Rule-based energy management strategies for hybrid vehicles

Theo Hofman* and Maarten Steinbuch

Department of Mechanical Engineering,
Faculty of Dynamics and Control Technology,
Section of Control Systems Technology, P.O. Box 513, Eindhoven,
The Netherlands
Fax: +31 40 246 1418 E-mail: t.hofman@tue.nl
E-mail: m.steinbuch@tue.nl
*Corresponding author

Roell van Druten and Alex Serrarens

Drivetrain Innovations B.V., MMP 1.42, Horsten 1,
5612 AX Eindhoven, The Netherlands
Fax: +31 40 247 5904 E-mail: druten@dtinnovations.nl
E-mail: serrarens@dtinnovations.nl

Abstract: The highest control layer of a (hybrid) vehicular drive train is termed the Energy Management Strategy (EMS). In this paper an overview of different control methods is given and a new rule-based EMS is introduced, based on the combination of Rule-Based – and Equivalent Consumption Minimization Strategies (RB-ECMS). The RB-ECMS uses only one decision variable and requires no tuning of many threshold control values and parameters. This decision variable represents the maximum propulsion power of the secondary power source (i.e. electric machine/battery) during pure electric driving. The RB-ECMS is compared with the strategy based on Dynamic Programming (DP), which is inherently optimal for a given cycle. The RB-ECMS proposed in this paper requires significantly less computation time with a similar result to DP (within 1% accuracy).

Keywords: automotive control; energy management strategy; hybrid electric vehicles; optimisation hybrid power systems; vehicle modelling, vehicle simulation.


Biographical notes: Theo Hofman received his MSc degree in Mechanical Engineering from the Technische Universiteit Eindhoven, in 1999. From 1999 until 2003 he was a development engineer at Thales-Cryogenics B.V. and worked on the development of small Stirling- and Pulse-Tube cryocoolers for various applications (Space and Defence industry). Since 2003, he has been a PhD candidate with the Control Systems Technology Group. The research programme, named ‘ImpulseDrive’ focuses on the design methodologies, and topology design for hybrid vehicle drivetrains with a significant reduction of fuel consumption, and emissions.

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1 Introduction

The hybridisation in vehicles implies adding a secondary power source with a reversible energy buffer (S) (i.e. an electric machine/battery) to a primary power source with an irreversible energy buffer (P) (i.e. an engine/filled fuel tank) in order to improve vehicle performance. The major desirable improvements are the vehicle’s fuel economy, emissions, comfort, safety and driveability. The fuel consumption of a vehicle can be reduced by down-sizing the engine, which results in less idle-fuel consumption, and a lower brake-specific fuel consumption. A second, though complementary, method is recuperation of the brake energy, and reusing this stored energy when momentary fuel costs are high, avoiding idle-fuel consumption and engine operation points with high brake-specific fuel consumption. The Energy Management Strategy (EMS) plays an important role in an effective usage of the drive train components, see, for example, Chen and Salman (2005), Delprat et al. (2004), Kessels et al. (2006), Koot et al. (2005), Paganelli et al. (2002), Rizonni et al. (2004) and Sciaretta et al. (2004). Control strategies may be classified into non-causal and causal controllers respectively. Furthermore, a second classification can be made among heuristic, optimal and sub-optimal controllers (Guzzella and Sciarretta, 2005). In the sections below some of these methods will be discussed in more detail.

1.1 Optimal control strategy – dynamic programming

A commonly used technique for determining the globally optimal EMS is Dynamic Programming (DP), see, e.g. Sciaretta et al. (2003), Koot et al. (2005) and Scordia et al. (2005). Using DP, the finite horizon optimisation problem is translated into a
finite computation problem (Bellman, 1962). Note that, although the DP solution may appear as an unstructured result, in principle the technique results in an optimal solution for the EMS. Using DP it is rather straightforward to handle nonlinear constraints. However, a disadvantage of this technique is the relatively long computation time, due to the relatively large grid density required. The grid density should be taken high, because it influences the accuracy of the result. Furthermore, it is inherently non-causal, and therefore not real-time implementable.

1.2 Sub-optimal control strategy – heuristic control strategy

Most of the described Rule-Based (RB) control strategies in the literature (Lin et al., 2003; Wipke et al., 1999; Wu et al., 2004; Zhu et al., 2006) are based on ‘if-then’ type of control rules, which determine, for example, when to shut down the engine or the amount of electric (dis-) charging powers. The electric (machine) output power is usually prescribed by a nonlinear parametric function. Each driving mode uses different parametric functions, which are strongly dependent on the application (drive train topology, vehicle and drive cycle), and need to be calibrated for different driving conditions. In Lin et al. (2003), Wu et al. (2004) and Zhu et al. (2006) the threshold values for mode switching and parameters are calibrated by using DP. Thereby, the power-split ratio between the secondary source $S$ and the vehicle wheels for each driving mode is optimised. To overcome the difficulty of calibrating a large number of threshold values and parameters, control strategies are developed based on optimal control theory, which will be discussed in the following section.

1.3 Sub-optimal control strategy – equivalent consumption minimisation strategy

In the literature Equivalent Consumption Minimisation Strategies (ECMS) are presented, see, for example, Guzzella and Sciarretta (2005), Paganelli et al. (2000, 2002), Musardo et al. (2005) and Sciarretta et al. (2004), which are based on an equivalent fuel flow rate $\dot{m}_{f, eq}(t)$ (g/s). The equivalent fuel flow rate uses an electric-energy-to-fuel-conversion-weight-factor, or equivalence (weight) factor $\lambda(t)$ (g/J) in order to weight the electrical power $P_s(t)$ (W) within the same domain at a certain time instant $t$. Basically, the $\lambda(t)$ is used to assign future fuel savings and costs to the actual use of electric power $P_s(t)$. Moreover, a well determined $\lambda(t)$ assures that the discrepancy between the buffer energy at the beginning and at the end of the drive cycle with time length $t_f$ is sufficiently small. The $\dot{m}_{f, eq}(t)$ is defined as,

