Modeling of Disturbance Forces of a \( x-y \) Manipulator on a Floating Platform

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Abstract—This paper describes a multi-body dynamics model for predicting disturbance forces and torques caused by a multi-stage manipulator (2 DOF) on a magnetically suspended platform (6 DOF). The manipulator consists of a beam attached to two parallel linear ironless actuators and a rotary arm underneath the beam. The model prediction can be used as a feed-forward in the control of the magnetic suspension of the platform. The model is verified with an experimental setup, that consists of a manipulator on an aluminium platform which is connected to the fixed world via a 6 DOF F/T sensor. The sensor is used to measure the forces and torques, that an ideal magnetic suspension will have to provide to compensate the for the disturbances given a certain movement of the manipulator. The measurements are compared with the predictions of the multi-body dynamics model.

I. INTRODUCTION

Most high-precision machines consist of several positioning stages which are often a cascaded set of long-stroke actuators with low precision and short-stroke actuators for high precision positioning [1], [2], [3]. In order to decrease production costs and time there is an increased demand for higher productivity of such machines. Several options are available in order to tackle this problem:

1) Faster machines could be built which would need more powerful actuators and, therefore, lead to increased mechanical and thermal stresses, resulting in a bulkier design.
2) The batch size of the production process could be increased which would lead to actuators with a longer stroke.
3) Parallel processing could be used. In this case another task will be performed while positioning. This way it is possible to improve performance without the need for increased machine size.

The first two options do not only result in heavier machines, but also compromise the accuracy of the machines. Parallel processing however does not have this drawback. At Eindhoven University of Technology such a parallel machine is currently under development (see Fig. 1). The goal of the project is to build a contactless planar actuator with a manipulator on top of it. Robots on the fixed world can place products on the planar actuator, which in turn can be used for transportation and positioning of the same product. While moving, the manipulator can be used for e.g. inspection or calibration of the product. Increased reliability and dynamics will result from removing all cables which connect it to the fixed world. Therefore, three different contactless techniques should be realized in this project:

1) Contactless movement of the planar actuator by using magnetic bearings and propulsion.
2) Contactless energy transfer by using inductive coupling.
3) Wireless control of the manipulator using a wireless low-latency data link.

The energy, which is necessary to operate the manipulator, is provided by the contactless inductive coupling. Furthermore, the manipulator is controlled from the ground via a wireless data link.

II. MECHANICAL DESIGN OF THE MANIPULATOR

The planar actuator consists of an array of stationary coils, above which a platform with permanent magnets is floating. The manipulator on top of the platform is an H-drive with two ironless linear actuators attached to a beam. In the center of the beam a rotary motor is assembled with an arm attached to it. The tip of this arm can be positioned anywhere in the \( x-y \) plane between the two linear motors by combining the translation of the linear actuators and the rotary movement of the arm.

The linear actuators are brushless 3-phase ironless actuators with a continuous force of 10 N and the rotary drive is a 3-phase slotless motor with a continuous torque of 0.28 Nm, and
therefore, all three motors have no cogging. The position of each linear actuator is measured with incremental encoders which have a 1 μm resolution. The angle of the rotary motor is measured with a 40 μrad resolution.

III. Model derivation using Lagrange’s equations

The derivation of the multi-body model is done by using Lagrange’s equations [4], [5]. These equations eliminate the need for computing all interacting forces between different bodies. Only external forces and constraint forces of interest have to be taken into account. Furthermore, all bodies are considered to be rigid.

Each body has a local coordinate system attached to it, which is used for describing the position and orientation with respect to the global coordinate system. The expressions for kinetic energy, \( T \), and potential energy, \( V \), are derived in terms of the global coordinate system. These energy expressions are used in Lagrange’s equations to obtain the equations of motion:

\[
\left( \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} \right)^T = Q^{nc} + W\lambda, \tag{1}
\]

where \( q \) is a column of generalized coordinates and \( Q^{nc} \) a column of generalized non-conservative forces. Furthermore, \( W \) and \( \lambda \) together represent the constraints on the system.

