Iterative learning control with wavelet filtering

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SUMMARY

The tracking performance of systems that perform repetitive tasks can be significantly improved using iterative learning control (ILC). During successive iterations, ILC learns a high performance feedforward signal from the measured tracking error. In practical applications, the tracking errors of successive experiments contain a repetitive part and a non-repetitive part. ILC only compensates for the repetitive part, while the non-repetitive part also enters the learning scheme and deteriorates the performance of ILC. In this paper, analysis of the tracking error of ILC shows the influence of non-repetitive disturbances. The disturbances of the last two iterations appear to have the largest influence on the tracking error. In order to remove the non-repetitive disturbances from the tracking error, a wavelet filtering method is proposed, which identifies and removes the non-repetitive disturbances by a comparison of the time–frequency content of two error realizations for each iteration of ILC. The wavelet filtered error signal contains only the repetitive disturbances and is used as input for ILC. Both simulations and experiments show that with wavelet filtering, a better tracking performance is obtained together with a feedforward signal that contains significantly less disturbances. Copyright © 2007 John Wiley & Sons, Ltd.

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1. INTRODUCTION

Systems that perform repetitive tasks are likely to make the same errors over and over again. A high performance feedforward signal for such systems can be derived using iterative learning control (ILC). The feedforward signal is updated through successive iterations and improves the tracking performance of the system significantly.

During repetitive tasks, two kinds of errors occur: repetitive errors which are identical every cycle and non-repetitive errors which vary every cycle. The derived feedforward signal of ILC

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only compensates for the repetitive part of the error. The non-repetitive errors, e.g. noise and other disturbances, deteriorate the performance of ILC [1, 2].

The first work on ILC was formulated in 1978 by Uchiyama [3]. The idea was developed further by Arimoto in the mid-1980s [4]. Since then, ILC studies have been performed for various types of systems and applications. The influence of disturbances already received attention in the past. In [5], a disturbance analysis is performed for the open-loop case and for measurement disturbances only. Disturbance analyses for the closed-loop case are performed in [1]. A recursive formulation of the tracking error is used in combination with a disturbance-free error signal obtained without any ILC input. This paper gives a non-recursive formulation for the tracking error of a feedback controlled system with ILC. Both measurement and load disturbances are included in the analysis.

Several methods to improve the performance of ILC in the presence of non-repetitive disturbances have been developed in the past. These methods can be divided into two categories: methods that perform prefiltering on the input signal of ILC and methods that adjust the ILC scheme itself. The first category includes the use of a disturbance observer under the assumption that the pattern of the non-repetitive disturbances is known [6]. In [7, 8], the non-repetitive disturbances are identified using a wavelet transform. The repetitive disturbances are assumed to be completely located at low frequencies which kills one of the great properties of ILC, namely that repetitive disturbances can be reduced far beyond the closed-loop bandwidth. The power of wavelet analysis as both a time and frequency decomposition is also not exploited, since non-repetitive disturbances are cancelled by removing the higher frequency bands, and since no time information is taken into account.

Methods of the second category include a phase lead compensated ILC scheme [9], parameter estimation of the Markov parameters [10], and optimization techniques such as multi-objective optimization [3] and buffer-state ILC [2]. Another method is to use a time-varying robustness filter which can adapt to the time–frequency content of the feedforward signal. For this, a time–frequency analysis can be used in the form of a Wigner–Ville transform [11, 12] or in the form of a wavelet transform [13]. Changing the cut-off frequency of the learning filter based on the present non-repetitive disturbances also prevents learning of repetitive errors above that frequency. By preventing non-repetitive disturbances from entering the learning scheme, the complete frequency band of interest can be included in the learning scheme, as will be shown in this paper.

A common approach to eliminate non-repetitive components from the measured error signal is to average a number of realizations of the error signal [14]. This approach generally requires a high number of realizations, cannot be applied on-line, and assumes the non-repetitive components to behave stationary. In this context, on-line implementation means updating the feedforward signal during normal operation. The proposed wavelet filtering method only needs two subsequent realizations and can therefore be applied on-line. Furthermore, varying signal properties of the non-repetitive components can be dealt with.

