Rotor Dynamics of a Centrifugal Pump

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1 Objective

Several forces act on a centrifugal pump when it is operational; like fluid-induced forces but also forces with a mechanical background. These forces originate due to imperfect balance (like a slight bent in the shaft or a mass eccentricity), design faults, and bearings. These forces may grow to an unacceptable level when the rotor is operational near its natural frequency. Every system has its own natural frequency. A characteristic of this frequency is that little energy is required to give rise to enormous amplitudes.

The centrifugal pump, which is considered in this report, is built in an experimental setup at the Technical University of Eindhoven to investigate the influence of a suction flow on the forces on the shaft of a mixed-flow water jet pump. It is a part of the investigation of Dr. B.P.M. van Esch. Former tests resulted in unexpected outcome. When blade excitation forces are scaled and plotted for different rotational speeds it is expected that the forces are coinciding with each other. Only this was not the ease (see figure 1). One possible cause is the influence of the dynamical response of the shaft on the measurements.

When steady forces in a stationary frame are drawn in a similar figure, the forces correspond better with each other. Apparently, in this range of the rotor speed (0 - 11.7 Hz / 0 - 700 rpm) the dynamic response is constant. Only one resonance is expected to originate at rotational speed with amplitude matching to the gravity force of the impeller (~130 N). To account for an explanation of the dynamic response of the blade excitation forces the resonances at higher frequencies is to be sought after.

The response of a centrifugal pump is usually a result of different mechanical sources. It gives rise to oscillations in the rotor system. Lots of study is performed to describe the dynamic response of rotating machinery, compiled under the name 'Rotor Dynamics'. This can be used to give insight, interpret and explain the originated vibrational motions.

The subject of this report is determining, by using force measurements, the dynamic response of a centrifugal pump in operation. To what extent does the dynamic response of the shaft influence the measurements of the suction flow and how can results be corrected for this. Subsequently the question is how the resonance frequency changes in attendance of fluid.

To archive the object, chapter two compiles a simple model of a rotor system. This includes whirl movements, equations of motions of a simplified centrifugal pump model, torsional vibrations and the influence of bearings on the response of a rotor system. Insight and understanding is created of possible arising movements of a shaft.

Chapter 3 is devoted to define the dynamic response of the centrifugal pump. The chapter is subdivided into three parts. First the used method ‘Modal Analysis’ is explained. After this a test plan is given how to perform experiments with which the full dynamic response of a centrifugal
pump can be assessed. In the last paragraph measurements are made and discussed. First, experiments are made with a centrifugal pump operating in air. To research the influence of liquid, these results are compared with the results obtained by experiments in water. A Cascade plot is a common representation to visualize the results of the experiments. Finally a conclusion is formulated which will answer the objective of this report.
2 Whirl of simple rotors

In rotor dynamics a common phenomenon is whirling; it is one of the most important self-excited vibrations. Often it can be observed in practice, which makes this physically easy to interpret.

Self-excited vibrations are related to the instability of the rotary system. If a rotor operates near its natural frequency, the oscillations swept up and the system becomes unstable. Self-excited vibrations therefore may lead to destructive stresses. These will finally result into fatigue failure of materials.

Whirling is a lateral vibration and originates due to an energy supply by a uniform source. One well known cause is unbalance. Unbalance creates a centrifugal force which has an excitation frequency corresponding to the rotor speed. It is generally present since it is practically impossible to produce a perfectly balanced rotor. Another source is gravity. The presence of this source can be noted when, for instant, the rotor-bearing has an asymmetric stiffness. In this case gravitational forces will influence the dynamic response of the rotor system.

To get insight into the dynamic response of rotor system, it is important to fully understand the whirling effect. First a simplified rotor model is discussed by introducing a Jeffcott rotor. Then the model is adapted to a centrifugal pump and equations of motions are determined. Whirling motions are not the only self-excited vibrations, also torsional vibrations may not be neglected in rotor-systems. Therefore paragraph three is devoted to these oscillations. Finally the dynamic response of a centrifugal pump depends on the behavior of suspending bearings. The last paragraph will discuss the influence of these bearings.

2.1 Basic Principles of Rotor Dynamics

This paragraph will treat a simple model of a Jeffcott rotor. With this model the main motions are clarified. First the model shall be discussed after which an interpretation of the response is given. The next sub-paragraph deals with the influence of gravity of the whirling motion of the disk. Finally orbit movements are considered.

2.1.1 Jeffcott rotor

A Jeffcott rotor model is often used to explain the whirling effect. A rotor system is modeled as a massless, elastic shaft supported by two non-flexible bearings. A disk with a known mass unbalance is placed in the middle. The rotor system is rotating with a constant rotational speed, \( \Omega \). (See figure 2)
Due to the rotating unbalance, the geometrical centre of the disk traces a path, the so-called whirl orbit. This whirling motion has the same frequency as the rotational speed and is called therefore synchronous. Sometimes non-synchronous vibrations also occur in rotor dynamics. This can be interpreted as if the disk with a mass unbalance is rotating with its own frequency which does not coincide with the frequency of the orbit. This is illustrated in figure 3. In this figure represents 'O' the origin of the shaft, 'G' the geometric centre of the disk and the black dot the mass unbalance. Non-synchronous vibrations are independent of the rotational frequency. Most forced vibration responses are created by a non-synchronous excitation.

![Figure 3 Difference between synchronous whirl (a) and Non-synchronous whirl (b)]

In a synchronous motion the angle $\beta$ between the mass unbalance and the rotating whirl-axis remains constant. The rotational speed of the whirl, $\phi$, is equal to the angular velocity of the rotor, $\Omega$. This is not the case for a non-synchronous vibration. The angle $\beta$ is changing in time. Here the rotational speed of the rotor is the sum of the whirl-speed $\dot{\phi}$ and $\dot{\beta}$.

### 2.1.2 Interpretation response

Typical plots of amplitude of the dynamic response of a Jeffcott rotor are given in figure 4. The horizontal and vertical axis represents the dimensionless rotational speed and amplitude respectively. The angular velocity is normalized with the natural frequency, $\omega_0$. The whirl amplitude is made dimensionless by dividing it by the radius of the unbalance, $e$. First the amplitude of the synchronous whirl is growing with increasing rotational speed until a maximum is reached. The rotational speed that corresponds with this maximum is called as the critical rotor speed. This frequency corresponds with the natural frequency of the system. At higher shaft speed the rotor becomes supercritical. The amplitude decreases continually till a stationary point is reached. This point is equal to the radius of the mass unbalance, $e$. In practice this can be used for balance a rotor. It implies that for high rotor speed the synchronous whirl can be made arbitrarily small by reducing this unbalance radius $e$ by putting an additional mass opposite to the mass unbalance. A more attractive method is of course trying to smooth out the unbalanced mass.

![Figure 4 Amplitude response, influence of increasing damping-ratio $\zeta$]
Damping acts on the synchronous whirl amplitude, for increasing damping diminishes the amplitude around the critical rotor speed. This is shown in figure 4. The peak is more smoothed.

In the phase diagram the angle changes when the rotor reaches the critical speed. For low frequencies the response is in phase, the angle $\beta$ is zero and the mass unbalance is in the prolongation of the line OG. At the critical point the angle becomes 90 degrees. The mass unbalance is now tangent to the orbit. By further incrementing of the angular velocity, the mass unbalance point, M, falls inside the orbit of the geometrical centre G. When the phase attains 180 degrees, the point M coincides with the centre of the orbit. It appears the geometrical centre G now is rotating around the mass centre M. This is called the critical speed inversion. See figure 5.

![Figure 5 Interpretation phase response](image)

Here the dimensionless rotational speed is given by: $\Omega^* = \frac{\Omega}{\omega_n}$.

Three methods exist to decrease the whirl amplitude:

1. Balancing the rotor
2. Adapting the operating frequency range (the operating speed must be sufficiently away from the critical speed)
3. Adding damping to the rotor-bearing system

Damping can be internal or external. Internal damping includes hysteresis in the shaft and friction in the bearings. This kind of damping may just lead to instabilities. There is a change these have a negative influence and may lead to self-excited, non-synchronous vibrations.

### 2.1.3 Gravity

The dynamic response also depends on gravity. This constant radial force acts continuously on the rotor. In the rotating whirl frame the excited vibrations will respond in the same frequency as the revolutions of the shaft. When the force is transformed into stationary frame, the expected responses can be split up into two parts: a static deflection and a vibrational component with a frequency equal to twice the revolution speed due to mathematical results.

The influence of gravity is especially important when the stiffness in the rotor is anisotropic. This means that the rotor system has a different stiffness in the x and y direction. Anisotropic bearings, asymmetric shaft or a shaft with a crack are examples which will lead to typical responses. Gravity force will interfere with the anisotropic rotational stiffness of the rotor. In a rotor vibrational spectrum this will cause a higher harmonic resonance at twice the shaft frequency. This will be especially noted during transition speed of the rotor. When the rotor reaches half the main critical speed the amplitude will be swept up.

When a centrifugal force (as consequence of a mass unbalance) and a gravitational force act at the same moment on the anisotropic rotor system, a second curl will originate in the whirl motion as observed from a stationary frame (see figure six). The combined influence is obtained by addition of the values of the separate forces.
Where the first term represents the response caused by the unbalance force and the second term due to gravity forces. So the geometrical interpretation of the combined influence of gravity and mass unbalance is sum of three vectors: a vector (OP) of constant displacement, a vector (QR) belonging to the mass unbalance which rotates with the rotational speed, $\Omega$, and finally a vector (PQ) with a frequency of twice the rotational speed corresponding with the gravity force. The size of the orbits is determined by the relationship between the lengths of these vectors and so by the amplitude of these forces.

Remark:
- The originated extra loop emerges for every constant radial force. Misalignment of the rotor, cracks and also fluid related forces are sources that originate vibrations at twice the rotational speed.
- Generally the constant radial force generates a dominant vibration at a frequency of twice the rotational speed in the stationary frame. This is not always the case, sometimes also vibrations of three or four times the rotational speed originate. (For an overview of possible frequencies originated in rotor systems a troubleshooting chart is given in appendix D)
2.1.4 Orbits

An orbit reflects the actual geometric path of the rotor centerline when the rotor is operational and/or when external forces are working on the rotor system. These can be ‘clockwise’ or ‘counterclockwise’ with the rotational speed. This is also called a forward whirl, where the movement of the orbit is in the same direction as the rotor speed, and backward whirl, where the whirl rotational is in the opposite direction.

‘Forward’ and ‘backward’ motion can clearly be explained in figure 7. Here a Jeffcott model is presented with two anisotropic bearings. This will lead to two responses in a bode diagram. The critical speeds are represented in the figure as $\omega_{nx}$ and $\omega_{ny}$. The stiffness in the x-direction is here lower than the stiffness in the y-direction, hence $\omega_{nx} < \omega_{ny}$. When the rotor speed is smaller than $\omega_{nx}$ or exceeds $\omega_{ny}$, the whirl response will give a forward whirl. When the rotor operates between those critical frequencies, the geometric center will rotate opposite to the rotational speed, resulting in a backward whirl.

When damping is included in the model the backward and forward responses remain intact, but the transition speed from forward to backward, or vice versa, will deviate slightly from the two critical speeds due to the more erased peak.

![Figure 7 Unbalance response of a simple rotor with anisotropic bearings](image)

Remark:
- This phenomenon of transition from forward to backward whirling is not generally obligate to happen when the rotational speed transfers over a critical speed.

