Comparison of electric series-parallel transmissions

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Introduction

Hybrid cars are a popular item nowadays. The number of cars using a hybrid transmission, using both electric and chemical energy sources, has increased a lot the last couple of years. One of the many hybrid concepts used is the series-parallel concept, which consists of an electric variator and a power-split device. The series-parallel type of hybrid drivetrain is also known as the power split concept. The series-parallel concept is used by several manufacturers because of its relative high efficiency and relative good fuel economy.

Each manufacturer gives has its own view on the series-parallel concept. Therefore there are many differences between the several series-parallel transmissions, like the one mode series-parallel transmission (e.g. Toyota Prius) and the two mode series-parallel (e.g. Renault Laguna IVT). The two mode Renault uses multiple planetary gear sets and brakes to switch modes in order to decrease the electric variator power. This report tries to clarify the differences between the several concepts of series-parallel transmissions and to make a fair comparison.

In this report the subjects for research are:

- An introduction to the electric series-parallel transmission concept, defining its components and its operating modes.
- A comparison between vehicles using the series-parallel concept, namely the Toyota Prius, Ford Escape Hybrid, Renault Laguna IVT, Opel Astra GTC hybrid and the Lexus RX400h. these cars will be compared using several criteria for performance and functionality. The criteria for performance are component power, fuel economy, vehicle acceleration, negative recirculation and ratio coverage. The criteria for functionality are complexity and the ability to use all operating modes.
- An analytic comparison in efficiency and fuel economy between to of the transmissions, namely the Toyota Prius and the Renault Laguna IVT using Matlab. In this comparison the influence of the speed ratio and the electric efficiency on the overall transmission efficiency will be the main topic.

A conclusion will be drawn using the comparison on literature and the comparison in Matlab.
Chapter 1: Introduction to series-parallel transmissions

§ 1.1 General introduction

Series-parallel transmissions are a kind of hybrid transmissions that are well known and used these days. These transmissions are in fact a combination of a series hybrid transmission and a parallel hybrid transmission. The series-parallel transmissions take the benefits of each of these two concepts and combine them. The three concepts are shown below.

In figure 1.1 can be seen that in the series concept all the power from the combustion engine is transferred to the wheels via the generator and the electric motor. Advantages of this concept are the ability to store excessive power in the battery and the ability to recover brake energy. Main disadvantage is the low efficiency at high vehicle loads.

In the parallel transmission the power to drive the wheels comes from the combustion engine or from the electric motor, which is powered by the battery. Advantages of this concept are that the electric motor can assist the combustion engine when needed, in example for acceleration and the higher efficiency. Disadvantage is the lack in ability to store excessive power from the combustion engine in the battery.

The series-parallel concept combines the advantages of series and parallel. The power delivered by the combustion engine is divided and excessive power can be stored if necessary. Furthermore the electric motor can assist the combustion engine when needed.
§ 1.2 Components of the series-parallel transmission

The series-parallel transmission consists of several components. The transmission consists of a series-parallel device that divides the incoming power. The series-parallel device divides the power. One part goes to the so-called cinematic chain, the other part to the variator chain. The cinematic chain is in fact the direct link between the ICE (internal combustion engine) and the wheels. The variator chain is the part that determines the transmission ratio.

In the next paragraphs each part of the series-parallel device will be discussed separately.

§ 1.2.1 Cinematic chain

The cinematic chain is in fact the direct link between the internal combustion engine and the wheels. The cinematic chain consists of one or more planetary gear sets which are coupled to each other. The planetary gear set divides the power and determines the speed of each branch connected to it.
The following equations can be derived for the planetary gear set:

\[ z = \frac{R}{R_s} \]  

(1.1)

\[ \omega_s + z \cdot \omega_r - (1 + z) \cdot \omega_c = 0 \]  

(1.2)

\[ T_r = z \cdot T_s \]  

(1.3)

\[ T_c = -(z+1) \cdot T_s \]  

(1.4)

with the subscript \( s = \) sun, \( c = \) carrier and \( r = \) ring. \( R \) isthe radius of the gear.

§ 1.2.2 Variator

The variator is the part of the series-parallel transmission that determines the transmission ratio. The variator chain usually consists of to electro machines connected to each other but also a continuous variable transmission can be used. In this paper only electric variators will be discussed.

During operation, one of the two electro machines has the function of a generator, the other one operates as electromotor. Between these two machines a battery or a supercapacity is placed. This is done to make the electro machines cooperate in a good way. The power generated by the generator is given to the electromotor, or is stored in the battery. In order to make this possible, power electronics are necessary. The power electronics convert the power delivered by the generator from DC to AC or the other way around. This is necessary for making it possible to store the power in the battery. The electromotor is again connected to the wheels by means of the planetary gear set.

Because the speeds and torques of both electro machines don not have to be the same, the two machines can work as a variator. The generator rotates at a certain speed, depending on the speeds in the planetary gear set. At this speed the generator delivers a certain power. This power is used to drive the electromotor. The difference in amount of power asked by the electromotor and the power delivered by the generator is stored or taken from the battery. The variator chain is drawn in the figure below.

![Figure 1.4: Variator](image-url)
§ 1.3 Functionality of the series-parallel transmission

The overall strategy for the series-parallel transmission is to keep the engine running on the E-line, the most efficient points. This means low engine speeds and high torques. In order to improve the fuel economy even more, often an engine using the Atkinson cycle is used. Such an engine has thermodynamic combustion efficiency, which is about 10 percent higher than a normal engine.

Another important goal is to keep the power flow through the electric variator chain as low as possible. The efficiency of the variator chain (maximum 85%) is much less than the efficiency of the cinematic chain (constant 95%).

The functionality of the series-parallel transmission can be shown, by pointing out the operating modes of the transmission. For a series-parallel transmission the following operating modes hold.

§ 1.3.1 Electric drive and regeneration mode

In this mode the engine and the generator are shut off and only the electric motor is working. In electric drive the electric motor is working as a motor, during regeneration as a generator. This mode is also called series mode.

In the figure below the blue arrows indicate the power flows in the transmission.

![Figure 1.5: Electric drive and regeneration mode](image)

§ 1.3.2 Normal driving mode

In normal drive mode or cruise mode the engine is on and the power from the engine is split. One part is given to the wheels mechanically and the other part flows via the electric path.

The electric power circulates from the engine via the generator to the electric motor, where it sums with the mechanical power.
§ 1.3.3 Battery charge mode

This mode is used when the vehicle is parked or during an idle stop. The engine is running but the vehicle requires no power, so all the power from the engine is delivered to the generator and then stored in the battery.

