Tyre/road friction modeling

Literature survey

R. van der Steen

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Coaches: I. Lopez
          B. de Bruijn
          A.J.C. Schmeitz

Supervisor: H. Nijmeijer

Eindhoven University of Technology
Department of Mechanical Engineering
Dynamics and Control group

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Contents

List of symbols 5

1 Introduction 7

2 Materials 11
  2.1 Elastic ................................................................. 11
  2.2 Visco-elastic ......................................................... 13

3 Friction models and surface roughness 15
  3.1 Amontons-Coulomb friction ......................................... 15
  3.2 Friction of viscoelastic materials ................................... 16
  3.3 Surface roughness ..................................................... 18

4 Finite Element Method 21
  4.1 Contact and friction ................................................ 22
  4.2 Literature related to FEM and tyres ............................... 26

5 Validation and modeling methods 31
  5.1 LAT100 and Tekscan ................................................ 31
  5.2 Eplexor and DIK ...................................................... 31
  5.3 Flat plank and tyre test trailer .................................... 32
  5.4 Tyre modeling in ABAQUS ........................................... 32

6 Conclusions 35
  6.1 Friction ................................................................. 35
  6.2 FEM ................................................................. 36

A 32th Tire Mechanics Short Course 39
  A.1 Strength, wear and friction of rubber. ............................ 39
  A.2 The tire as a vehicle component. .................................. 42
  A.3 Tire materials and manufacturing. .................................. 43
  A.4 Rules and regulations governing tires. ............................ 44
  A.5 Advanced tire modeling. ............................................ 44
  A.6 Tire wear, traction and force generation. ......................... 47
  A.7 Tire stresses and deformation analysis. ........................... 50
### List of symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$\alpha$</td>
<td>[rad]</td>
<td>Slip angle</td>
</tr>
<tr>
<td>$\gamma_{cr}$</td>
<td>[m]</td>
<td>Elastic slip</td>
</tr>
<tr>
<td>$\dot{\gamma}_{cr}$</td>
<td>[m/s]</td>
<td>Elastic slip rate</td>
</tr>
<tr>
<td>$\delta$</td>
<td>[rad]</td>
<td>Loss angle</td>
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<tr>
<td>$\theta$</td>
<td>[ ]</td>
<td>Field variable</td>
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<td>Longitudinal slip</td>
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<tr>
<td>$\lambda$</td>
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<td>Length scale</td>
</tr>
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<td>$\nu$</td>
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<td>Poisson’s ratio</td>
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<tr>
<td>$\mu_f$</td>
<td>[-]</td>
<td>Maximum friction coefficient</td>
</tr>
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<td>$\mu_r$</td>
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<td>$\phi_T$</td>
<td>[Hz]</td>
<td>Segmental jump rate at temperature $T$</td>
</tr>
<tr>
<td>$\omega$</td>
<td>[rad/s]</td>
<td>Spinning velocity</td>
</tr>
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<tr>
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</tr>
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<td>[m/s]</td>
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<tr>
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<td>[m]</td>
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<tr>
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<td>[Pa]</td>
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<tr>
<td>$z$</td>
<td>[m]</td>
<td>Distance to wheel center</td>
</tr>
<tr>
<td>Symbol</td>
<td>Unit</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-----------</td>
<td>------------------------------------------</td>
</tr>
<tr>
<td>$A$</td>
<td>[m$^2$]</td>
<td>Contact area</td>
</tr>
<tr>
<td>$E$</td>
<td>[Pa]</td>
<td>Young’s modulus</td>
</tr>
<tr>
<td>$E^*$</td>
<td>[Pa]</td>
<td>Complex Young’s modulus</td>
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<tr>
<td>$F_n$</td>
<td>[N]</td>
<td>Normal force</td>
</tr>
<tr>
<td>$F_x$</td>
<td>[N]</td>
<td>Longitudinal force</td>
</tr>
<tr>
<td>$F_t$</td>
<td>[N]</td>
<td>Tangential force</td>
</tr>
<tr>
<td>$F_r$</td>
<td>[N]</td>
<td>Rolling friction force</td>
</tr>
<tr>
<td>$G$</td>
<td>[Pa]</td>
<td>Shear modulus</td>
</tr>
<tr>
<td>$G'$</td>
<td>[Pa]</td>
<td>Dynamic shear modulus</td>
</tr>
<tr>
<td>$G''$</td>
<td>[Pa]</td>
<td>Dynamic loss modulus</td>
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<td>[Pa]</td>
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<td>[N/m]</td>
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<tr>
<td>$T$</td>
<td>[K]</td>
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</tr>
<tr>
<td>$T_g$</td>
<td>[K]</td>
<td>Glass temperature</td>
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<td>$V_s$</td>
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<td>$W$</td>
<td>[N]</td>
<td>Normal load</td>
</tr>
<tr>
<td>$B$</td>
<td>[-]</td>
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</tr>
<tr>
<td>$C$</td>
<td>[-]</td>
<td>Magic formula coefficient</td>
</tr>
<tr>
<td>$D$</td>
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<tr>
<td>$E$</td>
<td>[-]</td>
<td>Magic formula coefficient</td>
</tr>
<tr>
<td>$F_f$</td>
<td>[-]</td>
<td>Slip tolerance</td>
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<tr>
<td>$G$</td>
<td>[Pa/m]</td>
<td>Penalty coefficient</td>
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Chapter 1

Introduction

This literature report has been written in the scope of the CCAR project FEM Tyre Modelling. This project aims to realize a thorough base in modeling of a tyre using a finite element method. An import aspect is the development of a robust friction model to include tyre/road interaction. The report has been written to obtain a basic understanding in the field of (rubber) friction and the finite element method. An overview is presented with respect to friction models and surface roughness, contact and friction in the finite element method, and tyre analyses using the finite element method. It can therefore be used as a reference guide for future steps within the CCAR project.

Since the patent on a pneumatic tyre, by John Boyd Dunlop in 1888, the development of pneumatic tyres started and is still going on today. The worldwide tyre production is now greater than 1 billion units per year with a market value exceeding US $100 billion. Around 73% are passenger car tyres and 70% of these tyres are replacement tyres. There are basically two types of tyres, the bias ply and, since 1948, radial ply tyres. Nowadays most of the passenger car tyres are radial tyres. The main difference is the orientation of the body plies, see figure 1.1.

![Bias tyre and Radial tyre](image)

Figure 1.1: Difference between bias and radial tyres (picture taken from Gent (2007)).

The tyre construction, such as aspect ratio, belt construction and tread compound, depends on the size and target market (speed rating). This information is printed on every tyre, e.g., 215/55 R 16 97 V. The first number (215) is the nominal section width in mm, the second number (55) is the percentage of the height/width ratio of the cross-section. The R (radial) stands for the tyre construction code. Then the rim diameter (16) is given in inches and 97 is the load capacity index, which indicates the maximum load capacity. The last symbol (V) is the speed
symbol, which stands for the maximum speed. On winter tyres an additional logo is printed together with the mud and snow (M&S) symbol. See table 1.1 for an overview of load indices and the speed symbols for cars.

Table 1.1: Load indices and speed symbols for car tyres.

<table>
<thead>
<tr>
<th>Load index</th>
<th>Load (kg)</th>
<th>kg index</th>
<th>kg</th>
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<th>kg</th>
<th>Symbol</th>
<th>Speed (km/h)</th>
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<td>290</td>
<td>79</td>
<td>437</td>
<td>93</td>
<td>650</td>
<td>107</td>
<td>975</td>
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<td>66</td>
<td>300</td>
<td>80</td>
<td>450</td>
<td>94</td>
<td>670</td>
<td>108</td>
<td>1000</td>
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<tr>
<td>67</td>
<td>307</td>
<td>81</td>
<td>462</td>
<td>95</td>
<td>690</td>
<td>109</td>
<td>1030</td>
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<tr>
<td>68</td>
<td>315</td>
<td>82</td>
<td>475</td>
<td>96</td>
<td>710</td>
<td>110</td>
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<td>69</td>
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<td>83</td>
<td>487</td>
<td>97</td>
<td>730</td>
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<td>98</td>
<td>750</td>
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<td>71</td>
<td>345</td>
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<td>515</td>
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<td>103</td>
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<td>600</td>
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<td>900</td>
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<td>77</td>
<td>412</td>
<td>91</td>
<td>615</td>
<td>105</td>
<td>925</td>
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<td>630</td>
<td>106</td>
<td>950</td>
<td>120</td>
<td>1400</td>
</tr>
</tbody>
</table>

Basically a tyre should carry load and transmit forces. However the design of a tyre is a complex trade-off problem, because a tyre needs to perform under varying conditions. In figure 1.2 the main components of a radial tyre can be seen. All these different components provide specific properties.

For example the tread should have good traction for all weather conditions, low rolling resistance and low wear. But low rolling resistance and a long lifetime require a hard compound, which has less traction. For good wet traction groves in the tread are required to drain off water, but this means less dry traction since the contact area is decreased. And softer compounds, which provide more traction than harder compounds, also wear faster.

Besides the engineering aspects, economical factors, (limitations of) the manufacturing process (Walter, 2007a) and government regulations (Walter, 2007b) have to be taken into account.

To model a tyre and the interaction with the environment different modeling approaches can be used: analytical models, empirical models or physical models. In the following the focus is on physical models to be used in a Finite Element Analysis (FEA). Information of the used materials and geometry of the tyre and an interaction with the road is required for the Finite Element Method (FEM).

The basic materials used in tyres and material models are described in chapter 2. In chapter 3 an overview of different friction models is given and the relation with surface roughness is shown. Chapter 4 covers the basics of FEM and numerical methods related to contact. Some aspects of contact in ABAQUS are mentioned and an overview of different types of tyre problems is given. Validation possibilities are given in chapter 5 and finally conclusions are presented in chapter 6.