Due to combined forces the orbit may deviate from a cylindrical or elliptical shape to more complex orbits. This was shown before when gravity and mass unbalance acts on the rotor system. Here an additional loop arises in the whirl motion. With aid of the equations of motions, which are written in independent equations, the motion of each independent normal mode can be determined. When these two motions in the x- and y-direction are plotted, a common figure may rise, known as Lissajous figures. The motions may also be written in polar coordinates. This form represents important families of curves.

$$ r = r_f e^{j\theta} + r_p e^{-j\theta} $$

(3.2)
Examples are the epicycloids and hypotrochoids. These are singled out below. In Appendix A more different families of curves are given which can be observed in practical rotors.

a) Epitrochoids and Hypotrochoids

A cycloid motion is obtained by an imaginary fixed point $P$ on the circumference of a small circle (radius $a$) which is rotating without slipping in another circle (radius $A$). If the circle ‘$a$’ is rotating outwards circle ‘$A$’, an epicycloids result. If the circle rolls inward circle $A$ the curve becomes a hypocycloid. When the fixed point translates on the radius (inward or extended), the curve develop a loop. These obtained curves are entitled as epitrochoid and hypotrochoid. \[6\] The new originating movements are described respectively as:

Epitrochoid:  
\[ r_e = (A+a)e^{j\Omega t} - ce^{j\left(\frac{A+a}{a}\right)\Omega t} \]  

Hypotrochoid:  
\[ r_h = (A-a)e^{j\Omega t} + ce^{j\left(-\frac{A-a}{a}\right)\Omega t} \]

Where $c$ is the distance calculated from the centre of circle ‘$a$’ to the fixed point $P$.

\[ \text{Figure 8 (a) Epitrochoid and (b) Hypotrochoid} \]

A general expression of this equation is  
\[ r = r_1e^{j\Omega t} + r_2e^{j\left(\frac{n}{m}\right)\Omega t} \]  

Here the ratio $\frac{n}{m}$ is a rational faction, $m$ and $n$ having no common factor.
From these equations some observation can be drawn [4]:

1) The equation (3.3) or (3.4) can be interpreted as a combined whirl motion of a circle of radius \((A+a)\) or \((A-a)\) rotating forward at the rotational speed \(\Omega\) and another circle of radius \(c\) with an angular velocity of \(\Omega' = \frac{n}{m}\Omega\). The whirl direction of this second movement depends on the sign of \(\Omega'\).

2) An epitrochoid is obtained from equation (2.3) when a forward whirl motion is combined with a forward synchronous whirl. The rotational speed \(\Omega'\) is larger than the rotational speed of circle with radius \((A+a)\); the relative speed \(\Omega' - \Omega\) is forward.

3) If the angular velocity of radius \(c\) is opposite to the rotational speed \(\Omega\), the obtained curve from equation (2.4) results in a whirl motion called hyp trochoid. The relative speed \(\Omega' - \Omega\) is a backward motion.

4) If the ratio \(\frac{n}{m}\) is a rational factor, the whirl motion with \(|m-n|\) cusps is closed after \(m\) cycles. If the ratio is irrational, the curve never closes.

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**Bentley Nevada**

Bentley Nevada is a company which provides instrumentation special for testing mechanical condition (e.g. proximity transducers). They pay special attention in resource of orbiting. Three times a year Bentley Nevada publish a technical magazine ORBIT in which several aspects are published which are interesting for rotor dynamics; for instance the practice of condition monitoring, diagnostics, and performance optimization. [10]

### 2.2 System model

A Jeffcott rotor model is a simple representation of a rotor system. A disadvantage is that gyroscopic and inertia effects are not taken into account. In fact, due to gyroscopic and inertia effects rotational movements are possible. The Jeffcott rotor is more a model of non rotating simple structures subjected to rotational excitation. The rotational related terms in the equation of motion are the result of coordinate transformation and are therefore mathematical. A Jeffcott model may not be used to predict the motions of a rotorsystem.

In this paragraph theoretical equation of motions for a simplified system are defined by use of the Lagrangian equations. With this equation it is possible to define a lumped parameter model and determine the theoretical critical frequency. [13] [14] [5].

First the centrifugal pump is approximated by a simple model. Thereafter the Lagrangian equation is solved step by step. The result is a common equation of motion which represents a lumped parameter model.

#### 2.2.1 Simplified model

To define the equations of motions, first a rotor of a centrifugal pump is approximated by a simple model. The pump is modeled as shaft with an overhung rigid disk. The system is supported by two bearings. (See figure 9). In the rigid disk is an unbalance recognized. Furthermore it is supposed that the shaft will rotate at constant speed and is symmetric.
2.2.2 Lagrangian equations

Many mechanical systems contain oscillations. These vibrations are small but have influence on the dynamics of the total system. The differential equations of motion for a vibrating system can be derived in terms of energy, like kinetic and potential energies. These formulations are better known as the Lagrangian equations. This formula is represented in equation (2-6).

\[
\frac{d}{dt} \frac{\partial T}{\partial q_n} - \frac{\partial T}{\partial \dot{q}_n} + \frac{\partial V}{\partial q_n} = (Q^{nc})^T
\]

\( (3.6) \)

Where

- \( T \) = total kinetic energy of system
- \( V \) = total potential energy of system (like gravity and spring forces)
- \( q_n \) = generalized coordinate
- \( Q^{nc} \) = non-conservative forces, forces not directly related to the potential energy of the system
2.2.3 Kinetic energy

The total kinetic energy of a rotor system involves the motions of the shaft, the disk and the possible unbalance. Yet it is unknown where the unbalance is located exactly. Therefore, in this theoretical model, only the additional originating force is taken into account. The kinetic energy is only estimated by the dynamic motion of the shaft and the disk.

\[ T = T_{\text{disk}} + T_{\text{shaft}} \]  \hspace{1cm} (3.7)

The motion of the rotor is expressed in figure 9. Where \( u, v, \) and \( w \) reflects the motion in \( e_x, e_y \) respectively \( ez \)-direction.

Besides pure radial or axial movements the disk is also tilted due to the bending shaft. Looking to a body reference frame, the motion of a tilted disk can be defined as two superimposed rotations; \( \theta_x \) and \( \theta_y \). The kinetic energy of a disk can be expressed as:

\[ T_{\text{disk}} = \left\{ \frac{1}{2} M (u^2 + v^2) + \frac{1}{2} I_1 (\dot{\theta}_x^2 + \dot{\theta}_y^2) + \frac{1}{2} I_p L \Omega^2 - I_p \Omega (\dot{\theta}_x \dot{\theta}_y) \right\} \]  \hspace{1cm} (3.8)

Where \( M \) is the mass of the rigid disk, \( I_1 \) the diametric mass moment of inertia and \( I_p \) the polar mass moment of inertia. These are supposed to be constant.

The first and second terms are the classical expressions for the energy due to the pure translational and rotational kinetic energy of the rigid body. The term \( \frac{1}{2} L \Omega^2 \) defines the kinetic energy due to the disk rotates at the rotational speed, \( \Omega \). Because it is assumed that the rotational speed of the shaft is constant, this term will appear later to have no influence in the equation of motion. Finally, the last term represents the gyroscopic effect.

The kinetic energy of the shaft can be derived in the same manner:

\[ T_{\text{shaft}} = \int_0^L \frac{1}{2} \rho z A (u^2 + v^2) dz + \int_0^L \frac{1}{2} J_1 (\dot{\theta}_x^2 + \dot{\theta}_y^2) dz + \frac{1}{2} J_p L \Omega^2 - \int_0^L J_p \Omega (\dot{\theta}_x \dot{\theta}_y) dz \]  \hspace{1cm} (3.9)

Where \( \rho \) is the density, \( A \) in the cross-sectional area of the shaft, and \( J_1 \) and \( J_p \) the diametric and polar inertia of mass from the shaft respectively.

The first integral of the kinetic energy reflects the energy due to bending of the shaft. The second term is known as the secondary effect of rotatory inertia, also noted as a Timoshenko beam [3]. The last term also originates from the gyroscopic effect.

When equations (2.8) and (2.9) are summed, the total kinetic energy of the system is calculated.

2.2.4 Potential energy

Besides kinetic energy also potential energy has an influence on the dynamics of the system. The potential energy is the sum of all internal energy of the system and the energy of conservative forces; like gravity, spring or magnetic field forces. For a rotor system the potential energy is expressed as the strain energy of the deformation of the shaft, the shear energy (Timoshenko beam) and the spring forces from the bearing working on the shaft.

The potential energy can be expressed as:
\[ V = U^{in} + V^{ex}, \] (3.10)

Where
\[ U^{in} = \frac{1}{2} \int_0^L \left( \frac{\partial^2 u}{\partial z^2} \right)^2 + \left( \frac{\partial^2 v}{\partial z^2} \right)^2 \, dz + \frac{1}{2} L kG \left( \varphi_x^2 + \varphi_y^2 \right) \]

\[ V^{ex} = \left( -F_{1x}u - F_{1y}v - M_{1x} \frac{\partial v}{\partial z} - M_{1y} \frac{\partial u}{\partial z} \right) \delta(z - l_1) + \left( -F_{2x}u - F_{2y}v - M_{2x} \frac{\partial v}{\partial z} - M_{2y} \frac{\partial u}{\partial z} \right) \delta(z - l_2) \]

In the equation of internal energy, \( U^e \), \( E \) represents the modulus of elasticity, \( G \) the shear-modulus and \( k \) the shear coefficient. These are assumed to be constant. \( A \) and \( I \) are the cross-sectional area and moment of inertia of the shaft. The first term is related to the shaft bending, the second to shearing. The shear angle is given by \( \varphi \).

Usually the bearings are modeled as a couple of springs and dampers. The forces and moments which arise due to the spring force together form the external conservative forces in the potential energy, \( V^{ex} \). In the equations of external conservative forces, \( \delta \) represents the Dirac delta function.

### 2.2.5 Non-conservative forces

When a centrifugal pump is operational, different hydrodynamic forces work on the impeller as consequence of the fluid. These hydrodynamic forces are non-conservative forces. Also dissipative energy from the damping force in the two bearings is non-conservative. Finally mass unbalance generates an additional centrifugal force. Yet these non-conservative forces are unknown and shall be represented as \( F_{\text{hydrodynamic}} \), \( F_{\text{unbalance}} \), and \( W_{\text{damping}} \). Here the \( W \) stands for the virtual work. Since the virtual work is given by \( W^{nc} = \delta q^T \dot{Q}^{nc} \), the non-conservative force due to damping can be calculated. (See paragraph 3.2.5. for the denotation of \( q \)). All these non-conservative forces have a value and a direction. Therefore they are rendered in a force vector.

\[ Q^{nc} = F_{\text{hydrodynamic}} + F_{\text{damping}} + F_{\text{unbalance}} \] (3.11)

### 2.2.6 Generalized coordinates

The generalized coordinates are given by \( u, v, \theta_x \) and \( \theta_y \). The lateral displacements are divided in a part only depending on the z-coordinate and a part depending on time. The angular displacements \( \theta_x \) and \( \theta_y \) are assumed to be small, therefore they are approximated by the derivative of \( u \) respectively \( v \) in the z-direction. Thus the only independent coordinates are \( u \) and \( v \). These are finally used as the generalized coordinates: \( q_1 \) and \( q_2 \).

\[
\begin{align*}
\dot{u}(z,t) &= f(z)q_1(t) \\
\dot{v}(z,t) &= f(z)q_2(t) \\
\theta_x(z,t) &= \frac{\partial u}{\partial z} = -\frac{df(z)}{dz}q_1 = -g(z)q_1 \\
\theta_y(z,t) &= \frac{\partial v}{\partial z} = \frac{df(z)}{dz}q_2 = g(z)q_2
\end{align*}
\]

The coordinates are depending on the position \( z \) on the shaft. The oscillation motion is approximated by a sinusoidal function. Normally different orders of vibration act at the same time. It is demonstrated that these vibrations are linear and can be superimposed \[4\] \[8\]. A dynamical motion of the rotor-bearing system is expressed in figure 11.
This approximated sinusoidal motion of the shaft can than be prescribed as:

\[ f(z) = A \sin\left(\frac{n \cdot \pi}{l_2 - l_1} z - l_1\right) + s \]  

(3.12)

Where \( n \) stands for the higher order vibration and \( s \) is for pure radial displacement.

The velocity can also be represented in generalized coordinates:

\[
\begin{aligned}
\dot{u}(z,t) &= f(z)q_1(t) \\
\dot{v}(z,t) &= f(z)q_2(t) \\
\dot{\theta}_x(z,t) &= g(z)q_1(t) \\
\dot{\theta}_y(z,t) &= g(z)q_2(t)
\end{aligned}
\]

Where

\[ g(z) = \frac{\partial f(z)}{\partial z} = \frac{n \cdot \pi}{(l_2 - l_1)} \cdot A \cos\left(\frac{n \cdot \pi}{l_2 - l_1} z - l_1\right) \]  

(3.13)

The acceleration is given as:

\[
\begin{aligned}
\ddot{u}(z,t) &= f(z)\ddot{q}_1(t) \\
\ddot{v}(z,t) &= f(z)\ddot{q}_2(t) \\
\ddot{\theta}_x(z,t) &= h(z)\ddot{q}_1(t) \\
\ddot{\theta}_y(z,t) &= h(z)\ddot{q}_2(t)
\end{aligned}
\]

Where

\[ h(z) = \frac{\partial^2 f(z)}{\partial z^2} = -\left(\frac{n \cdot \pi}{l_2 - l_1}\right)^2 A \sin\left(\frac{n \cdot \pi}{l_2 - l_1} z - l_1\right) \]  

(3.14)

### 2.2.7 Equations of motions

To solve the Lagrangian equation some assumptions are made. This theoretical model supposes a perfectly state although in normal use this can not be guaranteed. For instant due to friction or internal deformation of the shaft, the disk would not have a constant rotational speed. Although, in this model it is presumed that the shaft is rotating at constant speed. Furthermore
the deformation in the $e_z$-direction will be much lower than the deformation in the $e_x$ and $e_y$-direction and is therefore neglected.