The error analysis of ILC as presented in this paper leads to an error expression which shows the influence of non-repetitive disturbances on ILC. The proposed wavelet prefiltering method filters out the non-repetitive disturbances in both the time and the frequency domain. The improvement of the performance with wavelet filtering will be shown by both simulations and experiments on a real-life motion system. The proposed method can deal with both stochastic disturbances as well as deterministic disturbances that are non-repetitive over the learning interval. Also, incidental pulse-like disturbances are removed from the learning scheme.
This paper is organized as follows. In Section 2, the ILC concept is discussed in more detail. A disturbance analysis is performed in Section 3, leading to the expression for the tracking error of ILC. Section 4 introduces the wavelet transform which is the basis of the wavelet filtering method of Section 5. Simulations and experiments are presented in Section 6. Finally, conclusions are drawn in Section 7.

2. ITERATIVE LEARNING CONTROL

ILC reduces the repetitive part of the error signal by updating the feedforward signal during several successive iterations. All iterations have an equal finite-time length $t \in [t_0, t_e]$. A block diagram of a system $P(s)$ with feedback controller $C(s)$ and ILC added is shown in Figure 1. Zero initial conditions are assumed for each iteration. The tracking error $e(t)$ is defined as the difference between the reference signal $r(t)$ and the measured output $y(t)$. The feedforward signal of ILC is denoted by $f(t)$. The signals $d(t)$ and $n(t)$ denote, respectively, the measurement and the load disturbances. Iterations are denoted by an index, $e_k$ being the error of the $k$th iteration.

The feedforward signal is updated off-line, i.e. between successive iterations. For this, the error $e_k$ is filtered by a learning filter $L(s)$ and added to the feedforward $f_k$. The sum of filtered error and the feedforward is applied to a robustness filter $Q(s)$ to obtain the feedforward signal for the next iteration $f_{k+1}$. The learning update of the feedforward signal equals

$$f_{k+1} = Q(f_k + Le_k) \tag{1}$$

Using the closed-loop transfer function from the feedforward to the error signal, i.e. the process sensitivity $S_P = P(1 + PC)^{-1}$, the propagation of the error from iteration to iteration can be written as

$$e_{k+1} = Q(1 - LS_P)e_k \tag{2}$$

The error reduces between successive iterations if

$$||Q(1 - LS_P)||_\infty < 1 \tag{3}$$

Equation (3) is based on the fixed point theorem and gives a sufficient, but not necessary condition for the convergence of ILC [15, 16]. For convergence, a suitable choice for the learning filter would be $L = S_P^{-1}$. In most cases, especially for motion systems with the position as

![Figure 1. Feedback controlled system with ILC.](image)
output, the process sensitivity $S_P$ is strictly proper and cannot be inverted. Furthermore, if $S_P$ contains non-minimum phase zeros, the inverse will contain unstable poles. Therefore, often an approximated $S_P^{-1}$ is used [17, 18]. The filter $Q$ provides robustness against modelling errors and is designed such that convergence criterion (3) is satisfied for all frequencies. The filtering by $Q$ is implemented as a forward–backward algorithm, such that the phase of the original signal is preserved.

3. DISTURBANCE ANALYSIS

The ILC concept assumes that the disturbances, the system dynamics, the uncertainties, and the trajectories are iteration independent. However, in general there are also non-repetitive inputs which vary every iteration, e.g. measurement and load disturbances. These non-repetitive disturbances limit the performance of the ILC scheme [1, 2].

Lemma 3.1
The error of iteration $k$ with $k \in Z^+$ can be written as a function of the reference signal $r$ and the disturbances of the $k$th and previous iterations as

$$e_k = (1 - S_P Q L) S r - \sum_{j=1}^{k-2} S_P Q (Q (1 - L S_P))^{k-j-1} L S r$$

- $S d_k + S_P Q L S d_{k-1} + \sum_{j=1}^{k-2} S_P Q (Q (1 - L S_P))^{k-j-1} L S d_j$

- $S_P n_k + S_P^2 Q L n_{k-1} + \sum_{j=1}^{k-2} S_P^2 Q (Q (1 - L S_P))^{k-j-1} L n_j$ (4)

Proof
The proof of above equation will be given by means of construction. The error of iteration $k$ can be written as a function of the reference signal $r$, the disturbances $d_k$ and $n_k$ and the feedforward signal $f_k$ as

$$e_k = S r - S d_k - S_P n_k - S_P f_k$$ (5)

where the sensitivity $S = (1 + P C)^{-1}$ and $S_P = P S$. The reference $r$ is not varied over the iteration process. The feedforward of iteration $k$ can be written with the use of the error $e_{k-1}$ and (1) as

$$f_k = Q L S r - Q L S d_{k-1} - Q L S_P n_{k-1} + Q (1 - L S_P) f_{k-1}$$ (6)