An additional appendix, which is a short summary of the 32th tire mechanics short course is also included.
Figure 1.2: Components of a radial tyre (picture taken from Gent (2007)).
Chapter 2

Materials

Different materials are used in tyres. The rubber compounds are the most complex parts of the tyre. The principal tyre elastomers are natural rubber (NR), polyisoprene (IR, this is synthetic NR), styrene butadiene rubber (SBR), butyl rubber (IIR) and butadiene (BR). A compound can be defined as the formulation of a mixture of rubber and additives which meets the needs of the tyre component application. Usually a compound consists of one or more polymers, vulcanizing agents, accelerators, reinforcing fillers, antidegradents, plasticizers, softeners and tackifiers (used to bond pieces together). This explains why the properties of a compound can deviate from a pure elastomer. Bead wires are usually made of steel. Belt ply cords consist of steel or rayon, while body plies cords can be made of nylon or polyester. For a FE model the geometry of the parts, the applied loading and boundary conditions and the material behavior of each of the different materials is required. In this chapter the basics of elastic and visco-elastic material behavior are mentioned.

2.1 Elastic

2.1.1 Small strains

For small strains the material properties can be defined by the shear modulus \( G \) and the modulus of bulk compression \( K \), which are related to the tensile or Young’s modulus \( E \) and Poisson’s ratio \( \nu \) as follows:

\[
E = \frac{2(1 + \nu)G}{(2 + \nu)} \quad (2.1)
\]

\[
\nu = \frac{1}{2} \left( \frac{3K - 2G}{3K + G} \right) \quad (2.2)
\]

When a material behaves elastically, there is no permanent deformation and dissipation. This means that if the stress is released the strain will become zero. Furthermore there is no time or path history, thus the stress can be calculated if the strain is known.

The strains in the belt and body plies cords, under normal conditions, are in the linear elastic range, which means that they can be described by either \( G \) and \( K \) or \( E \) and \( \nu \).

2.1.2 Large strains

Rubber is an elastic, soft and virtually incompressible material, but it can usually be stretched more than 500%. Its molecular structure consists of long, linear flexible molecules forming random coils. The molecular segments are mobile, so-called Brownian motion (rate of segmental jumps,
which depends strongly on temperature), and the molecules are interlinked into a 3D network. Chemical crosslinks are usually made by sulfur linkages also known as vulcanization. The rubbery state of a polymer is determined by the so-called glass transition temperature $T_g$. If the temperature is above $T_g$ the polymer is rubbery, below $T_g$ the polymer is glassy, see figure 2.1. For rubbers the $T_g$ is below room temperature.

For rubber typically $K \gg G$ and it follows that $E = 3G$ and $\nu = 0.5$. If a rubber block with $\nu = 0.5$ is compressed, it is sheared outwards and generates an internal pressure. This effect causes a compression modulus, which is generally greater than $E$. To model this behavior other constitutive models have to be used. Hyperelastic material models are based on energy functions, which define the strain energy stored in the material per unit of reference volume, instead of the Young’s modulus ($E$) and poisson ratio ($\nu$) for isotropic linear elastic materials. In ABAQUS several hyperelastic material models are available, an overview can be seen in table 2.1. A specific material model is usually chosen based on available test data, however it is also possible to enter specific material constants directly. It is possible to fit material models on experimental data, such as uniaxial, biaxial, planar and volumetric test data. For more details see the ABAQUS Analysis user’s manual, chapter 17.

### 2.1.3 Filled rubbers

In tyres rubbers are usually filled with particles like carbon black or silica. This creates two additional effects. The so-called Payne effect and Mullins effect, see e.g. Diani et al. (2006), which are both softening effects. The Payne effect is a softening effect for small strains (0.1%), attributed to breaking apart aggregates of filler particles. The Mullins effect is a substantial softening effect at higher strains, attributed to progressive detachment of rubber molecules from filler particles. The Mullins effect can be modeled in ABAQUS, but it cannot be used together with visco-elasticity or hysteresis options. In Bergstrom (2005) a new constitutive material model is presented, which is compared with the standard ABAQUS material models. In Becker et al. (1998) a material model
for filled rubbers is presented, which is implemented into an in-house FE code and compared to experiments.

## 2.2 Visco-elastic

The constitutive laws for large strains cannot be used to fully describe the stress-strain relation, since rubbers do not follow reversible stress-strain relations. When rubber is dynamically stretched and released the returned energy is less than the energy that is put into the rubber. This visco-elastic effect cannot be described by the perfect elastic shear modulus $G$, but a dynamic shear modulus $G'$ and a dynamic shear loss modulus $G''$ is introduced to describe this hysteresis. Another term that is frequently used is the loss angle, which is defined as $\tan \delta = \frac{G''}{G'}$. For an elastic solid the strain is in phase with the stress, while for a visco-elastic solid the strain lags behind the stress with a delay of $\frac{\delta}{\omega}$, where $\omega$ is the frequency (rad/sec).

The universal WLF relation, named after Williams, Landel and Ferry, (Williams et al., 1955) describes the dependence of the segmental jump rate on temperature,

$$\log \frac{\phi_T}{\phi_{T_g}} = \frac{17.5(T - T_g)}{52 + T - T_g}, \quad (2.3)$$

where $\phi_{T_g}$ is defined as $0.1/sec$. Some polymers, such as butyl rubber, behave according to a different relation

$$\log \frac{\phi_T}{\phi_{T_g}} = \frac{17.5(T - T_g)}{100 + T - T_g}. \quad (2.4)$$

This is possibly due to unusually long moving segments. The WLF relation can by used to predict results at other temperatures, e.g. the frequency $f$ at temperature $T$ is equivalent to a frequency $f^*$ at $T^*$, using the following relation

$$\frac{f^*}{f} = \frac{\phi_{T^*}}{\phi_T} = a_T, \quad (2.5)$$

where $a_T$ is known as the shift factor, this equivalence also holds for strain rates or velocities.

To create master curves of material properties of most (pure) polymers this transformation can be used. Normally experiments can only be done for a limited range of frequencies. If the same experiments are conducted at different temperatures, a master curve can be made for a chosen reference temperature by shifting the measurements using (2.5). An example is shown in figure 2.2.

Details of a FE implementation of 3D visco-elastic behavior for small and finite strains can be found in an article of Kaliske and Rothert (1997).
Figure 2.2: Creation of master curve for $E^*$ using WLF shifts (picture taken from Gent (2007)).
In this chapter several friction models are discussed. Friction is present in all mechanical systems and originates at an interface between bodies, which are in contact with each other. This contact enables solids to transmit forces through the surfaces. The resultant force can be resolved into a normal and a tangential force. This is referred to as the normal force and the friction force respectively. Friction can now formulated as the ratio between the tangential and normal contact force. Forces can only be transmitted if there is contact, therefore the contact area plays an important role in describing friction aspects. Several theories have been developed to describe surface roughness and the relation to the real contact area.

It is well-known that contaminations have a strong effect on friction, e.g. lubricants. One of the classical works on friction and lubrication is the book of Bowden and Tabor (2001).

### 3.1 Amontons-Coulomb friction

This is the most used friction model, usually referred to as the Coulomb friction model. It is based on the two basic laws of friction (Bowden and Tabor, 2001). When a block is pressed upon a plane with a normal force and is pulled in the horizontal direction, there is a threshold value such that the block starts to move if the tangential force is bigger than this value. This threshold value is proportional to the normal force $F_t = \mu F_n$, with $\mu$ the so-called coefficient of friction. The friction force always works in opposite direction of the moving velocity. Coulomb also showed that kinetic friction could be significantly lower than static friction, which is sometimes referred to as the third friction law.

#### 3.1.1 Deviations from Coulomb friction

Experiments often show deviations from the basic Coulomb friction model. Variants of the basis model are the difference between a static $\mu_s$ and a kinetic $\mu_k$ friction coefficient, see figure 3.1 for some examples. This transition can be discontinuous or continuous, using an exponential decaying function between the static and kinetic coefficient. More general, friction varies with sliding velocity. An example is the so-called Strubeck curve, first the coefficient decreases to a minimum value and after that it increases with higher sliding velocities. The review paper of Olsson et al. (1998) describes several static and dynamic friction models. The static models include Coulomb friction, viscous friction and the models of Karnopp, where a zero velocity interval (deadzone) is introduced, and Armstrong, where stiction and sliding are decoupled in two separate equations. The dynamic models include the reset integrator model, the Dahl model and two variants of this
3.2 Friction of viscoelastic materials

Friction of polymers is closely related to their viscoelastic behavior. Generally speaking, the coefficient of friction increases with sliding velocity until a maximum value is reached at a certain speed, followed by a decrease of the friction coefficient. This is caused by the flexibility of the polymer chains.