First the kinetic and potential energy are specified in generalized coordinates. Subsequently the derivate is taken in the generalized coordinates $q_1$ and $q_2$. This is presented in appendix B.

The result is a common general representation of equation of the dynamic response of a rotor. This general formulation is a lumped parameter model which can describe almost every system.

$$
M\ddot{q}(t) + D\dot{q}(t) + Kq(t) = \ddot{Q}^{nc}
$$

(3.15)

Where:
- $q(t)$ = Generalized coordinates, the structural degrees of freedom
- $Q^{nc}(t)$ = Non conservative forces
- $M$ = Mass matrix
- $D$ = Damping matrix
- $K$ = Stiffness matrix

$$
q(t) = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} u \\ v \end{bmatrix};
$$

$$
M = \begin{bmatrix} \hat{M} & 0 \\ 0 & \hat{M} \end{bmatrix};
D = \begin{bmatrix} 0 & \Omega \hat{G} \\ \Omega \hat{G} & 0 \end{bmatrix};
K = \begin{bmatrix} \hat{k} & 0 \\ 0 & \hat{k} \end{bmatrix};
$$

$$
\ddot{Q}^{nc} = \begin{bmatrix} F_{\text{hydrodynamic}} + F_{\text{damping}} + F_{\text{unbalance}} + \hat{F}_x\hat{M}_y \\ F_{\text{hydrodynamic}} + F_{\text{damping}} + F_{\text{unbalance}} + \hat{F}_y\hat{M}_x \end{bmatrix}
$$

Note that the matrices are depended on rotor speed. The damping matrix is explicitly dependent on this quantity, but also the stiffness and mass matrix are implicitly dependent. As in the article of K.W. Wang et all [13], the rolling element bearing stiffness will appear to decrease as the rotational speed increases. Furthermore also the gyroscopic effect and Inertia effect changes when the rotor speed varies. This will lead to a complicated model.

Evaluation of this model is only possible by a numerical solution. An approach is the Finite Element Method which can determine the response of complex structural mechanics problems. In Matlab such system can be implemented. Dr. ir. Bram de Kraker developed a numerical toolbox entitled RoDy. The program approaches the rotor-bearing system linearly, which for a large class of practical situations is representative. With aid of this program the eigenvalues, eigenmodes, a Campbell plot, Bode- and Nyquist diagram or a animation can be calculated.

____________________

Timoshenko beam theory [3]

When the shaft is subjected to bending also lateral shear force affects the lateral vibration. The Timoshenko beam theory is an analysis of beam vibrations that requires consideration of this shear deflection. The shaft can be divided into several elements. Each element will rotate slightly in addition to its lateral motion when the shaft deflects. Forces must be acting to cause this rotational and for a complete analysis these forces must be considered. In any beam, except one subject only to pure bending, a deflection due to the shear stress in the beam occurs.

The effect of shear forces is small when the curvature of a beam is large relative to its thickness. For a centrifugal pump the influence is only noticed when the shaft is vibrating in higher order modes. The shear angle $\varphi$ can be expressed in the generalized coordinates and finally will include the equations of motions. But for this simplified model it is assumed that this effect is small and may be neglected in the potential energy of the rotor system.

Technical University Eindhoven, February 2006
2.3 Torsional vibrations

As said before, lateral vibrations are not the only secondary motions of the shaft, also torsional vibrations must be taken into account. This paragraph will discuss these important oscillations.

Torsional vibrations are oscillatory angular motions causing twisting of the shaft. It is superimposed on the steady rotational motion. Often these kinds of vibrations are overseen because they are ‘quiet’. Unlike lateral vibrations torsion motions can not be transferred to the machine corpus via supports. Even transmission by air as acoustic waves is not possible. Therefore they are not detected by noise and they can only be observed by specially developed measurement instruments.

Vibrations by torsion are dangerous. The main reason is that torsional vibrations have extremely low damping (roughly 10 times as low as lateral vibrations). Damping is only caused from internal material damping. As consequence amplitudes can grow enormously when the rotor is subjected to periodic excitations. A torsional impulse is long-lasting. Extreme stresses occur and material fatigue will ultimately lead to collapse of the rotor. It is recommended to measure the torsional vibrations especially during transient behavior of the system, like starting-up period. In such case torsional moments and therefore also shear stresses are larger than when the rotor is operating in normal condition.

Torsion motion is related to transmission of torque from the driving load to the end of the rotor. More different causes exists which give rise to torsional vibrations; among which imperfect transmissions, poor bearings, or resistance of the impeller due to hydrodynamic forces.

A property of torsional vibrations is that they are linked with lateral modes. This coupling arises due to rotor unbalance, constant lateral forces, tangential forces (when the rotor is surrounded by liquid for instance) and gyroscopic effects. Therefore not only pure torsional excitation will lead to torsional vibrations, but also typical common lateral excitations. Due to these effects torsional vibrations are always present in rotating machinery.

Dynamical models exist which describe torsional modes. These models have similarities which the equations of motions of lateral vibrations (which are treated earlier). Even with these models the eigenvalues and modes can be calculated and resonance frequencies can be predicted. Often responses of torsional vibrations are related to the rotational speed. Centrifugal forces may lead for instance to once-per-revolution pulsating torsional force when a disk is rotating with a mass unbalance.

The analysis of torsional vibration in rotors is not complicated, only the results are not always satisfactory when a strong coupling predominates between lateral and torsional vibrations. These give rise to non-linear behavior. Sometimes the rotor system may become unstable. As a consequence of the non-linear nature of lateral and torsional coupling, torsional resonance arises not only at the frequency corresponding to the natural frequency, but also at even fractions of these frequencies. For an overview of different sources of excitation of torsional vibrations with their corresponding amplitude and frequency is given in table 1.
### Table 1 Sources of Excitation of Torsional Vibration [3]

<table>
<thead>
<tr>
<th>Source</th>
<th>Amplitude in terms of rated torque</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mechanical</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gear runout</td>
<td></td>
<td>1x, 2x, 3x rpm</td>
</tr>
<tr>
<td>Gear tooth machining tolerances</td>
<td></td>
<td>No. gear teeth x rpm</td>
</tr>
<tr>
<td>Coupling unbalance</td>
<td>1x rpm</td>
<td></td>
</tr>
<tr>
<td>Hooke’s joint</td>
<td>2x, 4x, 6x rpm</td>
<td></td>
</tr>
<tr>
<td>Coupling misalignment</td>
<td>Dependent on drive elements</td>
<td></td>
</tr>
<tr>
<td><strong>System function</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Synchronous motor start-up</td>
<td>5-10</td>
<td>2x slip frequency</td>
</tr>
<tr>
<td>Variable-frequency induction</td>
<td>0.04-1.0</td>
<td>6x, 12x, 18x line frequency (LF)</td>
</tr>
<tr>
<td>motors (six-step adjustable frequency drive)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Induction motor start-up</td>
<td>3-10</td>
<td>Air gap induced at 60 Hz</td>
</tr>
<tr>
<td>Variable-frequency induction</td>
<td>0.01-0.2</td>
<td>5x, 7x, 9x LF, etc.</td>
</tr>
<tr>
<td>motor (pulse width modulated)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Centrifugal pumps</td>
<td>0.10-0.4</td>
<td>No. vanes x rpm and multiples</td>
</tr>
<tr>
<td>Reciprocating pumps</td>
<td></td>
<td>No. plungers x rpm and multiples</td>
</tr>
<tr>
<td>Compressors with vaned diffusers</td>
<td>0.03-1.0</td>
<td>No. vanes x rpm</td>
</tr>
<tr>
<td>Motor- or turbine-driven systems</td>
<td>0.05-1.0</td>
<td>No. poles or blades x rpm</td>
</tr>
<tr>
<td>Engine geared systems with soft</td>
<td>0.15-0.3</td>
<td>Depends on engine design and operating conditions; can be 0.5n and n x rpm</td>
</tr>
<tr>
<td>Coupling</td>
<td></td>
<td>Operating conditions</td>
</tr>
<tr>
<td>Engine geared system with stiff</td>
<td>0.50 or more</td>
<td>Depends on engine design and operating conditions</td>
</tr>
<tr>
<td>Coupling</td>
<td></td>
<td>Operating conditions</td>
</tr>
<tr>
<td>Shaft vibration</td>
<td>N x rpm</td>
<td></td>
</tr>
</tbody>
</table>

### 2.4 Bearings

Because dynamic response of a centrifugal pump is particularly determined by the behavior from bearings, this paragraph is devoted to bearings.

The centrifugal pump exists of a main shaft supported by a revolving outer shaft. Both are supported on separate axial-radial pairs of bearings. (See figure 12). Close to the impeller a radial bearing and a self-aligning single roller thrust bearing is placed. On the other site the shaft is attached by a pure thrust and pure radial bearing and by a double roller, self-aligning thrust bearing. In a double thrust roller the force is established in the axial and radial directions, in the single thrust roller bearing the force can only be transferred in axial direction.
The behavior of bearings is important for the performance of the dynamic response of the total system. Major reason is that the stiffness of a rotor-bearing system is mainly determined by the stiffness of the bearings in series with the bending stiffness of the shaft. Furthermore, the damping is mainly determined by the dampers and sealing. A right choice for the bearing stiffness minimizes the forces that are lead through the shaft. Also it can be chosen to generate so much damping that the system is stabilized, despite unwanted internal damping which is inevitable.

Sometimes bearings can make the system non-linear. This can be created by cross coupling terms. Another reason is that some peculiarities of bearing are influenced by the rotor speed. This depends on which bearing is used. In squeeze film bearings for instance, time is needed to build up a pressure in the thin film layer. Therefore, the damping differs during starting up and during constant speed. Bearings containing liquid depend more on the rotational speed than rolling element bearings, but still these bearings will influence the dynamical response.

The bearings which are used here are rolling-element bearings. An advantage of these kinds of bearings is that they have low friction (even at low speeds), have a high supporting power, are very stiff, have a high exactness and finally indicate the declining condition in advance. Far before failure it can be noticed that the bearings have to be replaced, for instance by occurrence of high frequent vibration. A negative characteristic is that they have low damping.

The forces that act in a roll bearing are caused by (elastic) deformation of the spherical or cylindrical elements in the inner and outer cage. These stresses are Hetz stresses. In practice the roller bearings are modeled as two perpendicular springs. As a consequence no cross couplings terms originate. Often these springs are approximated linearly but a better correspondence with reality is to model the two springs with different stiffness, non-linear or even rotary dependent. Damping is minimal, which normally is not beneficial.
3 Experiments

Vibrations are oscillating motions in mechanical systems. Often these will lead to reduction of performance and are therefore unwanted. Several sources exist which give rise to vibrations in machinery; like mass unbalance, gravity, hydraulic forces, misalignment, etc. For a vibration two factors are important: the source where the dynamic forces are generated and the path which transmits the energy.

Vibrations in system are related to resonance. When a system is subjected to a frequency close to its eigenvalues, the amplitudes of the vibration can swept up. This is due to the high transmitted power of a force; the energy is converted into vibrational energy of various modes. Another possible cause of resonance is the dynamical weakness of the system about this frequency.

In this chapter first a background is given of a common method to define the dynamic response of a system experimentally. Because several ways exist to force the system into vibrations, the second paragraph is devoted to excitations. Subsequently a plan of measurement is treated which is especially aimed at rotor systems. Finally experiments are implemented and results are presented. Conclusions are drawn on the dynamics of the centrifugal pump.

3.1 Background Structural Testing

Structural testing is concentrated to gather lumped parameters (see paragraph 2.3) to describe the dynamics of the system considered. In this paragraph a background is given for this test.

