Hot rolling multivariable model verification and QFT robust looper control

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Chapter 1

Preface

The work described in this report is part of an assignment on MIMO control of a hot strip finishing mill. The assignment is carried out by student from the control group of the Facultad de Ingeniería Mecánica y Eléctrica at the Universidad Autónoma de Nuevo León, Monterrey, Mexico under supervision of Dr. A. Cavazos. The assignment is carried out in order of Hylsa steel company, Monterrey, Mexico. The mill for which the controller will be designed is situated at the Hylsa plant in Monterrey. It’s an old mill for which a complex controller is needed to be able to reach the increasing demands in dimensional accuracy.

A lot of work on hot strip finishing mill modeling and control has been done in the past [6] [2] [1]. For the last decades the attention has lain on MIMO controllers. Several successful demonstrations of applying MIMO control have been made, especially in the Japanese industry [7] [8]. Also several attempts have been made to apply (MIMO) Quantitative Feedback Theory control on hot strip finishing mills [11]. The work described in this paper is the first step to an attempt to design, for the first time, a hot strip finishing mill controller for a, both input and system uncertain, $4 \times 4$ MIMO system with the Quantitative Feedback Theory.

The majority of the work described in the system modeling part of this paper has been done by Alma Rosa Obregón Zamudio as part of her graduation report on hot strip finishing mill modeling.
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Chapter 2

Introduction

In the hot strip mill process in the iron and steel industry, reheated slabs are rolled to the required strip thickness and width at the roughing and finishing mill.

![Diagram of the hot strip mill process](image)

In the finishing mill the strip thickness is reduced while the strip passes through each of the 6 stands by controlling the gauge. Between each two stands there is a looper which is a motor driven arm. The functions of the looper are to keep the strip at a reference tension and to isolate the operation of the stand by absorbing any mass flow error. Keeping the tension of the strip constant helps to maintain dimension accuracy of the strip gauge, width and crown especially when higher grade steels are being rolled at high speed. Controlling the looper angle, which is a measure of the strip mass flow prevents dangerous cobbles in the strip forming and decoupling the gauge control of each stand from another.

Due to the lack of space in the mill, the hostile environment and the high costs of a x-ray meter, usually the strip tension and strip gauge cannot be measured directly. The only measurements available are the rolling force and the looper angle which in conventional control are used in two separate loops. Rolling force is used to control the gauge (Automatic Gauge Control) by manipulating the hydraulic capsules, the looper angle in used to control the mass flow by adjusting the upstream stand’s main drive motor. The strip tension is controlled by use of an open loop model to supply the reference for the looper motor torque.
Conventional control ignores the interactions between the strip gauge, tension and the looper angle. The looper angle and the strip tension are linked directly through the mechanics of the looper system while the tension works as a disturbance on the stand gauge. Also a change by the Automatic Gauge Control changes the massflow at the entry and exit of the stand producing a disturbance to the strip tension and looper angle. [1][2]

In conventional control the loops are not decoupled sufficiently to reach the demands for increasing dimensional accuracy. Therefore in the last decades several successful attempts have been made to design and implement MIMO controllers who can better deal with the interactions.

In this paper the linear multivariable dynamic model of two stands and the intermediary looper system of a hot strip finishing mill is presented. Furthermore, a start has been made in developing a MIMO controller. Quantitative Feedback Design (QFT) has been chosen as the control technique. QFT is a powerful robust control technique developed by Horowitz [9], it can deal with large uncertainties and provides a transparent design process. First a QFT controller has been designed to control the angle of the looper. Moreover, the extension of the control technique to a MIMO system is explained with the description of the $2 \times 2$ MIMO looper system case.
Chapter 3

System Modeling

In this section the linear multivariable dynamic model, of two stands and the intermediary looper system of a hot strip finishing mill as shown in figure 3.1, is described. As a mathematical model the linearly approximated model around a set-up point is derived by using the nonlinear static relations rolling process of a hot strip finishing mill, and hence it is only valid in steady state under small perturbations.

