Mesh modeling of Angle-ply Laminated Composite Plates for DNS

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Abstract

Laminated composite materials find their applications in aerospace engineering and a lot of effort has been made to understand their mechanical behavior. At microscopic level some important mechanisms are present, e.g. damage initiation and delamination. With Direct Numerical Simulation (DNS) it is possible to understand and visualize the stress-strain response of the composite materials at microscopic level and macroscopic level.

In this study a mesh model for angle-ply laminated composite plates for DNS is presented. With DNS the laminated composites are modeled without any homogenization, i.e. the matrix and fibers are modeled separately. Former studies have shown that DNS is a reliable numerical approach, which can be used for predicting elastic properties of composite materials for several cases, like modeling fiber misalignments and fiber breakage. They also presented simulations of low-velocity impact experiments with the same mesh model.

To verify the accuracy of the proposed mesh for \( (0, \pm 45, 90) \), and for \( (45, -45) \) composites, virtual experiments are performed to predict the elastic properties. A short review is given on the theory for angle-ply laminates for determining these properties. Compared with the theory, DNS led to accurate predictions of the elastic behavior of angle-ply laminates without making any assumptions, like plane stress conditions.
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Chapter 1

Introduction

Due to the high specific strength, stiffness and damage tolerance at elevated temperatures, Metal Matrix Composites (MMC) are widely applied in aircraft components and space systems. With these superior properties engineers are able to reinforce structures with a minimum of weight increase. Still determining elastic properties with real experiments are limited the complexity of the experiments and by the ASTM standards [2].

In this study the properties of Titanium Matrix Composites (TMC) are used, because its relatively ease to manufacture and low operating costs [3]. This makes TMCs an attractive material for aerospace structures. To investigate the mechanical behavior of structures made out of composite materials, reliable values for elastic properties are required. A lot research effort has been made to identify these material constants. Several experiments are performed [4–6] and analytical methods are proposed [7–9] for both unidirectional and angle-ply laminates. Especially for angle-ply laminates assumptions are made to develop analytical models, like plane stress assumption and configurations with only one angle, like $\langle \theta, -\theta \rangle_s$ [7, 10].

Current analysis of composite structures are done one both macroscopic and microscopic level. Homogenized properties are used in the macroscopic analysis and the materials are assumed to be continuums. However with this approach it is difficult to verify the internal stresses caused by the different properties of the composite constituents. Microscopic analysis have been performed to predict the mechanical behavior using Representative Volume Elements (RVE) [11]. In this case the constituents are modeled separately with their corresponding elastic properties. However, this microscopic approach can not explain the full behavior of laminates, because there is no interaction considered between micro-stresses and interlaminar stresses.

In order to analyze the mechanical behavior of composite materials on microscopic and macroscopic level, Direct Numerical simulation (DNS) is used. DNS is an approach of modeling the whole composite structure, through modeling the fiber and matrix separately [1]. To fully model the composite structure with DNS only 3D elements should be used to prevent any assumption like plane stress. Because there is no assumption made, one can obtain the distribution of interlaminar stresses and elastics properties. Due to the full modeling of the composite structure, the number of DOFs increases rapidly, so large computing recourses are needed as well as an efficient parallel FEM analysis program. For this study the FEM program Internet Parallel Structural Analysis Program (IPSAP) [12] is used for analysis.
Chapter 2

Direct Numerical Simulation

To fully understand the mechanical behavior of composite materials and structures, both microscopic and macroscopic stresses should be investigated. Macromechanical approaches are based on the composite material behavior wherein the material is presumed homogeneous and the effects of the constituent materials are implemented only as averaged properties of the composite. Micromechanical approaches on the other hand describe the interaction of the constituent materials on a microscopic scale. With DNS the stresses on both levels are determined. The DNS models can be used for several loading conditions, like determination of the elastic properties or impact analysis. In this study only the determination of the elastic properties is performed to verify the accuracy of the proposed mesh for arbitrary angle-ply laminates.

2.1 Concept

The principle of DNS is to discretize the macroscopic composite structures at microscopic level, as presented in Fig. 2.1. This means that for DNS only 3-dimensional elements could be used for the direct modeling of composite structures at microscopic scale. In this case there are no artificial assumptions on the structure, so displacements or stresses in the through thickness direction, like interlaminar stresses, can be determined. This can only be achieved by enough discretization through the thickness. In order to work in an efficient way, the total mesh model is made of multiple unit cells. By transforming and coupling these unit cells, a macroscopic structure structure is obtained with microscopic details.

2.2 Utilizing DNS

To characterize the elastic properties of angle-ply laminated composite plates with virtual experiments using DNS, standard computer resources (like a standard PC) aren’t sufficient due to the large number of DOFs. To overcome this problem parallel computing is needed in order to perform these virtual experiments.
In this research the Internet Parallel Structural Analysis Program (IPSAP) is used as the parallel FEM-code, developed by Kim et al. [12]. This program makes use of a domain-wise multifrontal solver, an efficient direct solver for large-scale parallel FEM analysis. Kim et al. showed that performance of IPSAP using this solver is better than that of commercial FEM-codes, like ABAQUS and MSC Nastran. The simulations of the virtual experiments are executed on the Pegasus cluster system which consist of 400 Intel Xeon 2.2/2.4/2.8 GHZ processors [12].

