ACOUSTIC EXCITATION OF MECHATRONIC SYSTEMS

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Abstract

Within the specialty mechatronics a plurality of disciplines such as mechanics, electronics, software and control are combined to develop extremely accurate precision machinery. Examples are ultra-precise measuring equipment with nanometer accuracy, stages for lithography applications and stages for electron microscopes. The accuracy of mechatronic systems is rapidly increasing. Key in the development of such highly precise machinery is to control the disturbances affecting the accuracy of the machine. A systematic way to do so, is to define a “dynamic error budget” which is divided amongst the different disturbances. Many different disturbances need to be considered. To mention a few: floor vibrations, vibrations generated internally by the machine, acoustic excitation due to flow and/or clean-room air-conditioning systems, etc. The latter disturbance, acoustic excitation, claims a significant part to the error budget, especially for extremely accurate precision machinery.

In order to estimate the contribution to the dynamic error budget already in the design phase of the machine, it is necessary to predict the response of the system to acoustic excitation. In the design phase of a machine only approximate dimensions are available, which calls for approximate estimates of the machines sensitivity to acoustic excitation. The paper discusses such an approximate method, which considers rigid body motion of the machine only, excited by plane acoustic waves. The method uses an analytical model. The analytical model is derived, and the theory is validated by means of experiments.
INTRODUCTION

Within the specialty mechatronics a plurality of disciplines such as mechanics, electronics, software and control are combined to develop extremely accurate precision machinery. Examples are ultra-precise measuring equipment with nanometer accuracy, stages for lithography applications and stages for electron microscopes. The accuracy of mechatronic systems is rapidly increasing. Key in the development of such highly precise machinery is to control the disturbances affecting the accuracy of the machine. A systematic way to do so, is to define a “dynamic error budget” which is divided amongst the different disturbances (ref [1]). Many different disturbances need to be considered. To mention a few: vibrations generated internally by the machine like setpoint-motion related vibrations, flow noise, pneumatic noise etc. Other sources that usually claim a significant part of the error budget are external disturbances like floor vibrations and environmental acoustical noise. The site dependent behaviour of these disturbance sources makes it hard to quantify them precisely. Commonly, depending on the application, number of machines and the number of applicable environments, an envelope disturbance specification is required. The envelope disturbance specification specifies the worst-case conditions of floor vibrations and environmental acoustical noise at which the machine still performs well.

During the conceptual design stage of highly precise machinery an essential step is to evaluate different conceptual design studies with respect to external disturbances. Despite the lack of details in the design it is possible to make calculations that deal with floor vibrations and acoustics. Predicting the influence of floor vibrations is quite accurately possible by using simple 1D models consisting of just a few rigid bodies. The main compliances in these models usually include the floor vibration isolation system and the first resonance frequency. The power of these calculations is that they provide insight and that they are very time efficient. The science of predicting the influence of acoustical environmental noise at the early machine design stage is less developed than for floor vibrations. The explanation for that is twofold: Firstly, the relation between floor vibrations and machine vibrations is straightforward, where the relation between acoustics and machine vibrations appears more complicated. In this paper is explained that also for the acoustic excitation of machinery straightforward relations exist. Secondly, floor vibrations usually dominate in the 1-50 Hz frequency region where highly precise machines behave like rigid bodies. Acoustic excitation usually dominates in the low to medium frequency region. For instance, the acoustic noise spectrum of clean room air-conditioning systems usually dominates at frequencies around the 125 Hz $1/3^{rd}$ octave band (ref. [2]). For many mechatronic applications this implies that both the rigid body motion and the first internal resonance of a machine are excited. This paper focuses upon the rigid body behavior of machines subjected to acoustic excitation. Reference is made to the literature to deal with resonant behavior.

In the next section, an analytical derivation of the sensitivity of a rigid body to acoustic excitation is given. Next, the experimental tests that validate the analytical model are discussed.
ACOUSTIC SENSITIVITY OF NON-RESONANT RIGID BODIES

Basically, two frequency regimes of practical importance can be considered. The first frequency regime is above the suspension resonance frequency and below the first internal resonance of the structure, where the structural response is governed by inertia. This situation will be treated analytically in this section. A second frequency regime of practical importance is at structural resonance, where damping controls the structural response.