![Figure 3.1: Two stands and the intermediary looper system of a hot strip finishing mill](image)

3.1 Rolling Force

In this section the physical equations describing the rolling force and the corresponding parameters are explained.
The rolling force \([1]\) is given by:

\[
P = \sqrt{R'(H-h)[KwQ - m_1\sigma_{in} - m_2\sigma_o]}
\] (3.1)

with:

- \(H\) = Entry strip thickness [\(mm\)]
- \(h\) = Exit strip thickness [\(mm\)]
- \(m_1, m_2\) = Strip tension effect factors related to entry and exit tension respectively [\(mm\)]
- \(\sigma_{in}\) = Entry tension [\(Kgf/mm^2\)]
- \(\sigma_o\) = Exit tension [\(Kgf/mm^2\)]
- \(w\) = Strip width [\(mm\)]
- \(R'\) = Deformed roll radius [\(mm\)]
- \(K\) = Yield stress [\(Kgf/mm^2\)]
- \(Q\) = Geometric factor [-]

Hence, rolling force deviation from a set-up point is described as follows by the linear approximation:

\[
\Delta P_i = \frac{\partial P_i}{\partial H_i}\Delta H_i + \frac{\partial P_i}{\partial h_i}\Delta h_i + \frac{\partial P_i}{\partial \sigma_{i-1}}\Delta \sigma_{i-1} + \frac{\partial P_i}{\partial \sigma_i}\Delta \sigma_i
\] (3.2)

For the calculation of the partial derivative terms in expression [3.2], \(R'\) and \(K\) are kept constant for reasons of simplicity.

In the following subsections expressions for the deformed roll radius, geometric factor, yield stress and strip tension effect factors are given.

### 3.1.1 Deformed roll radius

The rolling force causes a deformation of the working roll. The expression for the deformed working roll radius [3] is given by:

\[
R' = R[1 + \frac{(16)(0.91)(1000)F}{\pi E_r w(H-h)}]
\] (3.3)

with:

- \(R\) = Undeformed roll radius [\(in\)]
- \(P\) = Working force [\(tonf\)]
- \(E_r\) = E-modulus of working roll [\(lbf/in^2\)]

Note that in this expression the parameters are in inche, tonf and poundforce, \(R'\) is also in inche. \(R'\) is defined as the effective radius at the strip-roll contact point. The roll force used to calculate \(R'\) is calculated with the undeformed roll radius.
3.2 Strip thickness

3.1.2 Geometric factor

The geometric factor [6] used in equation 3.1 is given by:

\[ Q = \frac{\pi}{4} + \frac{1}{4} \sqrt{\frac{R'}{h} \frac{r}{1-r}} \] (3.4)

with:

\[ r = \text{Reduction in pass} = \frac{(H - h)}{H} \quad [-] \]

3.1.3 Yield stress

No explicit expression for the yield stress in 3.1 with satisfying results is found. Therefore the yield stress is calculated with 3.1 itself, this is possible because the value of \( P \) is known apriori. With 3.1 the expression for the yield stress becomes:

\[ K = \frac{P}{\sqrt{R(H-h)wQ}} + \frac{(m_1\sigma_i + m_2\sigma_o)}{wQ} \] (3.5)

3.1.4 Strip tension effect factors

In 3.1 \( m_1 \) and \( m_2 \) are defined as the strip tension effect factors related to entry and exit tension respectively. So these factors are proportionality constants for the influence of the entry and exit strip tension on the total rolling force. Although \( m_1 \) and \( m_2 \) are used in expressions for the rolling force in different literature, no expression for \( m_1 \) and \( m_2 \) is given in any of these documents.

In some literature the expression for the rolling force is given without the strip tension part. From this we may conclude that the influence of the tension on the total rolling force is low. From [3] we can see that the tension part is approximately 1% of the total rolling force. Furthermore from [3] we can conclude that \( m_1 \approx m_2 \). With these two assumptions, 3.1 and the values of \( R', H, h, \sigma_i, \sigma_o \) and \( P \), the values for \( m_1 \) and \( m_2 \) are calculated.

