displayed in figure 2.1. The origin of the coordinate system is located at the same x- and z-coordinates as where the centres of the front tyres are in the complete vehicle. The y-coordinate of the origin is located in the centre of the vehicle. The x-axis in this system is in the longitudinal direction. This is also the direction in which the vehicle drives forward. The y-axis in this system is in the lateral direction and the z-axis is in the vertical direction. These directions are used throughout the remainder of this report.

![Figure 2.1 Schematic representation of the model](image)

In figure 2.1, the Body in White is divided into three parts. Between these parts are two connections, represented by black circles $A$ and $B$. These connections have two rotational degrees of freedom each. One of the degrees of freedom is rotation around the x-axis, which represents torsion of the Body in White. The second degree of freedom is rotation around the y-axis, which represents bending of the Body in White. Because there are two connections with two degrees of freedom, the system has four degrees of freedom. As a result, there are also four eigenmodes: two bending modes and two torsion modes.

Extra masses are added to the Body in White in the measurements that are used for validation of the multi-body model. These extra masses cause the Body in White to act like the complete vehicle in terms of the eigenmodes and therefore make the comparison between Body in White and complete vehicle easier. These masses are not added physically to the model, but are implemented by increasing the mass and moment of inertia of the vehicle body parts. In total, eight masses are added to the Body in White. In the front of the Body in White, one mass of 20 kg is placed on both the left- and right-hand sides. In the middle, one mass of 20 kg and one mass of 15 kg are placed on the left-hand side. This is also done for the right-hand side. In the rear, one mass of 20 kg is placed on both the left- and right-hand sides.

For this multi-body model, the following assumptions are made:

* The Body in White is divided into three parts.
* The centres of gravity of the vehicle body parts are assumed to be exactly at the centre of the vehicle in the y-direction.
* The joints are located exactly on the middle of the line connecting the centres of gravity of the bodies that the joint connects. This is a rough estimation and is not based on any measurements.
* The added masses are assumed to be placed 60 centimetres from the centre of the vehicle body in the y-direction and placed at the same x- and z-coordinates as the centres of gravity of the vehicle body part that they are connected to. Furthermore, they only affect the mass and moment of inertia around the x-axis of the vehicle body parts. The moment of inertia of a mass itself is neglected.
* The products of inertia of the vehicle body parts are typically 10% or less of the moments of inertia and therefore are neglected. However, the effect of neglecting these products of inertia to the accuracy of the model has not been investigated.
* The Body in White is connected to the ground through soft air springs connected to the middle of the Body in White. This is imitated in the model by using a series of joints with three degrees of freedom connected to the middle body. The stiffness and damping of this connection are low compared to the overall stiffness and damping of the Body in White. This is to keep the interference of this connection with the resonance frequencies of the model to a minimum. The three degrees of freedom are a rotation around the x-axis to allow torsional motions and a rotation around the y-axis and translation in the z-direction to allow bending motions.
* The torsional stiffness and damping is equal for both connections in the Body in White. This is also the case for bending stiffness and damping.
* The stiffness and damping behaviour is assumed to be linear.

**2.2 SimMechanics**

The model that is used in section 2.1 can be modelled using Matlab/SimMechanics and is shown in figure 2.2.

![SimMechanics model](image)

Figure 2.2 SimMechanics model
Figure 2.2 shows the three vehicle body parts connected to each other with a joint that can rotate around the x- and y-axis. The input noise signal is applied to the front body on the right-hand side, similar to the situation from the measurements that are used for validation. The relevant signals are saved into the workspace of Matlab. These signals include the input noise signal, the acceleration at four points on the vehicle body and the time signal. The acceleration is measured on the left- and right-hand sides of both the front and rear body. Finally, the vehicle body is connected to the ground by two rotational joints and one translational joint that allow bending and torsion of the vehicle body.

The connection between the vehicle body parts is shown in figure 2.3. Because it is not possible to connect two joints to each other directly, a mass is placed between the two rotational joints which itself has neither mass nor moment of inertia. This connection can also be replaced by a custom joint to reduce the complexity of the model but has not been done due to time restrictions.

![Figure 2.3 Connection between vehicle body parts](image)

As can be seen from figure 2.3, both rotational joints have their own spring and damper.

The connection between the model and the ground is shown in figure 2.4.

![Figure 2.4 Connection between model and ground](image)

This connection has three degrees of freedom: two rotational degrees of freedom around the x- and y-axis and one translational degree of freedom in the z-direction. Each joint has its own spring and damper coefficient which are kept to a minimum to avoid interference with the resonance frequencies of the vehicle body itself.
2.3 Parameters

2.3.1 Coordinates

The coordinates of the centres of gravity of the three bodies are obtained from the finite element model for the Body in White discussed in section 1.2 and are given in table 2.1. These coordinates are based on the coordinate system described in section 2.1 and figure 2.1. Because of the use of the finite element model, the actual lengths of the vehicle body parts are not necessary. The finite element model computes the mass and moment of inertia of specified parts of the vehicle body which is exactly what is needed for the SimMechanics model.

<table>
<thead>
<tr>
<th>Body</th>
<th>x-coordinate [m]</th>
<th>y-coordinate [m]</th>
<th>z-coordinate [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front body</td>
<td>0.033</td>
<td>0</td>
<td>0.146</td>
</tr>
<tr>
<td>Middle body</td>
<td>1.251</td>
<td>0</td>
<td>0.122</td>
</tr>
<tr>
<td>Rear body</td>
<td>2.356</td>
<td>0</td>
<td>0.268</td>
</tr>
</tbody>
</table>

The coordinates of the joints are given in table 2.2.

<table>
<thead>
<tr>
<th>Joint</th>
<th>x-coordinate [m]</th>
<th>y-coordinate [m]</th>
<th>z-coordinate [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.675</td>
<td>0</td>
<td>0.134</td>
</tr>
<tr>
<td>B</td>
<td>1.804</td>
<td>0</td>
<td>0.195</td>
</tr>
</tbody>
</table>

2.3.2 Stiffness and damping

The values for bending and torsional stiffness are obtained from the finite element model as well.

The torsional stiffness is obtained from a simulation with the finite element model where a torque around the x-axis is applied to the Body in White. The difference in angles between the front and rear suspension mounts then yields the torsional stiffness. The torsional stiffness obtained from this simulation is 8.8236x10^5 Nm/rad. Since there are two joints in the model, the stiffness of the joints is taken to be twice as large to obtain a stiffness of 8.8236x10^5 Nm/rad. The torsional stiffness of the joints therefore becomes 1.7647x10^6 Nm/rad. The torsional damping is obtained from the measurements on the real vehicle. However, the damping is expressed as a damping ratio in the measurements. For the first torsion resonance frequency, this damping ratio is 0.0035. This damping ratio can be converted to a damping coefficient as described in appendix A. A damping coefficient of 59 Nms/rad for both joints causes the damping ratio to become 0.0035.

In a similar way, the bending stiffness and damping coefficients can be computed. The bending stiffness is obtained from a simulation with the finite element model where the Body in White is fixed at the place where the suspension is mounted on the vehicle body in a complete vehicle and a force in the z-direction is applied evenly to the bottom of the passenger compartment. The biggest deflection on the bottom of the passenger compartment is then used to obtain the bending stiffness. The bending stiffness obtained from this simulation is 8700 N/mm, which is a translational stiffness. Since the stiffness in the model is a rotational stiffness, the stiffness obtained from the simulation with the finite element model has to be converted first. This is done by doing a similar simulation with the multi-body model. A bending stiffness of 1380000 Nm/rad for both joints in the model accomplishes the overall bending stiffness obtained from the simulation with the finite element model. The damping ratio is again obtained from the measurements on the real vehicle and is 0.0070 for the first bending resonance frequency. This damping ratio can again be converted to a damping coefficient as described in appendix A. A damping coefficient of 103 Nms/rad for both joints causes the damping ratio to become 0.070.
2.3.3 Mass and moments of inertia

The mass and moment of inertia are obtained from the finite element model as well and are displayed in Table 2.3. Since the vehicle body is divided into three parts, the total mass of the vehicle body has to be divided for the three parts. This is impossible using measurements on the real Body in White without damaging the Body in White. The use of the finite element model makes obtaining these parameters easier. As stated in the assumptions, the products of inertia are neglected.

