Dynamic buckling of a thin shallow arch

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DCT 2006.31

Bachelor final project report

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Eindhoven, April, 2006
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Chapter 1

Introduction

Thin walled structures are often used in light weight constructions, which are for example favorable for space applications. The light weight is important for costs and the capacity of the rocket. During the launch, the acceleration can cause a thin walled structure in the form of a shallow thin arch to 'jump' from it's original position to it's hollow position. This jump, or snap, is not desirable in many constructions, so they have to be designed with enough strength and stability.

The acceleration is not constant, but mostly has small high frequent variations. The objective of this research is to investigate the influence of these variations on top of a constant acceleration for the thin shallow arch. For the design of the light weight structures, this influence of the acceleration on the stability of the arch should be known. In this report, the dynamic stability of a thin shallow arch will be investigated for this kind of time dependant accelerations.

Chapter 2 pays attention to the geometry of the arch, what kind of arches will be investigated, and the model will be explained which is used for the later analysis. In chapter 3, the arch will be subjected to a constant acceleration only. This gives a better understanding of the behavior of the arch, and shows the influence of some important parameters in our model. In chapter 4, the influence of the dynamic acceleration on the stability of the arch is analyzed. The analysis give us an good insight in the behavior of the arch when it’s subjected to frequent or relatively large distortions in the acceleration.
Chapter 2

Modeling of a thin shallow arch

For designing light weight structures, the effects of (harmonic) accelerations on their stability should be known. In order to investigate this, a model is needed. In this report, only thin shallow arches are investigated. The arches can snap from their initial shape to their hollow shape, a very undesirable effect. The snap is caused by large transversal accelerations. The arch is shown in figure 2.1 (b), where the acceleration is given by the movement of the endpoints.

![Diagram of a thin shallow arch](image)

Figure 2.1: (a) : Arch geometry, (b) : pinned-pinned arch with prescribed transversal end-point motion.

The accelerations investigated, are not constant. They have a disturbance, which is modeled as an harmonic component of the acceleration. The acceleration is given by

$$\ddot{y}(t) = Ac(1 + r_d \sin(2\pi ft)).$$  \hspace{1cm} (2.1)

The disturbance is considered to be small, so $r_d \ll 1$. The acceleration will result in an inertia force on the moving arch, which can cause the arch to buckle. The considered shal-
low arch has the same properties as the shallow arch considered in [1], and the same series of arch shapes are investigated. The initial shape of the arch is a half-sine. The arch has a thickness \( d \), an initial height \( h \) in the center, a cross section area of \( A \), and an width \( L \). The arch has the material properties of steel. The properties of the arch can be found in table 2.1.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>( 2.056 \times 10^{-5} )</td>
<td>([m^2])</td>
</tr>
<tr>
<td>( EI )</td>
<td>0.232</td>
<td>([Nm^2])</td>
</tr>
<tr>
<td>( h )</td>
<td>( 38.4 \times 10^{-3} )</td>
<td>([m])</td>
</tr>
<tr>
<td>( L )</td>
<td>0.8315</td>
<td>([m])</td>
</tr>
<tr>
<td>( d )</td>
<td>( 0.803 \times 10^{-3} )</td>
<td>([m])</td>
</tr>
<tr>
<td>( \rho )</td>
<td>7850</td>
<td>([kg/m^2])</td>
</tr>
</tbody>
</table>

Table 2.1: Parameter values of the thin shallow arch

For analyzing the effects of a load on the arch, not one, but series of arch shapes will be used. All these shapes are symmetric with respect to \( x = L/2 \), en are described by the shape-variation parameter \( a \), see figure 2 (a).

As stated before, the arches with a half-sine shape are examined. But in reality, the shape of the arch or the suspension is never perfectly symmetrical. Therefore, an imperfection is introduced. This imperfection is the first asymmetrical mode with amplitude \( e \), see figure 2 (b). The imperfection in the arch is considered to be small, \( e/d < 1 \).