A. Coordinate systems

The manipulator consists of three separate bodies. The platform, magnet tracks and bearings together are the first body with mass, \( m_1 \). The second body is the beam and has a mass, \( m_2 \). The rotor of the rotary motor together with the arm form the last body with mass, \( m_3 \). Each body has a local coordinate system attached to it (see Fig. 2), which is used for describing the position and orientation with respect to the global coordinate system. Each coordinate system consists of three mutually orthogonal unit vectors:

\[
\vec{e}_i^0 = \left[ \begin{array}{c} \vec{e}_x^0 \\ \vec{e}_y^0 \\ \vec{e}_z^0 \end{array} \right] \quad \text{for } i = 0, \ldots, 3. \tag{2}
\]

\( \vec{e}_0^0 \) is fixed to the world and can, therefore, not move. \( \vec{e}_1^0 \) is placed in the center of the manipulator platform, \( \vec{e}_2^0 \) is located in the center of the beam and \( \vec{e}_3^0 \), finally, is attached to the rotary arm.

B. Position and orientation of the bodies

The position of the center of mass of each body with respect to the fixed world is written as:

\[
\vec{r}_{CM_i} = \left[ \begin{array}{c} x_i \\ y_i \\ z_i \end{array} \right] \vec{e}_i^0. \tag{3}
\]

The orientation of each body is described by means of Tait-Bryant angles. The orientation of a body is the result of subsequent rotations \( \theta_i \), \( \psi_i \) and \( \phi_i \), about, respectively, the local \( \vec{e}_1^i \), \( \vec{e}_2^i \) and \( \vec{e}_3^i \) axis. By the use of rotation matrices, \( A^i \), the transformation from one coordinate system to another can now be easily made:

\[
\vec{e}_j^i = A^{ij} \vec{e}_i^j. \tag{4}
\]

C. Generalized coordinates

In general, a body has 6 degrees of freedom (DOF) if it is not subject to any constraints. In presence of constraints, each constraint removes one degree of freedom. The minimum required set of coordinates to describe the position and orientation of each body are a set of \( n \) generalized coordinates, \( q \). So \( q \) is a \((n \times 1)\) column. The floating platform has 6 DOF due to its complete freedom with respect to the fixed world. Furthermore, the manipulator adds 1 DOF due to the translation of the beam and 1 DOF due to the rotational movement of the rotary motor. The beam is considered rigid and roller bearings on each side do not allow other movements of the beam than in the \( \vec{e}_2^1 \)-direction. The column of generalized coordinates for the manipulator on the floating platform is:

\[
\vec{q} = [ x_1 \; y_1 \; z_1 \; \theta_1 \; \psi_1 \; \phi_1 \; y_{LM} \; \phi_{RM} ]^T, \tag{5}
\]

where \( y_{LM} \) and \( \phi_{RM} \) denote, respectively, the movement of the beam and rotation of the arm. Now the position vectors, \( \vec{r}_{CM_i} \), can be written in terms of \( \vec{q} \).

D. Kinetic and potential energy

The next step in the Lagrangian approach is to define the total energy available in the system. The total kinetic energy of the system is the sum of the translational and rotational kinetic energy of each body:

\[
T = \frac{1}{2} \sum_{i=1}^{3} \left( m_i \vec{r}_{CM_i} \cdot \vec{\omega}_{CM_i} + \vec{\varepsilon}_i \cdot \vec{J} \cdot \vec{\varepsilon}_i \right), \tag{6}
\]

where \( m_i \) denotes the mass of body \( i \), \( \vec{\varepsilon}_i \) is the angular velocity of body \( i \) with respect to the fixed world and \( \vec{J} \) is the inertia tensor of body \( i \) with respect to the point of rotation \( O \).

The total potential energy of the system is the sum of the potential of the gravitational forces acting on each body and the energy stored in the system in spring-elements. Because there are no springs in the manipulator and no flexibility is included in the model the potential energy equation reduces to:

\[
V = - \sum_{i=1}^{3} m_i \vec{g} \cdot \vec{r}_{CM_i}, \tag{7}
\]
with \( \vec{g} \) the gravitational acceleration vector.