Substitution of (6) in (5) gives

$$e_k = (1 - S_P Q L) S r - S d_k + S_P Q L S d_{k-1} - S_P n_k + S_P^2 Q L n_{k-1} - S_P Q (1 - L S_P) f_{k-1}$$ (7)

Equation (4) now follows from (6) and (7) by repeated substitutions. $\square$
3.1. Ideal learning filter

If an ideal learning filter \( L = S_P^{-1} \) can be derived, no \( Q \) filter is needed since the convergence criterion of (3) is already met. Tracking error (4) reduces with \( Q(s) = 1 \) to

\[
e_k = -Sd_k + Sd_{k-1} - S_Pn_k + S_Pn_{k-1}
\]

In this case, the tracking error only depends on the disturbances of the last two iterations. In the worst case scenario, the load and measurement disturbances have opposite sign with respect to the last two iterations. For an ideal learning filter, the error can become at most \( e_{\text{max}} = 2||S||_{\infty}d_{\text{max}} + 2||S_P||_{\infty}n_{\text{max}} \) due to the presence of the disturbances in the learning scheme. In the case of no disturbances and an ideal learning filter, the tracking error \( e_k \) becomes zero. In practice, disturbances are always present. By preventing the non-repetitive disturbances from entering the learning scheme, the error will become \( e_{\text{max}} = ||S||_{\infty}d_{\text{max}} + ||S_P||_{\infty}n_{\text{max}} \) and, thus, the final tracking error can be reduced by a factor 2.

3.2. Approximation learning filter

If \( L \neq S_P^{-1} \), a \( Q \) filter is generally needed for convergence and the error \( e_k \) depends on the disturbances of iteration \( k \) and all previous iterations. Compared to the filters of the disturbance signals at iteration \( k-1 \), the disturbance filters of older iterations have additional multiplications with the term \( Q(1 - LS_P) \). The number of multiplications depends on the iteration number, see (4). To guarantee convergence, \( Q(1 - LS_P) \) must have an amplitude smaller than 1 for all frequencies. Because of this, the summation terms converge to fixed transfer functions in the frequency domain for \( k \rightarrow \infty \). Due to the additional multiplication with a term smaller than 1, the disturbances older than iteration \( k - 1 \) have a small contribution to the tracking error. Therefore, the maximum worst case tracking error is expected to be in the same order of magnitude as \( 2||S||_{\infty}d_{\text{max}} + 2||S_P||_{\infty}n_{\text{max}} \).

4. WAVELET TRANSFORM

The disturbance analysis has shown that the disturbances present in the learning scheme deteriorate the ILC performance. In order to eliminate the disturbances from the tracking error, a filtering method is designed.

The wavelet transform makes it possible to analyse the time–frequency content of a signal by choosing localized waves (wavelets) as basis functions for the decomposition [19–21]. The wavelet transform is preferred over the short-time Fourier transform (STFT). With the STFT a constant time–frequency resolution is obtained. The wavelet transform enables a multi-resolution analysis, which is often more desirable (e.g. a high time resolution for low frequencies and vice versa).

The discrete wavelet transform (DWT) [19–21] uses filter banks to decompose the signal into wavelet coefficients, which represent the signal content in various frequency bands and at various time instants. By adjustment of the wavelet coefficients, the reconstructed signal of the DWT can be changed in comparison with the original signal. An example of a two-level filter bank is shown in Figure 2, where \( L(z) \), \( H(z) \), \( L'(z) \) and \( H'(z) \) denote, respectively, the low- and high-pass analysis and synthesis wavelet filters. Furthermore, \( \downarrow \) denotes downsampling by a factor two and \( \uparrow \) upsampling by a factor 2.
The filter bank can be expanded to an arbitrary level, depending on the desired frequency band resolution. The branch with only low-pass filters \( L(z) \) retrieves the approximation of the signal and the coefficients of this branch are called approximation coefficients \( c_A \). Details of the signal at various frequency subbands \( q \) are extracted by the other branches, the coefficients of which are called detail coefficients \( c_D \). The frequency bounds of the approximation subband \( c_A \) and detail frequency bands \( c_D \) can be calculated for a \( p \)-level filter bank as

\[
f_{c_A} = [0, 2^{-p-1}f_s]
\]

\[
f_{c_D} = [2^{-p+q-2}f_s, \quad 2^{-p+q-1}f_s], \quad k = 1, \ldots, p
\]