\[ \dot{m}_{f, eq}(t) = \dot{m}_f(P_s(t)) - \lambda(t)P_s(t), \quad \lambda(t) > 0 \forall t \in \{0, t_f\}, \tag{1} \]

where $\dot{m}_f(t)$ is the instantaneous (actual) fuel flow rate. Although, for example, during discharging $P_s(t) < 0$ the actual fuel flow rate $\dot{m}_f(t)$ is reduced, Equation (1) shows that the fuel equivalent of the electrical energy $-\lambda(t)P_s(t)$ is momentarily increased and vice-versa. The optimal momentary power set-point $P_s^*(t)$ for the secondary power source is the power, which minimises Equation (1) given a certain $\lambda(t)$:

\[ P_s^*(t) = \arg \min_{P_s(t)} (\dot{m}_{f, eq}(t)|\lambda(t)). \tag{2} \]
The $\lambda(t)$ depends on assumptions concerning the component efficiencies and chosen penalty functions on deviation from the target battery state-of-charge. Next, an overview on various approaches to this optimisation problem seen in the literature is given. Note that the first two methods require that the drive cycle is known.

1.3.1 Method 1

In Paganelli et al. (2002), average efficiencies of the energy paths from fuel tank to battery, denoted as $\bar{\eta}_{\text{tank} \rightarrow \text{bat}}$, and vice-versa, denoted as $\bar{\eta}_{\text{bat} \rightarrow \text{tank}}$, are used to compute $\lambda(t)$:

$$
\lambda(t) = \min \left( \lambda_{\text{dis}} \frac{P_S(t)}{|P_s(t)|}, \lambda_{\text{chg}} \frac{P_S(t)}{|P_s(t)|}, \frac{P_A(t)}{|P_s(t)|} \right),
$$

with $\lambda_{\text{dis}} = 1/(\bar{\eta}_{\text{tank} \rightarrow \text{bat}} h_{\text{ls}})$, $\lambda_{\text{chg}} = \bar{\eta}_{\text{bat} \rightarrow \text{tank}} / h_{\text{ls}}$. The chemical content of fuel (J/g) is represented by $h_{\text{ls}}$.

1.3.2 Method 2

In Sciaretta et al. (2004) the equivalence factors for (dis-)charging $\lambda_{\text{dis}}$ and $\lambda_{\text{chg}}$ are calculated using various constant values of the control variable by running different simulations for all admissible control inputs, given the drive cycle, the vehicle parameters and the upper and lower bounds for the state-of-charge. Accordingly, linear functions are fitted through the values of the total fuel energy use and the electrical energy use during discharging and charging. The slopes of the straight lines that fit the data correspond with $\lambda_{\text{dis}}$ and $\lambda_{\text{chg}}$ respectively.

As mentioned by the authors (Musardo et al., 2005), a disadvantage may be the strong sensitivity of $\lambda(t)$ and thus the equivalence factors on the relative state-of-charge $\xi(t)$ deviations. The main reason is lack of feedback or adaptation of $\lambda(t)$ according to the actual storage energy level. For online application the equivalence factor $\lambda(t)$ needs to be tuned, estimated, or adapted online, because, e.g. referring to method 1, the average component efficiencies are not known a priori and are different for each drive cycle. Next, some methods are discussed that do not require drive cycle information a priori.

1.3.3 Method 3

In Paganelli et al. (2001) and Rizonni et al. (2004), a nonlinear penalty function is used to heuristically penalise the control power flow out of S using the discrepancy between the reference relative value of the state-of-charge of the battery $\xi_0 \triangleq \xi(t = 0)$ and the actual relative value $\xi(t)$. The shape of the penalty function

$$
\lambda(t) = \phi_1 \left( 1 - \left( \frac{\xi_0 - \xi(t)}{\xi_{\max} - \xi_{\min}} \right)^{2\phi_2 + 1} \right) + \phi_3 \int_0^t (\xi_0 - \xi(\tau)) d\tau,
$$

with $\phi_1 \in \mathbb{R}_0^+$, $\{\phi_1, \phi_3\} \subseteq \mathbb{R}_0^+$.
can be adapted via certain parameters, i.e. \( \phi_1, \phi_2 \), and \( \xi_{\text{min}}, \xi_{\text{max}} \) describing the desired relative state-of-charge \( \xi \) operation range. The integral term with the tune parameter \( \phi_3 \) keeps track of the state-of-charge of the battery, which mainly influences the amplitude of the low-frequent oscillation of \( \xi \). If \( \phi_3 \) is chosen too large, then \( \xi(t) \) may become unstable, i.e. \( \xi(t) \to \infty \).

### 1.3.4 Method 4

In Chen and Salman (2005) the instantaneous equivalent fuel rates are calculated a priori for all admissible control inputs, vehicle load torque, speed and equivalence factors. Then, the optimal engine torques (used as control input) and gear ratios are pre-calculated and are stored in look-up tables. Starting with an initial guess, \( \lambda(t) \) is adjusted online with a predetermined correction value \( \Delta \lambda \) over the drive cycle:

\[
\lambda(t) = \begin{cases} 
\lambda_u, & \text{if } \xi(t) > \xi_{\text{max}}, \\
\lambda_l, & \text{else if } \xi(t) < \xi_{\text{min}}, \\
\lambda' - \Delta \lambda, & \text{else if } \xi(t) = \xi_0 \wedge \lambda = \lambda_u, \\
\lambda' + \Delta \lambda, & \text{else if } \xi(t) = \xi_0 \wedge \lambda = \lambda_l.
\end{cases}
\]  

(5)

If this correction number is chosen too large the equivalence factor \( \lambda(t) \) may oscillate. The previous stored \( \lambda' \) is corrected for with \( \Delta \lambda \) when \( \xi \) exceeds the upper or lower boundary of \( \xi \). The \( \lambda(t) \) is ‘learned’ by using predetermined fuel equivalence factors, i.e. \( \lambda_u, \lambda_l \) that are used to bring back the \( \xi(t) \) to the reference value. Then, the \( \lambda(t) \) is updated. The value of these parameters determine how fast the \( \lambda(t) \) is learned. Practical experience is needed to tune or to optimise these parameters.