§ 1.3.4 Power boost mode

This mode is nearly the same as the normal drive mode, only the engine can’t deliver enough torque to accelerate or climb a hill. Therefore energy from the battery is taken and used by the electric motor. The power delivered by the electric motor is summed with the power that flows through the mechanic path.
§ 1.3.5 Negative split mode

In this mode the power in the transmission flows the other way around. The electric motor delivers power via the generator and the planetary gear set to the engine. This is done to keep the engine running at low speeds and thereby optimizes the fuel economy. A negative effect of this strategy is the overall efficiency of the transmission, which is much lower than in the other operating modes.
Chapter 2 Comparison of existing series-parallel transmission.

In this chapter existing series-parallel transmissions will be discussed and compared to each other. First the criteria for the comparison will be discussed, followed by an overview of each series-parallel transmission. Next these transmissions will be compared.

§ 2.1 Criteria for comparison

Each transmission concept will be valued on criteria on functionality and performance. In this paragraph the criteria will be discussed.

§ 2.1.1 Criteria on performance

- Power of the combustion engine and the electric machines. The ratio between these powers tells a lot about the strategy used for the transmission. The power of these components also determines the maximum vehicle speed.
- Fuel economy. Hybrid transmissions are mainly used to lower the fuel economy, so this is an important topic. It is also very often used in commercial campaigns to increase the sales of the vehicle.
- Acceleration. The acceleration is also a hot item for commercial campaigns and therefore important. The acceleration correlates with the power of the combustion engine and the electric machines used.
- Negative recirculation. Negative recirculation of power in the transmission lowers the efficiency of the transmission. Therefore it must be avoided as much as possible. This criterion will be mainly investigated in the simulation.
- Ratio coverage. The ratio coverage of a transmission is desired to be as high as possible to make a high range of speeds possible. This is also a criterion that will be investigated in the simulation.

§ 2.1.2 Criteria on functionality

- Operating modes: as was discussed in chapter 1 for hybrid series-parallel transmission five operating modes can be determined. Each mode requires some specifications for each component of the transmission. For each concept will be determined in which amount all operating modes are possible to use.
- Complexity: how complex is the transmission, and how much does it cost to realize the transmission. How much effort must be made to control the transmission in terms of actuators, clutches and brakes.

§ 2.2 Existing transmission concepts

In this paragraph, first all transmission concepts will be briefly discussed and next a comparison between the concepts will be made.
§ 2.2.1 Toyota Prius

The Toyota Prius was the first full hybrid to be brought on the market and therefore the trendsetter. Two models have been brought to the market, the Prius I and the latest Prius II. First the concept of the transmission will be shown.

The Toyota Prius uses one PGS (planetary gear set) to split the input power from the combustion engine. The engine is connected to the carrier of the PGS. The electric machine that mainly operates as a generator is connected to the sun gear and the other electric machine is connected to the ring gear. There are to inverters in the system to convert the electric power from AC to DC and backwards. Furthermore a battery and an ultra-cap are used to store the converted energy.

A layout of the transmission is shown below.

![Transmission Layout](image)

For the speeds and torques in the planetary gear set equations 1.1, 1.2 and 1.3 hold. From these, equations for the torques of the generator (Tg) and the electric motor (Tm) can be derived.

\[
T_m = T_{j1} - \frac{k}{k+1}T_e - \left( \frac{k}{k+1}J_e - J_g \right)\omega_e - \left( \frac{k}{k+1}J_g - J_m \right)\omega_m \tag{2.1}
\]

\[
T_g = \frac{1}{k}(T_e - J_e\omega_e - J_g\omega_m) \tag{2.2}
\]

Thus, in steady state the following equation holds

\[
T_{j1ss} = T_m + \frac{k}{k+1}T_e \tag{2.3}
\]

Next the specifications of the parts of the transmission will be shown. These specifications tell a lot about the strategy used for the transmission. The specifications are shown in the table below.
<table>
<thead>
<tr>
<th>Engine power</th>
<th>57 kW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electric motor power</td>
<td>50 kW</td>
</tr>
<tr>
<td>Generator power</td>
<td>20 kW</td>
</tr>
<tr>
<td>Fuel economy (km/l)</td>
<td>22.4</td>
</tr>
<tr>
<td>Mass</td>
<td>1313 kg</td>
</tr>
<tr>
<td>Acceleration 0-100 km/h</td>
<td>11.9 s</td>
</tr>
<tr>
<td>Battery power</td>
<td>21 kW</td>
</tr>
<tr>
<td>Maximum speed</td>
<td>170 km/h</td>
</tr>
</tbody>
</table>

Table 2.1: Specifications for the Toyota Prius transmission

Out of this specifications can be concluded which operating modes are possible and in which amount. The ratio between the power of the engine and the power of the electric motor tells that a relatively high amount of power flows through the electric part. The electric part has a low efficiency compared to the mechanic path. During electric drive, all power comes from the battery, which is 21 kW. This amount is high enough to reach speeds up to 35 km/h. The battery is also capable for storing a lot of energy generated during regeneration mode. The amount of power of the electric motor also tells that there is enough power for acceleration during power boost mode. Therefore all five operating modes are possible in the Toyota Prius transmission. A more thorough view on the Prius will be given in chapter 3.

§ 2.2.2 Ford Escape Hybrid

The Ford Escape hybrid was brought on to the market in October 2004. The concept is very similar to the concept used for the Toyota Prius, but there are some differences. The Ford escape concept has the addition of output gearing. This means that the electric motor is offset geared from the planetary gear set. This is done to make the ratio coverage bigger. The layout of the Ford escape hybrid is shown below.

![Figure 2.2: Ford Escape hybrid transmission layout.](image_url)
The following equations can be derived for the torques in the transmission

\[ T_{\beta\alpha} = \alpha T_m + \beta T_e - (\beta I_e - \varphi I_m) \dot{\omega}_e - (\beta I_m - \varphi I_m) \dot{\omega}_m \]  

(2.4)

with \( \alpha = \frac{N_2}{N_1} \)  \hspace{1cm} (2.5) \hspace{1cm} \beta = \left( \frac{k}{k+1} \right) \left( \frac{N_3}{N_2} \right), \hspace{1cm} (2.6)

\[ T_g = \frac{1}{k} (T_e - J_e \dot{\omega}_e - J_m \dot{\omega}_m) \]  \hspace{1cm} (2.7)

In steady state, the following equation holds:

\[ T_{\text{ JSX}} = \left( \frac{N_2}{N_1} \right) T_m + \left( \frac{k}{k+1} \right) \left( \frac{N_3}{N_2} \right) T_e \]  \hspace{1cm} (2.8)

For the escape the next specifications were found.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine power</td>
<td>99 kW</td>
</tr>
<tr>
<td>Electric motor power</td>
<td>65 kW</td>
</tr>
<tr>
<td>Generator power</td>
<td>28 kW</td>
</tr>
<tr>
<td>Fuel economy (km/l)</td>
<td>13.2</td>
</tr>
<tr>
<td>mass</td>
<td>1766 kg</td>
</tr>
<tr>
<td>Acceleration 0-100 km/h</td>
<td>10.8 s</td>
</tr>
<tr>
<td>Battery power</td>
<td>?</td>
</tr>
<tr>
<td>Maximum speed</td>
<td>164 km/h</td>
</tr>
</tbody>
</table>

Table 2.2: Specifications for the Ford Escape Hybrid transmission

§ 2.2.3 Renault dual mode e-CVT

Renault has used a concept that differs from the concept used by Toyota and Ford. Renault used the so-called dual mode concept, which they tested in a Laguna. The difference lies mainly in the use of not one but two combined planetary gear sets. This is called a quadripole arrangement. To make dual mode possible, two quadripole elements are needed. A quadripole arrangement is shown in the figure below.