To make an estimation of the dynamic response of a mechanical system, the eigenvalues and modes are necessary. A Frequency-Response measurement requires defining every natural frequency and the associated modal shape. For rotating machines usually the low frequencies are the important ones due to the major vibration problems occurring in this range of frequencies (0-200 Hz) [2].

In general a vibration can be defined as a frequency, phase and amplitude. This can be represented in graphs, called a frequency spectrum diagram. This represents the arising amplitude and phase belonging to a specific frequency.

For a rotor system three possible secondary motions exist which occur besides the main rotational movement: longitudinal vibrations, torsional vibrations and lateral vibrations. They can originate individually, but also a combination is possible (like the coupling of torsional vibrations with lateral motions). In comparison with lateral or torsional vibrations are longitudinal vibrations in a shaft much smaller and are therefore often neglected.

The number of natural frequencies of a dynamic system is similar to the number of degrees of freedom. A model for one degree of freedom is called a single degree of freedom model (SDOF). When the frequency is sub-critical, the mathematical model can be interpreted as a single spring. The static deflection is merely controlled by the spring stiffness. If the frequency is increased, the inertial force of the mass has more influence. At a particular frequency (the natural frequency; \( \omega_0 = \sqrt{\frac{k}{m}} \)) the mass and spring terms erase each other. The response is now only determined by the damping force. The amplitude rises extremely and the phase shifts -90 degrees. When the frequency increase further (supercritical), the dynamical behavior is dominated by the mass term. The phase shift to -180 degrees. (See figure 15).
This general approach to understand, describing and modeling structural behavior is called Modal Analysis. The main thought is to excite a system with a known external power which forces the system to vibrate. By measuring the response of the system a transmission can be defined. Thereby a frequency response function (FRF) is formulated and the natural frequencies determined.

A FRF measurement will show its response to be a series of peaks each responding to a resonance of a single degree of freedom. It is implicated that a structure behaves as if it is a set of substructures. This is the main basis of Modal Analysis; the different modes can be summarized due to the superposition property of linear systems. Then the dynamical behavior of a system can be analyzed by identifying and evaluating all the resonances.

The transmission of the response and excitations as a function of frequency is the frequency response function \( H(\omega) \). It is defined as the complex ratio between the output and input. (See figure 16)

\[
H(\omega) = \frac{X(\omega)}{F(\omega)}
\]  
\[
\text{(4.1)}
\]

Here represent \( X(\omega) \) the system response and \( F(\omega) \) the forced excitation function respectively.

A frequency response function is interpreted as the change of the sinusoidal input force with a frequency \( \omega \) to an output force with same frequency but a different amplitude of magnitude \( |H(\omega)| \) and a phase shift of \( \angle H(\omega) \).

For a SDOF the frequency response function \( H(\omega) \) can be expressed in terms of mass, damping and stiffness.

\[
H(\omega) = \frac{1}{-\omega^2 m + j\omega d + k}
\]  
\[
\text{(4.2)}
\]

The denominator of the equation is also known as the characteristic equation of the system and has the same form as the equation of motions prescribed in chapter two when a Laplace
operator is used; \( \frac{\partial}{\partial t} = s; s = j\omega \). The characteristic values of this equation are known as the complex roots or complex poles. The value corresponds with the modal frequency of the model.

Just as in the analytical case where the ultimate solution is derived from summarizing different modes of SDOF models, the frequency response functions between a multi-degree-of-freedom (MDOF) can similar be represented by a superposition of the single degree-of-freedom models. To define the frequency response function of these systems implies determining a lumped approximation of the mass, damping and stiffness matrix. From experimental viewpoint this can be done by defining the modal parameters like natural frequencies, damping, vectors and relative patterns of motion. Especially the relationship of these modal parameters with respect to the measured frequency response function is important.

In practice problems occur; the measurements may be influenced by external noise, the system has an extreme non-linear behavior (while the frequency response function approaches the system linearly), electrical noise in the instrumentation produce extra frequency peaks, the resolution is limited, etc. To minimize these problems statistical methods can be used, for example averaging the signal. Also methods exist to examine if the response is reliable, like the coherence function. To prevent any leakage effect the response can be multiplied with a window function.

### 3.2 Excitations

For implementing a Modal Analysis it is necessary to force the system to vibrate with a known frequency. Several vibration machines exist with can be used for this method. This paragraph treats the difference between these forced excitations.

The aim of an excitation is to create a response in a mechanical system. The choice of an input that forces the structure to oscillate depends on the character of the system. Especially the linearity of a system is of concern. Modal Analysis is a technique which is based on a linear approximation of a dynamical system. But every mechanical structure has a natural degree of non-linearity. For rotor dynamical systems the non-linearity is especially created by the bearings.

Various fashions exist to implement an excitation, also called perturbation, on the structure in the range of interest. Three main categories of waveform spectrum of excitations are; transient, random and swept sine.

A transient waveform can be obtained by implementing a short impulse on the structure. This technique is the most popular excitation for modal analysis. In general an impulse test is performed by a hammer-stroke, but it can also be created when the structure is loaded and suddenly released. An implemented impulse transports energy during a short time interval. The originated spectrum is continuous; it contains all frequencies. The accompanying amplitudes are not equal.

Random excitation computes a good frequency response spectrum that represents a linear reproduction of a system with clear non-linear properties. When the system characteristics prevails non-linearity, the system shall response differently each time when it is subjected to a similar excitation. Random excitation averages the responses, therefore the non-linear behavior can be smoothed and a good linear approximation is made. Another advantage of a random excitation is that the range is easy to be adapted. Disadvantage is the possibility of a leakage error. This can be solved by using a window function; a Hanning window for instance. Random signals are created by shakers and are, as well as hammer strokes, a commonly used excitation.

A swept sine is a deterministic sinusoidal signal with constant increasing frequency. Sufficient time is required to define the transient response of the system. Therefore the total time to complete a swept sine test is large. The required time depends on the range of interest and the system damping. A swept sine can also be performed by a shaker.

More waveform spectrums exist like periodic chirp, pseudo-random, burst random, etc. The advantages and disadvantages of these spectrums are summarizes in table 2.
A vibration machine can be used to subject a system to forced oscillations. In the last century many different shakers are developed; like direct drive vibrations machines in which oscillations are developed pure mechanical or by rotating unbalance. This category has disadvantages like transition of there own eigenfrequency and limited frequency range, but can be very suitable for measurements at low frequency.

An Electrohydraulic shaker is another vibration instrument which is recommended for measurements which require high amounts of force and relatively low frequencies. A quality of electrohydraulic shakers is its high energy transmission. It is ultimately appropriate for these kinds of measurements for low frequencies.

Finally a commonly used shaker is the electromagnetic shaker. This can be interpreted as a speaker without conus. The oscillation is driven by a Lorentz force. This force is induced when pulsating current flows through a coil. The arising force acts periodically with a known frequency. In the middle of the coil a bar is located. Hereby the bar is forced to move outwards due to this Lorentz force. The energy is guided by a small beam to the shaft. The shaft offers some resistance to deflect. If the frequency is close to the natural frequency only little energy is required to deform the shaft; the deflections are extremely. At other frequencies the shaft will be rigid and the amplitude becomes minor.

Care has to be taken that the vibration machine may not influence the dynamical behavior of the rotor-bearing system. Every additional mass, or restraint of movement of the system, results in a shifted natural eigenfrequency and its response is not reliable anymore.

To measure the response three different transducers can be chosen: displacement, velocity and accelerometers. Accelerometers are designed with the intention to define the deformation for high frequencies. Now-a-days these accelerometers are adapted to be applied for low frequencies (0-10 Hz). But displacement transducers are more suited to test the lateral and axial vibrations and positions. Besides, it can detect complete rotor orbits and centerline positions. Therefore Agnieszka Muszynska [2] prefers proximity transducers. These measurement instruments are displacement transducers which are mounted very close to the shaft and can measure exactly the orbit of a shaft. At the TU/e some proximity transducers are available.

Nodal points always exist. Nodal points are points on the shaft which do not deflect during a normal mode of vibration of the system. At low frequencies the nodal points corresponds often

---

Table 2 Characteristics of Excitation Signals Used in Experimental Modal Analysis

<table>
<thead>
<tr>
<th></th>
<th>Slow swept sine</th>
<th>Periodic chirp</th>
<th>Impact</th>
<th>Step relaxation</th>
<th>Pure random</th>
<th>Pseudo-random</th>
<th>Periodic random</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimize leakage</td>
<td>Yes/No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Signal-to-noise ratio</td>
<td>Very high</td>
<td>High</td>
<td>Low</td>
<td>Low</td>
<td>Fair</td>
<td>Fair</td>
<td>Fair</td>
</tr>
<tr>
<td>RMS-to-peak ratio</td>
<td>High</td>
<td>High</td>
<td>Low</td>
<td>Low</td>
<td>Fair</td>
<td>Fair</td>
<td>Fair</td>
</tr>
<tr>
<td>Test measurement time</td>
<td>Very long</td>
<td>Very short</td>
<td>Very short</td>
<td>Very short</td>
<td>Good</td>
<td>Very short</td>
<td>Long</td>
</tr>
<tr>
<td>Controlled frequency content</td>
<td>Yes*</td>
<td>Yes*</td>
<td>No</td>
<td>No</td>
<td>Yes*</td>
<td>Yes*</td>
<td>Yes*</td>
</tr>
<tr>
<td>Controlled amplitude content</td>
<td>Yes*</td>
<td>Yes*</td>
<td>No</td>
<td>Yes/No</td>
<td>No</td>
<td>Yes*</td>
<td>Yes*</td>
</tr>
<tr>
<td>Removes distortion</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Characterize nonlinearity</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

* Special hardware required
with the suspended places, like bearings. At higher order frequencies the originated nodes have generally complex pattern due to non-linear behavior. It is important to measure not directly at a node for amplitudes will not be depicted and as a result some resonances can not be detected. Therefore it is important to make good considerations of implementing the excitation and placing the transducer before testing.

3.3 Plan of measurements

In the next paragraphs the plan of measurements is discussed. The method is obtained in the book 'Rotordynamics' [2]. First the rotor dynamics dependency of speed is clarified. Thereafter the theoretical method of measurements is proposed. Due to lack of time this theoretical plan of measurements is not performed. The executed experiment in this project is presented in the third sub-paragraph. Finally the results are given and discussed.

3.3.1 Rotor speed dependency

Rotor dynamic measurements differ from static system measurements. Note that in the discrete system model the matrices have a dependency of rotor speed (paragraph 2.2), which prohibits direct use of a standard modal analysis to determine the frequency response of the system. As mentioned before, the natural frequency can be calculated by the complex eigenvalues of the characteristic equation of the system. Nevertheless change the values due to rotor speed dependable parameters (stiffness, damping), what mainly is engendered by gyroscopic effect, the properties of bearings and inertia effect. In the article of C.H. Chen and K.W. Wang [13] the effect of rotor speed is investigated. Centrifugal force strengthens as the rotary speed increases. This is also valid for gyroscopic effects. Furthermore the bearing stiffness decreases as the angular velocity rises. To fully understand the rotating machine working process and identify its operational characteristics, the perturbation test should be conducted to the rotor in its operating speed. A measurement on performing the frequency response on working systems is also known as Operational Deflection Modes (ODM).

3.3.2 Machine Operating Modes

In this paragraph the theoretical plan of measurements is presented. It is adapted to the considered centrifugal pump which is built in a test setup at the Technical University of Eindhoven. Forces in the pump are measured with a dynamometer. It consist four square posts, parallel to the axis, and equipped with strain gauges such that six generalized force components (radial forces, force moments, axial force and torque) can be measured at any time.

To collect sufficient data to define the dynamical parameters of a rotor system several test should be implemented during five different machine operating states; at rest, at slow roll, at start-up, at operational speed and finally shutdown.

1. At rest
   This test provides the rotors static position within the bearings. Also the presence of any external source of vibration may be discovered; the level of rustle is detected. Furthermore the dynamometer can reflect the value of the gravitation force.

Test setup measurement static deflection:
- Drift correction: due to temperature the voltage drift may change. Therefore the signal amplifier is turned on one hour before testing.
- Maximum Signal Bridge 1: The shaft is rotated until bridge 1 gives the maximum output signal.
- Minimum Signal Bridge 1: similar to maximum Signal Bridge but now for minimum output voltage.
- These voltages are converted into forces, after which the difference between those represents two times the gravitation force.
2. At slow roll
   The straightness of the shaft is checked. The dynamical response is mainly due to bending of the shaft and run out of electric or mechanical forces. The rotational speed is typical less than 10% of the first balance resonance speed.

