Experimental characterization of the stick/sliding transition in a precision mechanical system using the Third Order Sinusoidal Input Describing Function

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Abstract

In this paper a new, non-parametric frequency domain based measurement technique is introduced that enables capturing the stick to gross sliding transition of a mechanical system with dry friction. The technique is an extension of the Sinusoidal Input Describing Function theory (SIDF) to Higher Order Describing Functions. The resulting Higher Order Sinusoidal Input Describing Functions (HOSIDF) relate the magnitude and phase of the higher harmonics in the periodic response of a non-linear system to the magnitude and phase of the sinusoidal excitation. A non-linear mechanical system with dry friction is analyzed using both the classical Frequency Response Function (FRF) technique and the newly developed HOSIDF technique. Where the FRF technique is not able to identify the stick/sliding transition of the system, the third order SIDF clearly displays this transition. From the third order SIDF the pre-sliding displacement of the system is determined. The first order SIDF is used to generate information about the resonance frequency of the system due to the friction-induced stiffness. From the pre-sliding displacement and the friction-induced stiffness, the friction force is calculated which must be present in the stick-phase. Validation with force measurements shows excellent agreement.

Key words: Frequency domain analysis, nonlinear systems, harmonic distortion, describing function, system identification, friction, machine condition monitoring.

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1 Introduction

In many high precision positioning systems, position accuracy is an important aspect. During the last three decades, accuracy requirements changed from the micrometer range to the nanometer range. Examples of high precision systems are wafer steppers for lithographic applications, laser beam recorders for CD/DVD mastering, lathes for the production of contact lenses and also consumer products like hard disc drives and optical storage devices. A common problem in the design and operation of these dynamic systems is nonlinear behavior like friction. Friction has a strong influence on positioning accuracy because it can lead to tracking errors, large settling times and limit cycling [1–6]. Extensive work has been done with respect to both static and dynamic friction modeling, resulting in comprehensive models like the LuGre model and its modifications [7–9]. These friction models include the Stribeck effect, hysteresis, spring-like behavior for stiction and varying break-away force. As the required control performance of these positioning systems increases, incorporating the non-linear behavior due to friction into the design process becomes inevitable [10–13]. To do this successfully, either the dynamic friction model parameters and system states must be identified or the actual friction force must be determined. Often a time domain approach is chosen [14]. Alternatives are frequency domain approaches based upon Frequency Response Functions (FRF) measured with random noise excitation signals [15] or swept sine techniques [16–18]. A limitation of using FRFs is the implicit assumption of linearity of the system behavior. Every real life system is non-linear although the implications are not always noticeable in the operating range. So these FRF results have to be treated with caution because the characteristics of the excitation signal can have significant influence on the measured system characteristics [19]. Using carefully chosen multi-sine excitation signals, separating the linear system behavior from the non-linear behavior is possible [20–23]. Non-linear systems with an analytic response function can be described with the Generalized Frequency Response Functions (GFRF) [24–26]. This parametric approach is based on higher order Volterra Kernels [27–29]. The Volterra structure however is not able to describe systems with non-local memory like hysteresis, dead-zone and backlash. Both techniques can not be used for analyzing the non-linear phenomena associated with the stick/sliding transition of a system with dry friction. In this paper a new, non-parametric approach will be presented suitable for analyzing the frequency domain behavior of the class of stable, time invariant, non-linear systems with harmonic responses and as such applicable for analyzing the transition from stick to gross sliding. The new method is an extension to the well known concept of the describing function [30,31] as it produces the Higher Order Sinusoidal Input Describing Functions (HOSIDFs). The HOSIDFs describe the linear gain and phase relations of the individual harmonic components in a harmonic system response relative to the magnitude and phase of the excitation sinusoid [32]. The pa-
per begins with the explanation of the principles behind the HOSIDFs. The virtual harmonics generator is introduced as a concept required to extend the describing function to the generalized higher order describing functions. The mathematical framework is presented as well as a practical non-parametric measurement method for HOSIDFs. In Section 3 the stick/sliding transition in a linear bearing with added dry friction is analyzed using both the classical FRFs and the new technique. From the HOSIDF results the system characteristics like stick/sliding transition, pre-sliding displacement and frequency of the friction-induced resonance will be determined. From these results the friction-induced stiffness and tangential force [10] will be calculated as function of frequency. In Section 4 the applicability in the mechatronics field of both the HOSIDFs concept and the tangential force reconstruction technique are considered. The conclusions and ideas for future research are presented in Section 5.