3.2 Strip thickness

3.2.1 Exit strip thickness

The exit strip thickness calculation [7] is based on the gaugemeter principle using the roll gap deviation and the rolling force deviation.

\[ \Delta h_i = \Delta S_i + \frac{\Delta P_i}{M_i} \] (3.6)

with:

\[ \Delta S_i = \text{Roll gap deviation} \quad [mm] \]
\[ \Delta P_i = \text{Rolling force deviation} \quad [Kgf] \]
\[ M_i = \text{Mill modulus} \quad [Kgf/mm] \]
Mill modulus

When a rolling force is applied on the working rolls, the mill housing will deform, causing an increase in the gap. The relation between the applied rolling force and the corresponding gap between the rolls is called the mill modulus $M$. It is determined by measuring the zero force, which is the force when the gap is set to zero, on different positions on an eccentric working roll [3].

3.2.2 Interstand strip transport lag

The strip entry thickness of mill $i+1$ is equal to the exit strip thickness of mill $i$ with a transport delay equal to the time it takes for the strip to get from stand $i$ to stand $i+1$. This delay is approximated with a first order lag element.

$$\Delta h_{i+1} = \frac{1}{1 + s\tau_{Di}} \Delta h_i$$

(3.7)

with:

$\tau_{Di}$ = Strip transport time constant  \([-\] \]

3.3 Slip

During the rolling process the strip will slip between the work rolls, this means the input/exit velocity of the strip is not equal to the work roll peripheral velocity.

3.3.1 Forward slip

Forward slip [7] is defined as:

$$f \triangleq \frac{V_{out} - V}{V}$$

(3.8)

with:

$V_{out}$ = exit strip velocity  \([\frac{mm}{s}]\]
$V$ = work roll peripheral velocity  \([\frac{mm}{s}]\]

The equation for forward slip [3] is given by:

$$f = \left(\frac{2R'}{h} \cos(\phi_n) - 1\right) \left(1 - \cos \phi_n\right)$$

(3.9)

$\phi_n$ is the angle with the neutral plane as defined in figure 3.2

$$\phi_n = \left(\sqrt{\frac{h}{R'}}\right) \tan \left[\frac{\pi}{8} \sqrt{\frac{h}{R'}} \ln(1 - r) + \frac{1}{2} \tan^{-1}\left(\sqrt{\frac{r}{1 - r}}\right)\right]$$

(3.10)

Hence, forward slip deviation from a set-up point is described as follows by the linear approximation:
Figure 3.2: Neutral angle

\[
\Delta f_i = \frac{\partial f_i}{\partial S_i} \Delta S_i + \frac{\partial f_i}{\partial H_i} \Delta H_i + \frac{\partial f_i}{\partial \sigma_{i-1}} \Delta \sigma_{i-1} + \frac{\partial f_i}{\partial \sigma_i} \Delta \sigma_i + \frac{\partial f_i}{\partial h_i} \Delta h_i \quad (3.11)
\]

For the calculation of the partial derivative terms in expression (3.11), \( R' \) is kept constant for reasons of simplicity. This means the partial derivative terms \( \frac{\partial f_i}{\partial S_i}, \frac{\partial f_i}{\partial \sigma_{i-1}}, \frac{\partial f_i}{\partial \sigma_i} \) are zero and \( \Delta f_i \) only depends on entry and exit strip thickness.