3.2.1 Grosch

The article of Grosch (1963) is one of the early reports, which describes a relation between the dependence of the friction coefficient due to sliding velocity and temperature and the viscoelastic deformation on strain rate and temperature. In this work experimental results of different vulcanized rubbers are presented. Experiments are conducted at different temperatures and shifted to a master curve using the WLF equation. This gives a relation between the sliding velocity and temperature in a similar way as the rate and temperature dependence of viscoelastic polymers above their glass temperature, see figure 3.2. Grosch indicated that friction is due to energy dissipated when rubber is compressed and released by asperities. For dry sliding on a smooth surface the curve for $\mu$ vs. sliding speed resembles again the curve for $\tan \delta$ versus frequency $f$. But now it is displaced to much lower speeds than before. Friction in dry sliding on a smooth surface is due to energy dissipated as rubber sticks and slips on a molecular scale. In the case of sliding on a rough dry surface the two dissipative processes appear at different speeds, corresponding to
different length scales: asperities and molecules, see figure 3.3. Several contributions to \( \mu \) can arise at the same sliding speed from stick-slip processes occurring at different length scales. An early theory to describe the experiments of Grosch has been developed by Schallamach (1963). An unified approach to the entire subject of friction and wear in rubbers and tyres can be found in an article of Moore (1980).

\[
\mu_k(V_s) = \mu_s + (\mu_m - \mu_s) \exp \left( - \frac{h^2}{2} \log^2 \left( \frac{|V_s|}{V_{\text{max}}} \right) \right)
\]  

(3.2)

where \( \mu_s \) is the static coefficient of friction and \( \mu_m \) the maximal coefficient for \( |V_s| = V_{\text{max}} \). \( h \) is a dimensionless parameter reflecting the width of the speed range in which friction varies significantly. More details on dry adhesive friction can be found in (Savkoor, 1965, 1966, 1986) and in the thesis of Savkoor (1987). The advantage of (3.2) is that it can be evaluated easily for a given sliding velocity.
3.2.3 Rolling friction

Rolling friction $F_r$ can be seen as $\frac{M}{R}$, where $M$ is a torque and $R$ the radius of the rolling body. For an elastic wheel the reaction force acts through the wheel center and therefore the moment $M = 0$. For a visco-elastic wheel the reaction force moves towards the loading side, because stresses are lower in unloading, $M = Wz$ where $W$ is the normal load and $z$ the distance from the wheel center, see figure 3.4. Displacement of the center of pressure, and hence the coefficient of friction $\mu_r$, depend on speed and temperature in the same way as the loss factor $\tan \delta$. A formulation for $\mu_r$ as proposed by Greenwood et al. (1961),

\[
\mu_r = 0.33 \left( \frac{W}{G^* R^2} \right)^{\frac{1}{3}} \tan \delta,
\]

where $G^*$ is the complex shear modulus. Rolling friction is mainly due to energy dissipated as rubber is compressed and released in the contact patch.

![Figure 3.4: Rolling friction (picture taken from Gent (2007)).](image)

3.2.4 Persson

Persson has published many papers (Persson, 1993, 1995, 1997, 1998, 1999, 2001a,b, 2002a,b; Persson and Tosatti, 2000; Persson and Volokitin, 2000, 2002, 2006; Persson et al., 2002, 2003, 2005) on the subject of rubber friction and the role of the surface, which is in contact with the rubber. These theories are a continuation of the early studies of Grosch, taking into account that sliding friction of rubber has the same temperature dependence as that of the complex elastic modulus. He states that the friction force (under normal circumstances) is related to the internal friction of the rubber, which is a bulk property. The hysteretic friction component is determined by sliding of the rubber over asperities of a rough surface. These oscillating forces lead to energy dissipation. These contributions can be described with a fractal description of the surface. Every length scale $\lambda$, up to the largest particles of asphalt, can be related to a frequency: $f \sim \frac{1}{\lambda}$. As a result of energy dissipation heat is generated, in a recent article (Persson, 2006a) he also takes the local heating of rubber into account, since the viscoelastic properties are strongly temperature dependent. At very low sliding velocities the temperature increases are negligible because of heat diffusion, but already for velocities in the order of 0.01 m/s local heating plays a role. He shows that in a typical case the temperature increase results in a decrease in rubber friction with increasing sliding velocity, if the sliding velocity is above 0.01 m/s.

3.3 Surface roughness

There is a long history on this subject, since it is well-known that almost all surfaces are rough at a certain length scale. This has great influence on the real contact area and therefore it is closely related to friction. The real contact area is in most cases different from the apparent contact area.
and high local contact pressures can give rise to plastic deformation. The two often used methods to describe surface roughness are statistical and fractal methods.

In Archard (1957) it is shown that in order to obey the Coulomb friction law for elastic deformation a rough surface is actually needed. In elastic contact the area $A$ is related to the load $W$ by

$$A \propto kW^{2/3},$$

(3.4)

where $k$ is a constant. It is supposed that the friction force is proportional to the contact area, therefore it can not hold that the friction force is proportional to the load. For increasing load the friction coefficient has to decrease. To show this, Archard used a flat surface in contact with a spherical surface with large radius. However if instead of one spherical contact, several spherical protuberances on this initial sphere are modeled and thus multiple contact areas are created, the following relation is derived

$$A \propto W^{44/45}.$$  

(3.5)

He showed that $\mu$ is now constant for an increasing load until all the local protuberances are flat and than $\mu$ drops again if the load is still increased further. This explains that for soft rubber sliding on a smooth surface (perfect contact) the frictional force is more or less constant, independent of the load $W$. Thus the coefficient of friction decreases continuously as the pressure increases. However there is also a coefficient of friction, independent of pressure, for harder rubber compounds, sliding on a rough surface. Apparently in this case contact is incomplete. An increase in pressure creates a larger true area of contact and hence a larger frictional force.

The Greenwood and Williamson (GW) contact model (Greenwood and Williamson, 1966) was published in 1966 and since then many revisions to this model have been made by several researchers. It is now known that some of the assumptions of the original model were incorrect (Greenwood and Wu, 2001).

The GW model is based on contact between a rough surface and a flat rigid plane. It is a statistical description of a rough surface, where the surface is covered with asperities. All these asperities have a spherical shape with the same radius of curvature. Further the summit surface density, the surface roughness and the surface height need to be specified. The surface height is supposed to follow a Gaussian or exponential distribution. Relations for the number of contacts, the real contact area and the applied load are obtained.

The model of Bush, Gibson and Thomas (BGT) (Bush et al., 1975) uses elliptical paraboloids asperities with random aspect ratio and orientation, a comparison between these models can be found in an article of McCool (1986).

Fractal descriptions to describe a rough surface first appeared in the papers of Majumdar and Bhushan (1990, 1991) and Majumdar and Tien (1990). If a surface exhibits geometrical self-similarity, also referred to as self-affinity, the surface appears the same under different degrees of magnification.

This can be described by the Weierstrass-Mandelbrot function, which satisfies continuity, non-differentiability and self-affinity. The advantage of this approach is that the parameters, which determine the isotropic rough surface, are independent of the roughness length scale and the resolution of the measuring instruments. This is in contrast to statistical parameters where the height, slope and curvature of asperities depend on the the resolution of the measuring instrument. Fractal models are now widely used and compared by a great number of authors, e.g. Klüppel and Heinrich (2000); Gal et al. (2005); Morrow (2003); Jackson and Streator (2006); Ciavarella et al. (2006a,b); Zavarise et al. (2007).

Besides his work on rubber friction, Persson has also made a large contribution to the field of contact mechanics (Persson, 2002a; Persson et al., 2002; Peressadko et al., 2005; Persson, 2006b; Yanga et al., 2006; Persson, 2006c). An extensive overview of contact mechanics and rubber friction can be found in a review paper of Persson (2005). A comparison of experiments and the predictive capabilities of current physical theories is presented in Westermann et al. (2004). This demonstrates the close interaction between rubber friction and surface roughness.
Chapter 4

Finite Element Method

The development of non-linear Finite Element Method (FEM) started about forty years ago together with the development of the digital computer, with one of the first text books of Zienkiewicz and Cheung (1967). Although there are many books related to FEM nowadays, this book or later editions are still cited in almost every article related to FEM.

There are two distinctive methods to solve FE problems, implicit and explicit. The implicit method solves for a statical or dynamical equilibrium, while the explicit method solves transient dynamic response problems using an explicit direct-integration procedure.

Besides the geometrical, material and interaction issues additional computational problems arise. Since the start of non-linear FEM many different types of elements and solution algorithms have been developed and implemented in specialized software such as ABAQUS, MARC, ANSYS, LS-DYNA or NASTRAN. This raises the question why one should study non-linear FEM if it is already available in many commercial packages? The answer to this is that a commercial package is basically a black box program providing an interface for simulations. The user has to make several choices in the model formulation, i.e., the discretization of the mesh, the element types, the solution procedure, the solver tolerances etc. These selections (may) have a tremendous effect on the final result and can even lead to non-physical, thus meaningless, results (see Kouznetsova, 2006).

There are four different sources of non-linearity in solid mechanics, geometrical, material, force boundary conditions and displacement boundary conditions, see also figure 4.1. Unfortunately all four play a role in tyre modeling. One example is incompressible material behavior of rubber. When the material response is incompressible, the solution to such a problem cannot be obtained in terms of the displacement history only, since a purely hydrostatic pressure can be added without changing the displacements.

In ABAQUS this is solved by removing this singular behavior in the system by treating the pressure stress as an independently interpolated basic solution variable, coupled to the displacement solution through the constitutive theory and the compatibility condition, with this coupling implemented by a Lagrange multiplier. This independent interpolation of pressure stress is the basis of the so-called hybrid elements. More precisely, they are ‘mixed formulation’ elements, using a mixture of displacement and stress variables with an augmented variational principle to approximate the equilibrium equations and compatibility conditions. The hybrid elements also remedy the problem of volume strain ‘locking’. Volume strain locking occurs if the finite element mesh cannot properly represent incompressible deformations. Volume strain locking can be avoided in regular displacement elements by fully or selectively reduced integration (ABAQUS Analysis
4.1 Contact and friction

Starting points for this subject are the article of Doghri et al. (1998), the book of Laursen (2002) and especially the book of Wriggers (2006) on computational contact mechanics. Contact is split into normal contact and tangential contact.

4.1.1 Normal contact

Normal contact can be formulated as a geometrical constraint, which describes the non-penetration condition of two bodies in contact, also known as ‘low contact precision’. This is the so-called Signorini contact description, which models the non-penetrability of the bodies in contact with the following relations. For contact it holds that \( g_N = 0 \) and \( p_N < 0 \), where \( g_N \) is the gap distance and \( p_N \) is the normal pressure. If there is a gap between the bodies, than the pressure is equal to zero, see figure 4.2. This can be formulated as

\[
g_N \geq 0, \quad p_N \leq 0, \quad p_N g_N = 0. \tag{4.1}
\]

Figure 4.1: Schematic view of non-linearities in solid mechanics (picture taken from Kouznetsova (2006)).