2 Higher Order Sinusoidal Input Describing Functions

In [32] the concept of Higher Order Sinusoidal Input Describing Functions including the measurement techniques is described in detail. In this section a brief description will be presented.

2.1 Theoretical background

Consider a stable, non-linear time invariant system. Let \( u(t) = \hat{a}\cos(\omega_0 + \phi_0) \) be the input signal. The system response \( y(t) \) is considered to consist exclusively of harmonics of the fundamental frequency \( \omega_0 \) of the input signal \( u(t) \), i.e. we assume that the transient behavior has vanished. Response \( y(t) \) can be written as a summation of harmonics of the input signal \( u(t) \), each with an amplitude and phase, which can depend on the amplitude \( \hat{a} \), phase \( \phi_0 \) and frequency \( \omega_0 \) of the input signal (Fig. 1). This non-linear system can be modeled as a cascade of a virtual harmonics generator and a parallel connection of (non)linear subsystems. The virtual harmonics generator is defined as a non-linear component which transforms a sinusoidal input signal \( u(t) \) with frequency \( \omega_0 \), amplitude \( \hat{a} \) and phase \( \phi_0 \), (1) into a harmonic output signal \( \hat{u}(t) \). This output signal \( \hat{u}(t) \) consists of an infinite number of harmonics of the input signal \( u(t) \) with frequency \( n\omega_0 \), amplitude \( \hat{a} \) and phase \( n\phi_0 \) (2).

\[
u(t) = \hat{a}\cos(\omega_0 t + \phi_0)
\]

\[
\hat{u}(t) = \sum_{n=1}^{\infty} \hat{a}\cos(n(\omega_0 t + \phi_0))
\]
The individual harmonics components \( \hat{a} \cos(n(\omega_0 t + \varphi_0)) \) serve as virtual inputs of the respective (non)linear subsystems \( H_n(\hat{a}, \omega) \) resulting in output components \( A_n(\hat{a}, \omega) \cos(n(\omega_0 t + \varphi_0) + \varphi_n(\hat{a}, \omega)) \) (Fig. 1). In this paper the (non)linear subsystems \( H_n(\hat{a}, \omega) \) will be referred to as the Higher Order Sinusoidal Input Describing Function (HOSIDF). These functions can be defined as the complex ratio of the \( n \)th harmonic component in the output signal to the virtual \( n \)th harmonic signal derived from the excitation signal. Like the first order describing function [33], the higher order describing functions are calculated from the corresponding Fourier coefficients (3).

\[
H_n(\hat{a}, \omega) = \frac{A_n(\hat{a}, \omega)e^{j(n(\omega_0 t + \varphi_0) + \varphi_n(\hat{a}, \omega))}}{\hat{a}e^{jn(\omega_0 t + \varphi_0)}} = \frac{A_n(\hat{a}, \omega)e^{j\varphi_n(\hat{a}, \omega)}}{\hat{a}} = \frac{1}{\hat{a}}(b_n + ja_n)
\] (3)

\( H_n(\hat{a}, \omega) \) can be interpreted as a descriptor of the individual harmonic distortion components in the output of a time invariant non-linear system with a harmonic response as function of the amplitude and frequency of the driving sinusoid.

2.2 Non-parametric identification of HOSIDFs

In this paper, the HOSIDFs are determined using FFT techniques. Both the input signal \( u(t) \) and output signal \( y(t) \) (Fig. 2) are Fourier transformed with a transform size of \( 2m \). The resulting single sided spectra contain \( m+1 \) frequency
lines each with 0 Hz in frequency line 0. The frequency spacing is \( \Delta f = \frac{1}{T_b} \) with \( T_b \) the length of the data block. \( T_b \) is chosen a multiple \( p \) times the period \( T_0 = \frac{2\pi}{\omega_0} \) of the excitation signal. This assures that all the power of the excitation signal is concentrated in frequency-line \( p \). The power of the response signal is fully concentrated in the frequency lines \( n \cdot p \) with \( n \in \mathbb{N} \), so leakage is absent.