Since IPSAP is a FEM processing program, pre- and postprocessing should be done with different programs. The total processing path is shown in Fig. 2.2.

For preprocessing MSC Patran is used for modeling the DNS mesh of angle-ply composites. The by MSC Patran created Nastran input file is converted to an IPSAP input file [14]. As already mentioned IPSAP is used as the FEM processing program and logically the obtained output contains the stresses and strains after deformation.

For postprocessing the output file is converted to a General Mesh Viewer (GMV) input file. GMV is a program for visualizing the stress and strain fields present in the material, according to the values given in the IPSAP output file. The same values are used to determine the stresses and strains, from which the elastic constants are determined by using Matlab. To determine the effective elastic constants, the volume averaged quantities are determined according to:

\[
\bar{f} = \frac{1}{V} \int_V f(x, y, z) dV.
\] (2.1)
In equation (2.1) \( f(x, y, z) \) represents the elastic constant and \( V \) the total volume of the model [13].
In previous studies on composites using FEM analysis, only special cases are considered, namely unidirectional and cross-ply laminates [1, 15]. The latter of these laminates consist of fibers, which are orientated perpendicular in different layers to each other. Since no geometric difficulties arise in the case of unidirectional laminates, mesh modeling is relatively easy. Compared with unidirectional and cross-ply composite plates, mesh modeling of arbitrary angle-ply laminated composite plates is more complex. Especially around the corners of the unit cell the geometry of the matrix becomes complex, since fibers with orientations different from the main laminate orientation, coincide in the corners.

Figure 3.1: Architecture for \((0, \pm 45, 90)_s\) angle-ply laminates
3.1 Architecture

As mentioned before, the most efficient way to use DNS is by using unit cells, which are coupled to form the macroscopic structure. The architecture of the unit cell is fully determined by the orientations of the fibers for an arbitrary volume fraction. An example of an architectural layout for \( (0, \pm 45, 90) \) laminate is given in Fig. 3.1.

Now that the layout is determined the dimensions of the unit cell should be chosen in order to realize a FEM mesh, according to the architecture. Let’s first consider the top view of the 0/45 layers given in Fig. 3.2.

![Figure 3.2: Top-view of 0/45 layers](image)

In this case the length \( AA' \) is set to 1[-]. This is a dimensionless measure, because the final mesh is depended on the volume fraction (also dimensionless), which determines the fibre radius together with the cell dimensions. If \( AA' \) is defined, it can be shown that \( BB' \) also equals one, because the plane \( BB' \) has it’s normal vector parallel to the 45 fibre and in that case has the same cross section as \( AA' \). Know \( BB' \), the length of \( CC' \) equals \( \sqrt{2}[-] \) (\( \approx 1.41[-] \)). However to couple this small cell, to create the final unit cell as given in Fig. 3.1 the length of \( CC' \) should be taken 1.4 [-], which introduces a difference in the length \( BB' \) of 1%. To keep the volume fraction of the total unit cell equal to the initially chosen value, the fibre radius of the \( \pm 45 \) fibres are modified.

By choosing these dimensions of the small cell as discussed above, an unit cell with dimensions of \( 7 \times 7 \times 4 \) [-] can be created for \( (0, \pm 45, 90) \) angle-ply laminates.

3.2 Mesh

Now that the dimensions of the architecture are determined, the FEM mesh can be modeled. One possible approach is to couple the small unit cells, as given in Fig. 3.2 to create the final unit cell as given in Fig. 3.1. Then one has to fill the boundary in order to create a closed unit cell. However, this is very time consuming since filling the boundary, requires multiple new cells.

A more straightforward method, is using boolean operations. MSC Patran is capable of executing
Boolean operations with geometry objects. First the fibers are modeled as octagonal cross sections and then a solid geometry is created by extruding the cross section. By applying rotations, the preferred orientation is assigned to the fibers. When all the fibers are modeled, each with its orientation, they are subtracted from the matrix and the final matrix geometry is created. The final geometry is shown in Fig. 3.3.

Figure 3.3: The geometry of the matrix (left) and the fibers (right) after boolean operations

As can be seen in Fig. 3.3 each fiber is made of 12 solids each. This provides a proper element distribution during mesh modeling of the fibers.

To create a FEM mesh, the geometry model given above should be transformed into a mesh. It is preferable to use hexahedron elements, because they provide a high accuracy during large deformations. However, mesh modeling of complex geometries, like angle-ply laminates, is a difficult task. In this case, it is due to the complex geometry of the matrix, which is caused by the different orientations in each layer. Another important and logical demand, which the mesh of both fiber and matrix should meet, is connectivity of all elements at the interface of the matrix and fiber. For that reason tetrahedron elements are used in order to fulfill this demand. The main advantage of tetrahedron elements is that they can describe the surfaces of complex geometries accurately, which in this case is desired.

A disadvantage of tetrahedron elements is that they can cause locking and overstiffening of the model during large deformations [16]. As mentioned before this study only concerns deformations in the elastic regime, which usually are small.