3. Start-up
   The data collected during this test is important; it reproduces information about range of slow-roll speed, resonance speed, vibration modes, effective modal damping and detect presence of self-excited vibrations. If the rotor is stable and operates at steady state then some free vibrations may not be detected as consequence of averaging the signal. These free vibrations becomes important however when a sudden force is applied to the system. Therefore measurements during transient state of the rotor-bearing system are essential. Data from this test is often displayed in a cascade spectrum plot.

Test setup transient measurement:
- Continuous increment of rotational speed motor until maximum rotor speed is obtained
- Rate of change: the rate of change of angular velocity is constant. A reliable value has to be ascribed; the acceleration has to be small enough for good resolution of data versus rotational speed.
- Process the measurement data: The data shall be divided into overlapping sections. It is supposed that every section has a constant rotational speed. The spectrum is separately determined of the sections and plotted in a cascade plot. Use has to be made of the calculation program Matlab.
- Spectrum: Two manners of approach exist. The output voltage of the six different bridges can be taken and for each bridge a response spectrum is created. These convey information about the real obtained frequencies. Also first the output voltage is converted into forces and moments and thereafter the spectrum is defined. This reflects information about the originated forces in the centrifugal pump.

4. Operating speed
   The constant operating rotor speed represents the dynamical equilibrium of the rotor machine. It is expected a high resonance arise coinciding with the rotor speed. Vibrations due to mass unbalance resonate at a zero frequency and therefore correspond with the rotational speed. Machine Operating test are meaningful when the system is time depended.

Restraint is the possible frequency range of this test due to the limited rotor speed. The centrifugal pump which is tested has a maximum operational speed round 700 rpm which corresponds with ~11 Hz. Former test data indicates an interesting region of 0–100 Hz. To achieve these frequencies and get a reliable response due to rotor speed dependency, a non-synchronous perturbation test can be performed. A hammer stroke, for instance, can be implemented during the rotor is working at his operational speed. This implementation causes an extra excitation on the rotor.

First it was deliberated to define the rotor natural frequencies by hammer strokes on the steady rotor-bearing system. Unfortunately the test gave no satisfactory results for a frequency range of 0 – 100 Hz. One cause could be unfit accelerometers. This may be solved using proximity transducers. Second restraints are the dynamical behavior of bearings. Yet it is unknown if the bearings absorb possible frequencies due to the friction inside the bearing. Enough energy has to be generated for impact testing to force the bearing outside these frictions. When a linear approximation of the rotor system is made two extreme values exist. One is assuming there are no suspensions; the bearings have no influence on the dynamical response of the rotor system. In practice, this means that the rotor has to be built out of the test setup. Secondary supposition is assuming the bearings as fixed points; here the bearings are totally fixed. The actual solution is somewhere between those extremes. For this reason it is also advised to test on operational rotor for it gives the closest approximation of the reality. Problem is how to excite the additional perturbation. With a hammer it is difficult to hit the steady rotor system on the same place (with the similar implementation). When the rotor is operational care has to be taken how a constant impact can be performed. An option is to use a shaker instead of a hammer stroke. Note that the dynamical response of the rotor-bearing system may not change when use is made of this vibration machine. No additional friction or damping may be generated, and also it may not act as if it is a fixed point. Good considerations have to be made how to fit the shaker on the rotating shaft. Other options are pendulum impactors, snap-back cables, external eccentric
rotating mass, or extreme devices such as explosions or guns. The last group are a little exotic and is not very reliable. But which type of vibration is chosen, care has to be taken how to implement the additional excitation.

Test setup steady rotation rotor machine:
- Rotor is operational at constant speed
- Data is measurement for about 40 seconds
- Measurement data is filtered, averaged, corrected, and processed with aid of the computer program Matlab. This will not be discussed in this report. For each bridge a spectrum plot is made.

5. The shutdown / coast down
First the rotor system is brought to his maximum rotational speed after which the electrical supply is cutoff and the rotor is freely damped out. This test has similarities with the run-up test for it measures the rotor dynamic response during transient period. The main difference however is the absence of driving torque.

3.3.3 Measurement

By reason of lack of time only a transition test is performed for this will provide most information of the dynamical response of the rotor system. For different rotational speeds the response of the dynamometer is defined. The responses were obtained by constant operational speeds.

One sub-objective of this project is to examine experimentally the influence of liquid on the dynamical response of the rotor-bearing system. To define the difference, first the test is performed on the centrifugal pump without water. Thereafter the test is repeated when liquid is circulating. The obtained outcomes are compared with the first test.

Step by step the transitions test
1. Drift correction: the signal amplifier is turned on one hour before testing
2. Maximum and minimal signal for bridge 1 are obtained.
3. The rotor speed is incremented until the rotor operates at steady state. The speed of the pump is checked and possible corrected using the tachometer.
4. Data is measured for next 40 seconds.
5. Rotational speed is increased with fixed step size; 25 rpm is chosen.
6. This is repeated until rotor reaches maximum rotor speed of 700 rpm.

Subsequently the obtained data are processed in a program file in Matlab. A special script is made to visualize the results in a Cascade diagram. First the zero frequency is removed by subtracting the averaged signal of the Voltage file. A distinction is made between the voltage originating in the strain gauges and the converted signals to calculate the six generalized force components.

A discrete Fourier transformation is used to convert the time depended signal to a frequency depended signal. This is done for every signal obtained by bridges 1 to 6 of the dynamometer for different shaft speeds. For a force spectrum, the voltage data is first converted to calculate the corresponding forces. Thereafter a Fourier transformation is made. To acquire a cascade plot, every spectrum is plotted in the same figure with an offset to create a 3D-effect. This offset is linked to the revolution speed of the shaft. The amplitude of the voltage spectrum is given in log scale. The amplitudes in the force spectrum are represented in normal scale. (The used Matlab file is given in Appendix E)

In a cascade diagram different spectra of revolutions speeds are represented. On the x-axis the frequency is plotted in Hz. On the left vertical axis the amplitude of the generated vibrations corresponding to a frequency are given and on the right the rotational speed of the centrifugal pump. In a cascade diagram easily two different originated vibrations can be distinguished; rotary independent frequencies and vibrations that are coupled to the natural bending modes of a system. The first category the resonance frequency increases with the rotor speed. The second group will arise only at one specific frequency.
The natural frequency of the centrifugal pump can be noticed when the rotary dependent resonances corresponds with this frequency. The amplitude will rise enormously. For an example of a cascade plot see figure below.

![Cascade diagram](image)

**Figure 15 Cascade diagram**

### 3.3.4 Expected Results

It is expected that fluid influences the dynamical response of the system. Especially surrounded fluid in bearings, seals, blade-tip clearances, and shrouded impellers causes this phenomenon. The attendance of fluid reduces the natural frequency and strengthens the damping effect. Furthermore, it is possible that additional vibration may rise due to the fluid flow, especially with an originating frequency corresponding with the blade excitations.

Fluid-induced vibrations can be subdivided by nature of there excitation. For example liquid can behave as additional mass. As consequence the mass moment of inertia will change and also the natural frequencies. This effect depends on the viscosity of the liquid. Also vibrations may arise due to unsteady flow, like random pressure difference as a consequence of turbulent flow. Finally vortex induced vibrations can occur. In a centrifugal pump additional mass mainly influences the dynamical response. Therefore it is expected that the natural frequencies ($\omega_0 = \sqrt{\frac{k}{m}}$) arise at a lower rotational speed than when measurements are made in air. Also damping effect shall increase, but this can not be measured in a cascade diagram.

The centrifugal pump may also become unstable. When the rotor speed exceeds a specific limit value, instable and non-synchronous whirl motions appear. The speed that belongs to this point is entitled as the threshold speed of instability. Mostly this is defined as rotor lateral vibrations with unacceptably high amplitudes.

Also fluid can be the cause to give rise to unacceptable amplitudes. Due to the motion of the impeller, the liquid is forced to rotate and generates as consequence a strong circumferential flow. The liquid participates with the dynamics of the system. From the rotation energy new forces are risen which acts on the rotor. These forces are self-excitation and are know as fluid whip and fluid whirl.

Fluid whip is a sub-synchronous, fluid-induced, self-excited vibration that is coupled to a bending mode of natural frequency. It arises at one specific frequency. Fluid whirl is also a
sub-synchronous, fluid-induced, self-excited vibration but this vibration is related to the shaft speed, \( \Omega \). Typically, this frequency arises at half the rotational speed and is therefore traditionally called “half-speed whirl”. Fluid whip and fluid whirl have the same characteristics as modes. Also second and higher order modes can be visualized in a spectrum diagram.

The synchronous resonance is for low rotational speed mainly dominated by forces due to mass unbalance. The exciting force as consequence of eccentric masses is a centrifugal force and works on the rotating shaft in the radial direction. Because this force is rotating at the shaft speed, it is expected that this response originate at a frequency corresponding with the rotational speed in the cascade plot. Another possible result of unbalance is a rocking motion and consequently vibration in both radial and axial directions. Therefore also in the axial direction a line of first resonance speed is expected to be drawn. At a specific frequency the amplitude will grow, the frequency cross the threshold speed of instabilities. The increment is continually until the amplitude reaches a constant value. On the moment when the rotor speed reaches the threshold of instability, fluid whirl originate at half revolution speed. When the rotational speeds increases further to roughly two times the balancing resonance, a smooth transition of fluid whirl into fluid whip originates. The amplitude will grow exponential and corresponds with the natural frequency. See also the cascade diagram in figure 17.

3.4 Results and conclusions

Overview of all results of the experiments is given in the cascade diagrams in Appendix C.

VOLTAGE SPECTRUM OF CENTRIFUGAL PUMP: OPERATIONAL IN AIR (figure 25)

At a first glance at the voltage response spectrum of the considered centrifugal pump, a straight line of high amplitude is seen. These synchronous responses correspond with the rotational speed of the centrifugal pump. This line was expected to get. Furthermore non-synchronous responses are marked in the voltage spectrum at 50 Hz, 150 Hz, and sometimes at 250 Hz (spectrum of bridge 5). As one knows the net frequency is 50 Hz, therefore it is expected that the responses can probably be devoted to this phenomena.

Furthermore clearly higher peaks are visible around a frequency of roughly 15X and 27X rpm, especially in bridge 3, 4 and 6. In minor extend but still present are these vibration in bridge 1 and 2. The cause of this rising amplitudes is yet unknown. In bridge 5 a slight bandwidth of non-synchronous peaks between 226 and 238 Hz are noticed. Also the cause of these rising frequencies is unknown.

Note that in the cascade diagram the y-axis is relative; the actual amplitude is determined to subtract the offset.

FORCE SPECTRUM OF CENTRIFUGAL PUMP: OPERATIONAL IN AIR (figure 18-20, 26)

Minor rising amplitudes at the blade passing frequency where expected to be seen in the force spectrum of a centrifugal pump, measured when the test-set up was not filled with liquid. But barely resonances occur beside the resonance-peaks originating at a frequency corresponding with the rotational speed (1X). Probably these are presented but are very small in comparison with the raised amplitude of 1X. It can be considered what the influences of these oscillations are and therefore may be neglected. For torsional and axial vibrations distinct higher order responses are visible in the range the of 0-100 Hz.

In the torsion spectrum (figure 19) three conspicuous things have to be remarked. First, the highest raised amplitudes are not always corresponding with the frequency corresponding with the rotational speed. Secondly, in the spectrum a second order synchronous response is seen where for low rotational speed slightly a second peak originates. Obviously the resonance of 4.5X dissipates when the rotor operates with a revolution speed higher than 250 rpm. This phenomenon can be explained due to the received energy, which the system uses to vibrate, is translated to another mode of vibration (for example axial or at higher orders). The fact that the line originate at 4.5X, instead of a whole number of first resonance speed, is diagnosed before in research in textbooks [3]. In Appendix F a table of characteristic ratio of whirl frequency and rotor speed is given for torsional vibrations. Finally the varying amplitude at 1X has to be marked. First the amplitude increases until the rotor speed approaches 250 rpm (4.1 Hz), thereafter the response falls. This could be a first natural frequency of torsional vibrations. This
will be explained in detail later. More concerned is another enlargement of amplitudes after further increment of the rotational speed. After 475 rpm (7.9 Hz) the amplitude continually increases till it reaches a steady value. At the same moment a resonance emerges at half the resonance speed. This development was clarified earlier with fluid whirl. Probably instability originates for torsional vibrations at 475 rpm. This phenomenon was observed when the test setup was not filled with water. This instability can therefore not be induced by any flow. Presumably this instability is originated by another self-excited vibration.