5. NON-REPETITIVE ERROR REJECTION USING WAVELETS

In order to remove the non-repetitive part of the error, a DWT-based filtering method is designed. For two realizations of the error signal in a single iteration of ILC, repetitive errors will lead to equal wavelet coefficients, whereas non-repetitive errors will result in different wavelet coefficients. By comparing the wavelet coefficients of the two realizations, the repetitive part of the error can be distinguished from the non-repetitive part. The proposed wavelet filtering method calculates an adjusted set of wavelet coefficients corresponding to only the repetitive part. From these adjusted wavelet coefficients, a filtered error signal is reconstructed.

5.1. Wavelet function

The various families of wavelet filters all have specific properties [19, 21]. Each family consists of several wavelet functions with different numbers of vanishing moments. The larger the number of vanishing moments, the better the frequency localization of the decomposition, but also the larger the computational costs.

The success of a wavelet decomposition depends strongly on the choice of the wavelet filters \( w \). Both the chosen wavelet family and the number of vanishing moments influence the results. For the suppression of non-repetitive disturbances of the polynomial type, the number of vanishing moments should exceed the degree of the polynomial [22]. The best wavelet function for the suppression of deterministic and stochastic non-repetitive disturbances in combination with ILC is determined using an objective function as follows. The error signals \( e_c \) of an ILC simulation without disturbances are compared to the reconstructed error signals \( e_w \) for a simulation with known non-repetitive disturbances and wavelet filtering. The optimal wavelet function is determined by optimizing

\[
\min_w ||e_c - e_w(w)||_\infty
\]
where \( w \) are the wavelet filters of various wavelet families and with various numbers of vanishing moments. The optimization is performed for different kind of disturbances, i.e. both deterministic and stochastic disturbances, by means of simulations since then both the error signals with and without disturbances can be compared.

5.2. Decomposition level

The decomposition level is chosen such that the approximation coefficients \( c_A \) contain most of the frequency content of the reference signal since this is the largest intentionally applied disturbance. The desired maximum frequency of the approximation coefficients \( f_{\text{des},A} \) is determined using a power spectral density (PSD) of the reference signal. The decomposition level can be designed for a sampling frequency \( f_s \) as

\[
p = \text{floor}\left( \frac{\log(f_s/f_{\text{des},A})}{\log(2)} - 1 \right)
\]

where the floor function rounds the argument to the nearest integer towards minus infinity.

5.3. Coefficient adjustment method

In order to distinguish between the repetitive and non-repetitive part of the error, at least two error signals with an equal repetitive part have to be available. These error signals are obtained by performing two runs at a single iteration of ILC.

**Definition 5.1**

A run is defined as a simulation or experiment, unrelated to the number of feedforward updates. Runs will be indicated with a superscript. The number of iterations is defined as the number of feedforward updates in ILC. With these definitions, \( e^i_k \) denotes the \( i \)th run at iteration \( k \).

The coefficient adjustment method will be described in the following steps. For the sake of simplicity, the subscript indicating the iteration number \( k \) is omitted.

- Decompose the errors signals of the two runs into two sets of wavelet coefficients at various frequency bands. These wavelet coefficients contain an equal repetitive part \( c_r \) and different non-repetitive part \( c_{nr} \) as

\[
c^1(t) = c_r(t) + c_{1nr}(t)
\]

\[
c^2(t) = c_r(t) + c_{2nr}(t)
\]

- The difference \( \Delta c \) between the wavelet coefficients (13) and (14) contains only the non-repetitive parts. The mean \( \bar{c} \) of the wavelet coefficients (13) and (14) contains the repetitive part and an average non-repetitive part

\[
\Delta c(t) = c^1(t) - c^2(t) = c^1_{nr}(t) - c^2_{nr}(t)
\]

\[
\bar{c}(t) = c_r(t) + (c^1_{nr}(t) + c^2_{nr}(t))/2
\]
• The measure $\delta(t)$ for the non-repetitive part is defined as the relative difference between the two sets of wavelet coefficients

$$\delta(t) = \frac{\Delta c(t)}{\tilde{c}(t)}$$

(17)