Table 2.3 Mass and inertia of the multi-body model

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Front body</td>
<td>110</td>
<td>32.3</td>
<td>21.8</td>
<td>44.1</td>
</tr>
<tr>
<td>Middle body</td>
<td>117</td>
<td>50.1</td>
<td>29.1</td>
<td>63.2</td>
</tr>
<tr>
<td>Rear body</td>
<td>119</td>
<td>34.2</td>
<td>19.9</td>
<td>43.9</td>
</tr>
</tbody>
</table>

Because the added masses, discussed in section 2.1, are not included in the finite element model, the mass of the vehicle body parts in the multi-body model has to be increased. The moment of inertia increases as well because of the added masses. As stated in the assumptions, only the moment of inertia around the x-axis changes and can be computed by

\[ \Delta I_{xx} = m_w \cdot r^2 \]  

with

\[ \Delta I_{xx} = \text{the increase in moment of inertia around the x-axis [kgm}^2] \]

\[ m_w = \text{mass of the added weight [kg]} \]

\[ r = \text{distance from the axis of rotation [m]} \]

It is assumed that the masses are placed 60 centimetres from the centre of the vehicle body in the y-direction, so the new values for the masses and moments of inertia can be computed. As stated in the assumptions, the moments of inertia around the y- and z-axis do not change. The adjusted parameters are given in Table 2.4.

Table 2.4 Mass and inertia of the multi-body model with added masses

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Front body</td>
<td>150</td>
<td>46.7</td>
<td>21.8</td>
<td>44.1</td>
</tr>
<tr>
<td>Middle body</td>
<td>187</td>
<td>75.3</td>
<td>29.1</td>
<td>63.2</td>
</tr>
<tr>
<td>Rear body</td>
<td>159</td>
<td>48.6</td>
<td>19.9</td>
<td>43.9</td>
</tr>
</tbody>
</table>

2.4 Validation

The measurements that are used for validation are obtained from the modal analysis laboratory. In these measurements, the Body in White is fixed to the ground by a connection with a very low stiffness to avoid interference with the resonance frequencies of the Body in White itself. A force is applied to the vehicle body by means of a shaker and consists of a white noise signal with a frequency range from 0 to 100 Hz and an amplitude of 49 N. This force is applied to the front right-hand side of the Body in White. The acceleration is then measured on different locations on the Body in White using accelerometers. From the measurement, a frequency response function is calculated with acquisition software called LMS Testlab. The input for this function is the force signal and the output is the measured acceleration. As a result, there are as many frequency response functions as there are measurement points.

The model described in section 2.2 can be used for simulation. The acceleration signals can be transformed into the frequency domain in the form of a frequency response function using the
The imaginary parts of the frequency response functions are used to determine where the different measurement points are in or out of phase, and are a good way to locate the torsional and bending resonance peaks.

The first peak for the model is located at 28.88 Hz and as can be seen from figures 2.5 and 2.6, all measurement points are in phase. This represents the first bending resonance frequency. The second peak is located at 30.48 Hz. The measurement points on the front right- and rear left-hand sides are in phase as well as the measurement points on the front left- and rear right-hand sides. Therefore, this is where the first torsional resonance frequency is located. The third peak is located at 46.08 Hz. Here, the measurement points on the left- and right-hand sides are out of phase and this is where the second torsional resonance frequency is located. The fourth peak is located at 63.12 Hz. At this frequency, the
measurement points on the front are out of phase with the measurement points on the rear. This is the second bending resonance frequency.

A schematic representation of a side view in the y-direction of the modes is given in table 2.5. In this representation, a straight black line represents the left-hand side of a vehicle body part and a straight grey line represents the right-hand side of a vehicle body part. Therefore, a black line above a grey line represents a rotation of that vehicle body part around the x-axis (torsion). A grey line above a black line indicates the same rotation, but out of phase. If there is no grey line to be seen in the representation, there is no rotation around the x-axis. As can be seen in these representations, the complete vehicle body contains three vehicle body parts in series.

Table 2.5 Eigenmodes of the model for the Body in White

<table>
<thead>
<tr>
<th>Eigenfrequency [Hz]</th>
<th>Eigenmode</th>
<th>Schematic representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>28.88</td>
<td>First bending</td>
<td><img src="image" alt="First bending" /></td>
</tr>
<tr>
<td>30.48</td>
<td>First torsion</td>
<td><img src="image" alt="First torsion" /></td>
</tr>
<tr>
<td>46.08</td>
<td>Second torsion</td>
<td><img src="image" alt="Second torsion" /></td>
</tr>
<tr>
<td>63.12</td>
<td>Second bending</td>
<td><img src="image" alt="Second bending" /></td>
</tr>
</tbody>
</table>

To compare the multi-body model with the measurements, similar frequency response plots can be made for the measurements. Figures 2.7 and 2.8 show plots for the imaginary parts of the frequency response functions for the z-direction obtained from the measurements in the modal analysis laboratory. Figure 2.7 shows the imaginary parts of the frequency response functions for the two points on the front of the vehicle, whereas figure 2.8 shows the imaginary parts of the frequency response functions for the two points on the rear of the vehicle.

![Figure 2.7 Frequency response functions in the z-direction on the front of the Body in White (real vehicle)](image)
The first peak for the measurements is located at 28.12 Hz and as can be seen from figures 2.7 and 2.8, all measurement points are in phase. This represents the first bending resonance frequency. The second peak is located at 29.30 Hz. The measurement points on the front right- and rear left-hand sides are in phase as well as the measurement points on the front left- and rear right-hand sides. Therefore, this is where the first torsional resonance frequency is located. Furthermore, there are more peaks in a higher frequency range, but for the comparison with the multi-body model only the second bending and torsion are taken into account. The third peak is located at 39.26 Hz. Here, the measurement point on the left- and right-hand sides are out of phase and this is where the second torsional resonance frequency is located. The last peak that is taken into account is located at 81.84 Hz. At this frequency, the measurement points on the front are out of phase with the measurement points on the rear. This is the second bending resonance frequency. A summary of the results of the comparison are displayed in table 2.6.

As can be seen from table 2.6, the first bending and torsional resonance frequencies are close together. However, the second bending and torsional resonance frequencies are further apart from each other. Also, the measurements have more resonance frequencies than the multi-body model.

To compare the results from the measurements on the real vehicle and the model directly, both functions can be plotted in the same figure. These figures can be found in appendix B. To compare both magnitude and phase, Bode plots have been made instead of the normal frequency response functions.

The main differences, besides the frequency difference as pointed out before, can be found in the magnitude at the resonance frequencies. Table 2.7 shows the relative difference between the magnitudes at the four resonance frequencies for the four measurement points. The magnitude of the multi-body model is expressed in a percentage of the magnitude of the measurements in this table.
Table 2.7 Relative amplitudes for the multi-body model with respect to the measurements

<table>
<thead>
<tr>
<th>Place on the vehicle</th>
<th>Relative amplitude for first bending [%]</th>
<th>Relative amplitude for first torsion [%]</th>
<th>Relative amplitude for second torsion [%]</th>
<th>Relative amplitude for second bending [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front right-hand side</td>
<td>319</td>
<td>100</td>
<td>137</td>
<td>128</td>
</tr>
<tr>
<td>Front left-hand side</td>
<td>237</td>
<td>83</td>
<td>174</td>
<td>160</td>
</tr>
<tr>
<td>Rear right-hand side</td>
<td>448</td>
<td>142</td>
<td>133</td>
<td>611</td>
</tr>
<tr>
<td>Rear left-hand side</td>
<td>263</td>
<td>87</td>
<td>62</td>
<td>325</td>
</tr>
</tbody>
</table>

2.5 Conclusions

This chapter focuses on the validation of the bending and torsion properties of the multi-body model for the Body in White. For this validation, a model is built and it is compared to measurements on a body in white using frequency response functions. The stiffness and damping properties of the body can then be used for the complete vehicle.