The final mesh is created with the automatic mesh generator from MSC Patran. To prevent distorted or poor sized elements, the maximum length and aspect ratio of the tetrahedron elements are used as input for the automatic meshing program. Also the interface between matrix and fiber is specially assigned in order to describe this in an accurate way. The result of meshing is shown in Fig. 3.4.

Figure 3.4: Mesh for \(\langle 0, \pm 45, 90 \rangle\) angle-ply laminates
Chapter 4

Theory

To validate the proposed mesh, the results of the virtual experiments can be compared with real experiments or with analytical models based on the volume fractions of the constituents. In this research only analytical models are used for validation, therefore a review is given on the theory for angle ply laminated plates.

4.1 Stress-strain relations

4.1.1 Unidirectional lamina

In order to describe the mechanical behavior of the total angle ply laminate, first the stress-strain relation for an unidirectional lamina is described. The materials used as constituents of the composite are assumed to be isotropic materials, which makes the unidirectional lamina orthotropic. Assuming plane stress one can show that the stress-strain relation becomes,

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix}
= \begin{bmatrix}
Q_{11} & Q_{12} & 0 \\
Q_{12} & Q_{22} & 0 \\
0 & 0 & Q_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix},
\]

(4.1)

where \(Q_{ij}\) are the reduced stiffness coefficients, which are related to the elastic properties of the laminate as

\[
Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}}, \quad Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}}, \quad Q_{12} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}}\quad \text{and} \quad Q_{66} = G_{12}.
\]

(4.2)

With this relation one can describe the stress-strain relation for a angle ply lamina. The elastic constants \(E_1, E_2, G_{12}, \nu_{12}\) and \(\nu_{21}\) are given in literature, are determined with experiments or determined analytically. In section 4.2 a review is given on the determination of these elastic constants.
4.1.2 Angle Lamina

When fibers are placed at an angle with respect to the longitudinal direction 1, the stress-strain relation for an angle lamina is given as,

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} = \mathbf{T}^{-1} \mathbf{Q} \mathbf{R}^{-1} \begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix} = \begin{bmatrix}
\mathbf{Q}_{11} & \mathbf{Q}_{12} & \mathbf{Q}_{16} \\
\mathbf{Q}_{12} & \mathbf{Q}_{22} & \mathbf{Q}_{26} \\
\mathbf{Q}_{16} & \mathbf{Q}_{26} & \mathbf{Q}_{66}
\end{bmatrix} \begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix},
\]

(4.3)

where

\[
\mathbf{T} = \begin{bmatrix}
c^2 & s^2 & 2cs \\
s^2 & c^2 & -2cs \\
-2cs & 2cs & c^2 - s^2
\end{bmatrix} \quad \text{and} \quad \mathbf{R} = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 2
\end{bmatrix}.
\]

(4.4)

In equation (4.3) \(Q_{ij}\) are the components of the transformed reduced stiffness matrix \([\mathbf{Q}]\). Further shown in equation (4.4) are the transformation matrix \([\mathbf{T}]\), in which \(c = \cos(\alpha)\) and \(s = \sin(\alpha)\), and the Reuter matrix \([\mathbf{R}]\). See Appendix (A) for a detailed derivation of this relation.

4.1.3 Angle-ply Laminates

In this study angle-ply laminates are considered. These laminates consist of multiple angle lamina stacked on top of each other, each with different fiber orientations. To describe the stress-strain relation the classical lamination theory is used. The classical lamination theory invokes the following assumptions [10]:

- Each lamina is orthotropic.
- Each lamina is homogeneous.
- A line straight and perpendicular to the middle surface remains straight and perpendicular to the middle surface during deformation \((\gamma_{zz} = \gamma_{yz} = 0)\).
- The laminate is thin and is loaded only in its plane (plane stress) \((\sigma_z = \tau_{xz} = \tau_{yz} = 0)\).
- Displacements are continuous and small throughout the laminate \((|u|, |v|, |w| << h\), where \(h\) is the lamina thickness).
- Each lamina is elastic.
- No slip occurs between the lamina interfaces.

First the strain in the lamina is considered including the midplane strains \(\varepsilon_i^0\) and the midplane curvature \(\kappa_i\). One can derive the following relation for the laminate strain,

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix} = \begin{bmatrix}
\varepsilon_i^0 \\
\varepsilon_i^0 \\
\gamma_{xy}
\end{bmatrix} + z \begin{bmatrix}
\kappa_x \\
\kappa_y \\
\kappa_{xy}
\end{bmatrix}.
\]

(4.5)
Substituting equation (4.5) in equation (4.3) gives

\[
\begin{bmatrix}
\sigma_x \\ \sigma_y \\ \tau_{xy}
\end{bmatrix} =
\begin{bmatrix}
Q_{11} & Q_{12} & Q_{16} \\
Q_{12} & Q_{22} & Q_{26} \\
Q_{16} & Q_{26} & Q_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0
\end{bmatrix}
+ z
\begin{bmatrix}
B_{11} & B_{12} & B_{16} \\
B_{12} & B_{22} & B_{26} \\
B_{16} & B_{26} & B_{66}
\end{bmatrix}
\begin{bmatrix}
\kappa_x \\ \kappa_y \\ \kappa_{xy}
\end{bmatrix},
\]

(4.6)

However equation (4.6) only describes the stress-strain relation for a single layer. To verify the accuracy of the proposed mesh, virtual experiments are performed. Using the results of these virtual experiments, the effective elastic constants \(E_{\text{eff}}^x, E_{\text{eff}}^y, G_{\text{eff}}^{xy}\) and \(\nu_{\text{eff}}^{xy}\) are determined.