## E. External forces and torques

Since the platform will be floating, there is no friction except for negligible air-drag acting on the floating platform. In addition, there is friction in the bearings of the beam and the arm. However, the friction in the bearings is between the bodies, and does not affect the total system. Therefore, the magnetic suspension will only have to compensate the forces and torques due to accelerations of the beam and the arm. For these reasons, friction is not included in the model.

Usually actuator forces and torques are included in the dynamic equations as external non-conservative forces and torques. In this case, the forces of the linear motors and the torque of the rotary motor could be included as external non-conservative forces and torques. However, since the trajectories are known and the magnetic suspension only has to compensate the inertial forces and torques, it is more straightforward to include the forces and torque of the motors as constraint forces and torque. This results in an inverse dynamic analysis without a column of external forces and torques \( \tau \) and simplifies the calculation because only the constraint equation has to be solved. Therefore the column with non-conservative forces and torques is zero:

\[
Q^{nc} = 0
\]  

(8)

### F. Constraint equations

In the experimental setup, all DOF of the floating platform are fixed using a 6 DOF Force/Torque (F/T) sensor. These six DOF are thus constrained to be in the setup. Furthermore, the position of the beam and the arm are measured using encoders and following a prescribed trajectory in a feedback loop. Therefore, all motions are prescribed and can be included in the equations as driving constraints. From this formulation, the necessary forces to realize these trajectories are calculated. Finally, these are compared with the measured forces. All constraints are included as velocity constraints and are written as:

\[
h_{nh} = \begin{bmatrix}
\dot{x}_1 \\
\dot{y}_1 \\
\dot{z}_1 \\
\dot{\theta}_1 \\
\dot{\psi}_1 \\
\dot{\phi}_1 \\
\dot{y}_{LM} - \dot{u}_{LM}(t) \\
\dot{\phi}_{RM} - \dot{u}_{RM}(t)
\end{bmatrix} = 0,
\]  

(9)

where \( u_{LM} \) is the measured position of the beam and \( u_{RM} \) is the measured position of the arm. Because all velocity components appear as terms which are linear in the generalized coordinates, the constraints are rewritten as:

\[
W^T(q, t) \ddot{q} + \ddot{w}(q, t) = 0,
\]  

(10)

where \( W(q, t) \) is an \((8 \times 8)\) matrix which represents the velocity dependent components and has all ones on the diagonal in this case. The component \( \ddot{w}(q, t) \), an \((8 \times 1)\) column with the remaining components that do not depend on \( \ddot{q} \), i.e. \( \ddot{u}_{LM} \) and \( \ddot{u}_{RM} \)

### G. Lagrange multipliers

In order to incorporate the constraint equations in the equations of Lagrange, an \((8 \times 1)\) column, \( \lambda \), of so called Lagrange multipliers is introduced. The Lagrange multipliers represent the constraint forces and torques:

\[
\lambda = \begin{bmatrix} F_x, F_y, F_z, T_x, T_y, T_z, F_{LM}, T_{RM} \end{bmatrix}^T.
\]  

(11)

### H. Equations of Lagrange

The equations of motion now follow from the extended equations of Lagrange as described in [4]:

\[
\left( \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}} \right) - \frac{\partial T}{\partial q} + \frac{\partial W}{\partial \dot{q}} \right)^T = W\lambda.
\]  

(12)

The equations of Lagrange together with the constraint equations (Eq. 10) now completely describe the dynamics of the system. In order to solve the equations they are rewritten and combined. Therefore, first the equations of Lagrange without constraints (i.e. \( W\lambda = 0 \)) are put in the following form:

\[
\begin{bmatrix} M(q) \ddot{q} + H(q, \dot{q}) \end{bmatrix} = S(q)\zeta,
\]  

(13)

where \( M(q) \) is the mass-matrix, \( H(q, \dot{q}) \), a matrix with centripetal, Coriolis, friction and gravitational terms and \( S(q) \), the distribution of external forces and torques, \( \zeta \). In this case, there are no external forces and torques:

\[
S(q)\zeta = 0.
\]  

(14)

The next step is differentiating the constraint equations (Eq. 10) with respect to time:

\[
W^T(q, t) \ddot{q} + \ddot{w}(q, \dot{q}, t) = 0,
\]  