As an example, let us consider the calculation of the third order HOSIDF. According to (3) this HOSIDF is calculated from the third harmonic compo-

\[
\hat{A} = \sqrt{a_p^2 + b_p^2} \\
\tan(\phi_0) = -\frac{b_p}{a_p}
\]

Fig. 2. Determination of the third order HOSIDF using the FFT method.

In the spectrum of the system output signal \( y(t) \), the frequency line \( 3 \cdot p \) with complex value \( a_{3p} + j b_{3p} \) represents the output of the subsystem \( H_3(\hat{a}, \omega) \). From this line the amplitude \( A_3(\hat{a}, \omega) \) and phase \( \phi_{3out} \) are calculated using (6),(7).

\[
A_3(\hat{a}, \omega) = \sqrt{a_{3p}^2 + b_{3p}^2}
\]
\[ \tan(\varphi_{3_{\text{out}}}) = \frac{-b_{3p}}{a_{3p}} \] (7)

Phase angle \( \varphi_{3_{\text{out}}} \) is the sum of the phase of the third component of the harmonics generator \( \varphi_{3_{\text{in}}} \) and the system phase \( \varphi_{3}(\hat{a}, \omega) \), (7).

\[ \varphi_{3_{\text{out}}} = \varphi_{3_{\text{in}}} + \varphi_{3}(\hat{a}, \omega) = 3\varphi_{0} + \varphi_{3}(\hat{a}, \omega) \] (8)

From (4) and (6) the magnitude of the third order HOSIDF can be calculated as:

\[ |H_{3}(\hat{a}, \omega)| = \frac{\sqrt{a^{2}_{3p} + b^{2}_{3p}}}{\sqrt{a^{2}_{p} + b^{2}_{p}}} \] (9)

The phase \( \varphi_{3}(\hat{a}, \omega) \) of the HOSIDF is calculated from (5), (7) and (8).

3 Analysis of the stick/sliding transition in a linear bearing with friction.

3.1 Design of test set-up

To study the applicability of the HOSIDF method with respect to characterizing stick/sliding transitions, a small test set-up is designed which is exemplary for many motion systems. In this set-up, an electromechanical shaker is excited with a sinusoidal current. The resulting force drives a sledge which is subjected to dry friction, through its stick/sliding transition. This happens in a well controlled and reproducible way. Both the electrical stimulus to the shaker and the acceleration of the mass are measured. From these data the HOSIDFs are determined. The excitation force is also measured for validation purposes. The system consists of a supported mass with one translational degree of freedom. Although air bearings and hydrostatic bearings have very reproducible friction behavior, linear ball bearings were chosen because of their low complexity and similarity with numerous drive systems in industry. Additional dry friction is realized with a separate friction finger mounted onto the sledge. This finger slides over a very smooth and wear-resistant stationary surface to ensure reproducible and position independent friction behavior. The normal force in the contact point can be changed to adjust the friction force. This additional friction force is made significantly larger than the friction in the ball bearings. The overall system dimensions allow the use of commonly available piezo sensors for measuring force and acceleration. The light-weight and stiff design results in clean dynamic behavior in the frequency range of interest. In Figs. 3, 4 a picture and a functional drawing of the measurement
setup are shown. In appendix A the details of the measurement setup are presented.

Fig. 3. Picture of the system under test. Fig. 4. Layout of system under test.