The first strong point of the model is that a multi-body model can be easily adjusted. Not only can the layout of the model be adjusted, but also the system parameters. The resulting changes to the system behaviour can then be computed by doing a simulation with the adjusted model.

Another strong point of the model can be found in the resonance frequencies. As can be seen in the results from the validation, the differences in resonance frequencies for the first bending and torsion mode are only 0.76 Hz and 1.18 Hz for respectively the first bending and first torsion mode. Also, the modes for these resonance frequencies are similar.

There are also discrepancies between the model and real vehicle. One of these discrepancies is caused by the division of the vehicle body into three rigid bodies. This means that there are only two bending and two torsion modes, whereas a real vehicle has more bending and torsion modes. Furthermore, since the bodies are rigid, there cannot be any torsion or bending within one of the bodies in the multi-body model. This causes a difference in modes. However, the modes of the multi-body model and real vehicle have been compared by using animations, but no further analysis is done on the modes such as MAC (Modal Assurance Criterion).

The next thing that affects the accuracy is the linear spring and damping behaviour of the multi-body model. The stiffness for bending and torsion is obtained from static measurements. This yields a linear stiffness that can be used for the multi-body model. However, no non-linear effects are taken into account. This may not affect the accuracy of the multi-body model for the Body in White much, but for the complete vehicle it may cause discrepancies. Non-linear elements in the complete vehicle such as the suspension and engine mounts may cause problems.

Finally, several assumptions have been made for the system parameters. The distribution of the vehicle body mass between the three bodies and the moments of inertia for each body are obtained by dividing the finite element model into three parts. Also, the added masses in the measurements are assumed to be at 60 centimetres from the centre of the body in the y-direction and the masses do not affect the moments of inertia of the body masses around the y- and z-axis. Finally, the two connections are assumed to have the same stiffness and damping properties.
3. Modal analysis of the complete vehicle

The next step into the modelling and validation process is the complete vehicle. For the model for the complete vehicle, the basic idea from the Body in White model can be used. Although the stiffness and damping for the joints from the Body in White can be used, the mass and inertia have to be adjusted. This is to account for electronics, plate work, engine, tyres and fuel tank. Furthermore, new parameters have to be used for suspension, tyres, engine and windscreen.

3.1 The model

A schematic representation of the multi-body model for the complete vehicle is displayed in figure 3.1. The model consists of nine bodies. Furthermore, the black circles represent connections between masses and can consist of multiple joints. The dimensions are not representative for the actual dimension in the model.

![Figure 3.1 Side view of the model]

In figure 3.1, point A represents the origin of the system, which is located exactly between the two front tyres on the ground. Points B through F represent joints. The forward driving direction is defined in the negative x-direction.

Table 3.1 gives an explanation of the symbols and numbers used in figure 3.1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Origin of the coordinate system</td>
</tr>
<tr>
<td>B</td>
<td>Two translational joints in the z-direction (suspension and damper top mount)</td>
</tr>
<tr>
<td>C</td>
<td>Two translational joints in the z-direction (suspension and damper top mount)</td>
</tr>
<tr>
<td>D</td>
<td>Two rotational joints around the x- and y-axis</td>
</tr>
<tr>
<td>E</td>
<td>Two rotational joints around the x- and y-axis</td>
</tr>
<tr>
<td>F</td>
<td>Two rotational joints around the y-axis and an axis in the x,z-plane parallel to the plane of the windscreen</td>
</tr>
<tr>
<td>1</td>
<td>Front wheels</td>
</tr>
<tr>
<td>2</td>
<td>Rear wheels</td>
</tr>
<tr>
<td>3</td>
<td>Engine</td>
</tr>
<tr>
<td>4</td>
<td>Front body</td>
</tr>
<tr>
<td>5</td>
<td>Middle body</td>
</tr>
<tr>
<td>6</td>
<td>Rear body</td>
</tr>
<tr>
<td>7</td>
<td>Windscreen</td>
</tr>
</tbody>
</table>

In the rest of this section, a more detailed explanation is given of the connections that are not displayed properly in figure 3.1. Figure 3.2 shows a translational and rotational joint that are used in the
schematic representations further on in this section. Each joint has its own stiffness and damping coefficients.

Figure 3.2 Translational joint (left) and rotational joint (right)

Figure 3.3 shows a more detailed representation of the front suspension.

Figure 3.3 Front suspension

In figure 3.3, there are two disks which represent both front wheels and one rectangular block which represents the front body. There are five translational joints. The two joints in series between wheel and front body are two translational joints in the z-direction and represent the suspension (lower joint) and damper top mount (upper joint). This is done because of the static friction in the suspension. The applied force in the modal analysis measurements is not large enough to overcome the static friction of the suspension. In the next chapter, measurements are used where this is not the case. The suspension therefore has to be separated from the damper top mount in order to be able to lock it. The joint between the wheels on the left- and right-hand sides is a translational joint in the z-direction and represents the anti-roll bar. The rear suspension has a similar layout.

Figure 3.4 shows a more detailed representation of the engine mounts.

Figure 3.4 Engine mounts

In figure 3.4, the rectangular block represents the engine. There are four rotational joints and two translational joints. The line at the bottom represents the front body. As can be seen from this figure, there are two identical connections between engine and front body on the left- and right-hand sides. These connections consist from top to bottom of one translational joint in the z-direction and two rotational joints around the x- and y-direction respectively. As a result, the engine has three degrees of freedom.

For this multi-body model, the following assumptions are made
The model features nine masses: the engine, the windscreen, four wheels and three vehicle body parts.

The centres of gravity of the vehicle body parts, engine and windscreen are assumed to be exactly in the centre of the vehicle in the y-direction.

The vehicle body is divided into three parts with equal lengths in the x-direction.

There is a linear mass distribution between the vehicle body parts. This means that the mass of the middle body is equal to the total mass of both the front body and the rear body divided by 2.

The centre of gravity of the engine is positioned at the same x-coordinate as the engine mounts. Furthermore, it is in the centre of the vehicle in the y-direction and at an equal height as the centres of gravity of the vehicle body parts.

The tyres are modelled using the MF-tyre model from TNO. This model uses a tyre property file.

For the calculation of the moments of inertia, the masses are assumed to be rectangular blocks.

The stiffness and damping coefficients for torsion and bending in the vehicle body are assumed to be the same as for the Body in White.

The torsional stiffness and damping for the two connections in the vehicle body are equal. This is also the case for bending stiffness and damping.

The stiffness and damping behaviour for every joint is assumed to be linear.
3.2 SimMechanics model

The model that is discussed in sections 3.1 can now be built in SimMechanics. The complete model is shown in figure 3.5.

Figure 3.5 SimMechanics model

The relevant signals are saved into the workspace of Matlab. The signals in this model are saved in a structure called $s$.

3.2.1 Vehicle body

The vehicle body is shown in figure 3.6.
Figure 3.6 The vehicle body

Besides the mass and inertia of the vehicle body parts, the basic layout of this part is the same as for the Body in White. Body sensors have been placed on the four corners of each vehicle body part at an equal height as the centres of gravity. The connection to the ground gives the option to give the vehicle an initial condition. Finally, the input force is applied to the vehicle by a body actuator.

The connection between the vehicle body parts is the same as for the Body in White and is displayed in figure 2.3.

3.2.2 Engine

The engine of the vehicle is shown in figure 3.7.
The engine is connected to the front body by means of two engine mounts. As can be seen from the figure, each mount has three degrees of freedom. The acceleration on four places on the engine is measured and saved into the workspace of Matlab.

### 3.2.3 Suspension

The front suspension including anti-roll bar and damper top mount is shown in figure 3.8.

![Figure 3.8 Front suspension and anti-roll bar](image)

The rear suspension has a similar layout, so only the front suspension is discussed here.

The suspension connects the vehicle body to the wheel hub and the wheel hub is connected to the wheels by a revolute joint around the y-axis. This revolute joint represents the wheel bearing and is assumed to rotate without spring or damper forces. The hubs on the left- and right-hand sides are connected to each other with an anti-roll bar. The signal that is measured is the hub acceleration.