Finding analytical values for these effective constants, first the force resultant related to the midplane strains and curvature is considered.

Fig. (4.1) gives a schematic cross section representation of an angle-ply laminate with thickness \(h\), consisting of \(n\) layers. By integrating the global stresses in each lamina, the resultant force per unit length in the \(x - y\) plane through the laminate thickness is given as

\[
\begin{bmatrix}
N_x \\ N_y \\ N_{xy}
\end{bmatrix} = 
\int_{-h/2}^{h/2}
\begin{bmatrix}
\sigma_x \\ \sigma_y \\ \tau_{xy}
\end{bmatrix} dz = 
\sum_{k=1}^{n} \int_{h_{k-1}}^{h_k}
\begin{bmatrix}
\sigma_x \\ \sigma_y \\ \tau_{xy}
\end{bmatrix} dz.
\]

(4.7)

Substituting equation (4.3) in equation (4.6), considering that the components of the transformed reduced stiffness matrix \([Q_k]\) are independent of the \(z\)-coordinate gives

\[
\begin{bmatrix}
N_x \\ N_y \\ N_{xy}
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} & A_{16} \\
A_{12} & A_{22} & A_{26} \\
A_{16} & A_{26} & A_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0
\end{bmatrix}
+ \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k}
\begin{bmatrix}
B_{11} & B_{12} & B_{16} \\
B_{12} & B_{22} & B_{26} \\
B_{16} & B_{26} & B_{66}
\end{bmatrix}
\begin{bmatrix}
\kappa_x \\ \kappa_y \\ \kappa_{xy}
\end{bmatrix},
\]

(4.8)

where

\[
A_{ij} = \sum_{k=1}^{n} [(Q_{ij})](h_k - h_{k-1}), \quad i = 1, 2, 6; \quad j = 1, 2, 6.
\]

(4.9)

As already mentioned only symmetric laminates are considered, which means that the all the components of the coupling matrix \([B]\) are 0. To determine the effective constants the reduced equation (4.9) is used. Expressions for the elastic constant are given as
\[ E_{x}^{eff} = \frac{1}{h} \left( A_{11}A_{22} - A_{12}^2 \right), \]  
\[ E_{y}^{eff} = \frac{1}{h} \left( A_{11}A_{22} - A_{12}^2 \right), \]  
\[ v_{xy}^{eff} = \frac{A_{12}}{A_{22}}, \]  
\[ G_{xy}^{eff} = \frac{1}{h} A_{66}. \]  
\[ (4.10) \]
\[ (4.11) \]
\[ (4.12) \]
\[ (4.13) \]

### 4.2 Elastic properties

As was show in equation (4.2) the elastic constants of the unidirectional lamina are unknown since they are both depended on the properties of fiber and matrix. In order to describe the stress-strain behavior of angle-ply laminates these constants could be determined by experiments or analytically. Experiments for angle-ply laminates are only conducted for simple configurations, so in this research only analytical values are used. Both Khashaba [17] and McCartney [7] present a set of equations for the elastic properties of an unidirectional lamina. First the theory of Khashaba and after McCartney is presented.

#### 4.2.1 Khashaba

Khashaba et al. [17] uses the rule of mixture, which is well known, to describe the elastic properties of the unidirectional lamina depending on the volume fiber fraction \( V_f \). The rule of mixture is used only for the longitudinal modulus \( E_1 \) and the mayor Poisson’s ratio \( \nu_{12} \), which gives

\[ E_1 = E_f V_f + E_m (1 - V_f), \]  
\[ \nu_{12} = \nu_f V_f + \nu_m (1 - V_f), \]  
\[ (4.14) \]  
\[ (4.15) \]

where the subscript \( f \) and \( m \) refer to the elastic constants of the fiber and matrix respectively. For the transverse modulus the Halpin-Tsai equation is used, which is also used in FEM code of MSC Patran [18].

\[ E_2 = E_m \frac{1 + \xi \eta V_f}{1 - \eta V_f}, \]  
\[ (4.16) \]

In equation (4.16) \( \eta \) is defined as \( \eta = \frac{E_f/E_m - 1}{E_f/E_m + \xi} \) and \( \xi = 2 \) for circular fibers. For the shear modulus \( G_{12} \) the following expression is used

\[ G_{12} = G_m \left[ \frac{(1 + V_f)G_f + (1 - V_f)G_m}{(1 - V_f)G_f + (1 + V_f)G_m} \right]. \]  
\[ (4.17) \]
4.2.2 McCartney