In the axial force spectrum (figure 20 and 26) more resonances occur compared with other spectra. Most notable are roughly 14.5X and 28X. Also a slight resonance originates at a frequency corresponding with twice the rotational speed (2X). Compared with the originated forces Fx, Fy, the value of the force in axial direction is small. The area under each curve is similar to the energy which is transmitted to the system. Because the amplitude of the axial force at 1X is small, the resonances at higher orders could be larger, the energy is divided over the range of resonances.

A slightly non-synchronous response can be noticed in the axial force spectrum at 40 Hz. The cause of this is unknown.

Also the value of forces or moments in x and y direction are predominating torque moment. It is concluded that the dynamical behavior of the system is resolved by the forces and moments in respectively the x and y direction. Torsional and axial vibrations contribute little to the total dynamical motions of the centrifugal pump.

When the attention is paid to the values of amplitudes (figure 27) it can be noticed that the values of Fx, Fy, Mx, and My do not clearly change. No distinct development of the amplitudes can be concluded. The value of the forces in x respectively in y direction remains between 100 - 140 N, which roughly corresponds with the gravitation force of the system. Hereby it can be concluded that for centrifugal pumps operating in air the dynamical response of the forces are mainly depended due to rotating mass. No natural frequencies can be noted at a range of 0 - 11.7 Hz.

Hereafter fluctuations of amplitudes of torsional and axial vibrations are considered. For torsional oscillations, strong resonances arise at a bandwidth which coinciding with 1X and 2X. As mentioned before the highest amplitudes occur when the threshold speed is exceeded. Again for 4.1 Hz increasing amplitude is noted which may remark the first natural torsion frequency. More interesting are the risen oscillations for axial forces. The development of amplitudes of 14X gives a maximum for 139 Hz at a rotational speed of 598 rpm. It can be concluded that the system is very sensitive for this circumstance. A suggestion can be made that this frequency is related to a natural frequency of the system. Also for 28X a clear maximum is seen. These originate with a frequency of 221 Hz and an operational speed of 475 Hz.

A natural frequency can be found if the frequency coincides with a frequency corresponding with the rotational speed or with a number of it; amplitudes will be swept up. It is expected that the natural frequency is independent on rotational speed and therefore may been seen in the cascade plot as a non-synchronous vibration. A response peak would be seen at one specific frequency in every spectra (like the 50 Hz from the net frequency) From this argumentation the increased amplitudes in the torque and axial spectrum is not a natural frequency. But since the equations of motion are depending on the rotational speed (Chapter three) it may be considered that the natural frequency also change with the rotor speed. Therefore not a line perpendicular to the x-axis may represent natural frequency but a straight line at an angle with respect to the x-axis. When this line crosses a line in the cascade diagram which represents responses originating at a frequency which corresponds with the rotational speed, an increment of amplitudes is seen. Unclearly is yet that this response of high amplitudes is not seen in every separate spectrum of constant rotor speed. For a hammer stroke the excited force spectrum is ranged over all frequencies, but for the implemented experiment this may not be the case. No matter how, it can be concluded that the system is very sensitive to this frequency for a specific rotational speed. Therefore this result must be taken into consideration for performing experiments and formulate conclusions.
Figure 16 Cascade diagram of $F_x$, centrifugal pump operational in air

Figure 17 Cascade diagram of torque, centrifugal pump operational in air
FORCE SPECTRUM OF CENTRIFUGAL PUMP: OPERATIONAL IN AIR

SPECTRUM OF AXIAL FORCE

Figure 18 Cascade diagram of axial force, centrifugal pump operational in air

VOLTAGE SPECTRUM OF CENTRIFUGAL PUMP: INFLUENCE OF LIQUID (figure 28)

To define the influence of liquid on the dynamic behavior of the shaft of the system, the results obtained by an operational centrifugal pump in air are compared with the measurements in liquid. The last measurement deviates from the 700 rpm which was tried to obtain. In the cascade spectrum plot this is seen by a high offset of the last curve.

Comparing the voltage spectra for both experiments noticeable higher order resonance appears. In spectrum of bridge 1 and 2 clearly 5X, 6X and 7X rpm are seen. Note that these only arise as the rotational speed of the shaft is higher than 350 rpm. These also additionally originate in spectrum of bridge 3 and 4 besides the 15X and 27X (roughly).

FORCE SPECTRUM OF CENTRIFUGAL PUMP: INFLUENCE OF LIQUID (figure 21-23, 29)
The additional 5X, 6X and 7X are also seen in the force cascade diagrams. It is wondered why these higher order do not only originate at 6X, which corresponds to the blade excitation frequency. Obvious is this response non-linear, the values of amplitudes are mirrored in the line of 6X.

A first thing that is noticed is the lower amplitudes at 1X. Increasing damping can be a cause as well as the more arising responses (as mentioned before due to conservation of energy). Still no extreme amplitude of the forces and moments in the lateral direction originates in the first resonance speed, the values global remain the same and no natural frequency is obtained.

When the development of amplitudes of torsional vibration are taken (figure 30) a clear resonance occurs at 31 Hz. Reflection this in the cascade spectrum and using zoom-options the resonance corresponds with 4.7X at an operational shaft speed of 400 rpm. Also in the line of response frequency 13.6X an abrupt increment of amplitude arises at 17 Hz with is corresponding rotational speed of 75 rpm. Reflecting back to the results obtained from the centrifugal pump operating in air, these straight lines of frequency responses were also noticed (4.5X and 13.5X) only sighted shifted to the right. Different is 4.7X which is not dissipated to other oscillations anymore but leads to extreme sensitivity in this instance. The peak of incremented amplitude in 13.6X is displaced; it originates at a higher frequency. If these peaks are related to natural frequencies, the expectations do not meet with the results. It was expected that liquid result to additional mass and a lower natural eigenfrequency. The background of this obtained result is yet unclearly as well as the shifted line of responses.
Another conspicuous additional vibration appears at 2.4X in the torsional cascade diagram (figure 22). Here the amplitude still raise exponential when the rotational speed is increasing. It may be possible that a natural frequency originate if the rotor speed exceeds 750 rpm or it may be an originated instability. Finally a slight 1.1X is appearing. All these occurring resonances where not noticed by the experiment in air, they are a fluid-induced results.

The instability noticed in torsional vibration for experiments in air is still present; only in comparison with the extra originated fluid-induced oscillations is not very strong. Also the half speed whirl is noticed very small.

The origination of more additional higher order resonances also occurs in the spectrum of axial forces (figure 23). This was also seen in the spectrum of bridge 6 which is linked to axial force and \( M_y \). The most prominent risen amplitudes in the range of 0-50 Hz, besides the 1X, are the 2.4X, 4.7X and 13.6X. These lines of responses are similar to arisen resonances in the torsional direction. In addition the development of the value of the amplitudes is analogous; distinct increment in amplitude is again noticed at 31 Hz at 4.7X and 17 Hz at 13.6X as well as the exponential increment of amplitude of 2.4X. New for axial force spectrum is the origination of half speed whirl after a revolution speed of 150 rpm. Subsequently amplitudes grow in 1X. This result may relate again to a created instability but which is now fluid-induced.

Furthermore the non-synchronous vibrations originated at 40 Hz are disappeared. The cause of this phenomenon is still unknown, but it can be said that the influence of liquid fades this response. Concluding; the originated vibration is not depended on rotor speed, it acts in the axial direction and fluid evanesce the response.

Finally marked are the expanded amplitudes in the axial directions. This was expected due to the flow on the impeller.

![Figure 19 Cascade diagram of Fx, centrifugal pump operational in liquid](image-url)
Figure 20 Cascade diagram of torque, centrifugal pump operational in liquid

Figure 21 Cascade diagram of axial force, centrifugal pump operational in liquid
3.4.1 Conclusions

The main conclusions of this experiment with the considered centrifugal pump are:

- The dominated responses in the centrifugal pump arise from forces and moments acting in the lateral direction.
- The first response of the forces $F_x$, $F_y$ and moments $M_x$, $M_y$ do not vary in dynamical response. No distinct development of the amplitudes can be concluded in the range of 0 to 11.7 Hz. The dynamical response of the system in this range is mainly depended on the gravitation force of the total system.
- Torsional vibrations develop a strong dynamical response. A response was noticed at a frequency corresponding with twice the rotational speed. Furthermore a strong increment of the amplitude of 1X originates round 250 rpm. Attention has to be paid on the originated instability at 475 rpm. A threshold critical speed lay probably for torsional oscillations around this rotor speed.
- In the cascade spectrum of the axial force a high sensitivity is noted for 14X at a frequency of 139 Hz and rotor speed of 598 rpm. This development also arises for 28X corresponding with a frequency of 221 Hz and rotational speed of 475 Hz. Also a slight non-synchronous vibration was noticed at a frequency of 40 Hz.
- In the attendance of liquid arisen amplitudes of 1X faded and more and higher resonances occur. Especially the 5X and 7X (and slightly 6X) are appearing in the spectrum of the $F_x$, $F_y$ and $M_x$, $M_y$. These blade excited frequencies are probably non-linear.
- Liquid does change the dynamical behavior of the system. In torsional but especially in the axial force spectrum higher order resonances occur. Prominent development in the responses is the distinct increment in amplitude of 31 Hz at 4.7X and 17 Hz at 13.6X as well as the exponential increment of amplitude of 2.4X. These results where obtained in both diagrams.
- Liquid may also lead to instabilities as for vibrations in the axial direction. Amplitudes increased at a rotational speed of 350 rpm and a half speed resonance originated.
- Liquid shifts the resonances to higher frequencies. This result does not correspond with the expectations where a shift to lower frequencies where expected.
- No non-synchronous lines are found which may represent the natural eigenfrequency of the considered system. It may be suggested the natural frequencies also change with the revolution speed of the shaft. The obvious increment of amplitudes in the results may be related to natural frequency of the system.
- Probably the characteristic development of the resonances in the cascade diagrams of torque and axial forces are also presented in the later force spectra and moments, but are faded in comparison to the dominant first order resonance.
4 Abstract

4.1 Abstract

In this report it is tried to establish the dynamical response of the shaft of a centrifugal pump. The intention was to clarify the unexpected results from former experiments. The amplitude and direction of the blade excitation forces, from suction flow acting on the impeller, differs for several rotational speed of the shaft of the pump. This phenomenon can possibly be devoted to the natural frequencies of the system.

First, to obtain insight of the dynamical movements of a shaft, a Jeffcott Rotor is discussed. Due to a mass unbalance the disk traces a path, which in jargon is called as whirl orbit. This motion can be synchronous (dependent) with the rotational speed or non-synchronous (independent). Also the secondary motions of the system can be subdivided into forward whirl and backward whirl, which imply the clockwise or counterclockwise motion of the geometric centerline.

With the Jeffcott model an interpretation is given of the response of a simple rotor system. If the revolution speed is sub-critical, the amplitude of the oscillations will increase with increment of rotor speed. The location of the mass unbalance is outside the whirl circle. If the rotor speed coincides with the critical speed, the amplitudes are swept up and the phase shifts -90 degrees. This is also interpreted as the unbalance point is tangent to the whirl orbit. Further increment of the revolution speed of the shaft show a decrement of amplitudes and a phase shift of -180 degrees. The unbalance spot is now inward the orbit. When it corresponds with the origin of the orbit the motion is called critical speed inversion.

The development of the curve of an orbit can be influenced by constant radial forces acting on the shaft, like gravity. An additional curl may arise due to a combined effect of mass unbalance and gravity. These paths of whirl orbits can be written in equations in polar coordinates, for instance epitrochoid and hypotrochoid motions are taken in this report.

A Jeffcott rotor is a simple model and does not take gyroscopic and inertia into account. To determine the dynamical response of a simple model of the centrifugal pump an analytical model is represented which include these effects. This method makes use of the Lagrangian equations. The outcome is a general representation of equations of motions of a system. With aid of numerical programs the equation can be solved and natural frequencies calculated.