Figure 3.9 shows the anti-roll bar.

![Figure 3.9 Anti-roll bar](image)

The anti-roll bar connects the hub on the left- and right-hand sides to each other. It consists of a prismatic joint with a spring and damper acting on it.

The suspension and damper top mount are displayed in figure 3.10.

![Figure 3.10 Suspension block](image)

The suspension spring and damper have been eliminated in order to take the static friction of the damper into account. The damper top mount is modelled as a prismatic joint in the z-direction with a spring and damper acting on it. Because the two joints cannot be connected to each other directly, a mass is placed between them. This mass should be as low as possible in order to reduce the influence of this mass to the dynamic behaviour of the entire vehicle. On the other hand, the mass cannot be too small because the eigenfrequency of this mass can become too high which leads to long calculation times. A high eigenfrequency not only makes the computation time longer, but also requires a high sample frequency. A mass of 3 kg is used between the suspension and damper top mount.

### 3.2.4 Windscreen

The windscreen is shown in figure 3.11.
Figure 3.11 Windscreen

The windscreen has one connection with the middle body. This connection consists of two rotational joints. The acceleration signals on top of the windscreen on the left- and right-hand sides are measured and saved into the workspace of Matlab.

3.2.5 Wheels and tyres

The last part of the SimMechanics model that is discussed is the wheel and tyre. The wheel and tyre for the front left side are shown in figure 3.12.

Figure 3.12 Wheel and tyre for the front left side

The square in the middle in figure 3.12 represents the MF-tyre model. It uses a tyre property file in which all the different tyre parameters are stored which are necessary for the MF-tyre model. There are two inputs to this block and one output. The output is used to save relevant signals to the workspace. The inputs are used for the connection to the wheel bearing and for the road input signal. The road input signal consists of 18 signals, which include the position, the velocity and the angular velocity in three directions and the rotation matrix which consists of nine elements.

Figure 3.13 shows the different items when opening the four post actuator block.

Figure 3.13 Four post actuator block

This block adjusts a specified signal in the z-direction to a road input signal for the MF-tyre model. The differentiator/filter block makes the specified signal differentiable. This is necessary because the MF-tyre model requires both position and velocity signals.
3.3 Parameters

3.3.1 Coordinates

The coordinates of the centres of gravity for the masses are given in table 3.2. The origin is located between the tyre-road contact points of the front tyres (figure 3.1).

Table 3.2 Coordinates of the centres of gravity for the multi-body model

<table>
<thead>
<tr>
<th>Body</th>
<th>x-coordinate [m]</th>
<th>y-coordinate [m]</th>
<th>z-coordinate [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front body</td>
<td>-0.141</td>
<td>0</td>
<td>0.52</td>
</tr>
<tr>
<td>Middle body</td>
<td>1.223</td>
<td>0</td>
<td>0.52</td>
</tr>
<tr>
<td>Rear body</td>
<td>2.586</td>
<td>0</td>
<td>0.52</td>
</tr>
<tr>
<td>Front right wheel</td>
<td>0</td>
<td>0.737</td>
<td>0.319</td>
</tr>
<tr>
<td>Front left wheel</td>
<td>0</td>
<td>-0.737</td>
<td>0.319</td>
</tr>
<tr>
<td>Rear right wheel</td>
<td>2.495</td>
<td>0.762</td>
<td>0.319</td>
</tr>
<tr>
<td>Rear left wheel</td>
<td>2.495</td>
<td>-0.762</td>
<td>0.319</td>
</tr>
<tr>
<td>Engine</td>
<td>0.20</td>
<td>0</td>
<td>0.52</td>
</tr>
<tr>
<td>Windscreen</td>
<td>0.766</td>
<td>0</td>
<td>1.019</td>
</tr>
</tbody>
</table>

The coordinates of the vehicle body parts are based on the technical data from the BMW website [1]. The length of the vehicle in the x-direction is used to divide the total vehicle body length into three equal parts. The height of the centre of gravity is based on an estimation from the height of the vehicle. The coordinates of the wheels are also based on the technical data from the BMW website [1]. The y- and x-coordinates are given and the z-coordinate is based on the radius of the tyre.

The position of the centre of gravity of the engine is based on the assumptions that it is exactly on the same x-coordinate as the engine mounts, precisely in the middle in the y-direction and at an equal height as the centre of gravity of the vehicle body.

The coordinates of the centre of gravity of the windshield are based on the coordinates from measurement points. The measurement points are located at the four edges of the windshield. The centre of gravity is positioned in the middle of those four measurement points.

The coordinates of the joints are given in table 3.3.

Table 3.3 Coordinates of the joints

<table>
<thead>
<tr>
<th>Joint</th>
<th>x-coordinate [m]</th>
<th>y-coordinate [m]</th>
<th>z-coordinate [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front body-front right wheel (B1)</td>
<td>0</td>
<td>0.737</td>
<td>0.319</td>
</tr>
<tr>
<td>Front body-front left wheel (B2)</td>
<td>0</td>
<td>-0.737</td>
<td>0.319</td>
</tr>
<tr>
<td>Rear body-rear right wheels (C1)</td>
<td>2.495</td>
<td>0.762</td>
<td>0.319</td>
</tr>
<tr>
<td>Rear body-rear right wheels (C2)</td>
<td>2.495</td>
<td>-0.762</td>
<td>0.319</td>
</tr>
<tr>
<td>Front body-middle body (D)</td>
<td>0.541</td>
<td>0</td>
<td>0.52</td>
</tr>
<tr>
<td>Middle body-rear body (E)</td>
<td>1.904</td>
<td>0</td>
<td>0.52</td>
</tr>
<tr>
<td>Middle body-windscreen (F)</td>
<td>0.541</td>
<td>0</td>
<td>0.849</td>
</tr>
<tr>
<td>Front body-engine (right)</td>
<td>0.20</td>
<td>0.3</td>
<td>0.52</td>
</tr>
<tr>
<td>Front body-engine (left)</td>
<td>0.20</td>
<td>-0.3</td>
<td>0.52</td>
</tr>
</tbody>
</table>

The coordinates of the joints that connect the vehicle body to the wheels are positioned at the centre of the rims. The coordinates of the joints that connect the different vehicle body parts are positioned exactly on the middle of the line that connects the two centres of gravity of the vehicle body parts it connects. The windshield is attached to the middle body by a joint that is positioned based on the coordinates of a measurement point located at the lower edges of the windshield. Finally, the coordinates of the engine mounts are obtained by a finite element model.

3.3.2 Stiffness and damping

Because the amplitude of the force applied to the vehicle body by the shaker is only 50 N, the input force is not large enough to overcome the static friction in the damper of the suspension. Therefore, the
suspension is eliminated from the model in this chapter. However, the next chapter discusses a validation using an other measurement in which the suspension does not stick. The suspension consists of a prismatic joint that can translate in the z-direction. The value for the stiffness coefficient is based on a rough estimation and is equal to 55000 N/m. This value is comparable to the stiffness value of a sport suspension. The value for the damping coefficient is adjusted to fit the measurements in the next chapter and is equal to 4000 Ns/m.

The stiffness of the damper top mount is based on measurements using a vertical load on the damper top mount of 6000 N, which is a normal loading condition during driving on an average road. The stiffness is equal to 600000 N/m. The damping coefficient for the damper top mount is adjusted to fit the measurements and is equal to 5000 Ns/m.

The torsion and bending stiffness and damping values for the vehicle body are assumed to be equal to the Body in White. The bending stiffness and damping are 1380000 Nm/rad and 103 Nms/rad respectively for joints D and E. The torsion stiffness and damping are 1764700 Nm/rad and 59 Nms/rad for joints D and E.

The value for the stiffness coefficient of the anti-roll bar is based on an estimation and is equal to 55000 N/m for both the front and rear. The damping value is assumed to be equal to 0 for both the front and rear.