The effective elastic properties of angle-ply laminates presented by McCartney et al. [7] are depended on a correction factor, proportional to $\lambda_1$, for the mismatch in the axial Poisson’s ratio of matrix and fibre compared with the rule of mixture (4.14). The longitudinal modulus $E_1$ and the mayor Poisson’s ratio $\nu_{12}$ are given by

\begin{align*}
E_1 &= E_f V_f + E_m (1 - V_f) + 2 \lambda_1 (\nu_m - \nu_f)^2 V_f V_m, \quad (4.18) \\
\nu_{12} &= \nu_f V_f + \nu_m (1 - V_f) - \frac{\lambda_1}{2}(\nu_f - \nu_m) \left( \frac{1}{K_f} - \frac{1}{K_m} \right) V_f V_m. \quad (4.19)
\end{align*}

The transverse modulus $E_2$ is calculated according to

\[ \frac{1}{E_2} = \frac{\nu_{12}^2}{E_1} + \frac{1}{4} \left( \frac{1}{K_t} + \frac{1}{G_t} \right). \quad (4.20) \]

Finally an expression for the shear modulus $G_{12}$ is given, which is based on the concentric cylinder model

\[ G_{12} = V_f G_{12}^f + V_m G_{12}^m - \lambda_2 (G_{12}^f - G_{12}^m)^2 V_f V_m \quad (4.21) \]

The expressions for $\lambda_1$, $\lambda_2$, $K_t$ and $G_t$ are given in Appendix B.
Chapter 5

Material Characterization

In this chapter the results of the virtual experiments are given and compared with the analytical values, which are based on the expressions given in Chapter 4. As mentioned before TMCs are characterized in this research. For the constituents of these composites, logically an alloy of Titanium\(^1\) is chosen for the matrix material and Silicon Carbide as material for the fibers. The properties for both constituents are assumed to be isotropic and are given in the research of Bednaryck [15]. These can be found in table 5.1 and are measured at room temperature.

<table>
<thead>
<tr>
<th>Constituent</th>
<th>(E) [Gpa]</th>
<th>(\nu) [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Titanium 15-3</td>
<td>14.7</td>
<td>14.6</td>
</tr>
<tr>
<td>Silicon Carbide</td>
<td>14.2</td>
<td>14.2</td>
</tr>
</tbody>
</table>

The virtual experiments are used to determine the effective properties of the angle ply laminated composite plates. Since the angle ply lamina are considered to be orthotropic, only three experiments are required to determine the effective properties. An axial tensile test will be used to determine the effective axial Young’s modulus, \(E_{1}^{eff}\) and the effective Poisson’s ratio \(\nu_{12}^{eff}\) by applying a prescribed displacement. For the determination of the effective transverse Young’s modulus, \(E_{2}^{eff}\), an transverse tensile test is executed. At last a shear test is performed to determine the effective in-plane shear modulus \(G_{12}^{eff}\). All the effective properties are calculated according to volume average quantities, which are given in equation (2.1).

Two different angle ply laminates, both with different volume fraction, are used to show that this approach is valid for arbitrarily angle ply laminates. First the effective properties of a \(<45, -45>_s\) laminate are compared with both McCartney and Kashaba. The volume fraction of this laminate is chosen to be 55%. Hence that this volume fraction is chosen arbitrarily. Since Kashaba only considers cross-ply lamina, the results of the experiments for \(<0, \pm 45, 90>_s\) are only compared with McCartney who also considers angle-ply laminates. In the case of the \(<0, \pm 45, 90>_s\) laminate the volume fraction is set to 34%.

\(^1\)Ti 15-3 is an alloy of Titanium containing 15% Vanadium, 3% Chromium, 3% Aluminum and 3% Tin.
5.1 Characterization of $<45, -45>_s$ laminates

Since the FEM model is build of tetrahedron elements the deformations should remain small. For that reason only small strains are applied to the structures in all experiments. Beneath the applied strains the material is assumed to stay within the elastic regime as was shown in [15].

The boundary conditions applied for all the experiments provide that the composite can contract free in all directions, so that no constraints are implied.

5.1.1 Axial Tensile Test

To satisfy the demand that the deformations should stay small, a strain of 0.2 % is applied in the axial direction (in this case the z-direction). Since the DNS approach is used, the model is made of 12 unit cells, resulting in a large number of, roughly 1.2 million, elements. For that reason parallel computing was used, since ordinary desktop computers run out of memory for these number of elements. The results, visualized with GMV, are shown in Fig. 5.1.

![Figure 5.1: Axial stress in $<45, -45>_s$ laminate](image)

The results in Fig. 5.1 shows a natural distribution of the stress true the composite. The highest stresses are found in the fiber region in all layers. According to the higher modulus of the fibers, this is a logical result of the applied tension. The stress in the matrix material is highest in the region between the fibers, which are in the same layer. This can be explained by the orientation of the fibers present in the layers. If the top layer is considered, high stresses in the matrix are present between the fibers with an orientation of 45°, indicated with an I. If a line parallel to the axial direction is considered in the region of I, this line first will pass the matrix and after that the more stiffer fiber. To satisfy the applied strain in this region higher forces are required and for that reason the stresses will be higher. Compared with the region where only matrix material is present in axial direction, which is indicated with a II, lower stresses are present. This is due the lower stiffness of the matrix material.
5.1.2 Transverse Tensile Test

According to the axial tensile test, the applied strain and boundary conditions in this case are the same, only applied in the transverse direction, namely the x-direction. Also in this case 12 unit cells are coupled to create the final structure. The results of the transverse tensile test are shown in Fig. 5.2.