Different modes occur in a shaft when the pump is operational, under which torsional vibrations may not be neglected. A disadvantage of these oscillations is that they are quit and difficult to notice. Other quality of torsional vibrations is that they are coupled with lateral modes.

Bearings influence the dynamical response of the system. Sometimes non-linearity may origin, especially in squeeze film bearings.

To obtain the dynamical response experimentally, the technique of Modal Analysis can be applied. This method is based on the phenomenon that little energy is required to bring the system to oscillate in its natural frequency. A known external power force the system to vibrate in a range of frequencies. With the response of the system compared to the excitation force, a transition function can be made which exhibit information of the natural frequencies of the system.

Problems rise when the stationary centrifugal pump was excited with a hammer stroke. The used accelerometers were not adapted to measure the response in a range of 0-10 Hz. But more important was that the dynamical response of rotor systems changes with the rotational speed. This was also seen in the equations of motions; the lumped parameters depends implicit (and explicit for damping) on the rotational speed. Therefore Operational Deflection Modes are needful to describe the system.

A measurement setup is given where the centrifugal pump is implemented during five different machines operating states (at rest, at slow roll, start-up, operational speed, and shutdown). With these experiments it is possible to fully define the dynamical parameters. The transition speed during start-up and shutdown gave herein the most important information. In this report only the transition test is performed. During different constant operational speeds (with a step size of 25 rpm) the responses are measured and plotted in a cascade diagram with aid of the computer program Matlab. Difference is made between the output voltage of the six bridges and the forces and moments in the rotational axis.
To research the influence of liquid on the dynamical response of the centrifugal pump the test is performed twice; first the pump was operational in air, second in liquid. The results in the cascade diagrams are compared with each other. The conclusions from these experiments are drawing in paragraph 3.4.1.

4.2 Recommendations

This paragraph represents several remarks for future research.

Equations of motion are defined in Chapter two. In these equations some assumptions are made. The model can be more or less extended in detail if desired. The presented equation of motion is therefore not the only correct solution. Computer programs are necessary to solve the equations of motion. It is interesting for future study to solve the equation of motion numerical and compare the outcome with the results of the experiments.

The experiment is only performed on one of the five different machines operational states. To fully define the dynamical response of the system the experiment can be extended to the other states. Also the test can be improved when the outcome from the dynamometer is compared with results obtained from transducers. Proximity transducers therefore have to be placed closely to the dynamometer and the response of the shaft is measured. In addition a full cascade diagram can be obtained which also provides information of the forward of backward motion of the arising oscillation. [2]

Furthermore the experiment can be performed by implementing a perturbation on the operating shaft. Care has to be taken how to perform this excitation without influencing the dynamical response of the system and execute this with high exactness to get acceptable results.

The cascade diagram is obtained by calculating spectra of each signal and plotting these spectrums in one figure with a given offset. This offset is created by adding an accurate value, which is scaled with the revolution speed, on the measured values. By adding a constant value some resonances are faded because the value of the amplitude is smaller than the added value. Especially in the cascade diagrams of the lateral forces and moments this is happening. It can be considered what the influences of these oscillations are in comparison to the high amplitude occurring at the 1X. In this report it is chosen to assume these small oscillations can be neglected. Another option (instead of adding an accurate value) is to multiply every outcome with a constant scaled value. This will result in less readable cascade diagrams and is therefore not used in this report.

More research is recommended to the possible obtained natural frequencies from the cascade diagrams. The question arises if the natural frequencies are depended on the revolution speed of the shaft. To what extend does the rotor speed influence the natural frequencies? Also more inquiry is recommended to the cause of the origination of the resonances. Tables in textbooks may provide information.

Subsequently the research can be extended to take the flow volume of the liquid into consideration. How is the dynamical response of the shaft influenced by different flow volumes? Herewith the obtained results represented in figure 1 can be more clarified.

An answer of the objective is not entirely given. In this report the dynamical response of the system is clarified as well as the influence of liquid. How the dynamical response influences the measurements of suction flow and how possible corrections can be made is yet unclear.
References


LIST OF ABBREVIATIONS

FRF       Frequency Response function
ODM       Operational Deflection Mode
SFD       Squeeze Film Damper
PSFD      Porous Squeeze Film Damper
SDOF      Single degree of freedom

SYMBOLS

General

\(rmp\)   revolutions per minute
\(\Omega\) rotational speed
\(\omega\) frequency
\(H\)    frequency response function
\(X\)    higher order resonance
\(\delta\) speed parameter \(= \frac{\Omega}{\Omega_c}\)

Sub/ superscript

\(c\) critical
\(*\) dimensionless

Paragraph 3.1

\(\Omega\)  rotational speed
\(\varphi\) rotational whirl
\(r\)   radius of orbit
\(e\)   radius of unbalance
\(G\)   geometric centre disk
\(M\)   mass unbalance
\(\beta\) angle between mass unbalance and rotary axis
\(\omega_o\) natural frequency
\(k\)   spring stiffness
\(m\)   mass system
\(d\)   damping

Paragraph 3.2

\(T\)   Kinetic energy
\(t\)   time
\(V\)   potential energy
\(Q\)   not conservative forces
\(\bar{q}\) generalized coordinates
\(I_t\) diametric mass moment of inertia disk
\(I_p\) polar mass moment of inertia disk
\(\rho_{\text{shaft}}\) density shaft
\(A\)   cross section area of a shaft
\(J_t\) diametric mass moment of inertia shaft
\(J_p\) polar mass moment of inertia shaft
\(L\)   length shaft
\(u\)   displacement in the x-direction
\(v\)   displacement in the y-direction
\(\theta_x\) rotational of x-axis
\(\theta_y\) rotational of y-axis
\(M\)   mass disk
\(E\)   modulus of elasticity
\(G\)   shear modulus
\(K\)   shear coefficient
\(\beta\) shear angle
\(I\)   moment of inertia shaft
Rotor Dynamics of a Centrifugal Pump

\[ F \] forces
\[ W \] work
\[ M \] moment
\[ A \] amplitude
\[ M \sim \] lumped mass
\[ G \sim \] lumped damping
\[ k \sim \] lumped stiffness

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Appendix A, Orbits

Motions of shaft can be written in equations representing category of curves. In this appendix an overview of different families of curves are given. [4]

(a) Circle : 
\[ r = e^{i \theta} \]

(b) Ellipse :
\[ r = e^{i \theta} + 0.5 e^{i 2 \theta} \]

(c) Limaçon :
\[ r = e^{i \theta} + e^{i 2 \theta} \]

(d) Cardioid :
\[ r = e^{i \theta} + 0.5 e^{i 2 \theta} \]

(e) Petal Curve :
\[ r = e^{i \theta} + e^{i 3 \theta} \]

(f) Petal Curve 
\[ r = e^{i 2 \theta} + e^{i 4 \theta} \]

(g) Logarithmic Spiral :
\[ r = 2 e^{0.1 \theta} e^{i \theta} \]

(h) Epicycloid :
\[ r = e^{i \theta} - 0.2 e^{i 5 \theta} \]

(i) Hypocycloid :
\[ r = e^{i \theta} + 0.2 e^{i 5 \theta} \]

(j) Astroid :
\[ r = e^{i \theta} + \frac{1}{3} e^{i 3 \theta} \]

(k) Epitrochoid :
\[ r = e^{i \theta} + 0.3 e^{i 3 \theta} \]

(l) Hypotrochoid :
\[ r = e^{i \theta} + 0.3 e^{i 5 \theta} \]

Figure 22 Category of different curves
Appendix B, Equation of Motions

To complete the derivation of equations of motions, the kinetic energy and subsequently the potential energy are specified in generalized coordinates. Hereafter the results are substituted in the Lagrangian equations. The result is a general representation of an analytical dynamical response of a system.

**KINETIC ENERGY**

\[
T = T_{\text{disk}} + T_{\text{shaft}}
\]

\[
T_{\text{disk}} = \frac{1}{2} M (\dot{u}^2 + \dot{v}^2) + \left\{ \frac{1}{2} I_t (\dot{\theta}_x^2 + \dot{\theta}_y^2) + \frac{1}{2} I_p \Omega^2 - I_p \Omega (\dot{\theta}_x \theta_y) \right\} - I_p \Omega^2
\]

\[
T_{\text{shaft}} = \frac{1}{2} \rho A \int_0^L (\dot{u}^2 + \dot{v}^2) dz + \frac{1}{2} J_i \int_0^L (\dot{\theta}_x^2 + \dot{\theta}_y^2) dz + \frac{1}{2} J_p L \Omega^2 + \frac{1}{2} J_p \Omega \int_0^L (\dot{\theta}_x \theta_y) dz
\]

\[
\Rightarrow T = \frac{1}{2} \left[ M f^2(L) + I_t g^2(L) + \rho A \int_0^L f^2(z) dz + J_i \int_0^L g^2(z) dz \right] \left( \dot{q}_1 + \dot{q}_2 \right)
\]

\[
+ \frac{1}{2} \left[ I_p + J_p L \right] \Omega^2 + \left[ I_p \Omega g^2(L) + J_p \Omega \int_0^L g^2(z) dz \right] (\dot{q}_1 \dot{q}_2)
\]

\[
= \frac{1}{2} \left[ \tilde{M} \right] \left( \dot{q}_1 + \dot{q}_2 \right) + \frac{1}{2} \left[ \tilde{T} \right] \Omega^2 + \left[ \tilde{G} \right] (\dot{q}_1 \dot{q}_2)
\]

**POTENTIAL ENERGY**

\[
V = \frac{1}{2} \int_0^L \left\{ \frac{\partial^2 u}{\partial z^2} \frac{\partial^2 u}{\partial z^2} \right\} dz + \frac{1}{2} \int_0^L k G A \left( \beta_x^2 + \beta_y^2 \right) dz +
\]

\[
\left\{ -F_{1x} \frac{\partial v}{\partial z} + F_{1y} - M_{1x} \frac{\partial u}{\partial z} - M_{1y} \frac{\partial u}{\partial z} \right\} + \left\{ -F_{2x} \frac{\partial v}{\partial z} + F_{2y} - M_{2x} \frac{\partial u}{\partial z} - M_{2y} \frac{\partial u}{\partial z} \right\}
\]

\[
\Rightarrow V = \frac{1}{2} \left[ \tilde{k} \right] \left( \dot{q}_1 + \dot{q}_2 \right) + \frac{1}{2} \left[ k G A \right] \left( \beta_x^2 + \beta_y^2 \right) dz - \left[ \tilde{F}_x M_y \right] q_1 - \left[ \tilde{F}_y M_x \right] q_2
\]
LANGRANGE METHOD
\[
\frac{d}{dt} \frac{\partial \mathbf{T}}{\partial \mathbf{q}_n} - \frac{\partial \mathbf{T}}{\partial \mathbf{q}_n} + \frac{\partial \mathbf{V}}{\partial \mathbf{q}_n} = (\mathbf{Q}^\infty)^\top
\]

\[
\begin{bmatrix}
\frac{\partial}{\partial \mathbf{q}_1} \\
\frac{\partial}{\partial \mathbf{q}_2}
\end{bmatrix}
\begin{bmatrix}
\mathbf{T}
\end{bmatrix}
= \begin{bmatrix}
\mathbf{M} \ddot{\mathbf{q}}_1 + \mathbf{\Omega} \ddot{\mathbf{q}}_2 \\
\mathbf{M} \ddot{\mathbf{q}}_2
\end{bmatrix}
\]

\[
\ddot{\mathbf{q}}_1 = \ddot{\mathbf{q}}_2 = 0
\]

\[
\begin{bmatrix}
\frac{\partial \mathbf{V}}{\partial \mathbf{q}_1} \\
\frac{\partial \mathbf{V}}{\partial \mathbf{q}_2}
\end{bmatrix}
\begin{bmatrix}
\mathbf{q}_1
\end{bmatrix}
= \begin{bmatrix}
\mathbf{k} \mathbf{q}_1 - \mathbf{F}_x \mathbf{M}_x \\
\mathbf{k} \mathbf{q}_2 - \mathbf{F}_y \mathbf{M}_y
\end{bmatrix}
\]

LUMPED-PARAMETER MODEL
\[
\Rightarrow \begin{bmatrix}
\mathbf{M} & 0 \\
0 & \mathbf{M}
\end{bmatrix}
\begin{bmatrix}
\mathbf{q}_1 \\
\mathbf{q}_2
\end{bmatrix}
+ \begin{bmatrix}
0 & \mathbf{\Omega} \mathbf{G} \\
-\mathbf{\Omega} \mathbf{G} & 0
\end{bmatrix}
\begin{bmatrix}
\dot{\mathbf{q}}_1 \\
\dot{\mathbf{q}}_2
\end{bmatrix}
+ \begin{bmatrix}
\mathbf{k} & 0 \\
0 & \mathbf{k}
\end{bmatrix}
\begin{bmatrix}
\mathbf{q}_1 \\
\mathbf{q}_2
\end{bmatrix}
= \begin{bmatrix}
\mathbf{Q}^\infty \\
\mathbf{F}_x \mathbf{M}_x \\
\mathbf{F}_y \mathbf{M}_y
\end{bmatrix}
\]

\[
\Rightarrow \mathbf{M} \ddot{\mathbf{q}}(t) + \mathbf{D} \dot{\mathbf{q}}(t) + \mathbf{K} \mathbf{q}(t) = \mathbf{Q}^\infty
\]
Appendix C, Results Experiments

In this paragraph the obtained results from the experiments are presented in cascade diagrams. First the development of the voltage spectrum is given when the centrifugal pump is operational in air. Thereafter the force spectrums and the accompanying values of amplitudes is presented. Subsequently the same cascade plots are presented which are obtained by experiments when the centrifugal pump was operational in liquid.