The derivation given here applies to the first frequency regime. It follows the work of Fahy (ref. [3]), where Fahy gives a theory that applies to the resonant response of structures. The theory derived in this paper applies to a non-resonant rigid body motion, for which inertia effects govern its response. This is the case for frequencies above the suspension resonance frequency and below the first internal resonance of the structure. In many high-precision mechatronic applications this frequency range, which is typically between 10 and a few hundred Hertz, is most important with respect to acoustic excitation.

In situations that the resonant structural response needs to be determined (the second frequency regime), the reader is referred to the work of Fahy (ref. [3]).

The reciprocal problem; acoustic radiation of a vibrating structure

First, consider the reciprocal problem, i.e. a vibrating rigid body that radiates acoustic noise. Following Fahy (ref. [3]) the mean square acoustic pressure $p^2(r, \theta, \phi)$, measured at distance $r$ and at uniquely defined angles $(\theta, \phi)$ from the vibrating object, is as follows related to the radiated acoustic sound power $P_{\text{rad}}$:

$$p^2(r, \theta, \phi) = \frac{\rho_0 c P_{\text{rad}} D(\theta, \phi)}{4\pi r^2}$$

(1)

where $D(\theta, \phi)$ is the source directivity factor and $\rho_0 c$ is the characteristic acoustic impedance. The source directivity factor $D(\theta, \phi)$ is defined as

$$D(\theta, \phi) = \frac{I(\theta, \phi)}{I_0}$$

(2)

where $I(\theta, \phi)$ is the sound intensity in the far field, under the angles $(\theta, \phi)$ with the vibrating rigid body, and $I_0$ is the sound intensity produced at the same distance by a uniformly directional source of the same power, where $P_{\text{rad}} = I_0 4\pi r^2$.

Defining the acoustic radiation efficiency $\sigma$ as

$$\sigma = \frac{P_{\text{rad}}}{\rho_0 c S \left\langle v_n^2 \right\rangle}$$

(3)

with $\left\langle v_n^2 \right\rangle$ the spatially averaged square normal velocity of the rigid body, Eq. (1) can be rewritten as
Assume that the rigid body vibrates due to an external force $\vec{F}(\vec{r}_0)$ at point $\vec{r}_0$ of the rigid body. Assuming rigid body motion, and using the 2nd law of Newton, $\vec{F}(\vec{r}_0) = j\omega \vec{v}_0 \vec{m}$, where $\vec{m}$ is the mass of the rigid body and using equation (4) and where $\vec{v}_0$ is the velocity of the rigid body in the direction of the exciting force $\vec{F}(\vec{r}_0)$, the sound pressure radiated by the rigid body can be written as

$$p^2(r, \theta, \phi) = \frac{\rho_0 c^2 \alpha S (v_n^2) D(\theta, \phi)}{4\pi r^2}$$  \hspace{1cm} (4)

The factor $\frac{v_n^2}{v_0^2}$, which accounts for the surface mean squared normal velocity, given a velocity of the body as a whole, will be denoted by $\psi$:

$$\psi \equiv \frac{v_n^2}{v_0^2}$$  \hspace{1cm} (6)

$\psi$ is a constant, depending upon the shape of the body and the direction of vibration. Using definition (6), Equation (5) can be rewritten as

$$p(r, \theta, \phi) = \frac{\rho_0 c \sqrt{\frac{|\vec{F}(\vec{r}_0)| \alpha SD(\theta, \phi) v_n^2}{4\pi r^2}}}{{\sqrt{\psi}}}.$$  \hspace{1cm} (7)

In Figure 1 a schematic drawing is given. The left of Figure 1 represents the “reciprocal problem” which is considered now, and the right of Figure 1 represents the “direct problem of interest”, which is treated next.

**Figure 1** On the left the “reciprocal problem” (a force $F'$ exciting the structure mechanically, causing an acoustic pressure $p'$) and on the right the “direct problem” (a point source $Q$ exciting the structure acoustically, causing the structure to vibrate with a velocity $v$).
The direct problem: acoustic sensitivity of structure to acoustic excitation

Now, consider the direct problem of interest, i.e. a rigid body that is exposed to acoustic sound pressure field.