Figure 1.3: A quadripole element used by Renault. A1 and A2 are each a planetary gear set.
The main reason why they used the dual mode concept is to improve the efficiency. The efficiency of the cinematic chain is much higher than the efficiency of the electric variator, so a low power flow through the electric chain is desired. The dual mode concept makes sure that the amount of power that flows through the electric chains does not get too high for different speed ratios. This is done by applying brakes to the second quadripole element which affects the behavior of the first quadripole element. The layout is shown below

![Diagram](image)

Figure 2.4: Layout for the Renault transmission.

The specifications found for the transmission.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine power</td>
<td>95 kW</td>
</tr>
<tr>
<td>Generator power</td>
<td>25 kW</td>
</tr>
<tr>
<td>Electric motor power</td>
<td>25 kW</td>
</tr>
<tr>
<td>Fuel economy</td>
<td>Not known</td>
</tr>
<tr>
<td>(city/highway/combined)</td>
<td></td>
</tr>
<tr>
<td>mass</td>
<td>1305 kg</td>
</tr>
<tr>
<td>Acceleration 0-100 km/h</td>
<td>Not known</td>
</tr>
<tr>
<td>Battery power</td>
<td>Not known</td>
</tr>
</tbody>
</table>

Table 2.3: Specifications for the Renault Laguna hybrid.

A more thorough view on the Renault dual mode concept will be given in chapter 3.

§ 2.2.4 GM-Alisson advanced hybrid system

The fourth concept is a concept by General Motors in corporation with Alisson. The concept is very similar to the concept by Renault, a dual mode concept. This means that again two quadripole elements shown in figure 2.3 are needed to realize the series-parallel.
The layout of the transmission is shown below.

![Transmission Layout](image)

**Figure 2.5: Layout of the GM-Alisson advanced hybrid concept**

In the layout, the differences with the Renault concept can be seen. The electric machines are not connected to the same gears of the planetary gear set. This fact influences the behavior of the transmission because it determines the amount of power that flows through the electric chain and thus the efficiency, as was explained before.

The concept of GM and Alisson is used in an Opel Astra hybrid, which will be brought on the market soon. The specifications of the Opel Astra Hybrid are shown below. Some specifications are estimates given by Opel because the model has not been taken into production yet.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine power</td>
<td>92 kW</td>
</tr>
<tr>
<td>Generator power</td>
<td>30 kW</td>
</tr>
<tr>
<td>Electric motor power</td>
<td>40 kW</td>
</tr>
<tr>
<td>Fuel economy (km/l)</td>
<td>24</td>
</tr>
<tr>
<td>mass</td>
<td>1265 kg</td>
</tr>
<tr>
<td>Acceleration 0-100 km/h</td>
<td>7.8 s</td>
</tr>
<tr>
<td>Battery power</td>
<td>1.3 kW</td>
</tr>
</tbody>
</table>

**Table 2.4: Specifications for the Opel Astra hybrid**

§ 2.2.5 Lexus RX400h

The last concept is the concept by Lexus, which they used in the RX400 hybrid. They use a concept, with two planetary gear sets that forms a quadripole element, which is also used by Renault and General Motors. They do not use two modes but just one single mode.

Another main difference is the addition of an electric motor in the rear, which makes the Lexus a non-permanent four-wheel drive.

The layout is shown on the next page.
The layout differs a bit from the layout of the Renault and General Motors. The parts connected to the gears of the planetary gear set are a bit different. This, as was said before, has influence on the series-parallel behavior of the transmission. Furthermore, no component is connected to the carrier-gear of the second planet gear set. This carrier has therefore connection of the sun gear and ring gear as his only function.

The specifications for the Lexus:

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine power</td>
<td>155 kW</td>
</tr>
<tr>
<td>Generator power</td>
<td>123 kW</td>
</tr>
<tr>
<td>Electric motor power</td>
<td>110 kW</td>
</tr>
<tr>
<td>Electric motor power rear</td>
<td>50 kW</td>
</tr>
<tr>
<td>Fuel economy (km/l)</td>
<td>12.3</td>
</tr>
<tr>
<td>mass</td>
<td>1981 kg</td>
</tr>
<tr>
<td>Acceleration 0-100 km/h</td>
<td>7.3 s</td>
</tr>
<tr>
<td>Battery power</td>
<td>45 kW</td>
</tr>
</tbody>
</table>

Table 2.5: Specifications for the Lexus RX400h
§ 2.3 Comparison on performance

In this graph the concepts that were described before will be compared. For comparison, the criteria on performance and functionality described in graph 2.1 will be used.

§ 2.3.1 Power ratios of the components

The first criterion on performance is the amount of power of the combustion engine and the electric machines. Especially the ratio between the engine power and the power of the electric motor tells a lot about the series-parallel in the transmission. A high power ratio means that the power flow through the electric chain and the cinematic chain are about the same, so the power flow through the electric chain is relatively high. A result of this is a lower overall efficiency for the transmission. The ratios between the engine power and the power of the electric motor are shown below.

![Power ratio MG2/engine](image)

**Figure 2.7: Power ratio between electric motor 2 and the engine.**

In the figure can be seen that the ratios for the Toyota Prius and the Lexus are much higher then the others. The Prius, the escape and the Lexus are one-mode power transmissions; the other two are dual mode. In general the power ratio between electric motor and combustion engine for dual mode are lower then the power ratios for one mode. From this can be concluded that for dual mode transmissions relatively less power flows through the electric chain compared with one-mode transmissions. This means a higher efficiency for the dual mode transmissions.

Another parameter that tells a lot about the series-parallel technology is the power ratio between the two electric machines; the generator and the electric motor. This parameter tells a lot about the strategy during acceleration. During acceleration the electric motor assists the engine and the electric motor gets its power from the generator and the battery. The power ratio between the two electric machines is an indication of the amount of power that comes from the generator during acceleration and the amount that comes from the battery. The power ratios between the two electric machines are shown below:
Figure 2.8: Power ratio between the 2 electric machines; the electric motor and the generator.