3.2 Model of test set-up

Fig. 5 shows a model of the system. The various system components and their interconnections are modeled in order to explain the behavior of the system. The model has only one translational degree of freedom. The linear bearing is assumed frictionless. In the model the shaker, stinger (connection rod), sledge and friction finger can be recognized. The sledge has an acceleration of $\ddot{x}$, the excitation current is $i_m$. The shaker has an elastically suspended moving mass $m_1$. The axial stiffness and damping of the suspension are $b_1 = 1.6Ns/m$, $c_1 = 2.8e3N/m$. The magnetic force $F_1$ generated in the shaker is proportional with the drive current $i_m$. The shaker is coupled with the sledge $m_2$ through a massless stinger. The axial stiffness $c_2$ of the stinger is approximately $1.5e6N/m$. The effective excitation force of the shaker is $F_2$. For validation purposes this force is measured with a force sensor which mass must be taken into consideration. The mass between the sensor’s seismic plane

Fig. 5. Blockdragram of the system under test.
and its connection to the sledge is added to \( m_2 \). The remaining mass of the
sensor and the mass of the stinger are added to the moving mass \( m_1 \) of the
shaker. As a result of this mass distribution, \( m_1 = 0.045 \text{kg} \) and \( m_2 = 0.133 \text{kg} \).

In the friction finger, \( P \) is a generalized contact point representing the sum
of all the asperities of the friction finger between the static base of the set-up
and the sledge. Here the friction force is assumed to be generated. In the stick-
phase, the friction contact point \( P \) has a tangential stiffness \( c_3 \) and a damping
\( b_3 \) because the tangential force is less than the breakaway force [10]. In the
gross sliding phase, when the applied force exceeds the breakaway force, the
stiffness \( c_3 \) can be neglected. By imposing a high normal force in \( P \), the friction
force in \( P \) is high compared to the real friction force in the linear bearing.
Consequently, the pre-sliding displacement of the system will be determined
by \( c_3, b_3 \) and the tangential force in \( P \). The transition from stick to gross
sliding depends on the breakaway force in \( P \).

### 3.3 Measurements

The mechanical system will be analyzed using two different approaches. The
first technique is the classical FRF measurement with band limited white noise
excitation. The results will approximate the linear behavior of the system. The
alternative technique is the newly developed HOSIDF technique able to inves-
tigate its non-linear behavior. For both the FRF and HOSIDF measurements
the shaker current \( i_m \) will serve as the reference signal. First (i) because in
many motion systems, for example with a linear motor, a measurement of the
actual driving force is often impossible. Secondly, (ii) sinusoidal excitation is
a prerequisite for the HOSIDF technique. This is assured by taking the shaker
current, generated in a highly linear voltage to current converter, as reference.

#### 3.3.1 Measurement of \( H_1 \) FRFs using band limited white noise

In the first set of measurements the classical Frequency Response Function
(FRF) of system acceleration \( \ddot{x} \) as function of shaker current \( i_m \) is determined
using the \( H_1 \) estimator [34]:

\[
H_1 = \frac{G_{ab}}{G_{aa}}
\]

with \( G_{aa} \) the estimate of the single sided power spectrum of the input signal
\( i_m \) and \( G_{ab} \) the cross spectrum between system input \( i_m \) and measured output
\( \ddot{x} \). The excitation signal is band limited random noise in a frequency range up
to \( 1 \text{kHz} \). The RMS value of the signal is varied exponentially over 3.4 decades.
Figs. 6-8 show the magnitude, phase and coherence of the FRF as function
of the excitation frequency and the generator voltage. The dynamic behavior
of the system exhibits strong excitation level dependency. For low excitation
levels, the system is in the stick-phase and its Bode plot shows the friction-induced resonance [19] at approximately 200 Hz, see the $-50dB_{\text{rms}}$ traces in figures 6(b), 7(b). Using this information about the resonance frequency, and using the model as depicted in Fig. 5, and assuming $c_3$ independent of frequency, the friction-induced stiffness can be calculated from:

$$c_1 + c_3 = (2\pi f_{\text{res}})^2 \cdot (m_1 + m_2)$$

and is approximately $2.8e5 N/m$. For high excitation values, the system is in the gross sliding regime. Again, the system can be characterized as a second order system with additional dynamics, see the $+8dB_{\text{rms}}$ trace in figures 6(b) and 7(b). The low frequency resonance of approximately 20 Hz is caused by the axial stiffness of the shaker $c_1 = 2.8e3 N/m$ and the combined mass $m_1 + m_2 = 0.178 kg$. The coherence plot, Fig 8(a) indicates a region with a drop in magnitude, indicating non-linear behavior centered around an excitation level of $-25dBV$ ranging from 0 Hz to approximately 300 Hz. More specific information about the stick/sliding transition is not available from these FRF measurements due to the implicit assumption of linearity of the system to be described with the FRF technique.