Two aspects are considered here, the mathematical formulations of contact and friction and implementation issues in ABAQUS, and computational models for tyres found in literature to give an overview of current possibilities.
This formulation states that only compressive forces can be transmitted if there is contact (no
adhesion) and forms the basis of frictionless contact problems.

If for a contact problem knowledge of the micromechanical surface is essential to describe the phys-
ical phenomena (`high contact precision') an interface law is needed to capture this phenomena.
Instead of a geometrical constraint a constitutive equation is used, where the contact pressure is
given by a general non-linear function of the mean plane distance

\[ p_N = f(d). \]  

(4.2)

An example, taken from Wriggers (2006), of this general non-linear function:

\[ p_N = \frac{c_1(161764.152c_2c_4)}{5.589^{1+0.0.0711c_2}} \exp \left[ -\frac{1 + 0.0.0711c_2}{(1.363c_3)^2} - d^2 \right], \]  

(4.3)

where \( c_1 \) and \( c_2 \) are mechanical constants which express the nonlinear distribution of the surface
hardness and \( c_3 \) and \( c_4 \) are statistical parameters of the surface profile.

4.1.2 Tangential contact

Usually friction is present between two interacting bodies and this leads to tangential forces.
Tangential contact is described by constitutive laws. The response in tangential direction can be
divided into two cases. The first case is no tangential displacement in the contact zone (stick) and
in the second case there is relative displacement (slip).

Stick corresponds to a situation where the relative velocity (\( \dot{g}_T \)) is zero and thus a zero relative
displacement \( \dot{g}_T = 0 \), \( g_T = 0 \),

(4.4)

which poses a constraint on the motion in the contact interface.

If the tangential forces reach a certain limit the contacting bodies will no longer stick, but sliding
will occur. In the case of Coulomb friction this can be formulated as

\[ t_T = -\mu|p_N| \frac{\dot{g}_T}{||\dot{g}_T||}, \quad \text{if } ||t_T|| > \mu|p_N|, \]  

(4.5)

where \( t_T \) is the shear stress.

Since the friction coefficient usually depends on more parameters, a constitutive law can be written
as a non-linear function of several parameters,

\[ \mu = \mu(\dot{g}_T, p_N, \theta, \ldots), \]  

(4.6)

where \( \theta \) is defined as a field variable, e.g. temperature.

An example from Wriggers (2006) of a velocity and pressure dependent constitutive relation for
rubber friction is the following

\[ \mu(v_T, p_N) = c_5 \left[ \frac{p_N}{c_6} \right]^{c_7} + c_8 \ln \frac{V_s}{c_9} - c_{10} \ln \frac{V_s}{c_{11}}. \]  

(4.7)

Besides the Coulomb law (4.5), it is possible to formulate tangential constitutive equations by
regularization of the stick-slip behavior. This is to avoid the non-differentiability of Coulomb’s
law at zero velocity, which can prevent numerical problems, some examples of this are square
root, hyperbolic tangent of piecewise polynomial regularizations. Other formulations are in the
framework of elasto-plasticity, where the key idea is to split the tangential slip in an elastic (stick)
4.1. CONTACT AND FRICTION

Articles of Huemer et al. (2001a); Liu et al. (2000) and Huemer et al. (2001b) are related to frictional behavior of rubber, where a phenomenological friction law for rubber tread blocks on snow and concrete is presented for use in a macroscopic model. The coefficient of friction depends on normal pressure, sliding velocity and temperature. The friction coefficient itself is only a function of normal pressure and sliding velocity. Temperature effects are incorporated using the WLF transformation, i.e., if the current temperature is different than the reference temperature, an equivalent new sliding velocity for the reference temperature is calculated.

This research has been continued by Hofstetter et al. (2003), where a thermo-mechanical coupling has been introduced. Energy dissipation during sliding is converted into heat and this heat flux causes a temperature rise of the rubber and road. In (Hofstetter et al., 2006) also simulations of abrasion of the tread block are added.

Dorsch et al. (2002) derived a phenomenological friction law using data obtained from experiments on the LAT100.

The modeling and calculation of the transient rolling contact of tyres on road tracks is the subject of a large multidisciplinary research project (FOR492) at the University of Hannover. One of the projects focusses on computational homogenization procedures, to develop a friction law at a macroscopic level, based on effects on a microscopic level. More details on this subject can also be found in the book of Wriggers (2006).

4.1.3 Constraint handling in ABAQUS

The aforementioned constraints, imposed by contact and friction, can be handled by Lagrange multipliers, which satisfies the constraint exactly, or with penalty methods, which penalize violation of constraints. All the constraint and interaction methods of ABAQUS can be found in the ABAQUS Analysis user’s manual, chapters 28-32. One can find here that normal contact is handled by Lagrange multipliers, penalty methods or an augmented Lagrange method. Friction, in ABAQUS/standard, is normally handled by the penalty method.

While the surfaces are sticking, a certain elastic slip \( \gamma_{cr} \) is allowed and the magnitude of sliding is limited to this elastic slip. The allowable elastic slip is given as

\[
\gamma_{cr} = F f l, \tag{4.8}
\]

where \( F f \) is an user-defined slip tolerance and \( l \) is the characteristic surface length. The slope \( G \), which is a penalty stiffness, is adapted depending on the normal pressure, see figure 4.3. One can see that the smaller \( \gamma_{cr} \), the steeper the slope of \( G \). Exact sticking, \( G = \infty \), can be obtained by enforcing a Lagrange multiplier.

However there is one important exception: If a so-called Steady-State Transport Analysis (which is commonly used in tyre simulations) is performed, the penalty constraint is based on a maximum allowable slip rate

\[
\dot{\gamma}_{cr} = F f 2 \omega R, \tag{4.9}
\]

where \( \omega \) is the spinning velocity and \( R \) the radius of the object. This introduces discontinuities if a Steady-State Transport Analysis is combined with other ABAQUS standard analyses and should therefore be avoided.

4.1.4 Contact possibilities in ABAQUS

Several contact models are available in ABAQUS. In this section the import features are given. There are two contact discretizations, node-to-surface and surface-to-surface contact and two tracking approaches, small sliding and finite sliding.
Some of the mechanical contact property models available in ABAQUS/Standard include: Softened contact, friction and user-defined constitutive models for surface interaction. Surface interaction in thermal or coupled thermal-mechanical contact simulations can include heat exchange by conduction and radiation as well as the generation of frictional heat in coupled simulations.

During an increment in which the contact status has changed, ABAQUS/Standard will use the default hard contact criterion to determine whether the change should be reversed. In other words, if the contact status changes from open to closed during an increment, the contact pressure must remain positive for the changed status to persist. In subsequent increments the contact point can again sustain tensile pressures up to a specified value before the surfaces separate. This contact pressure-overclosure relationship is useful for cases where negative pressure values (such as surface cohesion) may be allowed physically. It can also be useful in overcoming numerical problems in difficult contact simulations and in obtaining solutions without excessive iteration. Another possibility to oppose the relative motion between the interacting surfaces is to specify damping.

The friction models available in ABAQUS:

- Include the classical isotropic Coulomb friction model, which in ABAQUS: in its general form allows the friction coefficient to be defined in terms of slip rate, contact pressure, average surface temperature at the contact point, and field variables; and provides the option to define a static and a kinetic friction coefficient with a smooth transition zone defined by an exponential curve.
- Allow the introduction of a shear stress limit, which is the maximum value of shear stress that can be carried by the interface before the surfaces begin to slide.
- Include an anisotropic extension of the basic Coulomb friction model in ABAQUS/Standard;
- Include a model that eliminates frictional slip when surfaces are in contact.
- Include a softened interface model for sticking friction in ABAQUS/Explicit in which the shear stress is a function of elastic slip.
- Can be implemented with a stiffness (penalty) method, a kinematic method (in ABAQUS/Explicit), or a Lagrange multiplier method (in ABAQUS/Standard), depending on the contact algorithm used.

Figure 4.3: Adapting penalty stiffness, based on a maximal allowable slip.
• Can be defined in user subroutine \textit{FRIC} (in ABAQUS/Standard) or \textit{VFRIC} (in ABAQUS/Explicit for the contact pair algorithm only), which allows modeling of very general frictional surface conditions.

Besides the friction models it is also possible to make use of user-defined interfacial constitutive behavior. This is provided so that any constitutive behavior across an interface can be added to the library of existing models. To use this the constitutive model (or a library of models) for the interface has to be programmed in user subroutine \textit{UINTER} (ABAQUS/Standard) or \textit{VUINTER}. It is available only for a surface-based contact definition involved in stress/displacement, coupled temperature-displacement, or heat transfer analysis. According to the manual the feature is very general and powerful, but it requires considerable effort and expertise and is intended for advanced users.

4.2 Literature related to FEM and tyres

Together with the development of non-linear FEM the use of FEM in tyre development started. There is a lot of literature, somehow related to tyres, on computational developments in the framework of FE and quite some work is also incorporated into ABAQUS. Examples are the works of Oden on friction and rolling contact (Oden and Pires, 1983, 1984; Oden and Martins, 1985; Oden and Lin, 1986; Oden et al., 1988; Faria et al., 1989), Laursen and Simo in the field of contact problems with friction (Simo and Laursen, 1992; Laursen and Simo, 1993a,b) and Padovan on rolling visco-elastic cylinders (Zeid and Padovan, 1981; Padovan, 1987; Kennedy and Padovan, 1987; Nakajima and Padovan, 1987; Padovan et al., 1992). A special issue (Vol 177, nr 3/4) of the journal Computer Methods in Applied Mechanics and Engineering is devoted to computational modeling of contact and friction. More recent are the works of Wriggers (Wriggers et al., 1990; Zavarise et al., 1992; Haraldsson and Wriggers, 2000; Bandeira et al., 2004; Wriggers, 2006) on constitutive interface laws with friction.