Following Verheij (ref. [4]), the following reciprocity relationship holds between the pressure \( p(r, \theta, \phi) \) at a distance \( r \) and at uniquely defined angles \( (\theta, \phi) \) from the vibrating structure, due to a force \( F(\tilde{r}_0) \) acting on the structure at position \( \tilde{r}_0 \) (our previously discussed “reciprocal problem”, see the left part of Figure 1), and the velocity response of the structure \( v(\tilde{r}_0) \) at point \( \tilde{r}_0 \) of the structure due to source \( Q(r, \theta, \phi) \) at a distance \( r \) and at uniquely defined angles \( (\theta, \phi) \) from the vibrating structure (the “direct problem of interest”, see the right part of Figure 1):

\[
\frac{v(\tilde{r}_0)}{Q(r, \theta, \phi)} = \frac{p(r, \theta, \phi)}{F(\tilde{r}_0)}
\]  

(8)

Using the reciprocity relation (8), the sensitivity of the structure to acoustic excitation can be determined from Eq. (7):

\[
\left| \frac{v(\tilde{r}_0)}{Q(r, \theta, \phi)} \right| = \frac{\rho_c c}{\omega m} \sqrt{\frac{c \sigma SD(\theta, \phi)}{4\pi r^2}} \sqrt{\psi}
\]  

(9)

Although this equation is valid for any position of the point source \( Q(r, \theta, \phi) \) relative to that of the structure, and for any acoustic environment, it is convenient to write the point source \( Q(r, \theta, \phi) \) in terms of the pressure at the location of the structure, but in its absence. Let’s denote this pressure by \( p'(\tilde{r}_0) \), where the apostrophe denotes the fact that the pressure is meant in the absence of the rigid body. Also assume that the point source \( Q(r, \theta, \phi) \) is located in a homogenous, free field, without flow, relatively far away from the rigid body (i.e. \( r >> L \) and \( r >> \lambda \), where \( L \) is a typical dimension of the rigid body), in which case the acoustic pressure at position \( \tilde{r}_0 \) can be written as

\[
p'(\tilde{r}_0) = Q(r, \theta, \phi) \frac{\rho_c c k}{4\pi}
\]  

(10)

Using this relation, Eq (9) can be written as

\[
\left| \frac{v(\tilde{r}_0)}{p'(\tilde{r}_0)} \right| = \frac{c}{\omega^2 m} \sqrt{4\pi c \sigma SD(\theta, \phi)} \sqrt{\psi}
\]  

(11)

Note that by assuming that the point source \( Q(r, \theta, \phi) \) is located far away from the structure, the structure will be excited by plane acoustic waves. Thus the relationship derived is valid for plane acoustic waves only.

This end result differs from that of Fahy (ref. [3]) at one main point. As we are considering a rigid body motion for which the inertia of the rigid body dominates, we have a term \( \omega^2 m \) in the denominator of Eq. (11), resulting in \(-12 \text{ dB/oct}\) for frequencies above the critical radiation frequency, while in Fahy’s result the denominator is reduced to a term \( \omega \), resulting in \(-6 \text{ dB/oct}\) for frequencies above the critical radiation frequency. This is because Fahy assumes a modal response, for which intertia effects cancel the stiffness effects and for which the response of the
structure is determined by damping of the structure only. This means that in the end expression of Fahy, the mass of the structure is not found, and the dependence on frequency is one order different (i.e. $\omega^{-1}$ instead of $\omega^{-2}$).

**EXPERIMENTAL VALIDATION**

In this section the analytical model is validated by means of experiments. In designing the experiment, a body was chosen for which analytical solutions of the source directivity factor $D(\theta, \phi)$ is available; a rigid sphere. The far-field directivity of an oscillating rigid sphere is given by

$$D(\theta, \phi = 0) = 3 \cos^2(\theta)$$

(12)

For an oscillating rigid sphere the factor $\psi$, which accounts for the varying normal velocity given a velocity of the body as a whole, as defined by Eq. (6), equals

$$\psi = 1/3$$

(13)

The acoustic radiation efficiency $\sigma$ of an oscillating rigid sphere is given by

$$\sigma = \begin{cases} 
(\omega/\omega_0)^4 & \text{for } \omega < \omega_0 \\
1 & \text{for } \omega \geq \omega_0 
\end{cases}$$

(14)

where $\omega_0$ is the critical angular frequency, which is given by

$$\omega_0 = \sqrt{2} \frac{c}{a}$$

(15)

and where $c$ is the speed of sound and where $a$ is the radius of the sphere.