As was observed in the case of the axial tensile test, also in this situation the highest stresses are found in the fibers, depicted with a III. The same holds for the stresses in the matrix region (indicated with a IV), where the stresses are the lowest. This is a logical consequence since the orientations of the fibers are only changed within the different layers, but the overall response of the composite does not change. The fibers with orientation of $45^\circ$ are not situated in the top and bottom layers but in the middle layers. The opposite holds for the fibers with orientations of $-45^\circ$. Therefore the response is similar to the case of axial tension.

5.1.3 Shear test

The boundary conditions of the shear test are chosen differently compared with the boundary conditions of the previous experiments. This model is also made of 12 unit cells, but is symmetric in the xz-plane. Therefore symmetric boundary conditions are applied in this plane. Further a shear strain of 0.15% is applied to the model. The result of this experiment is presented in Fig. 5.3.

Again the stresses in the fiber are the highest in the composite. In this case the fibers are loaded more in the direction of the fiber orientation. Consequently the stresses in the matrix region V are lower compared with the case of axial and transverse tensile test. Furthermore the stresses in the matrix region VI are the lowest in the composite. The same explanation can be given as in the axial and transverse tensile test.
5.1.4 Results

From all experiments just described the effective elastic properties are determined, using the volume averaged quantities given by 2.1. The numerically determined effective properties are given in table 5.2 together with the values determined by Khashaba and McCartney.

<table>
<thead>
<tr>
<th></th>
<th>$E_{1}^{eff}$ [Gpa]</th>
<th>$E_{2}^{eff}$ [Gpa]</th>
<th>$\nu_{12}^{eff}$ [-]</th>
<th>$G_{12}^{eff}$ [Gpa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Virtual Experiment DNS</td>
<td>14.7</td>
<td>14.6</td>
<td>0.53</td>
<td>12.2</td>
</tr>
<tr>
<td>Khashaba</td>
<td>14.2</td>
<td>14.2</td>
<td>0.59</td>
<td>12.7</td>
</tr>
<tr>
<td>McCartney</td>
<td>14.0</td>
<td>14.0</td>
<td>0.56</td>
<td>12.4</td>
</tr>
</tbody>
</table>

The results show good agreement with the results given by the analytical models. In the case of the effective axial and transverse moduli, a maximum difference is found with Khashaba of 3.6%. Notice that the effective axial and transverse moduli are approximately the same. This is caused by the fact that the fibre orientations in the composite are the same, but the position in the layer is different. This causes the overall behavior in these cases to be the same. In contrast with the analytical models where only effective constants are determined, it is possible with DNS to visualize the stresses caused by applied deformation. Even at microscopic level stresses in the constituents are determined.

In the case of the effective Poisson’s ratio a difference of almost 10% is found with Khashaba. A better agreement is found with the value given by McCartney. This is caused by the correction factor McCartney used for the mismatch in the constituents Poisson’s ratios.

At last the results of the effective shear modulus show good agreement with both Khashaba and McCartney, where a maximum difference of 3.7% is found. Again with DNS it is possible to visualize the stresses between the different layers and within the different constituents.
5.2 Characterization of $< 0, \pm 45, 90 >_s$ laminates

In this section the results of virtual experiments with $< 0, \pm 45, 90 >_s$ laminates are presented. The FEM model for this laminate was already shown in Fig. 3.4 and is even more complex than the mesh for $< 45, -45 >_s$ laminates. This is mostly contributed to the multiple fiber orientations in the composite. For all the experiments small strains are applied once more, since tetrahedron elements are used.

5.2.1 Axial Tensile Test

The applied strain of 0.2% is chosen, which is the same as in the case of $< 45, -45 >_s$ laminates. For this simulations 12 unit cells are coupled, which resulted in a lightly higher number of elements of roughly 1.3 elements. That means that also for these experiments parallel computing is used. The results of the virtual experiment is presented in Fig. 5.4.

![Figure 5.4: Axial stress in $< 0, \pm 45, 90 >_s$ laminate](image)

In this case the fibers with 0° orientation take most of the stress, because they are loaded in their direction. Also the higher stiffness contributes to the higher stress concentration. Around the fibers with 0° orientation, the stresses are low in the matrix region VII, caused by the lower stiffness and no presence of fibers in the loading direction. For the stress concentration in the layers containing the 45° and $-45°$ the explanation can be given as discussed in section 5.1.1. So the high stresses in the matrix region are caused by the presence of fibres in the loading direction.

The highest stressed in the matrix are found in the region IX. This high stress concentration is caused by the fibers with 90° orientation. Since there are fibers present in the loading direction in region IX, a higher force is required to deform the material. This causes the higher stress in the matrix region. For the region indicated with VIII, the stress is lower since in the loading direction only matrix material is present, which has the smallest stiffness.
5.2.2 Transverse Tensile Test

In this case the same boundary conditions are applied to the composite as was shown in the case of the axial tensile test, only applied in the transverse direction. Also this model contains 12 unit cells and the result of the transverse tensile test is shown in Fig. 5.5.