The engine mount stiffness in the z-direction is obtained from a measurement on a real engine mount. The static stiffness is 340000 N/m and is used for both engine mounts. The value of the damping coefficient is adjusted to fit the measurements and is equal to 3000 Ns/m. A joint with a rotational degree of freedom around the x-axis allows the engine to rotate around the x-axis as well. This joint does not have stiffness or damping. However, the stiffness and damping in the z-direction acts on the rotation of the engine around the x-axis as well. Finally, the third degree of freedom for the engine is around the y-axis. The stiffness and damping values for this joint are both adjusted to fit the measurements. The value for the stiffness coefficient is equal to 140000 Nm/rad and the value for the damping coefficient is 50 Nms/rad.

The value for the stiffness coefficients for the windscreen is obtained from a simulation with the finite element model. The stiffness around the y-axis as well as the torsional stiffness for the windscreen for the multi-body model is adjusted to fit the simulation with the finite element model. This yields a value for the stiffness around the y-axis of 302800 Nm/rad. The value for torsional stiffness is equal to 549825 Nm/rad. The damping coefficients are adjusted to fit the measurements and have values of 100 Nms/rad for the motion around the y-axis and 300 Nms/rad for the torsional motion.

The coefficients for the tyres are adjusted by TNO and can be found in the tyre property file. The tyre that the parameters are based on is the Bridgestone Turanza RFT with dimensions 205/55 R16.

Table 3.4 and 3.5 give a list of all the values obtained in this section for translational and rotational joints respectively.

### Table 3.4 Stiffness and damping values for translational joints

<table>
<thead>
<tr>
<th>Connection</th>
<th>Degree of freedom</th>
<th>Stiffness coefficient [N/m]</th>
<th>Damping coefficient [Ns/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Suspension</td>
<td>z-direction</td>
<td>55000</td>
<td>4000</td>
</tr>
<tr>
<td>Damper top mount</td>
<td>z-direction</td>
<td>600000</td>
<td>5000</td>
</tr>
<tr>
<td>Anti-roll bar</td>
<td>z-direction</td>
<td>55000</td>
<td>0</td>
</tr>
<tr>
<td>Engine mount</td>
<td>z-direction</td>
<td>340000</td>
<td>3000</td>
</tr>
</tbody>
</table>

### Table 3.5 Stiffness and damping values for rotational joints

<table>
<thead>
<tr>
<th>Connection</th>
<th>Degree of freedom</th>
<th>Stiffness coefficient [Nm/rad]</th>
<th>Damping coefficient [Nms/rad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front body - middle body</td>
<td>x-axis</td>
<td>1764700</td>
<td>59</td>
</tr>
<tr>
<td>Front body - middle body</td>
<td>y-axis</td>
<td>1380000</td>
<td>103</td>
</tr>
<tr>
<td>Middle body - rear body</td>
<td>x-axis</td>
<td>1764700</td>
<td>59</td>
</tr>
<tr>
<td>Middle body - rear body</td>
<td>y-axis</td>
<td>1380000</td>
<td>103</td>
</tr>
<tr>
<td>Engine mount</td>
<td>y-axis</td>
<td>1400000</td>
<td>50</td>
</tr>
<tr>
<td>Windscreen</td>
<td>y-axis</td>
<td>3028000</td>
<td>100</td>
</tr>
<tr>
<td>Windscreen</td>
<td>specified axis</td>
<td>549825</td>
<td>300</td>
</tr>
</tbody>
</table>
3.3.3 Dimensions

Since there is no finite element model to obtain data with respect to the moment of inertia for the vehicle body and engine, some assumptions have to be made. The bodies are assumed to be rigid blocks, so the moment of inertia can be described by the following set of equations

\[
I_x = \frac{1}{12} m \left( l_y^2 + l_z^2 \right) \\
I_y = \frac{1}{12} m \left( l_x^2 + l_z^2 \right) \\
I_z = \frac{1}{12} m \left( l_x^2 + l_y^2 \right)
\]

(3.1)

where

- \( I \) = moment of inertia [kgm²]
- \( m \) = mass of the object [kg]
- \( l \) = length [m]
- \((..)_{xx}\) = around the x-axis
- \((..)_{yy}\) = around the y-axis
- \((..)_{zz}\) = around the z-axis
- \((..)_{x}\) = in the x-direction
- \((..)_{y}\) = in the y-direction
- \((..)_{z}\) = in the z-direction

In order to calculate the moments of inertia, the dimensions have to be known. Table 3.6 gives the values for the dimensions that are used for the calculation of the moment of inertia. They are all rough estimations from the technical data from the BMW website [1].

Table 3.6 Dimensions for the vehicle body and engine

<table>
<thead>
<tr>
<th>Body</th>
<th>Length in the x-direction [m]</th>
<th>Length in the y-direction [m]</th>
<th>Length in the z-direction [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front body</td>
<td>1.364</td>
<td>1.781</td>
<td>0.69</td>
</tr>
<tr>
<td>Middle body</td>
<td>1.364</td>
<td>1.781</td>
<td>0.69</td>
</tr>
<tr>
<td>Rear body</td>
<td>1.364</td>
<td>1.781</td>
<td>0.69</td>
</tr>
<tr>
<td>Engine</td>
<td>0.650</td>
<td>0.450</td>
<td>0.450</td>
</tr>
</tbody>
</table>

3.3.4 Mass and moment of inertia

The masses for the different parts in the multi-body model are given in table 3.7.

Table 3.7 Masses of the different parts

<table>
<thead>
<tr>
<th>Body</th>
<th>Mass [kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine</td>
<td>330</td>
</tr>
<tr>
<td>Complete vehicle</td>
<td>1450</td>
</tr>
<tr>
<td>Tyre</td>
<td>11.75</td>
</tr>
<tr>
<td>Rim</td>
<td>8.81</td>
</tr>
<tr>
<td>Windscreen</td>
<td>21.3</td>
</tr>
</tbody>
</table>

The mass of the windscreen is obtained from the finite element model for the Body in White, the rest of the masses are obtained by measuring the mass of the real parts.

Because the vehicle body consists of three parts, the vehicle body mass has to be distributed between these parts. The maximal permitted axle load is 780 kg and 840 kg for the front and rear axle
respectively. This distribution is used to divide the total body mass: 48% of the mass is supported by the front tyres and 52% is supported by the rear tyres. This means that of the mass of the complete vehicle (1450 kg), 696 kg rests on the front tyres and 754 kg rests on the rear tyres. A schematic representation of this situation is shown in figure 3.14. The rims and windscreen have been neglected in computing the mass distribution of the vehicle body parts.

In figure 3.14, \( m_1, m_2 \) and \( m_3 \) represent the masses of respectively the front, middle and rear body and \( m_e \) represents the mass of the engine. The lengths \( v \) through \( z \) are given in table 3.8.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Length [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v )</td>
<td>0.141</td>
</tr>
<tr>
<td>( w )</td>
<td>0.200</td>
</tr>
<tr>
<td>( x )</td>
<td>1.023</td>
</tr>
<tr>
<td>( y )</td>
<td>1.272</td>
</tr>
<tr>
<td>( z )</td>
<td>0.091</td>
</tr>
</tbody>
</table>

In order to compute the mass distribution in the vehicle body, two moment equilibriums can be used. A moment equilibrium around the rear tyres yields

\[
(v + w + x + y) \cdot m_1 + (x + y) \cdot m_2 + y \cdot m_3 = (w + x + y) \cdot 696 + z \cdot m_e
\]  

(3.2)

with

- \( m_e \) = mass of the engine
- \( m_1 \) = mass of the front body
- \( m_2 \) = mass of the middle body
- \( m_3 \) = mass of the rear body

A moment equilibrium around the front tyres yields

\[
(w + x + y) \cdot 754 + v \cdot m_1 = (w + x + y + z) \cdot m_3 + (w + x) \cdot m_2 + w \cdot m_e
\]  

(3.3)

In order to solve the mass distribution problem, one more equation is needed since there are three unknowns and two moment equilibriums. As stated in the assumptions, the third equation is

\[
m_2 = \frac{1}{2} m_1 + \frac{1}{2} m_3
\]  

(3.4)
The mass of the engine $m_e$ is 330 kg, and combining (3.2), (3.3) and (3.4) yields

\[
m_i = 209.8 \text{ kg} \\
m_s = 373.3 \text{ kg} \\
m_v = 536.8 \text{ kg}
\]

The moments of inertia for the vehicle body and engine can be calculated using (3.1) combined with tables 3.6 and 3.7 and the masses obtained for the three vehicle body parts. The moment of inertia for the windscreen is obtained from the finite element model. The results are displayed in table 3.9.