To analyse the behavior of the arch, we use a 3 DOF (degrees of freedom) model so the deformed shape of the arch is described by 3 independent coordinates \( \mathbf{Q} = [Q_1 \ Q_2 \ Q_3]^T \). The deformation of the arch is described by three modes: the first 3 harmonic modes, see figure 2.2. The first harmonic mode, which is the first symmetric mode, is a half sine. The second harmonic mode, the first asymmetrical mode, is a full sine, and the third harmonic mode, the second symmetrical mode, is three half sine waves. Together they describe the deformed shape of the arch, this means that \( \mathbf{Q} = 0 \) always describes the initial upwarded...
shape of the arch, see figure 2.1. The corresponding equations of motion are (see [1]):

\[ M \ddot{Q} + C \dot{Q} + K(Q) = -B \ddot{y}(t), \]  

(2.2)

with \( M = \frac{\rho AL}{2} I, \ C = \frac{bL}{2} I, \ B = \frac{2\rho AL}{\pi} \begin{bmatrix} 1 & 0 & \frac{1}{3} \end{bmatrix}^T, \) where \( I \) is the identity matrix, and with:

\[ K(Q) = \begin{bmatrix} \frac{\pi}{2L} N(Q)(a + h + Q_1) + \frac{EI\pi^4}{2L^3} Q_1 \\ \frac{2}{\pi^2} N(Q)(18e\sqrt{3} + \pi^3 Q_2) + \frac{8EI\pi^4}{L^3} Q_2 \\ \frac{9\pi^2}{2L} N(Q)(Q_3 + a) + \frac{81EI\pi^4}{2L^3} Q_3 \end{bmatrix}, \]

(2.3)

with

\[ N(Q) = \frac{EA\pi^2}{4L^2} [Q_1(2a + 2h + Q_1) + 18aQ_3 + \frac{144\sqrt{3}}{\pi^3} eQ_2 + 4Q_2^2 + 9Q_3^2]. \]

As can be noted, the first asymmetrical mode described by \( Q_2 \), is only triggered if \( e \neq 0 \). The three coordinates are only coupled via \( K(Q) \).

Figure 2.2: The first three harmonic modes
Chapter 3

(Quasi-) static analysis

In this chapter the behavior of the arch subjected to a time-independent acceleration is analyzed, i.e. \( r_d = 0 \). The acceleration is constant in the upward direction. This results in an inertia load on the arch. With a constant load, the arch will deform until it finds a static equilibrium and then. For simplicity, the acceleration will be written as follows: \( \ddot{y}(t) = Ac \), so the static equilibrium can be written as:

\[
K(Q) = -BAc. \tag{3.1}
\]

In section 3.1, the behavior of a perfect shaped arch subjected to a static load will be investigated. For a better understanding of the results, the local stability and the eigenfrequencies are examined in section 3.2. In section 3.3 the behavior of an imperfect arch subjected to the static load will be investigated and the results are compared with the results of section 3.1. In section 3.4, the influence of different shapes on the behavior of perfect and imperfect arches is investigated.

3.1 Perfect arch

If the arch is considered to be perfect, \( (e = 0) \), and if we look at the initial shape of the arch, \( (a = 0) \), than the deformation of the arch for a specific load can be calculated. The static equilibrium, as described in equation 3.1, is calculated using the fsolve-function in Matlab. by taking the initial condition of the arch, \( Q = 0 \), and increasing the acceleration with a very small step. The new equilibrium, of the deformed arch, is then taken as initial condition to find an equilibrium with again a slightly larger acceleration. This cycle is repeated until a new equilibrium cannot be found. Here the arch 'snaps' through to the hollow shape, and finds a new equilibrium there.
These calculations result in a lot of equilibrium points, written in the 3 independent coordinates $Q = [Q_1 \ Q_2 \ Q_3]^T$ for an incremental increasing load $Ac$. To visualize the results, the 'load path' is drawn in figure 3.1. This load path is characterized by $W_{mid}$, which is a dimensionless measure for the displacement of the center of the arch, defined by:

$$W_{mid} = \frac{w(L/2) - w_0(L/2)}{\delta h}, \quad (3.2)$$

where $\delta h$ is the vertical distance between the unloaded initial shape, and the snapped hollow shape, measured at the center of the arch. The parameters are drawn in figure 2.1. This means that $\delta h = 2h$ for $a/h = 0 (a = 0)$. Because $Q$ is just a measure for the deformation of the arch, equation 3.2 can also be written as:

$$W_{mid} = \frac{Q_1(t) - Q_3(t)}{2h}. \quad (3.3)$$

Note that the asymmetric mode $Q_2$ is not triggered because the arch is modeled as a perfect arch here, ($e = 0$). With the maximum load, an acceleration of 59.3 $m/s^2$, the upwarded state of the arch snaps through (buckles) to the hollow shape.
3.2 Eigenfrequencies and stability