![Figure 5.5: Transverse stress in \(<0, \pm 45, 90>\) laminate](image)

The global response of the transverse tensile test is comparable with the response shown in the axial tensile test. The difference with the axial tensile test is that the position of the fibers has changed within the composite. The fibers oriented in the loading direction are no longer situated in the layers on the boundary, but lie in the middle of the structure. Clearly visible is that the maximum stress is present in these fibers. The matrix region X around these fibers take less stress, since in the loading direction only matrix material is present. This could also be seen in the results of the axial tensile test.

Fibers oriented perpendicular to the loading direction contribute to a higher stress in the matrix region XI, where in the region XII the stresses are lower since in that region only matrix material is present. This was also observed in the case of the axial tensile test.

5.2.3 Shear test

At last the result for the shear test are shown. Compared with the experiment discussed in section 5.1.3, the same boundary conditions are used for this experiment. So the boundary conditions applied, provide a symmetry condition. Also for this simulation 12 unit cells are coupled and the result of the shear test is shown in Fig. 5.6.

The results show that the fibers with 45 and −45 take most of the stress. In comparison with the shear test discussed for the case of the <45, −45> laminate this result is similar, since the fibers
are loaded nearly in the direction of their orientation. Consequently the matrix region XV takes less stress, which was also observed in the case of the $<45, -45>_s$ laminate.

Further presented in this figure is the high stress concentration in the top region XIII, where the $0^\circ$ fibers are present. This is natural since the fibers have a higher modulus compared to the surrounding matrix. In the layer containing the 90 oriented fibers, the stress distribution in the matrix is uniform. Since there is no constraint due to presence of fibers in the loading direction, the matrix deforms uniform in this region XIV.

5.2.4 Results

From the experiments described above, the effective elastic constant are determined using the volume average quantities as given in equation (2.1). The results of the virtual experiments and the results derived by McCartney are shown in table 5.3.

<table>
<thead>
<tr>
<th></th>
<th>$E_{1}^{eff}$ [Gpa]</th>
<th>$E_{2}^{eff}$ [Gpa]</th>
<th>$\nu_{12}^{eff}$ [-]</th>
<th>$G_{12}^{eff}$ [Gpa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Virtual Experiment DNS</td>
<td>153.8</td>
<td>153.7</td>
<td>0.26</td>
<td>57.6</td>
</tr>
<tr>
<td>McCartney</td>
<td>151.9</td>
<td>151.9</td>
<td>0.27</td>
<td>59.8</td>
</tr>
</tbody>
</table>

For both the axial and transverse modulus good agreement is found with McCartney. The maximum difference for both cases is 1.3%. Also in this case the axial and transverse modulus are roughly equal. This is due the fact that the global response is the same, caused by the same fibre orientations in the composite. The only difference is that the position of the oriented fibers is changed in the total composite for the different loading cases.

A difference of 5.1% is found for the Poisson’s ratio with McCartney. However, McCartney made assumptions in his model, so these experiments should describe the Poisson’s ratio more accurately.
For the shear modulus good agreement is found with McCartney, only a small difference of 3.6% is found. In contrast with the analytical values described by McCartney, with DNS it is possible to predict the elastic properties and also visualize the stress concentrations in the total composite on microscopic level.
Chapter 6

Conclusions and Recommendations

In this study an approach for mesh modeling of angle-ply laminated plates is presented. This approach is based on (1) modeling the complex geometry of the fibers and matrix using boolean operations and (2) using an automatic mesh generator (in this study provided by MSC Patran). This approach was used to create mesh models for \(<45, -45>\), and \(<0, \pm45, 90>\) laminated composite plates, both with different volume fractions.

To verify the virtual experiments, the effective elastic constants were determined and compared with analytical values given in literature. The results of these virtual experiments showed good agreement both with Khashaba and McCartney. In contrast with analytical models, with this DNS approach it was possible to visualize the stresses on microscopic level in the different layers. From these results it could be concluded that the influence of the fiber orientation was the major contribution in the matrix regions. Fibers orientated at an angle, different from the loading direction, caused higher stresses in the matrix region between these fibers. Also high stresses in the fibers orientated in the loading direction were visualized.

Compared with the analytical models, DNS gives a more accurate description of the elastic constants since no assumptions are made. Both Khashaba and McCartney assumed plane stress. Since DNS relies on direct modeling of the microstructure in 3D, naturally no assumptions like plane-stress are made. Due to this modeling of the full microstructure the number of elements increased to 1.3 million. To execute the simulations parallel computing was necessary, and in this research IPSAP was used as FEM program.
Recommendations

As presented in this study, the mesh of the complex geometries is based on tetrahedron elements. If larger deformation are involved these elements could cause problems, like overstiffening of the structure or locking. In order to provide these consequences the mesh should be made out of hexahedron elements. The automatic mesh generator of MSC PATRAN is not capable of meshing complex geometries with hexahedron elements, so a different mesh programm should be used.