• At those time instants where $\delta$ exceeds a predefined threshold level $\gamma$, the wavelet coefficients are set to zero. This leads to an adjusted set of wavelet coefficients $c_{adj}$ as

$$c_{adj}(t) = \begin{cases} \tilde{c}(t), & \text{if } |\delta(t)| < \gamma \\ 0, & \text{if } |\delta(t)| \geq \gamma \end{cases}$$

(18)

where $|\cdot|$ represents the absolute value operator.

• The threshold level $\gamma$ consists of two parts, a constant level $\gamma_c$ and a variable part $\gamma_{\text{var}}$

$$\gamma = \gamma_c \gamma_{\text{var}}$$

(19)

The wavelet coefficients of the repetitive part of the error will not be exactly equal due to numerical reasons or due to the presence of a small superposed non-repetitive part. The constant level $\gamma_c$ defines how much the wavelet coefficients may vary such that they are maintained in the adjusted wavelet coefficients. The variable part is adjusted based on the mean power, i.e. the root-mean-square (rms) values, of $\Delta c$ and $\tilde{c}$

$$\gamma_{\text{var}} = \frac{\text{rms}(\tilde{c}(t))^2}{\text{rms}(\Delta c(t))^2}$$

(20)

An illustrative example of the coefficient adjustment method is shown in Figure 3 for two sets of identical wavelet coefficients. At $t \in [0.5, 1]$ s a low frequent disturbance is added to $c^1$, at
If the non-repetitive disturbances have an opposite sign, they are passed by (18) since \(\Delta c\) equals zero. However, a disturbance free error signal is still obtained since the mean \(\bar{c}\) is taken in the adjusted wavelet coefficients.

Since the wavelet filter adjusts the wavelet coefficients at each time instant separately, the phase of the adjusted error signal is not changed with respect to the original error signals and the adjusted error signal can be used directly as input for the ILC algorithm.

A schematic representation of the control scheme with wavelet filtering and ILC is shown in Figure 4.

**Procedure 5.1**

The wavelet filtering ILC method procedure can be applied in the following steps:

1. Decompose two error signals \(e_1^k\) and \(e_2^k\) of a single iteration \(k\) of ILC into wavelet coefficients at various frequency bands using the DWT.
2. Calculate the deviation measure \(d\) and threshold level \(\gamma\) for each frequency band and for each time instant.
3. Adjust the wavelet coefficients of all frequency bands according to (18).
4. Reconstruct a filtered error signal using the synthesis filter bank of the DWT.

**5.4. Stability**

The addition of the wavelet filter \(W\) as shown in Figure 4 changes the behaviour of the original ILC scheme of Figure 1. The error reduction between subsequent iterations is also affected by the additional wavelet filter \(W\).

**Theorem 5.1**

The ILC scheme with additional wavelet filter \(W\) converges, if the convergence criterion (3) is satisfied.

**Proof**

The first step of the proof is to consider the adjustment of the wavelet coefficients. The filtering method poses a momentary threshold on \(\delta\) (17). According to (18), the coefficients are set to zero.
or the mean is taken for the construction of the adjusted set of wavelet coefficients. At each time instant, the amplitude of the adjusted coefficients is maximally the mean of the amplitudes of the coefficients of the original signal, which is always smaller than the amplitude of the coefficients of the largest signal. Using the $\mathcal{H}_\infty$-norm, this implies

$$||\tilde{e}||_\infty \leq ||(e^1 + e^2)/2||_\infty \leq \max(||e^1||_\infty, ||e^2||_\infty) \quad \forall f$$

(21)

Since the adjusted coefficients of the separate frequency bands are smaller than the maximum of the coefficients of the original signals, also the $\mathcal{H}_\infty$-norm of the reconstructed error signal $e_W = W(e)$ is smaller than or equal to the maximum original signal as

$$||e_W||_\infty \leq \max(||e^1||_\infty, ||e^2||_\infty)$$

(22)

Since the adjusted error signal is never larger than the largest original signal, it follows that the $\mathcal{H}_\infty$-norm of the wavelet filter $||W||_\infty \leq 1$, which implies

$$||WQ(1 - LS_p)||_\infty \leq ||Q(1 - LS_p)||_\infty < 1$$

(23)

So, the addition of the wavelet filter does not negatively affect the convergence of the ILC scheme.