The eigenfrequencies of the shallow arch are of interest for the understanding of the behavior of the shallow arch. The model has 3 generalized coordinates because it has 3 degrees of freedom so the system will have 3 eigenfrequencies. Furthermore, the stability of the found equilibrium points from the previous paragraph are evaluated by looking at the local linear stiffness matrix $dK(Q)/dQ$, see [2]. The vibrational eigenvalue problem of the form

$$(dK(Q)/dQ - \omega^2 M)\delta Q = 0,$$  \hfill (3.4)

gives one of the three eigenfrequencies $\omega [\text{rad/s}]$ with $\delta Q$ the corresponding mode.

![Figure 3.2: The first two eigenfrequencies of the shallow arch](image)

The first (lowest) 2 eigenfrequencies are plotted in figure 3.2. The first eigenfrequency becomes imaginary (not shown in figure 3.2) at an acceleration of $35.3 \text{ m/s}^2$ in this case. This means that the stiffness matrix becomes negative definite. The decreasing eigenfrequencies for an increasing load mean that the arch loses stiffness (note that the mass matrix $M$ is constant). Since the matrix $dK(Q)/dQ$ follows from the second derivative of the potential energy, $V$, with respect to $Q$: $V_{QQ} = dK(Q)/dQ$, see [2]. This means that above $Ac = 35.3m/s^2$ the equilibrium points are not stable. Section 3.3 points out that an imperfect arch snaps if the equilibrium point are not stable, while a perfect arch follows the load-path along the unstable points until the point where no equilibrium exists anymore. The first eigenfrequency belongs to the asymmetrical mode, the mode which corresponds to the imperfection in the model.
<table>
<thead>
<tr>
<th></th>
<th>$Ac = 0 \text{ m/s}^2$</th>
<th>$Ac = 35.3 \text{ m/s}^2$</th>
<th>$Ac = 59.3 \text{ m/s}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1^{st}$ eigenfrequency [Hz]</td>
<td>10.90</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$2^{nd}$ eigenfrequency [Hz]</td>
<td>24.52</td>
<td>17.59</td>
<td>2.1</td>
</tr>
<tr>
<td>$3^{rd}$ eigenfrequency [Hz]</td>
<td>319.08</td>
<td>328.93</td>
<td>333.69</td>
</tr>
</tbody>
</table>

Table 3.1: The eigenfrequencies for various loads

Table 3.1 shows some eigenfrequencies for interesting loads. The first eigenfrequency becomes imaginary when the acceleration exceeds $Ac = 35.3 \text{ m/s}^2$. The second eigenfrequency also goes to zero. An acceleration of $Ac = 59.3 \text{ m/s}^2$ is the maximum acceleration where the second eigenfrequency is still real positive.

### 3.3 Imperfect arch

In reality a perfect arch does not exist. There are always some imperfections in the construction, in the material or in the suspension of the arch. These kind of imperfections make the arch slightly asymmetric. To include these imperfections in the model, we have introduced an imperfection parameter $e$, see chapter 2. To study the effects of the asymmetry of the arch, we will vary the parameter $e$ from $e/d = 0$ to $e/d = 1$. The load paths are calculated in the same way as was done for the perfect arch. The load path ends at the acceleration where the arch snaps to the hollow shape. The results are shown in figure 3.3.

![Graph image](image-url)

(a) Load paths for varying $e/d$ and $a = 0$  
(b) Relationship between $Ac$ and $e$

Figure 3.3: Analysis of an imperfect arch
It is obvious from figure 3.3 (a) that the buckling load, the inertia load caused by the acceleration which causes the imperfect arch to snap, is much smaller than it is for the perfect arch. Where the perfect arch will snap at an acceleration of $59.3 \, \text{m/s}^2$, the imperfect arch snaps at an acceleration between $30 \, \text{m/s}^2$ and $35 \, \text{m/s}^2$, dependent of the ratio $e/d$. It should be noted that the buckling load in figure 3.3 (b) for $e/d = 0$ is $59.3 \, \text{m/s}^2$, but because this is the only point with this buckling load, this is not obvious from the figure.