**Table 3.9 Moments of inertia**

<table>
<thead>
<tr>
<th>Body</th>
<th>Moment of inertia around x-axis [kg.m$^2$]</th>
<th>Moment of inertia around y-axis [kg.m$^2$]</th>
<th>Moment of inertia around z-axis [kg.m$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front body</td>
<td>63.8</td>
<td>40.9</td>
<td>88.0</td>
</tr>
<tr>
<td>Middle body</td>
<td>107.0</td>
<td>68.5</td>
<td>147.6</td>
</tr>
<tr>
<td>Rear body</td>
<td>163.2</td>
<td>104.5</td>
<td>225.1</td>
</tr>
<tr>
<td>Engine</td>
<td>11.1</td>
<td>17.2</td>
<td>17.2</td>
</tr>
<tr>
<td>Windscreen</td>
<td>5.4</td>
<td>1.0</td>
<td>5.8</td>
</tr>
</tbody>
</table>

### 3.4 Validation

The measurements that are used for validation in this chapter are obtained from the modal analysis laboratory. The measurement has a similar setup as the Body in White measurement, only the vehicle is placed on its tyres. From the measurement, a frequency response function is calculated with acquisition software called LMS Testlab. The input for this function is a force in the z-direction applied to the front right-hand side of the vehicle body and the output is the acceleration of a point located on the vehicle. The force is applied to the vehicle body by means of a shaker and consists of a white noise signal with a frequency range from 0 to 100 Hz and an amplitude of 49 N. The acceleration is measured on different locations on the vehicle using accelerometers. As a result, there are as many frequency response functions as there are measurement points. The points that are used for this validation are the same four points on the vehicle body as discussed in section 2.2. The rest of the points include one point on each rim, four points on the engine and two points on the windscreen.

The model described in section 3.2 can be used for simulation. The acceleration signals can be transformed into the frequency domain in the form of a frequency response function using the `tfestimate` command in Matlab. Figures 3.15 and 3.16 show the imaginary parts of the frequency response functions of the four points on the vehicle body in the z-direction. Figure 3.15 shows the imaginary parts of the frequency response functions for the two points on the front of the vehicle body, whereas figure 3.16 shows the imaginary parts of the frequency response functions for the two points on the rear of the vehicle body.
Since the windscreen has a low mass compared to components such as the engine and vehicle body, the resonance frequencies of the windscreen itself cannot be seen in figures 3.15 and 3.16. Therefore, the imaginary parts of the frequency response functions of the windscreen are shown in figure 3.17.
Figure 3.17 Frequency response functions in the z-direction of the windscreen (multi-body model)

From figures 3.15 through 3.17 eleven modes can be distinguished. These modes include three ‘rigid body’ modes, two modes concerning engine movements, four modes concerning movement within the vehicle body and two modes concerning windscreen movements. The modes are explained in detail further on in this report.

To compare the multi-body model with the measurements, similar frequency response plots can be made for the measurements. Figures 3.18 and 3.19 show plots for the imaginary parts of the frequency response functions for the z-direction obtained from the measurements in the modal analysis laboratory. Figure 3.18 shows the imaginary parts of the frequency response functions for the two points on the front of the vehicle body, whereas figure 3.19 shows the imaginary parts of the frequency response functions for the two points on the rear of the vehicle.
As can be seen from figures 3.18 and 3.19, there are many more resonance frequencies for the real vehicle than for the multi-body model. By using LMS Testlab, animations can be made of the eigenmodes at different frequencies. This way it is easier to distinguish the eigenmodes. Table 3.10 shows the results of both the multi-body model and the measurement.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency multi-body model [Hz]</th>
<th>Frequency measurements [Hz]</th>
<th>Absolute difference [Hz]</th>
<th>Main deflections</th>
</tr>
</thead>
<tbody>
<tr>
<td>Translation of the vehicle in the z-direction</td>
<td>3.4</td>
<td>5.4</td>
<td>2.0</td>
<td>-Tyres -Damper top mounts</td>
</tr>
<tr>
<td>Rotation of the vehicle around the y-axis</td>
<td>3.8</td>
<td>6.1</td>
<td>2.3</td>
<td>-Tyres -Damper top mounts</td>
</tr>
<tr>
<td>Rotation of the vehicle around the x-axis</td>
<td>6.8</td>
<td>9.3</td>
<td>2.5</td>
<td>-Tyres -Damper top mounts</td>
</tr>
<tr>
<td>Translation of the engine in the z-direction</td>
<td>9.6</td>
<td>11.2</td>
<td>1.6</td>
<td>-Engine mounts</td>
</tr>
<tr>
<td>Rotation of the engine around the y-axis</td>
<td>15.2</td>
<td>17.0</td>
<td>1.8</td>
<td>-Engine mounts</td>
</tr>
<tr>
<td>First torsion</td>
<td>20.4</td>
<td>22.9</td>
<td>2.5</td>
<td>-Body</td>
</tr>
<tr>
<td>First bending</td>
<td>21.6</td>
<td>21.9</td>
<td>0.3</td>
<td>-Body</td>
</tr>
<tr>
<td>Second torsion</td>
<td>34.2</td>
<td>42.5</td>
<td>8.3</td>
<td>-Body</td>
</tr>
<tr>
<td>Second bending</td>
<td>37.6</td>
<td>36.8</td>
<td>0.8</td>
<td>-Body</td>
</tr>
<tr>
<td>Windscreen torsion</td>
<td>50.2</td>
<td>-</td>
<td>-</td>
<td>-Windscreen</td>
</tr>
<tr>
<td>Rotation of the windscreen around the y-axis</td>
<td>54.0</td>
<td>-</td>
<td>-</td>
<td>-Windscreen</td>
</tr>
</tbody>
</table>

In table 3.10, only the eigenmodes from the measurement that are comparable to the eigenmodes of the multi-body model are taken into account. However, the real vehicle has more resonance frequencies than that are displayed in table 3.10. Besides this difference, some of the modes are not completely similar to each other. This is discussed in section 3.5.
The differences displayed in table 3.10 can have several causes. These causes are discussed in the remainder of this section.

First of all, the resonance frequencies for the first three modes of the multi-body model are too low. These modes are mainly affected by the tyres and damper top mounts. This difference can partly be caused by the difference in used tyres. The tyres that are on the real vehicle are Bridgestone Potenza tyres with dimensions 225/45 R17, whereas the tyre property file that is made for the MF-tyre model in the multi-body model is based on a Bridgestone Turanza tyre with dimensions 205/55 R16. An other thing that can cause a difference is the stiffness of the damper top mounts. The value of the stiffness coefficient is based on a rough estimation. Finally, the assumptions made for the moment of inertia and mass distribution in the vehicle body can also cause differences.

Secondly, the resonance frequencies for the two modes of the engine are too low as well. This can be caused by the difference in how the engine is attached to the vehicle body. In the multi-body model, the engine is mounted to the vehicle body by means of two engine mounts. This is the same for the real vehicle. However, the engine in the real vehicle is attached to the driveline which is also attached to the vehicle body. This attachment is not taken into account in the multi-body model and can cause differences in both resonance frequencies as well as mode shapes.

The frequency difference of the second torsion and bending modes between model and real vehicle is larger than the frequency difference of the first torsion and bending modes. This can also be seen in the validation of the multi-body model for the Body in White. Secondly, the frequency difference for the two torsion modes between model and real vehicle is larger than for the two bending modes. Again, this can be due to the assumptions for the mass distribution and moment of inertia.

Finally, the windscreen in the multi-body model has two resonance frequencies but there is no data available of the resonance frequencies in the measurement.