6. APPLICATION RESULTS

The designed wavelet filtering method is tested by means of simulations and experiments on a mechanical motion system. In motion systems, the non-repetitive disturbances are particularly caused by actuator disturbances and measurement disturbances, e.g. quantization noise. These disturbances are mainly present at high frequencies. Therefore, the largest benefit of the wavelet filtering method can be found for these systems at higher frequencies. In classic ILC, when learning up to high frequencies, the error and feedforward signal become dominated by noise while the convergence criterion is still met. The wavelet filtering method makes it possible to learn up to high frequencies and derive a feedforward signal that contains almost no disturbances.

The used motion system is a printer set-up and is discussed in Section 6.1 together with the design of the feedback controller, the learning filters, and the wavelet filter settings. In Section 6.2, simulation results are presented to validate the disturbance analysis of Section 3. The wavelet filtering method is tested by means of simulations in Section 6.2 for both non-repetitive harmonic and stochastic measurement disturbances. The results of the experiments on the printer set-up are discussed in Section 6.3.

6.1. System and filter design

The printer set-up is shown in Figure 5. The printhead is driven using a motor and a driving belt. The position of the printhead is measured using a linear incremental encoder with a resolution of 40 $\mu$m. The measured FRF of the motor to the position of the printhead is shown in Figure 6. Based on this FRF a system model is identified using a least-squares fit, the FRF of which is also shown in Figure 6. The model is used as a system model for the simulations and for the design of the feedback controller and the learning filters.

In order to stabilize the system during the tracking experiment, a feedback controller $C(s)$ consisting of a lead filter is designed, which results in a bandwidth of the controlled system of
The designed controller can be written as

\[ C(s) = 8 \times 10^{-4} \frac{0.3s/\pi + 1}{s/(30\pi) + 1} \] (24)

The controller is discretized using a Tustin discretization method with a step size of 1 ms. The learning filter \( L \) of Figure 7 is designed using a discrete model of \( S_P \) and the zero phase error tracking control (ZPETC) method [18]. In order to satisfy the convergence criterion (3), an eight zero-phase order low-pass \( Q \) filter with a cut-off frequency of 120 Hz is used. The reference signal, used for both simulations and experiments, is a third-order reference trajectory that consists of two movements over 7000 encoder counts (28 cm), as shown in Figure 8.

The wavelet function is chosen as described in Section 5.1. For the motion system considered in this paper, the optimal wavelet function was found to be the ’db38’ wavelet function [21]. Simulations show that for the presented wavelet filtering method the chosen family of wavelet filters influences the results only slightly, provided the wavelet filters have enough vanishing moments. The PSD of the reference signal is shown in Figure 8, the desired maximum frequency of the approximation coefficients \( f_{\text{des},A} = 3 \) Hz. Based on (12) with \( f_s = 1 \) kHz, the
decomposition level $p = 7$. The adjustment of the wavelet coefficients according to (18) is performed with a constant threshold level of $\gamma_c = 5 \times 10^{-2}$, i.e. 5% deviation.

During successive iterations, the initial conditions for each run must remain the same. Here, for both simulations and experiments only zero initial conditions are considered.

### 6.2. Simulations

The performance of ILC without additional disturbances is tested in a simulation with 25 iterations. For the first iteration, a zero feedforward is used. The maximum absolute tracking errors of the 25 iterations are shown in Figure 9. The tracking error converges in five iterations to $|e|_{\text{max}} = 4.5486 \times 10^{-3}$ counts. The learned feedforward signal of the 25th iteration is shown in Figure 10.
6.2.1. Harmonic disturbances. The derived error expression (4) will be validated for harmonic measurement disturbances $d_k$ with a frequency of 45 Hz, which is chosen below the cut-off frequency of the $Q$-filter. The amplitude equals five counts and is chosen such that it exceeds the magnitude of the final tracking error of the ILC simulation without additional disturbances. The influence of measurement disturbances on the tracking error of ILC follows from (4) as

$$e_k,d = -Sd_k + S_PQLSd_{k-1} + \sum_{j=1}^{k-2} S_PQ[Q(1 - LS_P)]^{k-j-1}LSd_j$$  \hspace{1cm} (25)$$

Table I shows the maximum absolute calculated errors using the magnitudes $|S| = 1.0709$, $|S_PQLS| = 1.0254$ and $|\sum_{j=1}^{23} S_PQ[Q(1 - LS_P)]^{k-j-1}LSd_j| = 0.2622$ at the disturbance frequency of 45 Hz.