### 3.4 Effects arch shape variations

After analyzing the static buckling loads for a perfect arch and imperfect arches, a closer look on the effect of variation of the shape variation parameter $a$ is taken. This parameter is introduced in chapter 2. Only static loads are evaluated. The influence of the shape of the arch on the buckling load is investigated by taking a certain shape, and calculating the buckling load, using the same methods as were used before. Series of shapes are taken and a relationship can be found between the shape and the load where the arch will snap. Here, a distinction is made between the bifurcation load and the limit point load. The bifurcation load is the buckling load where the arch will snap. The model includes the imperfection $e/d = 0.5$. This imperfection is used because it represents an average imperfection as was taken in the previous section. The limit point load is the buckling load where theoretically, a perfect arch will snap. The limit point can, however, never be reached in experiments, because a perfect arch only exists on paper.

![Figure 3.4: Influence arch shape on bifurcation point](image)

Figure 3.4 shows us the relation between the bifurcation points $A_b$ and the limitpoint $A_{lp}$, and the shape variation, given by the rate $a/h$. It should be noted that the limit point load
at $a/h = 0$ matches the buckling load of the perfect arch we found earlier (see figure 3.1). Also, the bifurcation load matches the buckling load of an imperfect arch, as we found in figure 3.3. We can see that the bifurcation load doesn’t change very much with changes in shape. The limit point load, however, does have a strong relation with the shape variation, but is of no practical use. It is interesting to see that the maximum limit point load has a different $a/h$ rate than the maximum bifurcation load. The limit point load has a maximum at $a = 0.0384h$. 
Chapter 4

Dynamic analysis

In this chapter, the arch will be subjected to dynamic accelerations. We are particularly interested in accelerations where the dynamic component can be seen as a small disturbance, in other words $r_d \ll 1$ in (4.1). The acceleration is prescribed as:

$$\ddot{y}(t) = \begin{cases} 
0 & \text{if } t < 0 \\
A(1 + r_d \sin(2\pi f t)) & \text{if } t \geq 0 
\end{cases} \tag{4.1}$$

For the analysis of the dynamic behavior, the damping parameter $b$ must be determined to solve the equations of motion, see (2.2). In [1], a more detailed description of damping is given, here a realistic amount of damping is taken, $b = 2 \text{ Ns/m}$.

In section 4.1, the motion in time of the arch will be investigated by looking at the step response function. The dynamic component will be neglected here, i.e. $r_d = 0$, which will result in a step-function. In section 4.2 the effects of the parameter $r_d$ are considered. The dynamic snap load will be calculated for various frequencies and various values for $r_d$, all for $e = 0$. In section 4.3 the effects of $e \neq 0$ are given.

4.1 Step response

First, a constant load with the step response is investigated, $(r_d = 0)$. The second order equations of motion (2.2) are solved using Matlab by rewriting them as a system of first order differential equations (ODE’s). This results in:

$$\dot{x} = \begin{bmatrix} x_2 \\
-M^{-1}(CQ + K(Q) + B\ddot{y}(t)) 
\end{bmatrix}, \tag{4.2}$$

13
with \( \mathbf{x} = [ \mathbf{Q} \ \dot{\mathbf{Q}} ]^T \). The initial condition is considered to be \( \mathbf{x} = 0 \). Equation 4.2 can be solved using an ODE-solver in Matlab.

\[ \begin{align*}
0.005 & \quad 0.01 & \quad 0.015 & \quad 0.02 & \quad 0.025 \\
0 & \quad 0.1 & \quad 0.2 & \quad 0.3 & \quad 0.4 & \quad 0.5
\end{align*} \]

\[ \begin{align*}
\text{t [s]} & \quad \text{Wmid [-]} \\
0 & \quad -0.005 & \quad 0 & \quad 0.005 & \quad 0.01 & \quad 0.015 & \quad 0.02 & \quad 0.025
\end{align*} \]

(a) Perfect arch \((a = 0)\) and \((e = 0)\)

(b) Imperfect arch \((a = 0)\) and \((e = 0.1 \cdot d)\)

Figure 4.1: Position \(W_{mid}\) for an acceleration of \(Ac = 40\text{m/s}^2\)

Figure 4.1 shows the response of the arch with \(e = 0\) and \(e = 0.1 \cdot d\) for an acceleration of \(40.0\text{m/s}^2\). The response of both arches are calculated voor 0.5 seconds, using the integration method discussed earlier. The perfect arch, figure 4.1 (a), does not snap, even though the equilibrium points for this load-level are not stable. The imperfect arch, figure 4.1 (b), snaps after some time, and finds a new equilibrium position in the hollow position at \(W_{mid} \approx -1\).