The ratio is much lower for the Prius and the escape as can be seen in the figure. These have both one-mode transmissions. So in general a conclusion can be made that the one-mode transmissions have a lower power ratio between the two electric machines then the dual mode transmissions, the Laguna and the Astra. The Lexus is an exception in this.

The last parameter used to investigate the power is the power to weight ratio. The average of the power (in watts) of the combustion engine and the power of the electric motor was taken and divided by the vehicle mass (in kg). Results are shown below.

Figure 2.9: Power to weight ratios for the different vehicles.

§ 2.3.2 Fuel economy

The second criterion for comparison is the fuel economy. It is not realistic to compare the fuel economies shown in the tables in the previous graph, because all models are designed for a different segment in the market. To compare a SUV (Escape, Lexus) with a family car like the Laguna or the Prius wouldn’t be realistic.
Therefore for each car, first a comparison in fuel economy is made with a car that is from the same segment but not a hybrid. Some of the models investigated are also available in a non-hybrid version. Unfortunately no data were found for the Laguna hybrid; therefore no comparison could be made.

<table>
<thead>
<tr>
<th>Hybrid model</th>
<th>Fuel economy (km/l)</th>
<th>Fuel economy reference car (km/l)</th>
<th>Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opel Astra hybrid</td>
<td>24</td>
<td>19.5</td>
<td>23</td>
</tr>
<tr>
<td>Ford Escape hybrid</td>
<td>13.2</td>
<td>9.8</td>
<td>35</td>
</tr>
<tr>
<td>Lexus RX400h</td>
<td>12.3</td>
<td>9.4</td>
<td>31</td>
</tr>
<tr>
<td>Toyota Prius</td>
<td>22.4</td>
<td>16.8</td>
<td>33</td>
</tr>
</tbody>
</table>

Table 2.6: Fuel economy comparison with reference cars. The reference cars used were the Opel Astra GTC 1.7l diesel, the Ford Escape XLS 2.3l, the Lexus RX330 2.3l and the Toyota Avensis 1.8 l.

You can see that there is not one specific winner in this comparison. The change in fuel economy for the cars is about the same, except for the Opel Astra, which is a bit less.

§ 2.3.3 Acceleration

For the criterion acceleration holds the same as for fuel economy; it is not realistic to compare the accelerations of the hybrid cars because they are meant for a different segment of the market. Therefore again a comparison with a reference car is done for each hybrid car. The reference cars are the same as those used in the previous graph.

<table>
<thead>
<tr>
<th>Hybrid model</th>
<th>Acceleration 0-100 km/h (s)</th>
<th>Acceleration reference car 0-100 km/h (s)</th>
<th>Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opel Astra hybrid</td>
<td>7.9</td>
<td>10.7</td>
<td>+35</td>
</tr>
<tr>
<td>Ford Escape hybrid</td>
<td>10.8</td>
<td>7.7</td>
<td>-40</td>
</tr>
<tr>
<td>Lexus RX400h</td>
<td>7.3</td>
<td>7.7</td>
<td>+5</td>
</tr>
<tr>
<td>Toyota Prius</td>
<td>11.9</td>
<td>10</td>
<td>-19</td>
</tr>
</tbody>
</table>

Table 2.7: acceleration comparison with reference cars. The reference cars used were the Opel Astra GTC 1.7l diesel, the Ford Escape XLS 2.3l, the Lexus RX330 2.3l and the Toyota Avensis 1.8 l.

As can be seen in the table the differences are much bigger for acceleration than for fuel economy. The Opel Astra hybrid has a much higher acceleration than the non-hybrid version, the Ford Escape a much lower. The Lexus hybrid and the Toyota Prius don't differ much from the reference vehicles.

This is a criterion that is basically determined by the desires of the manufacturer. The acceleration also influences the choice of the power of the electric motor. If high acceleration is desired, much power is needed for the electric machine.
This also depends on the power of combustion engine. The Opel Astra has a turbo diesel engine and therefore has enough power reserve to accelerate so not much power for the electric motor is needed. This is again a choice made by the manufacturer.

§ 2.3.4 Maximum speed

For the maximum speed, again first a comparison with the references vehicles is made. Unfortunately there wasn’t a maximum speed given for the Opel Astra hybrid and the Renault Laguna hybrid.

<table>
<thead>
<tr>
<th>Hybrid model</th>
<th>Maximum speed(km/h)</th>
<th>Maximum speed reference car(km/h)</th>
<th>Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ford Escape hybrid</td>
<td>164</td>
<td>170</td>
<td>-4</td>
</tr>
<tr>
<td>Lexus RX400h</td>
<td>200</td>
<td>200</td>
<td>0</td>
</tr>
<tr>
<td>Toyota Prius</td>
<td>170</td>
<td>195</td>
<td>-15</td>
</tr>
</tbody>
</table>

Table 2.8: Maximum speed comparison with reference cars. The reference cars used were the Opel Astra GTC 1.7l diesel, the ford escape XLS 2.3l, the Lexus RX330 2.3l and the Toyota Avensis 1.8 l.

Except for the Toyota Prius, the maximum speeds don not differ much from those of the reference vehicles. This is again a choice made by the manufacturer, who usually aims at a maximum speed equal to that of a chosen reference vehicle.

The criteria negative series-parallel and ratio coverage will be discussed only in the simulation part in chapter 3, because there was very little information to find on these topics for the investigated hybrid cars.

§ 2.4 Comparison on functionality

In this graph a comparison is made on the criteria for functionality, which were named before.

§ 2.4.1 Complexity

The complexity of the series-parallel transmission depends on the components used in the transmission.

The more clutches, brakes and planetary gear sets are used, the more difficult is becomes to control the system. The complexity is also determined by the number of modes that each transmission has. Switching between modes requires accurate control.

Furthermore the number of components used determines the amount of space required to build in the transmission and it also determines the costs of the transmission.

An overview of the components used in the different transmissions is shown below.
Table 2.9: Number of different components used for each transmission

Note that the total number of components in the table is not in fact the real number of all the components used in the transmission but the total number of components that were used for the comparison.

§ 2.5 Overall comparison

In this graph all the results for the comparison from the previous graphs will be combined and shown in one figure.

The figure below

Table 2.10: Overall comparison of the different vehicles. $\Delta =$ average, $+$ = good and $-$ = poor

A real conclusion can not be made out of this table because to much data is missing for the Renault Laguna and the Opel Astra.