### 1.3.5 Method 5

In Koot et al. (2005, 2006) and Kessels et al. (2006) also the discrepancy between the actual value and reference value of \( \xi \) of the battery is used to calculate \( \lambda(t) \) online, in this case by using a PI-controller:

\[
\lambda(t) = \lambda_0 + K_p (\xi_0 - \xi(t)) + K_i \int_0^t (\xi_0 - \xi(\tau)) \, d\tau.
\]  

(6)

This solution seems rather straightforward and effective, however, the solution is sensitive to the initial value \( \lambda_0 \) and the choice of the control parameters \( K_p \) and \( K_i \). If the \( K_p, K_i \) control parameters are chosen large, the power flow out of the accumulator is strongly penalised, such that the fuel saving will be smaller compared to mildly tuned control parameters. However, when a good estimation of \( \lambda_0 \) is obtained only a relatively mild PI-controller is required, because the relative change of \( \lambda(t) \) over the drive cycle is usually very small.

### 1.4 Sub-optimal control strategy – RB-ECMS

In order to tackle the drawbacks of DP, RB and ECMS, which is the aim of this paper, a new and relatively simple solution for the EMS control problem is introduced, having the following main features:
the proposed method consists of a combination of methods, i.e. RB and ECMS (RB-ECMS)

the maximum propulsion power of the secondary power source (i.e. electric machine/battery) during pure electric driving is used as an unique decision variable

the predefined hybrid modes and rules are independent on the type of drive train topology.

Since a drive train topology defines the paths and the efficiencies of the energy flow between P, S and the vehicle wheels. However, a topology choice influences the optimisation of the decision variable. Summarised, an overview of the discussed methods is shown in Table 1.

<table>
<thead>
<tr>
<th></th>
<th>DP</th>
<th>RB</th>
<th>ECMS</th>
<th>RB-ECMS (this paper)</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ globally optimal</td>
<td>− sub-optimal</td>
<td>− sub-optimal</td>
<td>− sub-optimal</td>
<td></td>
</tr>
<tr>
<td>− apparently unstructured result</td>
<td>− tuning of many parameters, threshold values</td>
<td>+ few calibration parameters</td>
<td>+ few calibration parameters</td>
<td></td>
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<tr>
<td>− long computation time</td>
<td>+ relatively simple, engineering intuition</td>
<td>+ short computation time</td>
<td>+ short computation time</td>
<td></td>
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<tr>
<td>− off line strategy</td>
<td>+ on-/off line strategy</td>
<td>+ on-/off line strategy</td>
<td>+ on-/off line strategy</td>
<td></td>
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<tr>
<td>+ handling nonlinear constraints</td>
<td>− specific rules depend strongly on the topology choice</td>
<td>− $\lambda(t)$ sensitive to $\xi(t)$ deviations</td>
<td>+ modes/rules independent on the topology choice</td>
<td></td>
</tr>
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</table>

1.5 **Outline of paper**

The remainder of this paper is structured as follows: firstly, the general control optimisation problem of a hybrid drive train is discussed in Section 2. Then, the derived hybrid driving modes and the RB-ECMS are discussed in Sections 3 and 4 respectively. Furthermore, a physical background for not using all potentially available motoring power during pure electric driving is given. The relationship between $\lambda(t)$ as used in ECMS and the decision variable, as used in the proposed RB-ECMS is discussed. In Section 5, results of the proposed RB-ECMS will be compared for a specific application (Toyota Prius, model 1998) with results from DP and the vehicle simulation platform ADVISOR (Wipke et al., 1999). Finally, the conclusions are given in Section 6.
2 Problem definition

The optimisation problem is finding the control power flow $P_s(t)$, given a certain power demand at the wheels $P_v(t)$, minimising the cumulative fuel consumption, denoted by the variable $\Phi_f$, over a certain drive cycle with time length $t_f$, subject to several constraints, i.e.

$$\Phi_f(E_e(t), P_v(t), t) = \min_{P_s(t)} \int_0^{t_f} \dot{n}_f(E_e(t), P_s(t), t | P_v(t)) \, dt, \text{ subject to } \bar{h} = 0, \bar{g} \leq 0,$$

where $\dot{n}_f$ is the fuel flow rate in g/s. The state is equal to the stored energy $E_e$ in the secondary reversal energy buffer in J, and the control input is equal to the secondary power flow $P_s$ in W (see also Figure 1). The energy level in the battery is a simple integration of the power and is calculated as follows,

$$E_e(t) = E_e(0) + \int_0^t E_i(\tau) \, d\tau. \quad (8)$$

The main constraints on the secondary power source S are energy balance conservation of $E_e$ over the drive cycle, constraints on the power $P_s$, and the energy $E_e$:

$$h_1 : E_e(t_f) - E_e(0) = 0,$$

$$g_{1,2} : P_{s,\text{min}} \leq P_s(t) \leq P_{s,\text{max}},$$

$$g_{3,4} : E_{e,\text{min}} \leq E_e(t) \leq E_{e,\text{max}}. \quad (9)$$

Figure 1  Power flows for the different hybrid driving modes. S connected at the engine side of the transmission

The optimal solution is denoted $P_s^*(t)$. In this paper the value for the energy level instead of the charge level in the battery has been used. Note that, if the open-circuit voltage of a battery is assumed constant, then the relative state-of-charge $\xi$ is equal to the relative state-of-energy, i.e., $\xi(t) = E_e(t)/E_e(0)$. However, for battery systems the open-circuit voltage typically changes slightly as a function of $\xi$. This is not considered in this paper.
3 Hybrid driving modes

A hybrid drive train can be operated in certain distinct driving modes. In Figure 1, a block diagram is shown for the power distribution between the different energy sources, i.e. fuel tank with stored energy $E_f$, S with stored energy $E_s$, and the vehicle driving over a drive cycle represented by a required energy $E_v$. The efficiencies of the fuel combustion in the engine, the storage and electric motor S, and the transmission (T) are described by the variables $\eta_p$, $\eta_s$, and $\eta_t$ respectively. The energy exchange between the fuel tank, source S and the vehicle can be performed by different driving modes (depicted by the thick lines). The engine power at the crank shaft is represented by $P_p$. The power demand at the wheels ($P_v$) and the power flow to and from S ($P_s$) determine which driving mode is active. The following operation modes are defined:

- **M**: Motor only mode, the vehicle is propelled only by the electric motor and the battery storage supply (S) up to a fixed propulsion power (decision variable) for the whole drive cycle, which is not necessarily equal to the maximum available propulsion power of the electric machine. The engine is off, and has no drag – and idle losses.
- **BER**: Brake Energy Recovery mode, the brake energy is recuperated up to the maximum generative power limitation and stored into the accumulator of S. The engine is off, and has no drag and idle losses.
- **CH**: Charging mode, the instantaneous engine power is higher than the power needed for driving. The redundant energy is stored into the accumulator of S.
- **MA**: Motor-Assisting mode, the engine power is lower than the power needed for driving. The engine power is augmented by power from S.
- **E**: Engine only mode, only the engine power is used for propulsion of the vehicle. S is off and generates no losses.

During the M and BER mode the engine is off, and as a consequence uses no fuel. This is also referred to as the Idle-Stop (IS) mode. Since the electrical loads in vehicles are expected to increase in the near future it may be important to define more hybrid (charging) modes. However, the auxiliary loads are not considered in this paper. The reader is referred to Kessels et al. (2005) in which heating and air conditioner loads are discussed.

4 The RB-ECMS

The operation points for P and S, given certain driving conditions (drive cycle and vehicle parameters) can be found in certain distinct driving states, or modes. For ease of understanding, the modes are represented as operation areas in a static-efficiency engine map separated by two iso-power curves, as shown in Figure 2. The solid iso-power curve separates the M mode from the CH mode, and the E mode. The
dotted iso-power curve separates the operation points of the engine during the CH and the MA mode. The vehicle drive power values for which the secondary source during the M mode is sufficient (i.e. below the solid line in Figure 2) is given by,

$$P_v(t) = \max(0, P_i(t), P_M(t)) \eta_i(t) \eta_p(t),$$

(10)

with $P_{s,min} \leq P_M(t) < 0$ the largest possible motor only power. The minimum discharging power is denoted as $P_{s,min}$. So we also have that in M mode:

$$P_i(t) = -P_M(t) \eta_i(t) \eta_p(t),$$

(11)

which is shown as a solid line in Figure 2. Following from the EMS calculated with DP, the decision variable $P_M(t)$ determining when to switch between the M mode and the other modes, appeared to be approximately constant with the vehicle power demand $P_i(t)$, i.e. $P_M(t) \approx P_M \forall t \in \{0, t_f\}$. Whereas the (dis-)charging power and the mode switch between MA and CH mode varies with the vehicle power demand.

Figure 2  Contour plot of the engine efficiency as a function of the engine torque and speed. WOT = Wide-Open Throttle torque

In order to fulfil the integral energy balance constraint over the drive cycle (cf. Equation (9)), the energy required for the M and the MA mode needs to be regenerated during the BER mode or charged during the CH mode. To explain the basic principles of the RB-ECMS, which is a trade-off between energy balance and fuel consumption, consider the following two arbitrary cases. Both cases are schematically shown in Figure 3. The energy of the CH1 – or the MA1 mode is always balanced with the M or the BER mode, respectively. Either case represents a different choice for $P_M$ whereby the recuperated brake energy (BER) is: (1) not sufficient, and (2) more than sufficient for supplying the energy during the motor only mode (M) over a given drive cycle:
additional required energy for the M mode has to be charged during the CH1 mode resulting in additional fuel cost

redundant energy of the BER mode can be used for motor-assisting during the MA1 mode resulting in additional fuel savings.

Referring to case (1), if $-P_M$ is lowered, then the additional fuel cost becomes lower, due to a decrease in the required charging energy. However, the fuel saving due to the M mode is also reduced, and vice-versa, if $-P_M$ is increased. The same holds for case (2): the fuel saving during the MA1 mode is increased if $-P_M$ is lowered, but the fuel saving due to the M mode is reduced.

**Figure 3** Energy balance and fuel consumption, after completion of a whole drive cycle

For both cases, additional charging (CH2 mode) during driving and using this buffered energy for motor-assisting (MA2 mode) can be beneficial. The energy of the CH2 mode is always balanced with the MA2 mode. This is illustrated with an example as is shown in Figure 4. In this figure, the fuel flow rate $\dot{m}_f$ as a function of the engine power $P_p$ at a certain engine speed $\omega_p$ is shown. Typically, for engines the fuel flow rate increases more than linear (convex curvature) with the output power at any given speed (Koot et al., 2006). Therefore, if charging is done at a low engine power demand and used for assisting at a high engine power demand, then it can be seen that fuel is saved by subtracting the fuel saving minus the fuel cost. This fuel saving potential increases, if the progressiveness of the fuel flow rate, as a function of the engine power, increases.

However, it should be noticed that the additional fuel saving potential is relatively small and limited, because:

- the fuel flow rate as a function of engine power is usually quite linear, resulting in smaller fuel flow rate differences
- the conversion losses between $P$ and $S$, and the storage losses of $S$ further decrease the fuel saving potential
- the energy of the high drive power demands dependent on the driving conditions (drive cycle, vehicle parameters) is usually relatively small.