![Excitation amplitude dependency](image1.png)

![For 3 excitation levels](image2.png)

(a) Excitation amplitude dependency  
(b) For 3 excitation levels

Fig. 6. Magnitude of the $H_1$ FRF of the system.

### 3.3.2 Third order SIDF measurement design

In order to get a better understanding of the friction-induced non-linear behavior of this system, an HOSIDF analysis has been done. The measurement setup remains unchanged. The system is excited with a single sinusoid, which frequency is varied in 5 Hz increments from 10 Hz to 1 kHz resulting in a grid of 199 frequency point. In every point of the frequency grid, the excitation amplitude is varied exponentially in 50 discrete steps from $2.5V_{\text{rms}} (8dBV)$ to $1mV_{\text{rms}} (-60dBV)$. During every measurement the excitation level is kept constant. Between consecutive measurements a fixed waiting time of 500 msec is programmed to allow the system to settle. Time averaging over 5 time
records is applied to reduce the influence of non-synchronous disturbance signals. With the sampling frequency of 5120 Hz, the maximum frequency of the measured third order SIDF is 666 Hz. Since the measurement technique is based upon the assumption of single frequency sinusoidal excitation, the harmonic distortion of the excitation signal must be low. In Fig. 9 the harmonic distortion magnitudes of the shaker current \( i_m \) are shown. The values are calculated as the quotients of the power relations between the individual harmonics and the power of the base component. Values of \(-90\,\text{dB}\) indicate measurement points with a harmonics power less than the quantization noise of the instrumentation. The largest distortion component is the second order component at maximum drive level. Its magnitude is less than \(-55\,\text{dB}\) with respect to the base component. From these measurements it is decided that the shaker current can serve as input signal. In all subsequent third order SIDF measurements in this paper the shaker current \( i_m \) will be the excitation signal and acceleration \( \ddot{x} \) the response (Fig. 5).
Fig. 9. Magnitudes [dB] of the harmonic distortion components of the shaker current signal as function of frequency [Hz] and excitation amplitude [dBrel 1Vrms].

3.3.3 Results of the HOSIDF measurements

In Figs. 10 and 11 the magnitude plots and phase plots of the first order and third order SIDF are shown. The frequency axes have an upper limit depending on the order number. The excitation magnitude axes indicating the generator voltages are equal for all 3D plots.

(a) First order SIDF mag  
(b) First order SIDF phase

Fig. 10. Magnitude and phase of the first order SIDF.
Fig. 11. Magnitude and phase of the third order SIDF.

The plots show features with a strong excitation amplitude dependency. The first order SIDF, Figs. 10(a), 10(b) bears resemblance with the H₁ FFT method measurements Figs. 6(a), 7(a). The differences are mainly concentrated in the frequency region from 0Hz to 250Hz centered around an excitation level of −25dBVrms as can be seen in Fig. 12(a) and 12(b). This region coincides with the region of low coherence in the FFT measurements (Fig. 8(a)). An overview of the magnitude plot of the third order SIDF (Fig. 11(a)) clearly reveals the development of odd order non-linear system behavior. Two regions can be distinguished as function of the excitation level. For low excitation levels when the friction contact $P$ (Fig. 5) is in the stick phase, the magnitude of the third order SIDF is low. For high excitation levels, so $P$ is in the gross sliding phase, the magnitude increases by more than 20dB. The transition between the two regions is both dependent on excitation level and frequency. For increasing excitation levels in the gross sliding region, the magnitude of the third order SIDF decreases again (Fig. 12(a)). This can be explained by the fact that for increasing excitation levels, the influence of the stick/sliding transition of $P$ on a full period of movement will decrease. In the third order phase plots Figs. 11(b)-12(b), phase unwrapping is necessary. The unwrapping is done in such a way that the phase values for the highest excitation levels are set closest to zero and the derivatives of the phase to frequency and excitation magnitude are minimized over the stick/sliding boundary. The third order phase plot Figs. 11(b) also clearly show both the stick/sliding transition and the friction-induced resonance.