Literature specifically related to tyres usually does not provide detailed information. The main reason is that most tyre manufacturers use own (in-house) finite element codes and specific methods are kept confidential. Even in the publications where ABAQUS is used not much background information can be found. Besides this, most research at universities is done in cooperation with a tyre manufacturer and therefore still not much model details are given, more information can be obtained from articles where specific subjects related to tyres are treated. However the found literature provides insight in the trends and developments of tyre modeling throughout the past decades.

According to Padovan (2007), the following list of tyre problems are successfully handled by virtual CAE modeling methodologies

• Static rim mounting, inflation and axle load/deflection analysis
  – States of stress/deflection/strain in all tyre components
  – Force and moment, on center feel
  – Definition of footprint shape as a function of axle load state and pressure

• Rolling/handling models both quasi-static and dynamic including braking and cornering effects

• Thermal analysis/rolling resistance under rolling conditions

• Model cure press, and other manufacturing steps
4. FINITE ELEMENT METHOD

- Fatigue life prediction models under full duty cycle
- Natural frequencies, handling and critical speed analysis
- Hydroplaning models, snow traction, in mud performance
- Geometry, and cord layout optimization
- Tread lug design
- Acoustical analysis
- Suspension models incorporating tyres

In one of his presentations it is stated that the overall tyre performance is usually measured by global actions, such as force & moment, ride comfort or dynamic response and therefore gross FEA models can be employed to establish such characteristics. In contrast to this some aspects, such as tyre failure, require very localized models. In this sense a form of multi length scale methodology can be used, i.e., a global model is used to drive a local model.

A summary according to Padovan of advantages and disadvantages for global and local models: An one scale model has one contiguous mesh and a refined mesh is placed in locations of interest, transition meshes are needed to link refined and coarse zones. It is in general difficult to build a mesh with reliable local refinements. The models tend to be very large and time consuming and are not very adoptable making mesh modifications. The transition areas tend to yield unreliable results.

Multiple scales suffer the so-called localization syndrome: localization always requires further localization, so a localization cut-off criteria is needed. Besides that stress and strain take on a random character as localization brings on the probabilistic nature of such scale levels. While probabilistic variances can be very significant, generally the mean is close to the nominal design value and since stiffness of cord or metallic/plastic subcomponents are orders of magnitude higher than rubber, local actions of such subcomponents dominate; rubber is hence strictly a geometric placeholder. Also every new level should be correlated with experiments to assess accuracy and sensitivity to meshing.

Most of the mentioned tyre problems require a specific type of modeling approach, in this sense there is no universal tyre model. This totally depends on the kind of problem one wants to solve. An overview is given according to some points of the above list, although most articles fit in more than one category. Besides the journal of Tire Science and Technology, a good starting point are the two overview papers of Mackerle (1998, 2004) about rubber and rubber-like materials, finite element analyses and simulations, where an extensive reference list can be found.

4.2.1 Static analysis

One of the first overview papers, where FE models and the contact problem for tyres are described, is the paper of Noor and Tanner (1985). In this article, for a NASA research program for the space shuttle tyres, the current status and developments of computational models for tyres are summarized. This review has been made again by Danielson et al. (1996). Basically all early tyre models are axi-symmetric, with no tread pattern or only circumferential grooves due to the limits of computational resources.

Another popular way to reduce the number of degrees of freedom is the application of the global-local analysis. In this approach, an analysis of the complete structure is first performed with a coarse mesh. After that a part of the structure is meshed finer and interpolated displacements are applied at the boundaries of this region. Such an approach can be used to compute the forces in the contact area of a deflected tyre (see Gall et al., 1995 and Meschke et al., 1997). Although a
local model is able to give detailed numerical results corresponding to a tread block, the accuracy is strongly influenced by the simple global model.

With the ever increasingly computational power it is now possible to mesh a part of even the whole tread with a detailed pattern (e.g. Cho et al., 2004) and perform all kinds of static analysis such as footprint shapes as function of axle load state and pressure.

4.2.2 Force & Moment

In an article of Kabe and Koishi (2000) a comparison between ABAQUS standard, explicit and experiments for steady state cornering tyres is made. Although the results of both implicit and explicit are closer to each other than to the experiments, the implicit method is significantly faster, 6 hours and 8 days respectively. The prediction of tyre cornering forces is given in a paper of Tünnik and Ünlüsoy (2001), where MARC is used. An comparison with experiments is also presented.

The application of a new type of non-linear 3D finite element tyre model for simulating tyre spindle force and moment response during side slip is described by Darnell et al. (2002). The simulation model is composed of shell elements, which model the tread deformation, coupled to special purpose finite elements that model the deformation of the sidewall and contact between the tread and the ground. Despite the (seemingly) simple model the results corresponds quite well with experiments.

A FE simulation with ABAQUS, in which the effect of tyre design parameters on lateral forces and moments is studied is presented in a paper of Olatunbosun and Bolarinwa (2004). Parametric studies are performed on a simplified tyre (no tread, negligible rim compliance and viscoelastic properties, shear forces modeled with Coulomb friction with constant $\mu$).

Explicit simulations to predict tyre cornering forces, using PAM-SHOCK, are presented by Koishi et al. (1998). Besides a comparison with experiments, parametric studies on the effect of inflation pressure, belt angle and rubber modulus are performed.

Rao et al. (2003) simulated the dynamic behavior of a pneumatic tyre using ABAQUS/Explicit. A study on a passenger car radial tyre to simulate cornering behavior, braking behavior, and combined cornering & braking behavior is presented. Further the effect of camber angle and grooved tread on tyre cornering behavior is studied. To reduce computational load two tyre models are used, one treadless tyre and a tyre with five circumferential grooves.

A cleat test with a half rotational response treadless tyre model can be found in a paper of Olatunbosun and Burke (2002), where NASTRAN has been used. An comparison with experiments shows deviations as the traverse speed increases above 20 kph. The prediction of the radial force seems better than the longitudinal force. In an article of Cho et al. (2005) the dynamic response of a fully patterned tyre rolling over a cleat is presented, using ABAQUS/Explicit. To decrease the computational load the fiber-reinforced rubber is modeled with composite shell elements, instead of rebar elements.

The effect of plysteer for steady-state rolling tyres is investigated by Mundl et al. (2005), where a global-local analysis is used, and by Ohishi et al. (2002), where a detailed tread pattern is used in the contact area.

4.2.3 Thermo-mechanical approaches

More recently thermo-mechanical models for prediction of the temperature inside tyres have been presented (Allen et al., 1996a,b; Park et al., 1997; Futamura and Goldstein, 2004; Lin and Hwang, 2004). Besides the prediction of steady state temperatures for rolling tyres a small literature overview is given by Rao et al. (2006). Experimental results of the temperature rise in an actual contact area between the pavement and the tread of a cornering tyre can be found in an article of Fujikawa et al. (1994). It is shown that a significant temperature rise occurs for severe slip angles.
4.2.4 Hydroplaning

Another phenomenon is hydroplaning, where the interaction with fluid (water) is taken into account by coupling FEM and Computational Fluid Dynamics (CFD). For examples see Seta et al. (2000), Nakijima et al. (2000) and Cho et al. (2006).

4.2.5 Component modeling

Besides complete tyre models a lot of work focusses on the specific components of a tyre, e.g. the tyre cord reinforcements, which can be modeled using so-called rebar elements (Huh and Kwak, 1990; Heinwein et al., 1993; Meschke and Helwein, 1994). This is also incorporated in ABAQUS. Asperity modeling to simulate rubber sliding on rough surfaces is considered by Bui and Ponthot (2002) for the 2D case and by Faulkner and Arnell (2000) for two 3D hemispherical asperities. Recently a comparison between experimental and explicit FEM has been published (Kuwajima et al., 2006) to examine the interfacial phenomena between rubber and abrasive papers, such as the real contact length, partial slip, and apparent friction coefficient under vertical load and tangential force. Other examples are optimization of the carcass (Cho et al., 2002) and the optimum shape of the tread to avoid lateral slippage (Ahmed et al., 2005).
Chapter 5

Validation and modeling methods

5.1 LAT100 and Tekscan

The LAT100 (VMI) is a test machine originally developed by Dr. Grosch. It is designed to relatively rank tread compounds with respect to dry traction, wet traction, ice traction, abrasion and rolling resistance. Under certain test conditions it should be possible to relate these indoor experiments with outdoor tyre tests. On this machine a small rubber wheel, see figure 5.1(b), is placed on a spinning disk which represents a road surface. The temperature, speed, surface of the disk, the slip angle and normal load on the sample can be varied. More details of the LAT100 can be found in the documentation system of Vredestein (Doclib).

![ LAT100 machine. Sample wheel.](image)

Figure 5.1: The Laboratory Abrasion and skid Tester.

The Tekscan machine can be used to obtain the normal contact pressure distribution for a statically deformed or slow rolling tyre under zero slip angle. More details on the principle of Tekscan and possibilities of using small LAT100 samples on the Tekscan can be found in the report of Broeze (2007).

5.2 Eplexor and DIK

Material test data of rubber compounds can be acquired from the Deutsches Institut für Kautschuktechnologie (DIK). Visco-elastic material properties can be measured using the Eplexor available at Vredestein or at the university in cooperation with the section of Materials Technology.
5.3 Flat plank and tyre test trailer

The global tyre deformation characteristics and cleat experiments can be measured on the flat plank machine in the automotive lab at the TU/e. The global force and moments can be measured with the tyre test trailer of TNO. More details are presented in the report of Cremers (2005).