The optimal value of $P_M$ has to be a trade-off between the BER, M, CH (CH1+CH2), and the MA (MA1+MA2) mode. This requires some kind of optimisation in which all modes are included and will be explained in the following three sections.
Figure 4  The fuel flow rate as a function of engine power at a certain engine speed

4.1 Power flow during the BER and the M mode

Based on the EMS from DP, the optimal power set-point $P^o_s(t) = P^o_{s,I}(t)$ during the M and the BER mode is respectively,

$$P^o_{s,I}(t) = -\max \left( \frac{P_s(t)}{\eta_{s}(t)}, \frac{P_s(t)\eta_{s}(t)}{\eta_{h}(t)} \right).$$  \hspace{1cm} (12)

The subscript $I$ indicates the power flow during the BER and M mode. The minus sign in Equation (12) indicates that the source S is discharging during propulsion and charging during braking. Notice that if the source S is coupled at the wheel side of the transmission $\eta_{h}(t)$ in Equation (12) is left out. The power set-point is limited between the following constraints,

$$P_{s,\text{min}} \leq P^o_s(t) \leq 0 \leq P^o_{s,I}(t) \leq P_{s,\text{max}}.$$  \hspace{1cm} (13)

Braking powers larger than the maximum charging power $P_{s,\text{max}}$ are assumed to be dissipated by the wheel brake discs. If only the M and/or the BER mode are utilised, then the relative energy $\Delta E_{s,I}$ at the end of the drive cycle becomes,

$$\Delta E_{s,I} = \int_0^\gamma P^o_{s,I}(t)dt, \Delta E_{s,I} \in \mathbb{R}.$$  \hspace{1cm} (14)

In order to fulfil the equality constraint $h_1$ of Equation (9) this energy has to be counterbalanced with the relative energy $\Delta E_{s,II}$ at the end of the cycle during the MA and the CH mode as is shown in Figure 5, whereby,

$$-\Delta E_{s,I} = \Delta E_{s,II}.$$  \hspace{1cm} (15)
4.2 Power flow during the MA, the CH and the E mode

The fuel flow rate during the BER/M mode is \( \dot{m}_f(P_o) \). Therefore, the total fuel flow rate \( \dot{m}_f(t) \) can be written as the sum of the fuel flow rate only, depending on the drive power demand \( P_v(t) \) (engine only, E mode) and some additional fuel flow rate \( \Delta \dot{m}_f(t) \) depending on the (dis-)charging power \( P_s(t) \) during the MA and the CH mode,

\[
\dot{m}_f(t) = \begin{cases} 
0, & \text{if } -\frac{P_s(t)}{(\eta_f(t) \eta_t(t))} \geq P_M, \\
\dot{m}_f(P_v(t)) + \Delta \dot{m}_f(P_s(t)), & \text{elsewhere.} 
\end{cases}
\] (16)

If \( P_{s,HI}(t) = 0 \), then the vehicle is propelled by the engine only (E mode). During the MA and the CH mode the engine is on and the optimal motor-assisting or charging power \( P_{s,HI}(t) \) depends on the drive power demand \( P_v(t) \), the component efficiencies and the amount of energy \( \Delta E_{s,HI} \) that needs to be counterbalanced with the energy used during the BER/M mode \( \Delta E_{s,HI} \). The optimisation problem becomes finding the optimal power flow \( P_{s,HI}(t) \) during the CH and the MA mode, given a certain power demand \( P_v(t) \), while the cumulative fuel consumption denoted by the variable \( F \) over a certain drive cycle with time length \( t_f \) is minimised, subjected to the energy constraint of Equation (15):

\[
\Phi_f(P_{s,HI}(t), t) = \min_{P_{s,HI}(t)} \int_0^{t_f} \dot{m}_f(P_{s,HI}(t), t|P_v(t)) \, dt, \quad \text{subject to } \int_0^{t_f} P_{s,HI}(t) \, dt = \Delta E_{s,HI}. \] (17)

Finding a solution to this problem can be solved via an unconstrained minimisation of the Lagrangian function \( \Phi_f \) using a Lagrange multiplier \( \lambda(t) \).

\[
\Phi_f(P_{s,HI}(t), \lambda(t)) = \min_{P_{s,HI}(t)} \int_0^{t_f} (\dot{m}_f(P_{s,HI}(t), t|P_v(t)) - \lambda(t) P_{s,HI}(t)) \, dt + \lambda(t) \Delta E_{s,HI}. \] (18)

The optimal solution can be calculated by solving,

\[
\frac{\partial \left( \Phi_f(P_{s,HI}(t), \lambda(t)) \right)}{\partial P_{s,HI}(t)} = 0 \quad \text{and} \quad \frac{\partial \left( \Phi_f(P_{s,HI}(t), \lambda(t)) \right)}{\partial \lambda(t)} = 0. \] (19)
The solution is given by,

$$\frac{\partial (\bar{m}_f (P_{s,II}(t), \hat{t})|P_V(t))}{\partial P_{s,II}(t)} - \lambda(t) = 0 \quad \text{and} \quad \int_0^{\hat{t}} P_{s,II}(t)dt = \Delta E_{s,II}. \quad (20)$$

Since $\bar{m}_f (P_{s,II})$ is a convex function, there exists a unique solution for $\lambda(t)$ i.e. $\lambda(t) = \lambda_0$ and $P_{s,II}(t)$ imposed by the (in-)equality constraints (see for proof, Van den Bosch and Lootsma (1987)). The optimising solution $\lambda_0$ requires the a priori information of the complete drive cycle. If $\lambda_0$ is known, then the optimal accumulator power $P_{s,II}(t)$ can be calculated by solving at the current time instant $t$:

$$P_{s,II}(t) = \arg \min_{P_{s,II}(t)} (\bar{m}_f (P_{s,II}(t), t)|P_V(t)) - \lambda_0 P_{s,II}(t), \quad (21)$$

whereby the power set-point is limited between the following constraints,

$$P_{s,min} \leq 0 \leq P_{s,II}(t) \leq P_{s,max}. \quad (22)$$

Then $\Delta E_{s,II}$ is discharged (charged) at vehicle power demands where the fuel savings (costs), i.e. $\Delta \bar{m}_f$ are maximum (minimum). In addition, the energy quantities during the MA and the CH mode are in balance with the BER and M mode over the whole drive cycle.