### 4.2 Perfect arch

Next, the effects of \(r_d\), the harmonic component of the acceleration of the shallow arch given in 4.1, will be analyzed. This harmonic component of the acceleration will result in an harmonic excitation of the arch. This component must be seen as a little disturbance of the total acceleration subjected to the arch, in other words, \(r_d \ll 1\). In order to measure the magnitude of the vibration of the arch, a measure for the intensity of the vibration must be chosen. In some circumstances the maximum amplitude of the vibration does not change with changing frequency, but the minimum amplitude does. This is a result of the geometric boundaries of the arch. For larger amplitudes, the arch requires more length then it has. Therefore, the magnitude will be measured by taking the maximum amplitude, minus the minimum amplitude of the center of the arch:

\[ Ac = 40\text{m/s}^2 \]
\[ W_{exc} = \max(W_{mid}(t)) - \min(W_{mid}(t)), \quad (4.3) \]

with \( W_{mid} \) defined by 3.2. In figure 4.2, the intensity of the vibration is investigated for different loads, for \( e = 0, a = 0 \) and \( r_d = 0.1 \).

\[
\begin{align*}
0 & \quad 0.01 & \quad 0.02 & \quad 0.03 & \quad 0.04 & \quad 0.05 & \quad 0.06 & \quad 0.07 \\
0 & \quad 5 & \quad 10 & \quad 15 & \quad 20 & \quad 25 & \quad 30 & \quad 35 & \quad 40 \\
W_{exc} [-] & \quad \text{Frequencie [Hz]} \\
\end{align*}
\]

Figure 4.2: Excitation center of arch for perfect arch and \((a = 0)\)

The excitations were calculated by applying the acceleration on the arch, and solving the differential equations given by equation 4.2 for 50 periods. This makes sure that the vibration has reached a steady state vibration. After these 50 periods, 2 more periods are analyzed where the amplitudes are measured. First the sweep-up method is used. The first frequency is \(0.5 \text{ Hz}\). After 52 periods, the frequency is raised with \(0.5 \text{ Hz}\), so the transient effect is very small. After 52 more periods, the frequency is raised again, etc. After the sweep-up method, the sweep-down method is used. The frequency is lowered after every 52 periods with \(0.5 \text{ Hz}\). For most accelerations sweeping the excitation frequency up or down does not make a difference, but for \(A_c = 50 \text{ m/s}^2\) it does. The sweep-down method reaches a higher peak in the vibration, than the sweep-up method. This means that for very high accelerations the excitation depends on how the excitation frequency is reached. This is a typical nonlinear phenomena.

To show the effect of the peaks in figure 4.2, the response is plotted against time in figure 4.3. A frequency of \(15 \text{ Hz}\) and \(25 \text{ Hz}\) and an acceleration of \(A_c = 40 \text{ m/s}^2\) are chosen, because this shows the differences in vibrations very well. Figure 4.3 (a) shows that the maximum amplitude is reached earlier than the minimum amplitude. The sum of these amplitudes corresponds to the peak in figure 4.2.
The question whether the arch will snap at significantly lower load-levels when it is forced with a specific frequency is of great interest. The snap loads are calculated for different frequencies and for different values of $r_d$. The results are shown in figure 4.4 (a). Both the sweep-up and the sweep-down method are used. In this case, they gave exactly the same results. The calculations of the results shown in figure 4.4 (a) took approximately 8 hours on a computer with a 1.4 GHz processor. Figure 4.4 (b) shows an explicit case with $r_d = 0.1$ and $f = 7.5 \, Hz$.

Figure 4.3: The harmonic deflection of two frequencies

Figure 4.4: Influence harmonic acceleration on perfect arch
The maximum acceleration with \( r_d = 0 \) is constant, this is obvious because in this case there is no harmonic component. This snap load is slightly lower \((56.3 \ m/s^2 \) compared to \( 59.3 \ m/s^2 \)) as was calculated in section 3.1, as a result of the step in the dynamic load. For frequencies larger than \( 15 \ Hz \), the snap load remains approximately the same, so the harmonic component of the acceleration has no significant influence on the dynamic snap load. The snap loads become significantly lower for frequencies smaller than \( 15 \ Hz \). The snap load is maximal decreased with approximately \( r_d \cdot Ac \ [m/s^2] \). This is important, because this means that the induced vibrations itself do not contribute to a loss of stability, but the vibrations only mean a variation in acceleration.