In the experimental tests, a football with a radius of 0.1 m was used and a weight of 0.361 kg. Using Equations (11) through (15) then gives

$$\left| \frac{v(r_0)}{\omega_0^2 m} \right| = \begin{cases} 
\frac{c}{\omega_0^2 m} \frac{4\pi a}{cm} & \text{for } \omega < \omega_0 \\
\frac{c}{\omega_0^2 m} \frac{4\pi a}{cm} & \text{for } \omega \geq \omega_0 
\end{cases}$$

(16)

This relationship shows some typical properties of a rigid body excited by plane acoustic waves:

The critical radiation frequency $f_0 = \frac{\omega_0}{2\pi}$, which is commonly used to indicate the frequency above which a vibrating structure radiates acoustic noise efficiently, plays an important role for acoustic excitation also.

Below the critical radiation frequency, the sensitivity to acoustic excitation in terms of velocity over pressure, is frequency independent.

Above the critical radiation frequency, the sensitivity to acoustic excitation in terms of velocity over pressure, falls off with -12 dB/oct.

Experiments were performed using a football with a radius of 0.1 m and a weight of 0.361 kg. The experiments were conducted in an anechoic room. A loudspeaker is used to excite the football. The distance between the loudspeaker and the football was
large enough to create plane waves. The football was hanging on the ceiling by means of a string. The velocity of the football was measured by means of a laser vibro meter, which was located outside the anechoic room to avoid acoustic excitation of the laser vibro meter equipment. See Figure 2.

![Figure 2 Experimental test set-up in an anechoic room.](image)

The theoretical sensitivity of the football can be derived from Equation (16), giving:

\[
\frac{v(r_0)}{p'(r_0)} = \begin{cases} 
\frac{2\pi 0.1^3}{340 \cdot 0.361} = 5 \cdot 10^{-5} = -85dB \cdot \text{ref} \cdot \frac{1}{sPa} & \text{for } f < f_0 = 800Hz \\
\frac{c}{\omega^2 m} 4\pi a & \text{for } f \geq f_0 = 800Hz
\end{cases}
\]

(17)

The measurement results are shown in left subfigure of Figure 3. This figure also shows the theoretical sensitivity of the football.

![Figure 3 Structural response of the football to acoustic excitation, experiment & theory. On the left: football, a=.1m, 361 gram, Right subfigure: football filled with PUR, a=.0875 m, 258 gram.](image)

Below the critical radiation frequency of 800 Hz the correspondence between theory and experiment is sufficient. Above the critical radiation frequency the results are 'polluted' by an anti-resonance and a resonance of the acoustic sensitivity. This is caused by internal acoustic resonances of the football. Unfortunately (but logically)
these resonances are of the same order as the critical radiation frequency $f_c$, thus making it difficult to identify the -12 dB/oct fall-off.

To solve this problem, another experiment was conducted with another football that was filled with polyurethane foam (PUR), so to avoid acoustic resonances inside the football. The results are shown in the right subfigure of Figure 3. The resonant behavior of the football almost completely disappeared. Though there is some decrease of the acoustic sensitivity of the football above the critical radiation frequency, it is difficult to draw firm conclusions regarding the 12 dB/octave decrease as predicted by the theory.

CONCLUSIONS

An analytical model is derived to describe the acoustic excitation of a rigid body above its suspension resonance frequency. The model is based on the relation with the reciprocal case, i.e. the acoustic pressure resulting from a varying force exciting the rigid body. The model is applied to the acoustic excitation of a sphere. The resulting expression is experimentally verified in an anechoic chamber, using a speaker and a laser vibro measurer to determine the sphere's vibration amplitude.

While the model predicts a 12 dB/octave decrease of the acoustic excitation sensitivity above the critical frequency of the ball, this drop-off is not well confirmed by the experiments. Unfortunately, the result is 'polluted' by an anti-resonance and a resonance of the acoustic sensitivity just above the critical radiation frequency of the rigid sphere due to internal acoustic resonances. Additional experiments by filling the sphere with foam suppressed the resonant behavior of the sphere reasonably, and the measurements seems to give a decrease of acoustic sensitivity with frequency, but very firm conclusions can not be drawn from this. Below the critical frequency however, the model gives a rather accurate prediction of the acoustic sensitivity of the sphere.

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