(15)

where

\[
\ddot{w}(q, \dot{q}, t) = \frac{\partial \ddot{w}(q, t)}{\partial t} + \left( \frac{\partial W^T(q, t)}{\partial q} \right) \dot{q}.
\]  

(16)

The total dynamics is now written as:

\[
\begin{bmatrix} M(q) & -W(q, t) \\
W^T(q, t) & 0 \end{bmatrix} \begin{bmatrix} \dot{q} \\
\dot{\lambda} \end{bmatrix} + \begin{bmatrix} H(q, \dot{q}) \\
S(q)\zeta \end{bmatrix} = \begin{bmatrix} 0 \\
0 \end{bmatrix}.
\]  

(17)

As the goal of the model is to predict the forces and torques on the platform, now an expression is derived for the Lagrange multipliers. For a certain trajectory of the platform and manipulator, the Lagrange multipliers namely contain the forces and torques which keep the platform balanced. From Eq. 17 an expression for the generalized accelerations, \( \ddot{q} \), is now derived as:

\[
\ddot{q} = M^{-1}(q) \left( -H(q, \dot{q}) + W(q, t)\lambda \right).
\]  

(18)

Using this expression in the second line of Eq. 17 and solving for \( \lambda \) results in:

\[
\lambda = W^T(q, t)M^{-1}(q)W(q, t)^{-1} \left( W^T(q, t)M^{-1}(q)H(q, \dot{q}) - S(q)\zeta \right).
\]  

(19)

So an expression for \( \lambda \) is now available in terms of \( q, \dot{q} \) and \( t \).
The beam and the arm of the manipulator are moving and the trajectories and the resulting forces and torques on the sensor underneath the platform are measured. The measured reaction forces and torques are equal in magnitude to the forces and torques necessary to stabilize the platform in case it is floating. The derived multi-body dynamics model assumes that all bodies are rigid, therefore transfer function measurements of the three motors and of the 6 DOF F/T sensor are done. These measurements show that the first resonances due to flexibilities in the system occur at 80 Hz, which implies that the rigid body assumption is only valid up to this frequency. Therefore, all measurements are filtered offline using a fourth order butterworth filter with a cut-off frequency of 80 Hz and the anti-causal filtering function \textit{filtfilt} in Matlab to eliminate phase-lag by the butterworth filter.

The trajectories of $q$ are zero except for the movement $\dot{y}_{LM}$ of the beam and $\dot{\phi}_{RM}$ of the arm. The measured trajectories are now applied as inputs for the multi-body model. The model is kept to calculate the necessary constraint forces in order to keep the platform in the original position. Since the movement of the beam in the $\vec{e}_3$ direction, the force in the $\vec{e}_2$-direction ($F_2$) and the torque around the $\vec{e}_1$-axis ($T_1$) are dominant. The model predictions and the measured values of $F_1$ and $T_1$ are shown in Fig. 3 and Fig. 4, respectively. The torque around the $\vec{e}_3$-axis ($T_3$) is associated with the applied torque by the rotary motor that drives the arm. The predicted and measured values are shown in Fig. 5.

V. DISCUSSION

From the predicted and measured values in Figure 3 and Figure 4, it can be concluded that the model predictions and measurements match well. The remaining errors are caused by inaccurate model parameters such as mass and moments of inertia, since these parameters are estimated from the CAD model. Moreover, it was found that the 6 DOF F/T sensor was not properly calibrated, so the model parameters were not adjusted using the measurement data. The measured torque around the $\vec{e}_3$-axis looks very similar to the model predictions, however, the values are in the same range as the noise on the sensor signal in that direction. Therefore, it is not possible to draw strong conclusions from these measurements.

VI. CONCLUSIONS

A multi-body model was derived for a manipulator on a floating platform. It is used for predicting disturbance forces and torques on the platform. A manipulator was built and placed on a 6 DOF F/T sensor for verification purposes. The multi-body model was verified with measurement data. The model predictions match well with the measured values. The model will be used to calculate feed-forward control actions for the magnetic suspension of the floating platform.

REFERENCES