To compare the results from the measurements on the real vehicle and the model directly, both functions can be plotted in the same figure. These figures can be found in appendix C. To compare both magnitude and phase, Bode plots have been made instead of the normal frequency response functions.

From the figures in appendix C, it can be seen that especially the amplitude of the peak for the rotation of the engine around the y-axis is too large for the model. Adjusting the damping value for this degree of freedom does not give a solution because it affects the first bending resonance frequency as well. So it is a compromise between differences in the first bending mode and the mode where the engine rotates around the y-axis.

### 3.5 Conclusions

This chapter focuses on the validation of the multi-body model for the complete vehicle. For this validation, a model is built and it is compared to measurements on a complete vehicle using frequency response functions.

The first strong point of the model is that a multi-body model can be easily adjusted. Not only can the layout of the model be adjusted, but also the system parameters. The resulting changes to the system behaviour can then be computed by doing a simulation with the adjusted model.

Another strong point of this relatively simple model is the small difference between resonance frequencies. Except for the second torsion mode, the largest difference between resonance frequencies is equal to or below 2.5 Hz.

Also, the mode shapes of especially the rigid body modes and the torsion and bending modes of the vehicle body look similar. However, the modes of the multi-body model and real vehicle have been compared by using animations, but no further analysis is done on the modes such as MAC (Modal Assurance Criterion).
There are also discrepancies between the model and real vehicle. First of all, the vehicle body is divided into three rigid bodies. This means that there are only two bending and two torsion modes, whereas a real vehicle has more bending and torsion modes. Furthermore, since the bodies are rigid, there cannot be any torsion or bending within one of the bodies in the multi-body model. This can cause a difference in modes.

The next thing that affects the accuracy is the linear spring and damping behaviour of the multi-body model. The stiffness for bending and torsion is obtained from static measurements. This yields a linear stiffness that can be used for the multi-body model. However, no non-linear effects are taken into account.

Another thing that causes differences is the way the engine is mounted to the vehicle body. This causes differences in mode shapes between model and real vehicle. As a result, there are no modes for the rotation of the engine around the y-axis and translation of the engine in the z-direction that are exactly similar to the modes obtained from measurements on the real vehicle. The way that these modes are validated is to compare the modes from the model with modes from the measurement that look the most similar.

Not all the components of the real vehicle are modelled in the multi-body model. This can also cause differences. These components include for example the fuel tank and the systems for the exhaust, cooling and electrical equipment.

Finally, several assumptions have been made for the inertia parameters. The distribution of the vehicle body mass between the three bodies and the moments of inertia for each body are obtained by dividing the finite element model into three parts. Also, the added masses in the measurements are assumed to be at 60 centimetres from the centre of the body in the y-direction and the masses do not affect the moments of inertia around the y- and z-axis.

There are also causes for differences between measurement and model that are not due to the multi-body model itself. One of these causes is the accuracy of the provided data. The accelerometers are accurate above about 5 Hz. Especially for the rigid body modes, which are close to that frequency, nothing can be said about the accuracy of the data. Also, due to the conversion from time to frequency domain, effects such as leakage can cause differences.
4. Validation of the complete vehicle using hydropuls

The second validation for the complete vehicle is called a ‘hydropuls’ measurement. In this measurement, either the front or the rear tyres are excited. The input signal for the tyres is a sine and the left- and right-hand sides are excited out of phase. This is done for a frequency range of 5 till 30 Hz with a resolution of 0.2 Hz. The acceleration at different places on the vehicle is measured and a transfer function between the input acceleration and the acceleration of the measurement points is obtained using LMS Testlab.

The model that is used in chapter 3 can be used for the comparison with this measurement as well. However, some minor changes have to be made. Instead of exciting the vehicle body directly, the input signal is applied to the tyres. This is shown in figure 4.1 for excitation of the front wheels. The input acceleration signal is saved under the name \textit{noise} in the Matlab workspace.

![Figure 4.1 Excitation of the front wheels](image)

Another change is that the measurement point on the windscreen is put in the centre of the windscreen in the \$y\$-direction instead of on the corners. This is shown in figure 4.2

![Figure 4.2 Windscreen](image)

The last change is that the input force for this measurement is large enough to overcome the static friction of the damper in the suspension. Therefore, the suspension has to be modelled as well. This is shown in figure 4.3. The weld in figure 3.13 has been replaced by a translational joint that represent the suspension.

![Figure 4.3 Suspension](image)

Since the measurements in the hydropuls laboratory have only one measurement point on the engine, only one of the measurement points on the engine from the model will be used as well. This is the point on the front left-hand side of the engine. This point is chosen, because it is the closest to the measurement point from the hydropuls measurement.
4.1 Out of phase excitation of the rear wheels

The acceleration signals can be transformed into the frequency domain in the form of a frequency response function using the `tfestimate` command in Matlab. Figures 4.4 and 4.5 show the imaginary parts of the frequency response functions of two points on the rear of the vehicle body in the z-direction. Figure 4.4 shows the imaginary parts of the frequency response functions for the multi-body model, whereas figure 4.5 shows the imaginary parts of the frequency response functions for the real vehicle.

![Graph showing frequency response functions.](image)

**Figure 4.4** Frequency response functions in the z-direction on the rear of the vehicle body (multi-body model)

![Graph showing frequency response functions.](image)

**Figure 4.5** Frequency response functions in the z-direction on the rear of the vehicle body (real vehicle)

Furthermore, the Bode plots are again plotted in one figure for both the measurements on the real vehicle and the model. The plots can be found in appendix D. The measurement points include the four points on the vehicle body that are the same as in chapter 3, one point on the front of the engine and one point on the top of the windshield.

The first resonance frequency that is examined for the multi-body model has a value of 5.6 Hz. This resonance represents a mode where the vehicle body rotates around the x-axis (vehicle roll) and the biggest deflections are in the suspension. This eigenmode is located at 7.2 Hz for the measurement. The
The second resonance frequency for the multi-body model is located at 20.2 Hz and represents torsion of the body. This eigenmode is located at 21.0 Hz for the measurements. The same tendencies for these two modes can be seen in chapter 3.

Besides the resonance frequency difference, there are also large differences in the Bode plots of the engine and windscreen. The difference in the windscreen can be explained by the way the vehicle is excited. Due to the out of phase excitation of the left- and right-hand sides, the point in the centre of the windscreen does not translate much in the z-direction. This causes the magnitude to be low and also causes the Bode plots to be noisy. Because the eigenmodes of the real vehicle are more complex, this does not apply to the real vehicle. The difference in the frequency response functions of the engine can be explained by the differences in the way the engine is mounted to the vehicle body as discussed in section 3.5.

Finally, as can be seen from figures 4.4 and 4.5, the real vehicle has more resonance frequencies than the multi-body model. This can also be seen in the validation using the modal analysis measurements in chapter 3.

**4.2 Out of phase excitation of the front wheels**

The measurement that is used for this validation has only one difference with the previous measurement discussed in section 4.1. Instead of exciting the rear tyres, the front tyres are excited with the left- and right-hand sides out of phase. Figures 4.6 and 4.7 show the imaginary parts of the frequency response functions of two points on the front of the vehicle body in the z-direction. Figure 4.6 shows the imaginary parts of the frequency response functions for the multi-body model, whereas figure 4.7 shows the imaginary parts of the frequency response functions for the real vehicle.

![Figure 4.6](image_url)

**Figure 4.6 Frequency response functions in the z-direction on the front of the vehicle body (multi-body model)**
Figure 4.7 Frequency response functions in the z-direction on the front of the vehicle body (real vehicle)

The Bode plots of both the measurements and the model for the same six points as discussed in section 4.1 are displayed in appendix E.

From these figures, the same conclusions can be drawn as in section 4.1. The resonance frequencies of the rotation of the vehicle body around the x-axis and torsion of the vehicle body are too low for the multi-body model. The differences in Bode plots of the windscreen and engine are again larger.
5. Conclusions and recommendations

In this chapter, the main conclusions of the project are presented as well as some recommendations for further research.