What can be said is that the Lexus RX400h gets a good overall score compared to the others. The Ford Escape also gets a good score but less the then Lexus. The Toyota Prius stays behind with a score that is not to high and has a comfortable price as main advantage. More data have to be found for the Renault and Opel to get a complete picture.
Chapter 3 Simulation with Matlab

In this chapter two of the concepts that were discussed before will be modeled and simulated with Matlab. The concepts that will be simulated are the Toyota Prius and the Renault e-CVT concept. Reason for this choice is the big amount of information about the transmission lay-out on these two concepts, which makes is easier to obtain all the necessary specifications.

§ 3.1 Toyota Prius

First a schematic overview is made, which is shown below.

![Diagram of Toyota Prius layout](image)

Using figure 3.1, the equations for the Toyota Prius power train are derived. These equations can be found in appendix C.1.

To investigate the ratio coverage of the transmission and also the negative recirculation in the transmission, the so-called electric fraction $P_d$ is used.

This fraction is defined as the power flow through the electric path divided by the power input.

$$ P_d = -\frac{P_{vi}}{P_t} \quad (3.1) $$

Where $P_{vi}$ is noted as positive when the input of the variator (EM1) is working as a motor, which means negative recirculation in the transmission. $P_{vi}$ is noted negative when the input of the variator is working as a generator.

In the figure below the electric fraction $P_d$ is plotted versus speedratio $K$. 

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Figure 3.2: $P_d$ versus $K$ for the Prius transmission.

What can be concluded from this figure is that the power flow through the electric path is large; especially for small and large values of $K$. This means that the efficiency of the transmission is low for these values of $K$. The improved fuel economy is therefore mainly the result of using the combustion engine in an efficient operating point.

Furthermore the conclusion can be made that above a value of $K = 0.35$ negative recirculation takes place; for values of $K$ above 0.35 $P_d$ becomes negative.

Next thing investigated is the overall transmission efficiency as a function of speed ratio $K$, the input torque $T_i$ and input speed $\omega_i$. In order to do this, the equations in appendix C are used and different values of $K$, $T_i$ and $\omega_i$ are used.

There are two scenarios of power flows that can take place in the transmission. The scenarios are shown below together with the equation that holds for each scenario.

![Diagram showing positive and negative power splits](image)

\[ \eta_i = \left( \eta_{ps} \left( 1 - \frac{P_i}{P_{in}} \right) + \eta_i \right) \eta_{fd} \quad (3.2) \]

\[ \eta_i = \left( \eta_{ps} \left( 1 + \frac{P_{ps}}{P_{in}} \right) - \eta_i \right) \eta_{fd} \quad (3.3) \]

Figure 3.3: different scenarios for power flows in the transmission together with their transmission efficiency equation
The variator efficiency $\eta_v$ is a combination of the efficiency of the generator and the electric machine. These efficiency’s are determined by using efficiency maps where the efficiency of the electric machine can be determined as a function of the input torque $T_{in}$ and the input speed $\omega_{in}$. The efficiency maps of both the generator and the electric motor are shown below.

Using these efficiency maps, equations 3.2 and 3.3 and the transmission equations in appendix C the efficiency map of the total transmission is determined for different values of speedratio $K$.

As can be seen in figure 3.5, the efficiency is optimal for values of $K$ close to 0.35 (indicated with the red dotted line). This corresponds to figure 3.2; the smaller the amount of energy passing the variator, the higher the overall transmission efficiency. This is due to the relatively low variator efficiency.

Furthermore the limitations of the electric machines are visible. For each value of $K$, at a certain input speed $\omega_{in}$ the maximum speed of the generator(5500 rpm) will be exceeded. In this case the transmission efficiency is not defined.

Using the calculated efficiency map the transmission efficiency can be easily determined for any operating point. In graph 3.3 this will be used to simulate a drive cycle.
§ 3.2 Renault e-CVT

To derive the equations for the Renault e-CVT transmission the schematic overview used in graph 2.2.3 is used. As was mentioned before the Renault e-CVT power train uses two modes. Therefore two quadripole sets are necessary. The first quadripole consists of the planetary gear sets A1 and A2, the second of planetary gear sets B and C. Brake 1 and Brake 2 enable the mode changing. The gain factors $K_i$, $K_o$, $K_{vi}$, and $K_{vo}$ are in fact the ratios of gears in the transmission placed between the planetary gear sets.

In order to define these gain factors and the gear ratios in the four planetary gear sets that were used another schematic overview is needed. This overview is shown in the next figure.

![Figure 3.6: Renault e-CVT overview with gear ratios](image)

Using figure 3.6 the transmission equations are derived (see appendix C.2). With the transmission equations the electric fraction $P_d$ can be determined as a function of the speed ratio $K$ (equation 3.1).

The electric fraction is shown in the next figure.
As can be seen in figure 3.7 the mode change is performed at $K = 0.25$. This strategy is chosen because at this value of $K$ the power passing the variator is zero. The second mode makes sure that the electric fraction $P_d$ will stay relatively small, except for very low values of $K$.

Therefore the transmission efficiency will be low for low values of $K$.

Furthermore figure 3.8 shows that negative recirculation occurs for values of $K$ between 0.25 and 0.53 and for values of $K$ smaller than 0.05.

In order to determine the efficiency map of the total transmission efficiency again equation 3.2 is used. For the electric efficiency maps of the generator and electric motor are used. These maps have been created by scaling the generator map of the prius to the specifications of the Renault generator (maximum speed = 11000 rpm, max torque = 135 Nm, power = 25 kW).

Furthermore the map of the Renault motor is the same as the Renault generator map, because both have the same maximum power and both can deal with negative rotation speeds.
using these efficiency maps, equations 3.2 and 3.3 and the transmission equations in appendix C.2 the efficiency map of the total transmission is determined for different values of speedratio $K$.

![Efficiency map](image)

**figure 3.9:** efficiency for the renault for different values of $K$ (left) and a detailed view for a vehiclespeed of 40 km/h

### § 3.3 Comparison transmission concepts

In this graph a comparison between the two transmission concepts will be made. First a drive cycle test will be performed for both the concepts in order to compare the fuel consumption and efficiency. Next the influence of the variator efficiency and the transmission operating point on the transmission efficiency and fuel consumption will be investigated.

In order to compare the two transmissions a drive cycle test is performed for both transmissions. Using this drive cycle test the fuel consumption and the transmission efficiency of both transmission models can be compared. The drive cycle consists of a vehicle speed which is prescribed.

In the figure below the prescribed vehicle speed is shown.