4.3 Optimisation routine (offline) for calculating $P_M^o$

Summarised, the optimal power set-point for the secondary power source $S$, as discussed in the previous two sections, during the BER/M and the CH/MA mode becomes, respectively:

$$P_{s,II}(t) = \begin{cases} P_{s,II}(t) \ (\text{cf. Eq. (12))}, & \text{if } - P_V(t)/\eta_V(t) \eta(t) \geq P_{s,II}(t) \ \\ P_{s,II}(t) \ (\text{cf. Eq. (21)}), & \text{elsewhere.} \end{cases} \quad (23)$$

In Figure 6(a), a block diagram is shown of the offline optimisation routine suggested in this paper. The routine consists of two iteration loops. In addition, some example results of the iteration loops are shown in Figure 6(b), (c) and (d) respectively. The results are based on a hybrid vehicle, which will be discussed in more detail in the following section. In iteration loop 1, the value for $\lambda$ using a chosen fixed mode switch value of $P_M = [P_{s,min}, P_{s,max}]$ is determined, which ensures that, for the whole drive cycle, the energy during the BER/M modes is in balance with the energy during the CH/MA modes (see Figure 6(b)). The corresponding $\lambda$ is denoted as $\lambda_0$:

$$\lambda_0 \in \{ \Delta E_s = \Delta E_i(\lambda) | \Delta E_i(\lambda_0) = 0 \ \wedge \ \Delta E_s = \Delta E_{s,II} + \Delta E_{s,MA} \}. \quad (24)$$

In Figure 6(c) the resulting $\Phi_f$ as a function of different chosen fixed values for $P_M$ is shown. In iteration loop 2, the optimal value for $P_M$ is determined, which minimises the total fuel consumption $\Phi_f$:

$$P_M^o = \arg\min_{P_M} \Phi_f(P_M). \quad (25)$$

Then, simultaneously the corresponding value for $\lambda_0$, denoted as $\lambda_0^o$, is stored as shown in Figure 6(d). In the following section, based on the results with DP and the RB-ECMS, the relationship between $P_M^o$ and $\lambda_0^o$ will be discussed in more detail.
Figure 6  Numerical optimisation scheme for calculation of $P_M^\text{o}$ (offline) and example results of iteration loops 1 and 2

5 Simulation results

5.1 Component models

Simulations were done for a series-parallel hybrid transmission type (Toyota Prius, model 1998). In Figure 7 the control model, which is used to calculate the optimal control signal, is shown. The arrows indicate the direction of the power flow and the components, which are schematically represented as blocks, are modelled as static efficiency functions. The control model is a backwards facing or differentiating model. The input is the vehicle speed, which is assumed to be tracked exactly. The transmission technology under investigation consists of one planetary gear set and an electric variator (electric machines 1 and 2). The engine is connected to the carrier,
electric machine 1 is connected to the sun gear, electric machine 2 and the propulsion shaft are both connected to the annulus of the planetary gear set. In this case S is part of T. However, if only the components, that are used during the BER and M mode, are defined to be a functional part of S, then electric machine 2 (connected at the wheel-side of T) is part of S (see Figure 7).

Figure 7 Power flow in the hybrid vehicle drive train (backwards facing control model)

In Table 2 an overview of the component data (NREL, 2002) is given. The inertias of the electric machines, engine and auxiliary loads are, for simplicity, assumed to be zero. All simulations performed presented in this paper have been done for the JP10-15 mode cycle (JISHA, 1983). It is derived from the ten-mode cycle (maximum speed of 40 km/h) by adding another 15-mode segment of a maximum speed of 70 km/h. In Figure 8 the vehicle speed and the corresponding drive power demand at the wheels, as a function of time, is shown. Furthermore, the engine is assumed to be operated at its maximum efficiency operation points.

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Relevant component data</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Power-split CVT</strong></td>
<td></td>
</tr>
<tr>
<td>Electric machine 1 (engine side)</td>
<td>Manufacturer: Toyota; 15-kW (continuous) PM motor/inverter, Torque range: from 55 to 26 Nm (corresponding speed from 0 to 5500 rpm). The efficiency map includes the inverter/controller efficiencies.</td>
</tr>
<tr>
<td>Electric machine 2 (wheel side)</td>
<td>Manufacturer: Toyota; 30-kW (continuous) PM motor/inverter, Torque range: from 305 to 47.7 Nm (corresponding speed from 0 to 6000 rpm). The efficiency map includes the inverter/controller efficiencies.</td>
</tr>
<tr>
<td>Planetary gear set/final drive</td>
<td>The planetary gear set ratio and the final drive ratio are −2.6 and 0.2431 respectively. The efficiencies are both constant 0.98 assumed.</td>
</tr>
</tbody>
</table>
Table 2  Relevant component data (continued)

Energy storage system
Battery pack  Manufacturer: Panasonic; Type: Ni-MH, Nominal voltage 288 Vdc, Capacity 6 Ah, $\xi_{\text{low}} = 45\%$; $\xi_{\text{high}} = 75\%$; $\xi_{0} = 55\%$.

Vehicle data
Mass: 1368 kg, Air drag coefficient: 0.29, Frontal area: 1.746 m$^2$, Roll resistance coefficient: 0.9%, Maximum regenerative brake fraction: 0.5.

Engine data
Manufacturer: Toyota; Displacement and type: 43-kW (at 4000 rpm) 1.5-L SI Atkinson internal combustion engine. Maximum torque: 102 Nm at 4000 rpm

Figure 8  Vehicle speed and mechanical drive power demand at vehicle wheels

5.2 Control models
For comparison the control strategy based on measurement data as is implemented in ADVISOR (Wipke et al., 1999) is compared with the results from the RB-ECMS and DP. The control strategies, which will be compared, are listed in Table 3.

Table 3  Simulated strategies for comparison

<table>
<thead>
<tr>
<th></th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>RB1</td>
<td>Default ADVISOR control strategy</td>
</tr>
<tr>
<td>RB2</td>
<td>Optimised ADVISOR control strategy</td>
</tr>
<tr>
<td>RB-ECMS</td>
<td>RB-ECMS control strategy</td>
</tr>
<tr>
<td>DP</td>
<td>The strategy based on the outcome of the DP algorithm</td>
</tr>
</tbody>
</table>
5.3 Reference heuristic control model – ADVISOR

In Table 4 the rule-based conditions that define which hybrid mode is active are given. If the wheel torque demand is negative, i.e. $T_v(t) < 0$, then the BER mode is active.