5.4 Tyre modeling in ABAQUS

Guidelines for tyre analyses in ABAQUS can be found in the ABAQUS Example problems manual, chapter 3.1 and in the course notes on Tire modeling using ABAQUS. The way to obtain the global force and moment characteristics of a tyre under different driving conditions, such as free rolling or slip and chamber angles, in ABAQUS/Standard is the following:

- Start with a 2D cross-section of the tyre.
- Perform a static analysis in which the inflation pressure (and rim mounting) is simulated.
- Revolve the axi-symmetric cross-section about the axis of revolution, with a nonuniform discretization along the circumference, to obtain a 3D model.
- Apply a vertical load, which represents the weight of a vehicle.
- Perform a static full 3D footprint analysis of the tyre in contact with a flat road.
- Transfer these results to perform steady-state rolling analysis under different driving conditions.

The great advantage of the 'steady-state transport analysis' is that it uses a moving reference frame in which rigid body rotation is described in an Eulerian manner and the deformation is described in a Lagrangian manner (the so-called ALE-formulation), see also articles of Nackenhorst (2004); Ziefle and Nackenhorst (2005); Laursen and Stanciulescu (2006). This kinematic description converts the steady moving contact problem into a pure spatially dependent simulation. Thus the mesh need to be refined only in the contact region, the steady motion transports the material through the mesh. Frictional effects, inertia effects, and history effects in the material can all be accounted for.

The limitation of a steady-state transport simulation is that the circumference of the tyre must be continuous (in the current version). As a result only circumferential grooves can be used, instead of a detailed tread pattern.

If an uniform discretization along the circumference is used, it is possible to transfer steady-state rolling results to ABAQUS/Explicit and perform dynamic/impact simulations. This is computationally expensive and usually a coarse mesh and a simplified tyre model is used to handle these kind of simulations.

Contact must be divided into normal and tangential contact as mentioned in chapter 4. Both normal and tangential contact or friction can accounted for by using the available algorithms or by own user subroutines.

Temperature effects can be accounted for in different ways. A similar procedure for steady-state solutions can be followed as in Huemer et al. (2001b), where a reference temperature is used and WLF transformations are used to calculate equivalent sliding velocities at the reference temperature. In an implicit thermo-mechanical approach it is possible to convert energy dissipation due to friction into heat. If this option is used, one should be aware that material properties can be affected by temperature changes and ideally this should be taken into account as well. This also holds for the friction law, if temperature effects are taken into account the friction law should be an explicit function of temperature.
To incorporate transient dynamical effects, such as local temperature rise for severe slip angles, ABAQUS explicit can be used, although this probably takes a very long time to simulate and no records of this has been found in literature so far. Guidelines for heat-transfer problems can be found in the ABAQUS Example problems manual, chapter 5.
Chapter 6

Conclusions

To model a tyre and interaction with the environment using the finite element method not only knowledge of materials, geometry and interaction is required, but also a thorough knowledge of the computational mechanics behind the finite element method is required. However the most important conclusion is that the often used Coulomb friction model with a constant coefficient of friction is in general not realistic in the case of rubber friction.

6.1 Friction

Friction of polymers is closely related to their viscoelastic behavior. Generally speaking, the coefficient of friction increases with sliding velocity until a maximum value is reached at a certain speed, followed by a decrease of the friction coefficient. The viscoelastic properties of polymers depend strongly on temperature. Grosch showed the relation between the dependence of the friction coefficient due to sliding velocity and temperature and the viscoelastic deformation on strain rate and temperature.

The friction force for soft rubber sliding on a smooth surface (perfect contact) is more or less constant, independent of the load. Thus the coefficient of friction decreases continuously as the pressure increases. However there is also a coefficient of friction, independent of pressure, for harder rubber compounds, sliding on a rough surface. Apparently in this case contact is incomplete. An increase in pressure creates a larger true area of contact and hence a larger frictional force. This shows the influence of the temperature, sliding velocity and real contact area.

The current theory of rubber friction and surface roughness, which capture all these properties can be found in the publications of Persson, (see review paper Persson, 2005). The surface roughness is characterized by a fractal description, which exhibits geometrical self-similarity. These results are based on the early studies of Grosch, taking into account that sliding friction of rubber has the same temperature dependence as that of the complex elastic modulus. He states that the friction force is related to the internal friction of the rubber, which is a bulk property. The hysteretic friction component is determined by sliding of the rubber over asperities of a rough surface. These oscillating forces lead to energy dissipation. Every length scale up to the largest particles of asphalt, can be related to an excitation frequency. As a result of energy dissipation heat is generated, in a recent article this local heating of the rubber is also taken into account.

Although this theory provides knowledge of the physical origin of rubber friction it has some drawbacks. Exact knowledge of the contribution of each length scale is needed. Therefore it has, at the moment, limited practical use, since variable amounts of unknown foreign materials in the interfacial region makes it almost impossible to derive quantitative estimates of the absolute magnitude of friction from physical properties of the rubber and surface. So assumptions must still be made.
6.2 FEM

There are two distinctive methods to solve FE problems, implicit and explicit. The implicit method solves for a statical or dynamical equilibrium, while the explicit method solves transient dynamic response problems using an explicit direct-integration procedure. Although a lot of procedures are implemented in ABAQUS, the user still has to make several choices in the model formulation, i.e., the discretization of the mesh, the element types, the solution procedure, the solver tolerances etc. These selections may have a tremendous effect on the final result, so care must be taken.

Contact problems in FEM are divided into normal and tangential contact and interfacial laws must be chosen to solve the contact problem. If the tangential forces reach a certain limit bodies will no longer stick, but start sliding. Normally a local Coulomb law is used to calculate the shear stress in this case, but it is also possible to formulate tangential constitutive equations by regularization of the stick-slip behavior, e.g., to avoid the non-differentiability of Coulomb’s law at zero velocity. Besides a fixed friction coefficient in ABAQUS some build-in constitutive laws can used to make the coefficient of friction dependent on pressure, sliding velocity or other field variables. ABAQUS also provides the possibility to use own interfacial laws.

With the ‘standard options’ in ABAQUS it is nowadays possible to include a detailed tread pattern on a (part of a) tyre global model, instead of the global-local approach, and perform analyses to capture footprints as a function of axle load and inflation pressure. The stress and strain in all tyre components and the force and moment on a center feel can be calculated. The steady state option in ABAQUS makes it possible to efficiently calculate steady state solutions for different driving conditions, since the mesh need to be refined only in the contact region. With this option frictional effects, inertia effects, and history effects in the material can all be accounted for. This makes it a powerful approach to obtain steady state solutions and determine the global forces and moments for several variables, such as inflation pressure, normal load, velocity, slip and chamber angle.

A limitation of this so-called steady state transport simulation is that the circumference of the tyre and the road must be continuous. As a result only circumferential groves can be used in these kind of simulations. This also implies that a macroscopic friction law is required, in which surface roughness is embedded in the friction model. A phenomenological approach could be used to derive a macroscopic friction law, which depend on normal pressure, sliding velocity, temperature and surface roughness. The articles of Huemer et al. (2001a,b) are related to this context, a friction law for rubber tread blocks on snow and concrete is presented for use in a macroscopic model. The coefficient of friction depends on normal pressure, sliding velocity and temperature. The friction coefficient itself is only a function of normal pressure and sliding velocity. Temperature effects are incorporated using the WLF transformation. Hofstetter et al. (2003) introduced a thermo-mechanical coupling where energy dissipation during sliding is converted into heat and this heat flux causes a temperature rise of the rubber and road and this influences the friction coefficient. Dorsch et al. (2002) derived a phenomenological friction law using data obtained from experiments on the LAT100.

Despite the still increasingly computational power there are still limitations in the kind of simulations one can perform. Especially explicit simulations, where transient dynamic effects are captured, require a lot of time and as a result coarse meshes are often used to solve these problems. A way to perform explicit simulations is to use an uniform discretization along the circumference. It is than possible to transfer steady state rolling results to ABAQUS/Explicit and perform dy-
dynamic/impact simulations such as rolling over a cleat. To incorporate transient local temperature effects at severe slip angles, ABAQUS explicit can be used as well in theory, although this probably takes a very long time to simulate and no records of this has been found in literature sofar. However experimental results of the temperature in an actual contact area between the pavement and the tread of a cornering tyre show a significant temperature rise (Fujikawa et al., 1994).
Appendix A

32th Tire Mechanics Short Course

A.1 Strength, wear and friction of rubber.

The presentation of Gent (2007) deals with the mechanical properties of rubber. Four subjects are addressed: Elasticity and visco-elasticity, strength of rubber components, rolling and sliding friction, and abrasion of rubber and wear of tyres.

A.1.1 Elasticity and visco-elasticity

Elastic

Rubber is an elastic, soft and virtually incompressible material. Its molecular structure consists of long, linear flexible molecules forming random coils. The molecular segments are mobile (so-called Brownian motion) and the molecules are interlinked into a 3D network. Chemical crosslinks are usually made by sulfur linkages also known as vulcanization. The rubbery state of a polymer is determined by the so-called glass transition temperature $T_g$. If the temperature is above $T_g$ the polymer is rubbery, below $T_g$ the polymer is glassy. For rubbers the $T_g$ is below room temperature. The Brownian motion (rate of segmental jumps) depends strongly on temperature (sheet 6).