3.4 Determination of the stick/sliding transition

From the third order SIDF magnitude characteristics, the stick/sliding transition is determined as function of the measured amplitude/frequency grid. Fig. 11(a) shows that for a fixed frequency, the stick/sliding transition is a continuous, monotonous function of the excitation amplitude. Consider such
proximating the stick/sliding transition are displayed in Fig. 15 together with curves of constant displacement amplitude of the sledge (solid lines). Between the curve indicating 100nm displacement amplitude and the curve of 1µm displacement amplitude, lines representing a multiple of 100nm displacement amplitude are shown. The plot shows that the stick/sliding transition approximately coincides with the 200nm displacement contour. For frequencies
below the friction-induced resonance the stick/sliding transition is steep. Here a small variation in excitation magnitude will result in a large variation in displacement. For frequencies above the friction-induced resonance this excitation sensitivity is significantly smaller. At the friction-induced resonance, a lower excitation magnitude is required for the transition than for excitation signals outside the resonance region. Consequently, the excitation signal with the lowest magnitude causing a stick/sliding transition has a frequency equal to the actual friction-induced resonance frequency. Both the friction-induced resonance at approximately $180\,Hz$ and the steep stick/sliding transition region between $10\,Hz$ to $100\,Hz$ and $-20dBV$ to $-10dBV$ are also clearly visible in Fig. 16 which shows the displacement of the sledge in the measured amplitude/frequency grid-points. From the identified stick/sliding transition grid-points, the maximum displacement of the system in the stick regime can be calculated by double integrating the acceleration values in these grid-points. Fig. 17 shows that the maximum pre-sliding displacement of the system is approximately $200\,nm$ with a slight tendency to decrease with increasing frequency. Also indicated are the error bars on these displacement values. Due to the discrete excitation levels, the real stick/sliding transition per frequency is likely to occur between two excitation grid points $V_n$ and $V_{n\pm1}$. For every identified stick/sliding transition grid-point $(f_k, V_n)$, the real transition is certain to occur between $V_{n-1}$ and $V_{n+1}$. The very wide error bars for the frequencies below the friction-induced resonance can be reduced by increasing the density of the amplitude grid in this frequency range.

Because the friction-induced resonance frequency just before the stick/sliding transition is known, one is able to calculate the related stiffness to yield this resonance if the moving mass is assumed to be known. Using (11), $c_3$ is approximately $2.2e5\,N/m$, again assuming $c_3$ independent of frequency. Since this stiffness is realized in the stick-phase, the required tangential force $F_t(\omega)$ in

![Fig. 15. Excitation/frequency grid-points for stick/sliding transition.](image1)

![Fig. 16. Displacement of sledge.](image2)
contact point $P$ can be calculated with:

$$F_t(\omega) = -F_3(\omega) = c_3 \cdot x_{ps}(\omega)$$

(12)

where $x_{ps}(\omega)$ is the pre-sliding displacement of the system. In Fig. 18(a) the calculated maximum tangential force in $P$ is shown together with the uncertainty intervals due to the final resolution of the frequency-excitation grid. The uncertainty in the stick/sliding results propagates linearly in the calculation of the tangential forces. So again the large uncertainty up to 150 Hz is caused by the very steep rise of the third order HOSIDF in that frequency range as can be seen in Fig. 12(a). In order to validate the values of the calculated tangential force, the measured force signals $F_2$ (Fig. 5) are used:

$$F_3(\omega) = F_2(\omega) - m_2 \cdot \ddot{x}_{ps}(\omega)$$

(13)

where $\ddot{x}_{ps}(\omega)$ is the maximum pre-sliding acceleration. The results are displayed in Fig. 18(b) together with the uncertainty intervals. The data from the acceleration based method correspond very well with the results from the force measurements based method. The main difference is the width of the uncertainty intervals up to 150 Hz.

4 Applicability in mechatronics

The findings presented in Sections 2 and 3 are applicable in the field of mechatronics modeling, design and production.

- The concept of the harmonics generator will provide additional capabilities in frequency domain modeling of non-linear systems. The analytic formulations of HOSIDFs in models of common non-linear building blocks like
backlash, saturation, relay etc. result in exact harmonic responses to sinusoidal excitations. In friction-model validation, the influence of the rate of change of the excitation amplitude and frequency on hysteresis effects can be studied.