As was shown the modeling of the fibers, the cross section was made of an octagonal shape. To describe the interface between the fiber and matrix and global behavior of the composite more accurately, a circular cross section should be used to create the geometry of the fibers.

Finally if the both hexahedron and circular cross section are implemented, delamination and damage initiation could be analyzed under different loading geometries. Since TMCs are loaded under extreme conditions, this should be investigated in further research.
References


Appendix A

Derivation Stress Strain Relation for Angle Lamina

The stress-strain relation for an orthotropic unidirectional lamina is given as

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} =
\begin{bmatrix}
Q_{11} & Q_{12} & 0 \\
Q_{12} & Q_{22} & 0 \\
0 & 0 & Q_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix},
\]

where \(Q_{ij}\) are the reduced stiffness coefficients, which are related to the elastic properties of the laminate as

\[
Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}}, \quad Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}}, \quad Q_{12} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} \quad \text{and} \quad Q_{66} = G_{12}.
\]

(A.2)

With this relation one can describe the stress-strain relation for an angle lamina. To describe the stress-strain relation for an angle lamina, the stresses and strains should be transformed to their new state. Consider picture (A.1).

The global (x-y) and local (1-2) stresses are related to the angle of the lamina, \(\theta\)

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} = \mathbf{T}^{-1}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix},
\]

(A.3)

where \(\mathbf{T}\) is called the transformation matrix and is defined as

\[
\mathbf{T} = \begin{bmatrix}
c^2 & s^2 & 2cs \\
s^2 & c^2 & -2cs \\
-cs & cs & c^2 - s^2
\end{bmatrix}
\]

(A.4)
with $c = \cos(\theta)$ and $s = \sin(\theta)$. The relation between the global stress and local strain can be written as

$$
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} = T^{-1}Q
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_{12}
\end{bmatrix}.
$$

(A.5)

To describe the global stress-strain relation, first the relation between the global and local strains is described.

$$
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_{12}/2
\end{bmatrix} = T
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}/2
\end{bmatrix}.
$$

(A.6)

Equation (A.6) can be written as

$$
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_{12}/2
\end{bmatrix} = R T R^{-1}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}/2
\end{bmatrix}.
$$

(A.7)

where $R$ is the Reuter matrix and is defined as

$$
R = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 2
\end{bmatrix}.
$$

(A.8)

Substituting equation (A.7) in equation (A.5) gives

$$
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} = T^{-1}Q R T R^{-1}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix}.
$$

(A.9)
Working out the multiplication of the five matrices in equation (A.9) gives

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} = \begin{bmatrix}
\overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\
\overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\
\overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66}
\end{bmatrix} \begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix},
\] (A.10)

where \( \overline{Q}_{ij} \) are called the elements of the transformed reduced stiffness matrix \( \overline{Q} \), where

\[
\begin{aligned}
\overline{Q}_{11} &= Q_{11}c^4 + Q_{22}s^4 + 2(Q_{12} + 2Q_{66})s^2c^2, \\
\overline{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})s^2c^2 + Q_{12}(c^4 + s^2), \\
\overline{Q}_{22} &= Q_{11}s^4 + Q_{22}c^4 + 2(Q_{12} + 2Q_{66})s^2c^2, \\
\overline{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})c^3s - (Q_{22} - Q_{12} - 2Q_{66})s^3c, \\
\overline{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})cs^3 - (Q_{22} - Q_{12} - 2Q_{66})c^3s, \\
\overline{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{66})s^2c^2 + Q_{66}(s^4 + c^4).
\end{aligned}
\] (A.11)
Appendix B

Correction factors

As presented in section 4.2.2 McCartney used correction factors for determining the elastic properties of angle-ply laminates. The correction factor $\lambda_1$, which is used for calculations of $E_1$ and $\nu_{12}$, is given by

$$\frac{1}{\lambda_1} = \frac{1}{2} \left[ \frac{1}{G_m} + \frac{V_f}{K_f} + \frac{V_m}{K_m} \right].$$

(B.1)

In equation (B.1), $G_m$ is the shear modulus and $K_m$ the bulk modulus of the matrix material, where $K_f$ is the bulk modulus of the fibre material.

The correction factor $\lambda_2$ is used to describe the shear modulus $G_{12}$, where $\lambda_2$ is:

$$\frac{1}{\lambda_2} = G_m (1 + V_f) + G_f V_m.$$  

(B.2)

To determine the transverse modulus $E_2$ of a single lamina McCartney used the transverse bulk modulus $K_t$ and the transverse shear modulus $G_t$, according to,

$$\frac{1}{K_t} = \frac{V_f}{K_f} + \frac{V_m}{K_m} - \frac{\lambda_3}{2} \left( \frac{1}{K_f} - \frac{1}{K_m} \right)^2 V_f V_m,$$ 

(B.3)

$$G_t = V_f G_f + V_m G_m - \lambda_3 (G_f - G_m)^2 V_f V_m,$$ 

(B.4)

where,

$$\frac{1}{\lambda_3} = V_m G_f + V_f G_m + \frac{K_m G_m}{K_m + 2G_m}.$$ 

(B.5)