![Prescribed vehicle speed](image)

**figure 3.10:** prescribed vehicle speed for the NEDC drive cycle.

For both transmissions an identical reference vehicle is chosen. Using the parameters of the reference vehicle, the required vehicle torque and power can be calculated. In the table below the vehicle parameters are shown.
Table 3.1: vehicle parameters used for drive cycle test

<table>
<thead>
<tr>
<th>Vehicle parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle mass</td>
<td>1134 kg</td>
</tr>
<tr>
<td>Frontal area</td>
<td>2.2493 m²</td>
</tr>
<tr>
<td>Air drag coefficient</td>
<td>0.3025</td>
</tr>
<tr>
<td>Roll resistance</td>
<td>0.0118</td>
</tr>
<tr>
<td>Dynamic wheel radius</td>
<td>0.277 m</td>
</tr>
</tbody>
</table>

The transmissions will both be linked to a combustion engine, which will be the same for both transmissions. The engine used is a 64 kW engine. The map of the engine is shown in the next figure.

The required combustion engine power can be calculated using the vehicle power, which is equal to the output power of the transmission. Knowing the combustion engine input power, the engine speed and engine torque can be determined using the combustion engine map. The engine torque and speed are chosen on the optimal performance line (OOL-line) of the combustion engine. With the combustion engine torque and speed and the output torque and speed, the transmission efficiency can be determined for both transmission models using the transmission efficiency maps of figure 3.5 (prius) and figure 3.9 (renault). With the transmission efficiency the new combustion engine power can be calculated. The overview below is used to determine the transmission efficiency and the combustion engine power.
The input power and the transmission efficiency continuously affect each other. Therefore, an iteration loop is made to optimize the calculation of the input power and the transmission efficiency. The iteration loops continue as long as the difference in the calculated input power between two successive iteration steps is more than 100W. In this loop the transmission efficiency is also changed continuously during the iteration.

In the figure below the calculated transmission efficiency and the combustion engine power during the drive cycle are shown for both transmission models.

Figure 3.13 shows that the transmission efficiency of the Renault model and the Prius efficiency are about the same. Only for higher speed ratios, at the end of the drive cycle, the Renault efficiency is higher. Because of the very little difference in efficiency also the required engine power for the Renault and the Prius are almost equal, as can be seen in the right side figure.

With the combustion engine input power, calculated for each time step in the drive cycle, and the combustion engine map (Figure 3.11), the fuel consumption \( m_f \) for both transmissions during the drive cycle is calculated.

The next table shows the total fuel consumption and the average transmission efficiency over the drive cycle for both transmissions.
The slightly lower average efficiency for the Prius transmission leads to a little higher fuel consumption over the drive cycle.

Using the drive cycle test the relation between speedratio and the transmission efficiency is determined. This is done by plotting the transmission efficiency and speedratio at each operating point in the drive cycle.

The transmission efficiency versus the speedratio is plotted for both transmissions in the next figure.

Figure 3.14 shows that the efficiency of the Renault transmission is relatively more constant for speedratios above 0.3 and the Prius transmission has one peak in the transmission efficiency. Therefore, especially for large values of the speedratio $K$ the efficiency of the Renault transmission is higher than the Prius transmission efficiency. This is the result of the second mode that was added to the Renault transmission. Also for low values of $K$ the efficiency of the Renault is higher than the Prius efficiency.

When the relation between variator input and output power is studied an interesting phenomenon can be seen. This relation is shown in the figure below.
As can be seen in figure 3.15, the relation between the variator input power and the variator output power is very close to linear. This means that the variator efficiency determined with the electric machine efficiency maps can be replaced by a constant variator efficiency. This constant variator efficiency is then used to investigate the influence of the variator efficiency on the fuel consumption and the overall transmission efficiency. This is done by applying the calculated transmission efficiency maps with constant variator efficiency in the drive cycle test. This is then repeated for numerous variator efficiency’s. The influence of the variator efficiency on the fuel consumption is shown in the next figure.

Figure 3.16 shows that the influence of the variator efficiency on the fuel consumption is less for the Renault transmission than the Prius transmission. This is due to the relatively large amount of power passing the variator in the Prius transmission. The larger this amount, the bigger the influence of the variator efficiency. This can also be seen in the next figure where the influence of the variator efficiency on the overall transmission efficiency is shown.
The influence of the variator efficiency on the overall transmission efficiency is much less for the Renault than for the Prius. This corresponds to the influence of the variator efficiency on the fuel consumption; the fuel consumption corresponds directly to the transmission efficiency.

Next the influence of the operating points on the overall transmission efficiency is investigated. This is done by using a different drive cycle, the so-called JP10-15 drive cycle. The prescribed vehicle speeds in this cycle differ from the ones prescribed in the NEDC drive cycle that was used before.

In the figure below both drive cycle speeds are plotted

To investigate the influence of the operating point again the average transmission efficiency and the fuel consumption over the drive cycle is calculated with the drive cycle simulation, this time for both the NEDC cycle as the JP10-15 cycle. The results are shown in the next table.
Table 3.3: fuel consumption and transmission efficiency for both transmission for NEDC and JP10-15 drive cycle

<table>
<thead>
<tr>
<th></th>
<th>Fuel consumption [l/100km]</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prius</td>
<td>4.83</td>
<td>4.95</td>
</tr>
<tr>
<td>Renault</td>
<td>4.72</td>
<td>4.92</td>
</tr>
</tbody>
</table>

The difference in transmission efficiency and fuel consumption between both the transmissions gets smaller for the JP10-15 drive cycle. This drive cycle contains less high vehicle speeds (see figure 3.19) and thus less high transmission speedratios. This is beneficial for the Prius efficiency and fuel consumption (see figure 3.15). Furthermore the difference in fuel consumption between the NEDC cycle and the JP10-15 cycle is small. This is quite remarkable because the powers in the JP10-15 cycle are relatively low compared with the NEDC cycle; therefore the fuel consumption is also expected to be relatively low. Because of the characteristic specific fuel consumption map of the engine this is not the case. The specific fuel consumption for lower engine powers is relatively high compared to high engine powers. In the next figure the cumulative fuel consumption is plotted for both the transmissions and both the drive cycles.

Figure 3.20: Cumulative fuel consumption for both the transmissions and both drive cycles

§ 3.4 Simulation conclusion

Using all the simulation results some conclusion can be drawn.

The first conclusion is that the dual mode concept used by Renault gives a relatively more constant transmission efficiency and slightly lower fuel consumption compared with the single mode concept used in the Toyota Prius.

Next the conclusion can be drawn that the influence of the electric machines efficiency is less important for the Renault concept then for the Prius concept. Therefore is could be rewarding for Toyota to put effort in optimizing the electric machines efficiency for the Prius transmission (Of course an optimization electric efficiency is always beneficial, also for the Renault transmission).

Furthermore the operating points affect the overall transmission efficiency. The Prius transmission has a relatively high efficiency for the midrange of speedratios, the Renault transmission a relatively high and more constant efficiency for low and high speedratios.
The Prius transmission might therefore be more appropriate for city traffic, whereas the renault transmission is more suitable for highway traffic.