### 4.3 Imperfect arch

As was stated before, perfect arches only exist in theory. Therefore, the methods of section 4.2 are used here for an imperfect arch: \( e \neq 0 \). The step response function is used again. The response for different excitation frequencies is plotted in figure 4.5 (a). Both the sweep-up and the sweep-down methods are used, and gained the same results. The excitations for \( Ac = 40 \ m/s^2 \) and \( Ac = 50 \ m/s^2 \) are different from figure 4.2. This because the arch has snapped, and now vibrates in the hollow position. The amplitude of the response is smaller in the hollow shape than in the initial upper shape.

The influence of the harmonic load on the snap load is also examined for the case \( e = 0.5 \cdot d \). Figure 4.6 (a) shows the snap loads for varying frequencies and varying \( r_d \) and \( a = 0 \), figure
4.6 (b) shows them for $a = 0.0384h$. This shape, ($a = 0.0384h$), corresponds to the found maximum limit point load in section 3.4. The buckling load for $a = 0.0384h$ is slightly larger than the buckling load for $a = 0$. This also corresponds to the found results in section 3.4. These buckling loads are different than the buckling loads for perfect arches. Harmonic accelerations with frequencies larger than 4 Hz have no major influence on the snap load. For lower frequencies, the maximum decrease of $Ac$ is equal to $rd \cdot Ac$ [m/s$^2$]. This means that for low frequencies, the additional excitation has a very predictive influence on the snap load. For higher frequencies ($> 4$ Hz), the influence of the dynamic component of the acceleration becomes negligible, despite the excitation peak for $Ac = 30$ m/s$^2$ at 15 Hz in figure 4.5 (a).

In figure 4.5 (b), $Q_2$ is plotted for the same frequency range. The same scale is used: the maximum amplitude minus the minimum amplitude. When figures 4.5 (b) and 4.6 (a) are compared, then it is clear that $Q_2$ is not the explanation for the difference in the sensitivity for the harmonic acceleration between the perfect arch and the imperfect arch. The large excitation of $Q_2$ for low frequencies could cause the arch to snap earlier, but for frequencies between 4 Hz and 10 Hz this large excitation of $Q_2$ lost its influence.

![Graph](image1.png)

(a) Snap loads for $e = 0.5d$ and $a = 0$

![Graph](image2.png)

(b) Snap loads for $e = 0.5d$ and $a = 0.0384h$

Figure 4.6: Influence harmonic acceleration on imperfect arch
Chapter 5

Conclusions

In this article, the dynamic stability of a thin shallow arch is investigated. The arch, described and modeled in chapter 2, is first subjected to constant time independent accelerations and next to time dependent accelerations in the form of a constant component with on top a small harmonic component. A series of arches is analyzed, to determine the influence of arch shape parameters.

In chapter 3, the constant accelerations are analyzed, for a perfect arch that exist only theoretically, and for imperfect arches. For this case, the arch is subjected to time independent accelerations and static equilibria are determined. The stability and the eigenfrequencies of these equilibria are analyzed. The effects of the arch shape on the buckling load, the load-level which causes the arch to snap as a result of the acceleration, are large for a perfect arch, but not of real importance for an imperfect arch. The magnitude of the modeled imperfection does not have a large effect on the buckling load.

The dynamic accelerations, investigated in chapter 4, do not have a great influence on the behavior of the shallow arch. The buckling loads are investigated for an initial shaped arch, \((a = 0)\), and for \((a = 0.0384h)\), the shape which corresponds to the theoretical limit point load for a perfect arch found in chapter 3. For lower frequencies, the buckling load decreases with only the magnitude of the harmonic component of the total acceleration. For higher frequencies, the harmonic component of the acceleration loses its influence on the buckling load, the constant component of the acceleration determines when the arch will snap. The difference between a perfect arch and an imperfect arch only results in a slightly different frequency range where the harmonic component has no influence. The shape has a very small influence on the buckling load for harmonic accelerations. The disturbances in the acceleration have a very predictive effect on the stability of the arch, or no effect at all.
Bibliography