The x-axis represents the frequency in Hz of the corresponding resonances in the spectrums. On the right side of the diagrams the rotational speed of the shaft is given in rpm. The first resonance frequencies agree with the revolutions speed. For example, when the shaft was rotating at a speed of 600 rpm, a corresponding resonance occurs at 10 Hz.

The left y-axis the amplitude is given. Note that in the cascade diagram the actual value of amplitude is relative; to obtain the actual amplitude the offset has to be subtract.
Figure 23 Cascade diagram of voltage; centrifugal pump operational in air. Amplitudes are presented in logscale.
Figure 24 Cascade diagram of forces; centrifugal pump operational in air.
Figure 25 Overview of the arising amplitudes. Each peak corresponds with a revolutions speed of the rotor in the first four plots. In the spectrum of torque the increased resonance at 4.2 Hz is noted. The high amplitudes occurring round 10 Hz are due to the instability of the rotor when it exceeds the threshold speed. In the spectrum of axial force clear the sensitiveness of the system round 139 Hz (14X) and 221 Hz (28X) are seen.
Figure 26 Cascade diagram of voltage, centrifugal pump operational in liquid. Amplitudes are presented in logscale.
Figure 27 Cascadediagram of forces, centrifugal pump operational in liquid.
Figure 28 Overview of the arising amplitudes. Each peak corresponds with a revolutions speed of the rotor in the first four plots. In the spectrum of torque the sensitive resonances are given. The peak at 17 Hz (13.5X) is erased by the continual increased amplitudes of 4.7X. In the spectrum of axial force the extreme amplitude occurs due to instability. Exact order of higher resonance is unknown.
### Appendix D, Rotational Speed

For clarity of the report the revolution speed of the shaft is rounded to the established rotor speed. The actual measured rotational speeds are summed in the table below. The influence of the aberrance on the obtained results is nil, the offset between two rotational speeds is in Matlab adapted with the measured rotor speed.

#### Table 3 Measured revolution speed of the shaft during experiments

<table>
<thead>
<tr>
<th>Established revolution speed [rpm]</th>
<th>Measurement 1 Operational in air [Hz]</th>
<th>Measurement 2 Operational in liquid [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>0.427</td>
<td>0.4245</td>
</tr>
<tr>
<td>50</td>
<td>0.8282</td>
<td>0.8287</td>
</tr>
<tr>
<td>75</td>
<td>1.2553</td>
<td>1.2571</td>
</tr>
<tr>
<td>100</td>
<td>1.6564</td>
<td>1.6821</td>
</tr>
<tr>
<td>125</td>
<td>2.0621</td>
<td>2.0838</td>
</tr>
<tr>
<td>150</td>
<td>2.4886</td>
<td>2.5096</td>
</tr>
<tr>
<td>175</td>
<td>2.9125</td>
<td>2.9182</td>
</tr>
<tr>
<td>200</td>
<td>3.3431</td>
<td>3.3446</td>
</tr>
<tr>
<td>225</td>
<td>3.7607</td>
<td>3.7547</td>
</tr>
<tr>
<td>250</td>
<td>4.1658</td>
<td>4.1906</td>
</tr>
<tr>
<td>275</td>
<td>4.5928</td>
<td>4.6049</td>
</tr>
<tr>
<td>300</td>
<td>4.9643</td>
<td>4.9951</td>
</tr>
<tr>
<td>325</td>
<td>5.431</td>
<td>5.4282</td>
</tr>
<tr>
<td>350</td>
<td>5.8332</td>
<td>5.8332</td>
</tr>
<tr>
<td>375</td>
<td>6.2355</td>
<td>6.2487</td>
</tr>
<tr>
<td>400</td>
<td>6.6819</td>
<td>6.668</td>
</tr>
<tr>
<td>425</td>
<td>7.0737</td>
<td>7.0848</td>
</tr>
<tr>
<td>450</td>
<td>7.5095</td>
<td>7.5213</td>
</tr>
<tr>
<td>475</td>
<td>7.9242</td>
<td>7.9328</td>
</tr>
<tr>
<td>500</td>
<td>8.3185</td>
<td>8.3521</td>
</tr>
<tr>
<td>525</td>
<td>8.759</td>
<td>8.7455</td>
</tr>
<tr>
<td>550</td>
<td>9.1522</td>
<td>9.1666</td>
</tr>
<tr>
<td>575</td>
<td>9.5931</td>
<td>9.5832</td>
</tr>
<tr>
<td>600</td>
<td>9.9725</td>
<td>10.001</td>
</tr>
<tr>
<td>625</td>
<td>10.4425</td>
<td>10.4369</td>
</tr>
<tr>
<td>650</td>
<td>10.8284</td>
<td>10.89</td>
</tr>
<tr>
<td>675</td>
<td>11.2541</td>
<td>11.2642</td>
</tr>
<tr>
<td>700</td>
<td>11.7122</td>
<td>12.4711</td>
</tr>
</tbody>
</table>
## Appendix E, Vibration Troubleshooting Chart

An overview of possible troubleshooting in a rotary machine is summarized in the table below.

### Table 4 A Vibration Troubleshooting Chart

<table>
<thead>
<tr>
<th>Nature of fault</th>
<th>Frequency of dominant vibration Hz=rpm/60</th>
<th>Direction</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotating members out of balance</td>
<td>1x rpm</td>
<td>Radial</td>
<td>A common cause of excess vibration in machinery</td>
</tr>
<tr>
<td>Misalignment and bent shaft</td>
<td>Usually 1x rpm</td>
<td>Radial and axial</td>
<td>A common fault</td>
</tr>
<tr>
<td></td>
<td>Often 2x rpm</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sometimes 3 and 4 x rpm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Damaged rolling element bearings (Ball, roller, etc.)</td>
<td>Impact rates for the individual bearing components Also vibrations at high frequencies (2 to 60 kHz) often related to radial resonances in bearings</td>
<td>Radial and axial</td>
<td>Uneven vibration levels, often with shocks</td>
</tr>
</tbody>
</table>

Impact Rates $f(Hz)$:

For Outer Race Defect

$$f(Hz) = \frac{n}{2} f_r \left(1 - \frac{BD}{PD} \cos \beta\right)$$

For Inner Race Defect

$$f(Hz) = \frac{n}{2} f_r \left(1 + \frac{BD}{PD} \cos \beta\right)$$

For Ball Defect

$$f(Hz) = \frac{PD}{DB} f_r \left(1 - \left(\frac{BD}{PD} \cos \beta\right)^2\right)$$

$n$ = number of balls or rollers
$f_r$ = relative rps between inner and outer races
<table>
<thead>
<tr>
<th>Rotor Dynamics of a Centrifugal Pump</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Journal bearing loose in housing</strong></td>
</tr>
<tr>
<td>Oil-film whirl or whip in journal bearings</td>
</tr>
<tr>
<td>Hysteresis whirl</td>
</tr>
<tr>
<td>Damaged or worn gears</td>
</tr>
<tr>
<td>Mechanical looseness</td>
</tr>
<tr>
<td>Faulty belt drive</td>
</tr>
<tr>
<td>Unbalanced reciprocating forces and couples</td>
</tr>
<tr>
<td>Increased turbulence</td>
</tr>
<tr>
<td>Electrically induced vibrations</td>
</tr>
</tbody>
</table>
Appendix F, Table Self-Excited Vibrations

In the table below an overview of the risen frequency and the whirl direction is given for self excited vibrations.

Table 5 Diagnostic Table of Rotating Machinery Self-excited Vibrations [3]

<table>
<thead>
<tr>
<th>Whirling or whipping:</th>
<th>R, characteristic ratio: whirl frequency/rpm</th>
<th>Whirl direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hysteretic whirl</td>
<td>0.5</td>
<td>Forward</td>
</tr>
<tr>
<td>Fluid trapped in rotor</td>
<td>0.5 &lt; R &lt; 1.0</td>
<td>Forward</td>
</tr>
<tr>
<td>Dry friction whip</td>
<td>No functional relationship; whirl frequency a function of coupled rotor-stator system; onset rpm is a function of rpm at contact</td>
<td>Backward—axial contact on disc side nearest virtual pivot; Forward—axial contact on disc side opposite to virtual pivot; Backward—radial contact</td>
</tr>
<tr>
<td>Fluid bearing whip</td>
<td>R &lt; 0.5</td>
<td>Load-dependent</td>
</tr>
<tr>
<td>Seal and blade-tip-clearance effect in turbomachinery</td>
<td>Load-dependent</td>
<td>Forward</td>
</tr>
<tr>
<td>Propeller and turbomachinery whirl</td>
<td>Load-dependent</td>
<td>Backward—virtual pivot aft of rotor; Forward—virtual pivot front of rotor (where front is source of impinging flow)</td>
</tr>
<tr>
<td>Parametric instability:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Asymmetric shafting</td>
<td>R = 1.0, 2.0, 3.0, ...,</td>
<td>Unspecified</td>
</tr>
<tr>
<td>Pulsating torque</td>
<td>R = 0.5, 1.0, 1.5, 2.0, ...,</td>
<td>Unspecified</td>
</tr>
<tr>
<td>Pulsating longitudinal load</td>
<td>A function of pulsating load frequency rather than rpm</td>
<td>Unspecified</td>
</tr>
<tr>
<td>Stick-slip rubs and chatter</td>
<td>R &lt;&lt; 1</td>
<td>Essentially planar rather than whirl motion</td>
</tr>
<tr>
<td>Instabilities in forced vibrations:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bistable vibration</td>
<td>R = 1 with periodic square wave fluctuations in amplitude of frequency much lower than rotation rate</td>
<td>Forward</td>
</tr>
<tr>
<td>Unstable imbalance</td>
<td>R = 1 with slow variation in amplitude</td>
<td>Forward</td>
</tr>
</tbody>
</table>
Appendix G, Matlab file

The applied Matlab file, including all experiment data, is enclosed in the cd-rom.

MATLAB
Main file: All6_statistics_cascade.m
Sub files: plot_fft_cascade.m (Plot Voltage Cascade Diagram)
plot_fft_cascade_forces.m (Plot Force Cascade Diagram)
GetThreshold.m (Includes obtained measurements)

DATA
18-01-2005
A) Run-Up-Coast-Down test; Continual increment rotor speed until maximum speed is obtained, thereafter power supply is turned abruptly off
B) Centrifugal pump operational in air, step size 50 rpm

20-01-2005
A) Offset voltage (drift)
B) Centrifugal pump operational in air, step size 25 rpm. (Wrong sample size)

23-01-2005
A) Offset voltage (drift)
B) Centrifugal pump operational in air, step size 25 rpm. (Used for the obtained results)

25-01-2005
A) Offset voltage (drift)
B) Centrifugal pump operational in liquid, step size 25 rpm. (Error pulse at 350 rpm, 400 rpm, and 600 rpm)

26-01-2005
A) Offset voltage (drift)
B) Centrifugal pump operational in liquid, step size 25 rpm. (Bubbles arisen, tested till 500 rpm)
C) Centrifugal pump operational in liquid, step size 25 rpm. (Used for obtained results)
D) Down test; Rotor is operational at maximum rotor speed, abruptly turned power supply off. (Performed three times)