Final conclusion is that both transmission are simulated without all the hybrid functions (power assist, battery charge, electric drive, see chapter 1). To get a complete picture these hybrid functions have to be included in the simulation models.
Conclusion

The goal of this paper was to make a comparison of different series-parallel transmission concepts. First an introduction to series-parallel concepts was made, followed by a definition of the criteria used for the comparison. Next, the different concepts were discussed and an overall comparison was made.

The main conclusion is that the Prius has a low price and decent fuel economy as benefits, but shorts out in performance. The Lexus Rx400 and the Ford Escape hybrid both get a good score on performance and fuel economy, but they are more expensive.

Including the Renault laguna IVT and the Opel Astra hybrid in the comparison is difficult because there is a lack in information. This is due to the fact that these two cars haven’t been brought to the market yet. On the criteria that were investigated for these cars, they both got a good score.

The simulation in matlab was done for the Toyota Prius and the Renault Laguna. A comparison was made in transmission efficiency and the fuel consumption. This was done by calculating the efficiency maps for both transmission and applying these maps in a drive cycle test. In this drive cycle test both transmission where linked to a simular combustion engine.

Main conclusion from the drive cycle test is that the Renault transmission has a more constant efficiency then the Prius, and therefore higher for low and for high speedratio’s. The Prius has a higher efficiency in the midrange of speedratios. Furthermore the fuel economy of the renault transmission is slightly better then the Prius.

Next, the variator efficiency has less influence on the transmission efficiency and the fuel consumption of the renault transmission; the influence for the Prius is relatively big. Furthermore, the operating points affect the overall transmision efficiency. When a lowspeed drive cycle is used, the difference in transmission efficiency and fuel consumption between both transmission is smaller. The Toyota Prius could be designed more for city traffic then for highway use.

Final conclusion is that both transmission are simulated purely as a transmission, without all the hybrid functions. To get a complete picture these hybrid functions have to be included in the simulation models.
Appendix A Literature

Literature used for this report.

[1] Dual mode electric infinitely variable transmission - Arnaud Villeneuve - Renault


[3] Dynamic modeling and simulation of two-mode electric variable transmission - D. Zhang, J Chen, T Hsieh, J Rancourt, M R Schmidt - Department of Mechanical Engineering, Purdue School of Engineering and Technology, Indiana University Purdue, University at Indianapolis, Indianapolis, Indiana, USA


Appendix B Abbreviations

\( z \) = Planetary gear set ratio

\( P_i, T_i, \omega_i \) = Input power (W), input torque (Nm), input speed (rad/s)

\( P_o, T_o, \omega_o \) = Output power (W), output torque (Nm), output speed (rad/s)

\( T_a, \omega_a \) = Annulus gear torque (Nm), annulus speed (rad/s) in the planetary gear set

\( T_s, \omega_s \) = Sun gear torque (Nm), sun gear speed (rad/s) in the planetary gear set

\( T_c, \omega_c \) = Carrier gear torque (Nm), carrier gear speed (rad/s) in the planetary gear set

\( P_{vi}, T_{vi}, \omega_{vi} \) = Power (W), torque (Nm) and speed (rad/s) at the input of the variator

\( P_{vo}, T_{vo}, \omega_{vo} \) = Power (W), torque (Nm) and speed (rad/s) at the output of the variator

\( P_{ec} \) = Power cinematic chain (W)

\( \eta_{ec} \) = efficiency cinematic chain

\( \eta_{gen} \) = efficiency generator

\( \eta_{mot} \) = efficiency motor

\( \eta_{elec} \) = efficiency variator = \( \eta_{mot} \cdot \eta_{gen} \)

\( K \) = Transmission speed ratio

\( P_d \) = Series-parallel ratio

\( m \) = Vehicle mass (kg)

\( fr \) = Friction coefficient

\( \rho \) = Density of air (kg/m\(^3\))

\( C_d \) = airdragcoefficient

\( R_d \) = wheel radius (m)
Appendix C Transmission equations

§ C.1 Equations Toyota Prius

The next equations can directly be derived for the speeds in the transmission

\[ \omega_i = \omega_c \quad (C.1) \]
\[ \omega_{vi} = \omega_s \quad (C.2) \]

Furthermore, using the final reduction gear and the gear \( r_1 \).

\[ \omega_{jd} = -\omega_o \cdot fd \quad (C.3) \]
\[ \omega_o = -\omega_{jd} \cdot r_1 = \omega_o \cdot (fd \cdot r_1) \quad (C.4) \]
\[ \omega_{vo} = -\omega_{jd} \cdot r_1 = \omega_o \cdot (fd \cdot r_1) \quad (C.5) \]

And using the equations for the planetary gear set (see also chapter 1)

\[ \omega_s + z \cdot \omega_o = (1 + z) \cdot \omega_c \quad (C.6) \]

To get the set of equations complete 2 more equations are needed.

\[ K \cdot \omega_i = \omega_o \quad (C.7) \]
\[ \omega_i = 1 \quad (C.8) \]

For the torques in the transmission

![Figure C.1: Overview of the torques in the transmission](image)

Now the equations for the torques can be derived
\[ T_i = T_c \]  
\[ T_{vi} = T_i \]  \hspace{1cm} (C.9)  
\[ T_{vi} = T_i \]  \hspace{1cm} (C.10)

And using the equations for the planetary gear set

\[ T_a = z \cdot T_i \]  \hspace{1cm} (C.11)
\[ T_i = -(z+1) \cdot T_i \]  \hspace{1cm} (C.12)

And for gear r1

\[ T_{jl} = T_{vo} \cdot r1 + T_a \cdot r1 \]  \hspace{1cm} (C.13)
\[ T_a = T_{jl} \cdot fd \]  \hspace{1cm} (C.14)

\( T_{vo} \) depends on the efficiency of the variator and the variator input torque \( T_{vi} \).

Using the efficiency maps of the electric machines (see figure 3.4).

---

**Figure C.2: overview of variator**

using not battery power then \( P_2 = P_1 \).

\[ P_2 = P_1 = P_{vi} \eta_{gen} (T_{vi}, \omega_{vi}) \]  \hspace{1cm} (C.15)

\[ T_{vo} = \frac{P_{vo}}{\omega_{vo}} = \frac{P_{vo} \eta_{mod} (T_{vo}, \omega_{vo})}{\omega_{vo}} = \frac{P_{vi} \eta_{gen} (T_{vi}, \omega_{vi}) \eta_{mod} (T_{vo}, \omega_{vo})}{\omega_{vo}} \]  \hspace{1cm} (C.16)

but \( \eta_{mod} (T_{vo}, \omega_{vo}) \) can not determined because it depends on \( T_{vo} \), which is unknown.