The tracking errors of a worst case simulation with two iterations, i.e. the disturbances have opposite sign, are shown in Figure 11. The amplification of the error is caused by the presence of the non-repetitive disturbance signal in the learning scheme [23, 24]. The calculated errors show a good correspondence to the errors obtained by simulation as shown in Table I.
The maximum absolute errors for a simulation with 25 iterations are shown in Figure 12 by the solid line. The initial phase of the disturbance signals $\phi_{0,k}$ is chosen randomly between 0 and $2\pi$ rad. The largest maximum error of the iterations $k > 5$ equals 11.1537 counts, which agrees with the calculated value as can be seen in Table I. The limited influence of disturbances older than $k - 1$ can also be seen in Table I.

The results of the simulations with wavelet filtering are shown in Figure 12 by the dashed line. The wavelet filtering method removes the non-repetitive disturbances from the tracking error, leading to a maximum error which depends only on the disturbance signal of the current iteration, i.e. $|e_{\text{max}}| = ||S||_\infty|d|_{\text{max}} = 5.3545$ counts. The maximum absolute errors without wavelet filtering fluctuate due to the presence of the non-repetitive disturbances in the learning process. If the disturbance signals have a small phase difference, the disturbances are almost repetitive over the two iterations. The learned feedforward signal with the disturbances then cancels the disturbances in the next iteration and the resulting error can incidently become smaller than $||S||_\infty|d|_{\text{max}}$. The dotted line in Figure 12 shows the results for simply averaging the error signals of the two runs. With averaging the error still becomes larger than $||S||_\infty|d|_{\text{max}}$ since the non-repetitive disturbances are not completely removed from the learning scheme.

The PSDs of the learned feedforward signals, see Figure 13, show the reduction in the learned feedforward signals for the wavelet filtering case. The disturbance frequency of 45 Hz is removed by the wavelet filtering. The PSD of the learned feedforward signal with averaging shows that the non-repetitive disturbance of 45 Hz is still present in the learned feedforward.

### Table I. Maximum absolute errors with measurement disturbances calculated using (4) and obtained by simulation for various iterations $k$.

<table>
<thead>
<tr>
<th>Method</th>
<th>$k = 1$</th>
<th>$k = 2$</th>
<th>$k = 25$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculated</td>
<td>5.3545</td>
<td>10.4815</td>
<td>11.7925</td>
</tr>
<tr>
<td>Simulation</td>
<td>5.3563</td>
<td>10.3497</td>
<td>11.1537</td>
</tr>
</tbody>
</table>

Figure 11. Errors first iteration (solid) and second iteration (dashed) for the worst case simulation.
6.2.2. Stochastic disturbances. In practice, disturbances are often of a stochastic nature, i.e. random processes. The measurement disturbances used in the following simulations are uniformly distributed random signals in the interval $\frac{\text{max}(c_i)}{C_0} \leq \frac{\text{count}}{C_1}$. For the stochastic disturbances, the root-mean-square (rms) values are analysed instead of the maximum absolute errors. The rms values of the errors for the simulations with random disturbances are shown in Figure 14. With wavelet filtering the error converges to a smaller value than without wavelet filtering. The rms value of the error with wavelet filtering is also smaller than the rms value of the error obtained with averaging over two subsequent runs.

The learned feedforward signal without wavelet filtering (see Figure 15) is dominated by the random disturbances. The learned feedforward signal with averaging still contains a significant amount of the random disturbances, though less than without averaging. With wavelet filtering the learned feedforward signal shows a good correspondence with the optimal feedforward signal of Figure 10 and almost no random disturbances. Figure 16 shows the PSDs of the learned feedforward signals. It can be seen that the averaging method reduces the noise only by...
a small amount (dotted line in Figure 16). The noise reduction of the wavelet filtering method can clearly be recognized.