Small strains

For small strains the material properties can be defined by the shear modulus $G$ and the modulus of bulk compression $K$, which are related to the tensile or Young’s modulus $E$ and Poisson’s ratio $\nu$ as follows:

\[
E = 2(1 + \nu)G, \quad (A.1)
\]

\[
\nu = \frac{1}{2} \frac{3K - 2G}{3K + G}. \quad (A.2)
\]

For rubber typically $K \gg G$ and it follows that $E = 3G$ and $\nu = 0.5$. If a rubber block with $\nu = 0.5$ is compressed, it is sheared outwards and generates an internal pressure. This effect causes a compression modulus, which is generally greater than $E$. An example can be found on sheet 10.

Large strains

For large strains the deformation is usually described by stretch ratios $\lambda_1, \lambda_2, \lambda_3$ in the principle directions. Instead of a modulus a strain energy density function is used. Several forms (constitutive models) are available.
Filled rubbers

In tyres rubbers are usually filled with particles like carbon black or silica. This creates two additional effects. The so-called Payne effect and Mullins effect, which are both softening effects. The Payne effect is a softening effect for small strains (0.1%), attributed to breaking apart aggregates of filler particles. The Mullins effect is a substantial softening effect at higher strains, attributed to progressive detachment of rubber molecules from filler particles.

The constitutive laws for large strains cannot be used to fully describe the stress-strain relation, because rubbers do not follow reversible stress-strain relations.

A.1.2 Visco-elastic

When rubber is dynamically stretched and released the returned energy is less than the energy that is put into the rubber, see sheet 21. This visco-elastic effect cannot be described by the perfect elastic shear modulus \( G \), but a dynamic shear modulus \( G' \) and a dynamic shear loss modulus \( G'' \) is introduced to describe this hysteresis. Another term that is frequently used is the loss angle, which is defined as \( \tan \delta = \frac{G''}{G'} \). For an elastic solid the strain is in phase with the stress, while for a visco-elastic solid the strain lags behind the stress with a delay of \( \frac{\phi}{\omega} \), where \( \omega \) is the frequency (rad/sec).

The dependence of segmental jump rate on temperature is described by the universal WLF relation (Williams et al., 1955),

\[
\log \frac{\phi_T}{\phi_{T_g}} = \frac{17.5(T - T_g)}{52 + T - T_g},
\]

(A.3)

where \( \phi_{T_g} = 0.1/\text{sec} \). Some polymers such as butyl rubber behave according a different relation

\[
\log \frac{\phi_T}{\phi_{T_g}} = \frac{17.5(T - T_g)}{100 + T - T_g}.
\]

(A.4)

This is possibly due to unusually long moving segments. The WLF relation can by used to predict results at other temperatures, the frequency \( f \) at temperature \( T \) is equivalent to a frequency \( f^* \) at \( T^* \), where

\[
\frac{f^*}{f} = \frac{\phi_{T_g}}{\phi_T} = a_T
\]

(A.5)

\( a_T \) is known as the shift factor, this also holds for strain rates or velocities. An example is given on sheets 27-30. However the following remark should be taken into account: These shifts are only valid for pure rate (viscous) processes.

Strength of rubber components

In this section the tear strength and fracture energy are discussed. Also the visco-elastic contribution to strength effects of rate and temperature is treated. However this is not so important at this moment.

A.1.3 Friction

The coefficient of friction is defined as:

\[
\mu = \frac{F}{N},
\]

(A.6)

where \( N \) is the normal force and \( F \) the tangential force. For soft rubber sliding on a smooth surface (perfect contact) the frictional force \( F \) is more or less constant, independent of the load \( N \). Thus the ‘coefficient’ of friction decreases continuously as the pressure increases, see figure A.1. However, there is also a coefficient of friction, independent of pressure, for harder rubber compounds, sliding on a rough surface. Apparently in this case contact is incomplete. An increase in pressure creates a larger true area of contact and hence a larger frictional force.
A. 32TH TIRE MECHANICS SHORT COURSE

(A) Friction definition. (B) Friction law.

Figure A.1: Friction on smooth surface.

A.1.4 Rolling friction

Rolling friction $F_r$ can be seen as $\frac{M}{R}$, where $M$ is a torque and $R$ the radius. For an elastic wheel the reaction force acts through the wheel center and therefore the moment $M = 0$. For a visco-elastic wheel the reaction force moves towards the loading side, because stresses are lower in unloading, $M = Nz$ where $N$ is the normal load and $z$ the distance from the wheel center, see figure 3.4.

Displacement of the center of pressure, and hence the coefficient of friction $\mu_r$, depend on speed and temperature in the same way as the loss factor $\tan \delta$. A formulation for $\mu_r$ as proposed by Greenwood et al. (1961),

$$\mu_r = 0.33 \left( \frac{N}{G^* R^2} \right)^{\frac{1}{2}} \tan \delta,$$

where $G^*$ is the complex shear modulus. Rolling friction is mainly due to energy dissipated as rubber is compressed and released in the contact patch.

Figure A.2: Rolling friction.

Sliding friction

Grosch (1963) noted that the curve for $\mu$ vs. sliding speed on a lubricated rough surface resembles the curve for $\tan \delta$ vs. frequency $f$ (and they both obey WLF temperature shifts), see figure A.3. He claims that friction is due to energy dissipated as rubber is compressed and released by asperities. For dry sliding on a smooth surface the curve for $\mu$ vs. sliding speed $V_s$ resembles again the curve for $\tan \delta$ vs. frequency $f$. But now it is displaced to much lower speeds than before. Friction in dry sliding on a smooth surface is due to energy dissipation as rubber sticks and slips on a molecular scale. In the case of sliding on a rough dry surface the two dissipative processes appear
at different speeds, corresponding to different length scales: asperities and molecules, see figure A.4. Conclusion: Several contributions to \( \mu \) can arise at the same sliding speed from stick-slip processes occurring at different length scales.

A.2 The tire as a vehicle component.

The presentation of Potts (2007) consists of vehicle dynamics and can be compared with the book of Pacejka (2006) or the lecture notes on vehicle dynamics (Besselink, 2003). The presentation actually starts more or less at sheet 44 with the definition of a tyre: A Compliant roller bearing with tractive properties. Further load support and the so-called carpet plots are explained. The importance of vehicle understeer and the contribution of the tyre is treated in sheets 67-79. Testing equipment and results of forces and moments measurements as well as eigenfrequencies are presented in the remainder of the presentation. The tyre models used for vehicle handling are (semi-)analytical dynamic models.

Abrasions

Some general remarks can be found on sheets 64-65. This is not relevant at this moment.
A.3 Tire materials and manufacturing.

The materials and manufacturing processes used for pneumatic tyres are discussed in this presentation (Walter, 2007a). The worldwide tyre production is greater than 1 billion units with a market value exceeding US $100 billion. Around 73% of this is contributed by passenger tyres and 70% of these tyres are replacement tyres.

The presentation is divided in four subjects: the tyre components, tyre elastomers, the cord reinforcements and factory processing.

A.3.1 Tyre components

There are basically two types of tyres: bias and radial tyres, however the focus in this report will be on radial tyres, since these are manufactured at Vredestein. The main difference is the orientation of the plies, see figure A.5. Every rubber component in a tyre requires specific properties.

![Bias and radial tyres comparison](image-url)

(a) Bias tyre. (b) Radial tyre.

Figure A.5: Difference between bias and radial tyres.

An overview of these components can be found on sheet 10. Further the tyre construction, such as aspect ratio, belt construction and tread compound, depends on the target market (speed rating).

The principal tyre elastomers are natural rubber (NR), polyisoprene (IR, this is synthetic NR), styrene butadiene rubber (SBR), butyl rubber (IIR) and butadiene (BR). The reinforcement materials are: a) The fillers, such as carbon black and silica. b) The organic textile plies, such as polyester or rayon bodies, nylon cap or kevlar. And c) Steel, which is used in the belt cord and the bead wires. Some details and definitions are given on sheets 15–23.

The definition of a compound (sheet 24) is the following: the formulation of a mixture of rubber and additives which meets the needs of the tyre component application. Usually a compound consists of one or more polymers, vulcanizing agents, accelerators, reinforcing fillers, antidegradants, plasticizers, softeners and tackifiers (used to bond pieces together). This explains why the properties of a compound can deviate from a pure elastomer.
A.3.2 Tyre elastomers

In this part (sheet 29–64) the aforementioned rubbers are described in detail and the effect of fillers is discussed. Some sheets deal with the dynamic mechanical properties, this overlaps with Gent et al. (2007).

A.3.3 Cord reinforcements and Factory processing

In the last part the different cord reinforcements, textile and steel, are discussed. Different types of steel cord constructions are explained and finally the manufacturing processes is described. This will not be discussed here, however is should be noted that (limitations of) the manufacturing process greatly affects the final tyre. So it is worthwhile to scroll through this slides.

A.4 Rules and regulations governing tires.

This is an interesting presentation of Walter (2007b) about all the rules, which again affect tyre design. Although this presentation is focussed on the US federal government regulations, it provides a background on the fair amount of rules, which tyre manufactures have to cope with.

A.5 Advanced tire modeling.

The presentation of Padovan (2007) introduces the course goals of finite element analysis in tyre analysis. There are in total 12 presentations with over 600 sheets, so in the remainder of this chapter references to a presentation are made using the number (#) shown in table A.1. In this overview I’ll give some remarks to highlight interesting parts. In the introduction presentation

Table A.1: Presentations of Padovan.