5.1 Conclusions

In this report, a vibration model for a complete vehicle is built. The first step towards this model is to build a model for the Body in White to validate the torsion and bending properties of the vehicle body. The main assumptions for this model are that the model consists of three masses that are connected to each other with connections with two degrees of freedom. This connection allows bending and torsion in the body and the stiffness and damping for these connections is assumed to be linear.

The model is validated using measurements performed in the modal analysis laboratory. From this validation, the main conclusions are as follows:

* The difference in resonance frequencies between the model and the measurements are around 1 Hz for the first bending and torsion resonance. The difference becomes larger for the second torsion resonance (7 Hz) and second bending resonance (19 Hz).
* The difference in amplitudes of the imaginary parts of the frequency response functions between the model and the measurements is significant. The amplitudes of the model are between 62% and 611% of the amplitudes of the measurements when looking at the four matching resonance frequencies of both model and measurements.

Secondly, a model for the complete vehicle is built. The main assumptions for this model are that the model consists of nine masses. These include the engine, windscreen, four wheels and three vehicle body parts. The stiffness and damping of the connections between these masses is linear and the stiffness and damping between the vehicle body parts is the same as for the Body in White. Finally, the tyres are modelled using the MF-tyre model from TNO.

The model is validated using measurements performed in both the modal analysis laboratory and the hydropuls laboratory. From this validation, the main conclusions are as follows:

* The largest difference in resonance frequencies between the model and the measurements is 8.3 Hz. However, the rest of the differences in the compared modes is equal or less than 2.5 Hz.
* The amplitudes of the imaginary parts of the frequency response functions for both the model and measurements in the modal analysis laboratory are comparable. However, the amplitude for the resonance where the engine rotates around the x-axis is 4 times as high for the model.
* The outcome of the validation using the measurements in the hydropuls laboratory is similar to the validation using the measurements from the modal analysis laboratory.

5.2 Recommendations

The model for the complete vehicle presented in this report has some points that can be improved. From the validation it is shown that the worst part of the model is the connection between engine and vehicle body. Not only do the resonance frequencies differ, but the eigenmodes are different as well between the model and the real vehicle. Therefore, the improvement of the connection between engine and vehicle body is a good way to improve the overall accuracy of the model. However, this requires information about the exhaust system and driveline and the connections to the vehicle body.

Another point of improvement is the damping within the elements and components of the vehicle. Most of the values of the damping coefficients are adjusted to fit the measurements or based on assumptions. However, there are no available measurements to identify the damping characteristics. This also counts
for some stiffness coefficients such as suspension stiffness, anti-roll bar stiffness and rotational stiffness of the engine mounts.

The model obtained in this research is suited for a stationary modal analysis. The model is not validated for driving conditions and is therefore not suitable for simulating driving conditions. To adjust this model to driving conditions, some points have to be adjusted. One of these points is the tyre property file for the MF-tyre model. This file is not adjusted correctly for longitudinal slip of the tyre. Once the model is adjusted, other measurements on the real vehicle can be performed including measurements while driving.

Finally, some components that were ignored might improve the accuracy of the model. This includes for example fuel tank, electrical-, cooling- and exhaust systems and steering column. However, including these components requires information about these components such as stiffness, damping, inertia and dimensions.
References


Appendix A Damping ratio

A.1 Torsion

The damping ratio can be computed by using a fading oscillation [2]. Figure A.1 shows a result from a simulation of the model where a sine input on the front right-hand side of the vehicle with a frequency equal to the resonance frequency is suddenly turned off. This figure shows the damping of the torsional resonance.

A line through the minima and maxima can be used to compute the damping ratio. In case of an underdamped system, this line can be described as

\[ a = \pm A \cdot e^{-\zeta \omega_n t} \]  

(A.1)

with

- \( a \) = acceleration
- \( A \) = constant
- \( \zeta \) = damping ratio
- \( \omega_n \) = natural undamped frequency
- \( t \) = time

With the least squares method in Matlab (A.1) can be solved. This is displayed in figure A.2.
The dashed lines show the results obtained with Matlab. These lines can be described as

\[ a = \pm 67 \cdot e^{-0.6795t} \] (A.2)

Furthermore, the natural damped frequency for an underdamped system can be described as

\[ \omega_d = \omega_n \sqrt{1 - \zeta^2} \] (A.3)

with

\[ \omega_d = \text{natural damped frequency} \]

The natural damped frequency is obtained from section 2.4 and is equal to 192.14 rad/s. The results obtained so far yield the following two equations

\[ \omega_n \sqrt{1 - \zeta^2} = 192.14 \] (A.4)
\[ \zeta \cdot \omega_n = 0.6795 \] (A.5)

Substituting (A.5) in (A.4) gives a damping ratio of 0.0035.

### A.2 Bending

In a similar way, the damping ratio can be computed for bending. Figure A.3 shows the results of the lines through the maxima and minima.
Figure A.3 Fitting a line through the minima and maxima for bending

The dashed lines can be described as

\[ a = \pm 1960 \cdot e^{-1.2685t} \]  \hspace{1cm} (A.6)

The natural damped frequency is obtained from section 2.4 and is equal to 180.96 rad/s and together with (A.6) it yields the following two equations

\[ \omega_n \sqrt{1 - \zeta^2} = 180.96 \]  \hspace{1cm} (A.7)

\[ \zeta \cdot \omega_n = 1.2685 \]  \hspace{1cm} (A.8)

Substituting (A.8) in (A.7) gives a damping ratio of 0.0070.
Appendix B Bode plots of the Body in White using modal analysis

Figure B.1 Bode plot for the front right-hand side of the vehicle body in the z-direction

Figure B.2 Bode plot for the front left-hand side of the vehicle body in the z-direction
Figure B.3 Bode plots for the rear right-hand side of the vehicle body in the z-direction

Figure B.4 Bode plots for the rear left-hand side of the vehicle body in the z-direction
Appendix C Bode plots of the complete vehicle using modal analysis

![Figure C.1 Bode plot for the front right-hand side of the vehicle body in the z-direction](image1)

![Figure C.2 Bode plot for the front left-hand side of the vehicle body in the z-direction](image2)
Figure C.3 Bode plots for the rear right-hand side of the vehicle body in the z-direction

Figure C.4 Bode plots for the rear left-hand side of the vehicle body in the z-direction
Figure C.5 Bode plots for the rim on the front right-hand side in the z-direction

Figure C.6 Bode plots for the rim on the front left-hand side in the z-direction
Figure C.7 Bode plots for the rim on the rear right-hand side in the z-direction

Figure C.8 Bode plots for the rim on the rear left-hand side in the z-direction
Figure C.9 Bode plots for the front right-hand side of the engine in the z-direction

Figure C.10 Bode plots for the front left-hand side of the engine in the z-direction
Figure C.11 Bode plots for the rear right-hand side of the engine in the z-direction

Figure C.12 Bode plots for the rear left-hand side of the engine in the z-direction
Figure C.13 Bode plots for the right-hand side of the windscreen in the z-direction

Figure C.14 Bode plots for the left-hand side of the windscreen in the z-direction
Appendix D Bode plots of the complete vehicle using hydropuls with rear wheel excitation

Figure D.1 Bode plots for the front right-hand side of the vehicle body in the z-direction

Figure D.2 Bode plots for the front left-hand side of the vehicle body in the z-direction
Figure D.3 Bode plots for the rear right-hand side of the vehicle body in the z-direction

Figure D.4 Bode plots for the rear left-hand side of the vehicle body in the z-direction
Figure D.5 Bode plots for the engine in the z-direction

Figure D.6 Bode plots for the windscreen in the z-direction
Appendix E Bode plots of the complete vehicle using hydropuls with front wheel excitation

Figure E.1 Bode plots for the front right-hand side of the vehicle body in the z-direction

Figure E.2 Bode plots for the front left-hand side of the vehicle body in the z-direction
Figure E.3 Bode plots for the rear right-hand side of the vehicle body in the z-direction

Figure E.4 Bode plots for the rear left-hand side of the vehicle body in the z-direction
Figure E.5 Bode plots for the engine in the z-direction

Figure E.6 Bode plots for the windscreen in the z-direction