The control parameters $f_M\hat{P}_p(t)$ (engine-power-ratio threshold value) and $v_M$ (vehicle-electric-launch-speed threshold value) determine if the M mode is active. The battery is allowed to operate within a certain defined state-of-charge window, i.e. $\xi(t) = [\xi_{\text{low}}, \xi_{\text{high}}]$. If the state-of-charge $\xi(t)$ gets too low, then the battery is charged during driving (CH mode) with a certain charging power, which is the output of a proportional controller of which the input is the difference between $0$ and $T_p(t)$.

The motor-assisting (MA mode) is only performed if the engine torque demand is larger than the maximum available engine torque $T_{p,\text{max}}$, which is a function of the engine speed $\omega_p(t)$. The default control parameters $f_M$ and $v_M$, as implemented in ADVISOR (RB1), were optimised (RB2) to achieve the highest fuel economy, while the final $\xi(t_f)$ is maintained within a certain tolerance band $\pm 0.5\%$ from its reference value $\xi_0$.

Table 4 Rule-based control model as is implemented in ADVISOR

<table>
<thead>
<tr>
<th>Mode</th>
<th>Rule-based condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>BER</td>
<td>$\xi(t) &lt; \xi_{\text{high}}$ &amp; $T_v(t) &lt; 0$</td>
</tr>
<tr>
<td>M</td>
<td>$\xi(t) \geq \xi_{\text{low}}$ &amp; $P_p(t) &lt; f_M P_{p,\text{max}} \lor v(t) &lt; v_M$</td>
</tr>
<tr>
<td>CH</td>
<td>$\xi(t) &lt; \xi_{\text{low}} \lor \xi(t) &lt; \xi_0$ &amp; $P_p(t) \geq f_M P_{p,\text{max}}$</td>
</tr>
<tr>
<td>E</td>
<td>$\xi(t) = \xi_0$ &amp; $P_p(t) \geq f_M P_{p,\text{max}}$</td>
</tr>
<tr>
<td>MA</td>
<td>$\xi(t) \geq \xi_{\text{low}}$ &amp; $T_p(t) &gt; T_{p,\text{max}}(\omega_p(t))$</td>
</tr>
</tbody>
</table>

5.4 Reference optimal control model – dynamic programming

Using DP for solving the optimal control problem requires discretisation of Equation (7) with a time step $\Delta t$. Firstly, the continuous variables are mapped onto a fixed grid. The DP strategy is used with an input grid of 250 W and a state grid of 250 J. The relevant state variable is the energy level in the battery, which becomes

$$E_s(k + 1) = E_s(k) + P_s(k) \Delta t, \text{ for } k = [1, \ldots, n - 1] \in \mathbb{N}_0^+$$

with the constraints put on $P_s(k)$,

$$P_{s,\text{min}} \leq P_s(k) \leq P_{s,\text{max}},$$

and the bounds on $E_s(k)$ can be written as constraints on $P_s(k)$ in the following way,

$$E_{s,\text{min}} - E_s(1) \leq \sum_{k=1}^{n-1} P_s(k) \Delta t \leq E_{s,\text{max}} - E_s(1).$$

Furthermore, the energy balance conservation requires,
Then, a cost-to-go matrix \( \Phi^D_j \) is calculated, whereby each element represents fuel costs for reaching the final end-state. The DP algorithm holds,

\[
\Phi^D_j(E_s(k), k) = \begin{cases} 
0, & \text{for } k = n, \\
\min_{P(k)} \left\{ \Phi^D_j(E_s(k+1), P_s(k+1), k+1) + \bar{m}_f(P_s(k), k) \Delta t \right\}, & \text{for } k = [n-1, \ldots, 1]
\end{cases}
\]

Finally, at each time step \( k \) of the optimisation search, the function \( \Phi^D_j(E_s(k), k) \) is evaluated only at the grid points of the state variable. The recursive equation is solved backwards and the path with minimal costs represents the optimal trajectory (see Figure 9).

Figure 9 Illustration of feasible domain for battery energy and optimal trajectory resulting from DP algorithm along the drive cycle.

5.5 Results

In Table 5 the fuel economy results for the different strategies are listed. Note that the measured fuel economy reported by Toyota is 3.57 l/100 km (28 km/l). In Figure 10 the energy distribution over the different hybrid driving modes for each strategy is shown. In Figure 11(a) and (b) the relative energy over time \( \Delta E_s(t) \) and the fuel consumption \( \Phi_f(t) \) for the different strategies are shown, respectively. With the default control parameters, as implemented in RB1 \( (f_M = 0.20, \text{ which is equivalent to } P_M \eta_M(v_M) = -6 \text{ kW}, \text{ and } v_M = 12.5 \text{ m/s}) \), it was found, that, during propulsion at relative low \( P_s(t) \) and braking, the engine was not always allowed to shut off. This resulted in less idle stop and less effective regenerative braking power, due to additional engine drag torque losses respectively. The optimised control parameters for the RB2 are \( f_M = 0.116 \), which is equivalent to \( P_M \eta_M(v_M) = -5 \text{ kW}, \text{ and } v_M = 20 \text{ m/s} \). The optimal value for \( f_M \) is lower than the default value, which decreases the energy used during the M mode and the required additional charging cost during the CH mode (see Figure 10). Furthermore, if the threshold value \( v_M \) is
set to a larger value than the maximum cycle speed, then effectively more energy is charged during the BER mode, which reduces the required additional charging cost, during the CH mode, further.

Although electric machine 2 is specified at 30 kW, only approximately 4.9 kW is effectively used for propulsion during pure electric driving (see RB-ECMS in Table 5). The redundant machine power is mainly used for vehicle performance requirements. The discrepancy between the fuel economy results and the relative energy over time calculated with the RB-ECMS and DP is small (±1%). It can be concluded, that the fuel economy with the RB-ECMS can be calculated very quickly and with sufficient accuracy.