<table>
<thead>
<tr>
<th>Number</th>
<th>Title</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>Advanced tire modeling</td>
</tr>
<tr>
<td>2</td>
<td>Intro to FEA modeling</td>
</tr>
<tr>
<td>3</td>
<td>Multilength scale FEA model of tire</td>
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<tr>
<td>4</td>
<td>Dynamic elastomer properties</td>
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<td>5</td>
<td>Critical speed</td>
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<td>6</td>
<td>Rolling resistance</td>
</tr>
<tr>
<td>7</td>
<td>FEA modeling of tires stochastic effects</td>
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<tr>
<td>8</td>
<td>Optimization of cord spacing</td>
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<tr>
<td>9</td>
<td>Fracture mechanics</td>
</tr>
<tr>
<td>10</td>
<td>Modeling stochastic effects on durability</td>
</tr>
<tr>
<td>11</td>
<td>Overall fatigue analysis procedure</td>
</tr>
<tr>
<td>12</td>
<td>Hydroplaning analysis</td>
</tr>
</tbody>
</table>

there is, besides the outline of the other presentations, one sheet giving an overview of tyre problems, which are successfully handled by virtual CAE modeling methodologies (1):

- Static rim mounting, inflation and axle load/deflection analysis
  - States of stress/deflection/strain in all tyre components
  - Force and moment, on center feel
– Definition of footprint shape as a function of axle load state and pressure

• Rolling/handling models both quasi-static and dynamic including braking and cornering effects

• Thermal analysis/rolling resistance under rolling conditions

• Model cure press, and other manufacturing steps

• Fatigue life prediction models under full duty cycle

• Natural frequencies, handling and critical speed analysis

• Hydroplaning models, snow traction, in mud performance

• Geometry, and cord layout optimization

• Tread lug design

• Acoustical analysis

• Suspension models incorporating tyres

The second presentation is the introduction to finite elements, this presentation is more elaborate than the other presentations. These sketch the general problem and show the FE solution. It is important to determine the levels of deformation/strain to use appropriate stress-strain (linear vs. nonlinear) relations (2). In general tyre problems are large deformation/strain problems:

• Small deformation
  Deformation $<< \lll$ Characteristic length

• Small strain
  Strain = Deformation/Characteristic length $<< \lll 1$.  

• Large deformation
  Deformation = $O$( Characteristic length)

• Large strain
  Strain = Deformation/Characteristic length = $O(1.)$

In the third presentation it is stated that the overall tyre performance is usual measured by global actions, such as force & moment, ride comfort or dynamic response and therefore gross FEA models can be employed to establish such characteristics. In contrast to this some aspects, such as tyre failure, require very localized models. In this sense a multi length scale methodology can be used, i.e., a global model is used to drive a local model. A summary of advantages and disadvantages for global and local model is given in this presentation and also stated here.

A one scale model has one contiguous mesh and a refined mesh is placed in locations of interest, transition meshes are needed to link refined and coarse zones. It is in general difficult to build a mesh with reliable local refinements. The models tend to be very large and time consuming and are not very adoptable making mesh modifications. The transition areas tend to yield unreliable results.

Multiple scales suffer the so-called localization syndrome: localization always requires further localization, so a localization cut-off criteria is needed. Besides that stress and strain take on a random character as localization brings on the probabilistic nature of such scale levels. While probabilistic variances can be very significant, generally the mean is close to the nominal design value and since stiffness of cord or metallic/plastic subcomponents are orders of magnitude higher than rubber, local actions of such subcomponents dominate; rubber is hence strictly a geometric place holder. Also every new level should be correlated with experiments to assess accuracy and
sensitivity to meshing.
Sheets 30-36 give an example of a global-local approach to investigate local cord-rubber interaction. It is however stated here that with a global model the usual performance characteristics are usable.

Presentation 4 deals with material properties of elastomers, such as stress-strain and creep, and friction. The friction depends on interfacial pressure and dynamic effects, see figures A.6 and A.7.

![Figure A.6: Quasi static.](image)

![Figure A.7: Dynamic effect for ‘average roughness road’.](image)

Presentation 5 is about critical speeds of tyres and describes four methods to analyze this, but it is not relevant at the moment.

Rolling resistance is the subject of presentation 6 and is primarily caused (95%) by mechanical work stored in rubber that is partially converted to heat and secondary cause (5%) is the mechanical work stored in cords that is converted to heat and friction work induced in the footprint area during rolling, which generates heat in addition to wear. Two different approaches are given to determine the rolling resistance.

The other presentations go into specific subjects and are not relevant at the moment.
A.6 Tire wear, traction and force generation.

The footprint is the subject of the presentation by Pottinger (2007). A footprint is a complex object, since the tyre is a doubly curved surface and is pneumatically pre-stressed. And friction affects the deformation of the tyre.

Contact leads to a stress distribution across the interface, with normal component $\sigma_Z$, and shear components at each point, $\sigma_X$ and $\sigma_Y$. The nature of this distribution has a complex dependence on conditions of use. Two distinctive measurement techniques can be identified: Measure $\sigma_Z$ at ever finer resolutions or examine the entire stress field at somewhat coarser resolutions. A method for measuring $\sigma_Z$ is pressure sensitive film (like the Fuji-film) for a static case. Electronic pressure mats (like Tekscan) can be used for static and slow rolling cases. These cannot be used for large shears, which occur in cornering, braking or accelerating. The dramatic difference between static and slow rolling is shown in figure A.8, this shows why static footprints can be misleading. Sheets 18-30 give some examples of machines and concepts for measuring footprints.

Temperature plays a great role in the footprint. The surface temperature, in particular the change through contact, is related to wear. The bulk temperature determines the stiffness of the tread compound, see also chapter A.1. Both wear resistance and handling depend on tyre temperature in the footprint because it influences abradability, stiffness and also the coefficient of friction. In figure A.9 the effect of temperature rise on different surfaces is shown. It can be seen that the conductivity of the surface has great influence on the tread surface temperature, note that the slip angle is 6°. This slip angle is a very severe condition for wear and will cause gumming in racing tyres. Some years ago Michelin published data showing that over 99% of all driving in the United States occurred at less than 6° slip angle. The low slip angles involved in actual operation are why tyres have a reasonable wear life.

Figure A.10 shows basic passenger tyre footprint longitudinal and lateral stress characteristics. These figures are for zero slip and chamber angle. It can be seen that $\sigma_X$ is positive at the front and negative at the back. The shoulders have a larger effective radius than the crown in this figure. The lateral stresses are pointed outwards. The effect of slip angle can be seen in figure A.11. A
slip angle reshapes the footprint into a rough trapezoid. One of the shoulders becomes longer, for example, the right shoulder, when turning left, and the other grows shorter (the left shoulder in this case). The normal stress increases on the long shoulder and falls on the short shoulder.
(a) Longitudinal stress.  
(b) Lateral stress.

Figure A.10: Longitudinal and lateral stress.

Figure A.11: Effect of slip angle.
The tread pattern on tyres exist only as traction aids for contaminated surfaces, such as water or snow. These grooves cause uneven wear and noise. A remarkable citation from sheet 61: Industrial design of the pattern is artistic icing on an engineering necessity.

For good dry traction one should put as much rubber on the road as possible. The following traction summary shows the trade-off in tread design:

- **Dry**
  - Maximize contact area
  - Keep $\sigma_z$ low
  - Use soft compound

- **Wet**
  - Medium-hard compound
  - Maximize groove volume fraction
  - Minimize element size
  - Long narrow footprint

- **Ice**
  - Low $T_G$ rubber
  - Lots of element edges

- **Snow**
  - Large void projection times void width product perpendicular to direction of required force
  - Flexible tread elements to insure cleaning, so void doesn’t clog

Sheets 73-97 shows different aspects of wear. A relation between wear and shear energy intensity is presented. Some guidelines for outdoor and indoor testing are given. The last part (98-116) of the presentation focusses on the limits of force generation for several conditions, mostly based on experimental data, such as pavement texture, hydroplaning, compounds and snow traction.

This outlines again that a lot of environmental issues influence the force generation, and as a result friction and wear.

### A.7 Tire stresses and deformation analysis.

This last presentation given by Trinko (2007) gives an overview of the stresses and strains in tyres. It consists of three parts: Force topics, composite properties and stresses and strains.

#### A.7.1 Force topics

This section (sheets 3–49) is divided into 5 topics: Load transfer, cord tension, bead calculation, Purdy equations and air diffusion. Interesting part is air diffusion, which is equivalent to a heat transfer problem.

#### A.7.2 Composite properties

In this part of the presentation (sheets 51-86) the laminate theory is explained. This theory can probably be used to do some basic calculations to rubber/cord laminates and compare that to the so-called rebar elements in ABAQUS.
A. 32TH TIRE MECHANICS SHORT COURSE

A.7.3 Stress and strain

The last part gives some FE results about slip velocity and friction. Although this is another program than ABAQUS, it shows that a rolling tyre with friction and different slip angles is possible. In figure A.12 the slip and friction for $0^\circ$ and $1^\circ$ slip angle are shown. There are two approaches to rolling tyres. First a Lagrangian approach, where the mesh rolls. To use this approach equal segments around the whole tyre must be used. This allows a full tread pattern. The second one is a mixed Euler-Lagrange approach, where the material flows through the mesh. The mesh can deform, but does not roll. This can only be used for steady state solutions. The main advantage is that a fine mesh is only needed in the contact area, however this requires an axi-symmetric model with a smooth or circumferential grooved tread pattern. The proposed frictional stress for steady state rolling is

$$F = \mu(P, \Delta V) P,$$

where $\Delta V$ is $V_{\text{tyre}} - V_{\text{road}}$ and $P$ the normal pressure. Two friction laws are given, the first one as function of pressure,

$$\mu(P) = CP^{-\gamma},$$

where $C$ and $\gamma$ are constants and the second one as a function of $\Delta V$, see figure A.13. The power law (A.9), see also figure A.6, seems reasonable, according to Trinko, however the aligning moments are wrong compared to experiments. Trinko thinks this is due to a wrong friction law,
but he’s not aware of improved friction laws. Another remark is that there is still no conclusive correlation with wear. After the end of the presentation some additional sheets (143-169) are added. These describe some measurement techniques and explanations behind calculations.

![Figure A.13: Friction law as function of $\Delta V$ for rolling tyre.](image)

"$\mu$" vs "$\Delta V"


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