The wavelet filtering method is not able to remove the disturbances entirely from the error signal. The random disturbances have frequency components at all frequencies, so also in the frequency band where the repetitive part of the error is located. Because of the scaling of the threshold $\gamma$ with the variable part $\gamma_{\text{var}}$, the repetitive part is included in the reconstruction if the non-repetitive part does not dominate the error. The reconstructed error signal and consequently the learned feedforward signal will still contain a small part of the disturbances.

6.3. Experiments

Experiments are performed using the printer set-up of Figure 5 with the same feedback controller $C(s)$, learning filter $L(s)$, robustness filter $Q(s)$, and wavelet filter settings as used for
In order to achieve equal initial conditions for the different trials, a homing sensor is used.

The rms values of the errors obtained in the experiments with and without wavelet filtering are shown as a function of the iteration number in Figure 17. The error without wavelet filtering converges to a value of approximately eight counts. With wavelet filtering, the error converges to approximately three counts. The errors obtained in the experiments are larger than the errors of the simulations. This is probably due to the presence of friction in the experiments which is not taken into account in the model used for the simulations.

The learned feedforward signals with and without wavelet filtering are depicted in Figure 18. The learned feedforward signal without wavelet filtering is dominated by noise. With wavelet filtering significantly less noise is present in the feedforward signal. The PSDs of the feedforward signals (Figure 19) show that the reduction of the wavelet filtering is located mainly at high frequencies. The experiments show the practical applicability of the proposed method.
In this paper, the influence of non-repetitive disturbances on ILC is addressed. A wavelet-based prefiltering method is proposed to eliminate the non-repetitive disturbances from the learning scheme. The proposed method improves the tracking performance significantly.

The effects of non-repetitive disturbances on ILC are often not included in the analysis. However, the derived error expression shows that the non-repetitive disturbances, both load and measurement disturbances, can cause an amplification of the tracking error of up to a factor 2. The amplification is mainly determined by the disturbances of the last two iterations of ILC.

The wavelet-based prefiltering method identifies and removes the non-repetitive disturbances by a comparison of two realizations of the tracking error at an equal iteration of ILC. Simulations show that both deterministic and stochastic non-repetitive disturbances are almost entirely removed from the learning scheme.

Experimental results on an industrial printer set-up show a reduction of the tracking error of 62% with the proposed wavelet prefiltering method. Moreover, the learned feedforward signal contains significantly less non-repetitive disturbances.

Figure 18. Feedforward signals of the experiments without and with wavelet filtering.

Figure 19. PSDs of the learned feedforward signals in the experiments without wavelet filtering (solid) and with wavelet filtering (dashed).

7. CONCLUDING REMARKS

In this paper, the influence of non-repetitive disturbances on ILC is addressed. A wavelet-based prefiltering method is proposed to eliminate the non-repetitive disturbances from the learning scheme. The proposed method improves the tracking performance significantly.

The effects of non-repetitive disturbances on ILC are often not included in the analysis. However, the derived error expression shows that the non-repetitive disturbances, both load and measurement disturbances, can cause an amplification of the tracking error of up to a factor 2. The amplification is mainly determined by the disturbances of the last two iterations of ILC.

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Experimental results on an industrial printer set-up show a reduction of the tracking error of 62% with the proposed wavelet prefiltering method. Moreover, the learned feedforward signal contains significantly less non-repetitive disturbances.
The proposed method removes both low- and high-frequent disturbances which makes it possible to include higher frequencies in the learning process, i.e. the cut-off frequency of the $Q$-filter does not have to be lowered to eliminate non-repetitive disturbances.

The choice of the wavelet function can influence the results of the proposed method. For the suppression of the non-repetitive disturbances, a wavelet function with enough vanishing moments should be chosen. The relation between disturbance classes and the choice of the wavelet family will be studied further in the future. For the presented method in combination with deterministic and stochastic disturbances, simulations show that the chosen family of wavelet filters does not have a large influence on the results.

A drawback of the proposed method is that two runs are required in each iteration of ILC. Since the update of the feedforward signal by the $L$ and $Q$ filters is known, a recursive formulation can perhaps be made which eliminates the need for an additional run. This recursive formulation is subject for future research.

Future work also includes the search for alternative methods than wavelet analysis to identify the different disturbances. One possibility is the use of independent component analysis (ICA) to identify disturbances from different sources.

REFERENCES
ITERATIVE LEARNING CONTROL WITH WAVELET FILTERING