<table>
<thead>
<tr>
<th>Table 5</th>
<th>Fuel economy results</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Strategy</strong></td>
<td><strong>fuel economy (1/100 km)</strong></td>
</tr>
<tr>
<td></td>
<td><strong>City</strong></td>
</tr>
<tr>
<td>RB1</td>
<td>2.33</td>
</tr>
<tr>
<td>RB2</td>
<td>2.70</td>
</tr>
<tr>
<td>RB-ECMS</td>
<td>2.78</td>
</tr>
<tr>
<td>DP</td>
<td>2.78</td>
</tr>
</tbody>
</table>

Note: *Pentium IV, 2.6-GHz, with 512-MB of RAM

Figure 10  Energy balances for the different strategies
Figure 11  The relative energy and fuel economy over time for the different strategies

(a) Relative energy over time

(b) Fuel consumption over time
Rule-based energy management strategies for hybrid vehicles

In Table 6 the fuel saving for the different hybrid modes are shown. The reference fuel economy of 5.35 l/100 km is calculated with the same vehicle simulation model under the condition that \( P_s(t) = 0 \forall t \in \{0, t_f\} \). It can be seen, that the largest fuel saving improvement, 39.6\%, is realised with the BER and M mode. Additional charging (CH1 mode) and using this energy for the M mode, increases the relative fuel saving with approximately 43.6\% – 39.6\% = 4\%. The smallest fuel saving improvement 44.4\% – 43.6\% = 0.8\% is obtained by performing some additional charging (CH2 mode) and using this energy for motor-assisting (MA2 mode) during driving.

Table 6  Fuel economy results and relative improvements (RB-ECMS)

<table>
<thead>
<tr>
<th>Hybrid mode (active = x):</th>
<th>BER</th>
<th>M</th>
<th>CH1</th>
<th>CH2</th>
<th>MA2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>–</td>
<td>x</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>5.35</td>
<td>3.23</td>
<td>3.04</td>
<td>2.98</td>
<td></td>
</tr>
<tr>
<td></td>
<td>100.0%</td>
<td>60.4%</td>
<td>56.4%</td>
<td>55.6%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>–</td>
<td>39.6%</td>
<td>43.6%</td>
<td>44.4%</td>
<td></td>
</tr>
</tbody>
</table>

5.6 Evaluation of the motor only mode

The fuel flow rate of the engine can be approximated by the affine relationship (Koot et al., 2006),

\[
\dot{m}_f(t) \approx \dot{m}_{f0} + \lambda_1 P_p(t), \quad \dot{m}_{f0} \equiv \dot{m}_f(P_p(t) = 0) \quad (\text{idle fuel flow rate})
\]  \(31\)

The idle fuel flow rate at zero mechanical power is represented by \( \dot{m}_{f0} \). The slope of Equation (31) \( \lambda_1 \) is approximately constant and expresses the additional fuel flow rate over demanded engine power. If the optimal threshold power for the engine to switch on corresponds to \( P_{o,M}(t) = -P_{o,M}^0(t) \eta_p(t)/\eta(t) \), then the maximum fuel saving in the M mode is given by,

\[
\Delta \dot{m}_f(t) = \dot{m}_{f0} + \lambda_1 \cdot -P_{o,M}^0(t) \eta_p(t)/\eta(t).
\]  \(32\)

It is found with results from RB-ECMS and DP, that the engine switches on at the motoring power, where the equivalent fuel cost for charging is equal to the maximum fuel saving in the M mode:

\[
\lambda_0^* \cdot -P_{o,M}^0(t) = \dot{m}_{f0} + \lambda_1 \cdot -P_{o,M}^0(t) \eta_p(t)/\eta(t)
\]  \(33\)

\[
\Leftrightarrow P_{o,M}^0(t) = \frac{\dot{m}_{f0}}{\lambda_1 \cdot \eta_p(t)/\eta(t) - \lambda_0^*},
\]  \(34\)

describing the relationship between the optimal motoring threshold power \( P_{o,M}^0(t) \) and \( \lambda_0^* \). The optimal motoring threshold power is approximately constant, given that the
secondary source – and transmission efficiency are approximately constant for values
around $P_M^s$, i.e.,

$$P_M^s(t) \approx \left( P_M^s | \eta_s(t) \approx \eta_i \land \eta_t(t) \approx \eta_i \right),$$  \hspace{1cm} (35)

which is sufficiently accurate to be used with the RB-ECMS, as shown in the
previous section. For motoring threshold powers larger than $-P_M^s$ the fuel cost for
recharging becomes larger than the fuel saving, which is schematically depicted in
Figure 12.

Figure 12  Mode switch decision variable $P_M^s$

\[ \text{Figure 12} \quad \text{Mode switch decision variable } P_M^s \]

\[ \begin{array}{c}
\text{Fuel cost} \\
\lambda_0 \\
\lambda_1 \cdot \left( \eta_s / \eta_t \right) \\
\text{Fuel saving}
\end{array} \]

6  Conclusion

In this paper, an overview of different control methods is given and a new rule-based
EMS is introduced, based on the combination of Rule-Based – and Equivalent
Consumption Minimisation Strategies (RB-ECMS). The RB-ECMS consists of a
collection of driving modes selected through various states and conditions. In
addition, a graphical representation of the influence of the hybrid driving modes on
the energy balance and the fuel saving potential is discussed. The RB-ECMS uses
only one decision variable and requires no tuning of many threshold control values
and parameters. The main decision control variable is the maximum propulsion
power of the secondary power source (i.e. electric machine/battery) during pure
electric driving. The optimal maximum propulsion power will be a trade-off between
the fuel saving with pure electric driving (idle-stop), motor-assisting and the fuel
charging cost for counterbalancing the electric energy use. The fuel economy and
control strategy for the Toyota Prius, model 1998, calculated with the RB-ECMS has
been compared with results from the vehicle simulation platform ADVISOR and DP
strategy. The results show that the default strategy, as implemented in ADVISOR,
can be significantly improved (12%) and that the results of the RB-ECMS are very
close to the global optimal solution calculated with DP (±1%). The RB-ECMS
discussed is optimised offline very quickly, which can be used as part of a hybrid
drive train topology selection – and component specification tool, which is currently
under development by the authors. In future work the RB-ECMS implemented in an
online control application will be investigated.
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