Therefore the inverse of the efficiency map of the electric motor is used.

With this inverse the output power \( P_{vo} \) can be determined for a certain output speed \( \omega_{vo} \) and a certain input power \( P_1 \).

In the next figure the relation between the generator/motor input power and the generator/motor output power is shown for one certain output speed \( \omega_{vo} \).
when $P_{\text{vo}}$ is determined, $T_{\text{vo}}$ and $\eta_{\text{mot}}(T_{\text{vo}}, \omega_{\text{vo}})$ can also be determined.

§ C.2 Equations Renault Laguna IVT

Using figure 3.6 the equations for the transmission can be derived.

First for each planetary gear set the following equations hold:

\begin{align}
\omega_i + R_c \cdot \omega_a - (1 + R_c) \cdot \omega_c &= 0 \quad \text{(C.17)} \\
R_c \cdot T_c + (1 + R_c) \cdot T_a &= 0 \quad \text{(C.18)} \\
T_a - T_i \cdot R_c &= 0 \quad \text{(C.19)}
\end{align}

Rotational speeds in the transmission in mode 1:

For Planetary gear set A1:

\begin{align}
\omega_{A1} &= \omega_{i} \\
\omega_{cA1} &= \omega_{A2} = \frac{31}{55} \omega_i \\
\omega_{cA2} &= \omega_{cA2} = \frac{67}{41} \omega_i
\end{align}

For A2:

\begin{align}
\omega_{A2} &= \omega_{A2} = \frac{27}{51} \omega_i
\end{align}
\[ \omega_{cA2} = \omega_{dA1} = -\frac{67}{41} \cdot \omega_o \] 
\[ \omega_{dA2} = \omega_{cA1} = -\frac{31}{55} \cdot \omega_i \]  

For B:
\[ \omega_{sB} = \frac{30}{26} \cdot \omega_{sB} \] 
\[ \omega_{sB} = \omega_{sB} \]  
\[ \omega_{sB} = \omega_s = \omega_{F2} \]  

And for C
\[ \omega_{sc} = \omega_{sc} = \omega_{F2} \]  
\[ \omega_{sc} = \omega_{F1} = 0 \]  
\[ \omega_{dC} = \omega_{dC} = \frac{27}{51} \cdot \omega_{vi} \]  
\[ \omega_o = \omega_i K \]  

**Torques in the transmission in mode 1:**
\[ R_{cB} \cdot T_{cB} + \left(1 + R_{cB}\right) \cdot T_{aB} \] 
\[ R_{cB} \cdot T_{aB} - T_{aB} \]  

with \( i \) the planetary gear set \( A_1, A_2, B \) or \( C \)  
\[ T_{in} = -K \cdot T_{out} \]  

using equation (C.33) and equality of power and the equations for the rotational speeds
\[ \omega_i \cdot T_i + \omega_{cA1} \cdot T_{cA1} + \omega_{dA2} \cdot T_{dA2} = 0 \]  
\[ T_i + \frac{\omega_{cA1}}{\omega_i} \cdot T_{cA1} + \frac{\omega_{dA2}}{\omega_i} \cdot T_{dA2} = 0 \] 
\[ T_i - \frac{31}{55} \cdot T_{cA1} - \frac{31}{55} \cdot T_{dA2} = 0 \]  
\[ \omega_o \cdot T_o + \omega_{dA1} \cdot T_{dA1} + \omega_{eA2} \cdot T_{eA2} = 0 \]
with $\eta$ the transmission efficiency. Because this transmission efficiency depends on the speeds and the torques in the transmission, an iteration process is needed. In this iteration process the input torque is changed with the calculated transmission efficiency. For $T_{vo}$ again the relation between the motor input power and the motor output power is used (see figure C.3).

Next using $T_{vo}$, the motor efficiency $\eta_{mot}(T_{vo}, \omega_{vo})$ is determined.

### Rotational speeds in the transmission in mode 2

The basic of the transmission is the same, but there are some differences with the first mode.
In mode 1 brake 1 is closed and brake 2 is open, in mode 2 brake 1 is open and brake 2 is closed.
This will not influence planetary gear sets A1 and A2, for B and C there will be some changes, which will be listed.

For B:

\[ T_o + \frac{\omega_{a1}}{\omega_o} \cdot T_{a1} + \frac{\omega_{a2}}{\omega_o} \cdot T_{a2} = 0 \]
\[ T_o - \frac{67}{41} \cdot T_{a1} - \frac{67}{41} \cdot T_{a2} = 0 \]  (C.37)

\[ \omega_{s1} \cdot T_{vi} + \omega_{s2} \cdot T_{s2} + \omega_{oa} \cdot T_{oa} = 0 \]
\[ T_{vi} + \frac{\omega_{s2}}{\omega_{vi}} \cdot T_{s2} + \frac{\omega_{oa}}{\omega_{vi}} \cdot T_{oa} = 0 \]
\[ T_{vi} + \frac{27}{51} \cdot T_{s2} + \frac{27}{51} \cdot T_{oa} = 0 \]  (C.38)

\[ \omega_{vo} \cdot T_{vo} + \omega_{s2} \cdot T_{s2} = 0 \]
\[ T_{vo} + \frac{\omega_{s2}}{\omega_{vo}} \cdot T_{s2} = 0 \]
\[ T_{vo} + \frac{30}{26} \cdot T_{s2} = 0 \]  (C.39)

\[ T_{s1} + T_{c2} = 0 \]  (C.40)
\[ T_{s2} + T_{c2} + T_{f2} = 0 \]  (C.41)
\[ T_{c2} + T_{f1} = 0 \]  (C.42)
\[ T_{f2} = 0 \]  (C.43)
\[ T_i = \frac{T_i}{\eta_i} \]  (C.44)
\[ \omega_{3a} = \frac{30}{26} \cdot \omega_{ro} \quad (C.45) \]
\[ \omega_{e3} = \omega_{sA1} \quad (C.46) \]
\[ \omega_{e2} = \omega_{f2} = 0 \quad (C.47) \]

For C:
\[ \omega_{sc} = \omega_{rn} = \omega_{f2} = 0 \quad (C.48) \]
\[ \omega_{ec} = \omega_{f1} \quad (C.49) \]
\[ \omega_{ac} = \omega_{sA2} = \frac{27}{51} \cdot \omega_{vi} \quad (C.50) \]

**Torques in the transmission in mode 2**

There is only one difference compared to mode 1
\[ T_{F1} = 0 \quad (C.51) \]

for \( T_{ro} \) again the relation between the motor input power and the motor output power is used (see figure C.3).
With \( T_{ro} \) then the motor efficiency \( \eta_{mot} (T_{ro}, \omega_{ro}) \) is determined.