• In the design phase of (precision)mechatronics systems, measured HOSIDFs reveal essential information about the non-linear behavior of the system. This allows for fingerprinting of specific system characteristics like the friction induced resonance and pre-sliding displacement. HOSIDFs can be determined with well-controlled and low-risk experiments because of the very low crest factor of the excitation signals. The experiment shows that relatively simple provisions in the machine design like a current sensor and an acceleration sensor create valuable opportunities for the detection of important machine characteristic.

• In the production phase of a (precision)mechatronics system, tuning of the servo controllers often comprises adjusting the friction feedforward parameters. Over-compensation may lead to limit-cycling and must be prevented. With the approach described in Subsection 3.4, online determination of the actual tangential force becomes possible. Incorporated in auto-tuning software, this approach can contribute to an intelligent controller that achieves optimal performance over time of a precision positioning machine. The history of the reconstructed tangential force can be used as an indication of wear, allowing machine condition monitoring.

5 Conclusion and further research

A new frequency domain based measurement method was presented, able to successfully characterize the stick/sliding transition of a mechanical system with dry friction. The method uses Higher Order Sinusoidal Input Describing Functions (HOSIDFs) [32], an extension of the theory of Sinusoidal Input
Describing Functions. The HOSIDFs were determined with FFT techniques yielding auto spectrum and phase information. As test setup, a small mechanical system with reproducible dry friction behavior was used. By an optimized design, its dominant friction behavior originated from a well defined sliding contact point. Initial FRF measurements with various levels of random noise excitation revealed a significant input level dependency of the system dynamics. Due to the inherent limitations of this linear measurement technique, only limited information about the non-linear behavior could be gathered. With the newly developed measurement technique, the HOSIDFs of the system have been successfully identified. In contrast with the first order SIDF, the third order SIDF clearly showed the stick/sliding transition as function of frequency and excitation level. From this information, the maximum pre-sliding displacement was determined. The first order SIDF was used to determined the frequency of the friction induced resonance and the value of the friction-induced stiffness. From these results the maximum value of the tangential force, the breakaway force, was calculated. The data showed excellent agreement with the values derived from a force measurement. The new measurement method was successfully applied for determining important characteristics of the friction behavior of a mechanical system without the need of a separate force measurement. The applicability of the newly developed theory of HOSIDFs in the mechatronics field was discussed together with the use of the tangential force reconstruction technique. The HOSIDFs open interesting new research opportunities in the field of automated machine condition monitoring and hybrid control strategies for precision positioning systems. Although less fundamental, the translation of the proposed HOSIDF measurement theory into a realtime online analysis system using FPGAs is challenging. An interesting theoretical and experimental research item is the question of relevance of the HOSIDF technique for the analysis of bifurcation processes.

A Measurement setup

The applied linear bearings are Schneeberger type 2045. This type of bearing consists of two V-shaped runways which are separated by cylindrical rolling elements. The axes of rotation of two neighboring elements are mutually perpendicular. The rolling elements are separated by a cage. Although kinematically over-constrained, this setup assures low rolling friction and high stiffness. The bearings are lubricated with Molykote, a lubricant containing Molybdenum Disulfide (MoS\(_2\)). The sliding table is the mounting base of a friction finger which adds dry friction to the system. This finger is a beryllium bronze cantilever and slides over a very smooth surface (\(R_a \leq 25 \text{nm}\)) made of silicon carbide which is mounted to the fixed world. The friction finger is pre-loaded in z-direction due to its own stiffness. The system is excited with an electro-
A magnetic shaker type LDS 201 is coupled through a stinger. The driving force is measured with a piezo sensor type B&K8200. The acceleration of the sliding table is measured with a piezo accelerometer type B&K4326. Both charge signals are conditioned with charge amplifiers type B&K2626. The current through the shaker is measured with a current transducer type LA25-NP. All three signals are sampled synchronously with a dynamic signal analyzer type SigLab2024. Its signal generator is used to excite the shaker through a linear voltage to current converter with adequate dynamic range.

References