### 3.3.2 Backward slip

Backward slip \([7]\) is defined as:

\[
b \triangleq \frac{V_{in} - V}{V} \quad (3.12)
\]

with:

- \( V_{in} \) = entry strip velocity \[ \text{[mm/s]} \]

The equation for backward slip \([4]\) is given by:

\[
b = \frac{h (1 + f)}{H \left( \cos \left( \sqrt{\frac{H-h}{R'}} \right) \right)} - 1 \quad (3.13)
\]

Hence, backward slip deviation from a set-up point is described as follows by the linear approximation:

\[
\Delta b_{i+1} = \frac{\partial b_{i+1}}{\partial S_{i+1}} \Delta S_{i+1} + \frac{\partial b_{i+1}}{\partial H_{i+1}} \Delta H_{i+1} + \frac{\partial b_{i+1}}{\partial \sigma_{i+1}} \Delta \sigma_{i+1} + \frac{\partial b_{i+1}}{\partial \sigma_{i+1}} \Delta \sigma_{i+1} + \frac{\partial b_{i+1}}{\partial h_{i+1}} \Delta h_{i+1} \quad (3.14)
\]

For the calculation of the partial derivative terms in expression (3.14), \( R' \) is also kept constant for reasons of simplicity. This means the partial derivative terms \( \frac{\partial b_{i+1}}{\partial S_{i+1}}, \frac{\partial b_{i+1}}{\partial \sigma_{i+1}}, \frac{\partial b_{i+1}}{\partial \sigma_{i+1}} \) are zero and \( \Delta b_{i+1} \) only depends on entry and exit strip thickness.
3.4 Strip velocity

3.4.1 Exit strip velocity

Rewriting 3.8 gives the exit strip velocity:

\[ V_{\text{out}} = (1 + f)V \] (3.15)

Hence, exit strip velocity deviation from a set-up point is expressed as follows

\[ \Delta V_{\text{out},i} = (1 + f_i)\Delta V_i + \Delta f_i V_i \] (3.16)

3.4.2 Entry strip velocity

Rewriting 3.12 gives the exit strip velocity:

\[ V_{\text{in}} = (1 + b)V \] (3.17)

Hence, entry strip velocity deviation from a set-up point is expressed as follows

\[ \Delta V_{\text{in},i+1} = (1 + b_{i+1})\Delta V_{i+1} + \Delta b_{i+1} V_{i+1} \] (3.18)

3.5 Strip tension

The tension model \[1\] is based on longitudinal stress and strain as defined by Young’s modulus.

\[ \sigma(t) = E \left[ \frac{L_\theta(t) - L_s(t)}{L_s(t)} \right] \] (3.19)

where

\[ L_\theta = l_1 + l_2 \]

\[ l_1 = \sqrt{\left[(r_l - y + l \sin \theta)^2 + (a + l \cos \theta)^2\right]} \] (3.20)

\[ l_2 = \sqrt{\left[(r_l - y + l \sin \theta)^2 + (L - a - l \cos \theta)^2\right]} \]

\[ L_s = L + \int_{t_0}^{t} (V_{\text{out},i}(\tau) - V_{\text{in},i+1}(\tau))d\tau \] (3.21)

with:

\[ \theta \quad \text{= Looper angle} \quad [\text{rad}] \]

\[ E \quad \text{= Young’s modulus} \quad [\text{Kgf/mm}^2] \]

\[ L \quad \text{= Distance between stands} \quad [\text{mm}] \]

\[ r_l \quad \text{= Looper roll radius} \quad [\text{mm}] \]

\[ l \quad \text{= Looper arm length} \quad [\text{mm}] \]

\[ y \quad \text{= Vertical distance between looper pivot and center between working rolls} \quad [\text{mm}] \]

\[ a \quad \text{= Horizontal distance between stand i and looper pivot} \quad [\text{mm}] \]
3.6 Looper mechanics

The strain depends on the amount of material in the interstand region which depends on the velocity difference of the strip leaving stand $i$ and entering stand $i+1$. The extension is the difference between the geometric strip length and the length of the strip in the interstand. The looper dimensions are shown in figure 3.3.

![Looper system dimensions](image)

Figure 3.3: Looper system dimensions

Hence, strip tension deviation from a set-up point is described as follows by the linear approximation:

$$\Delta \sigma_i = \frac{E_i}{sL_i} \left( \Delta V_{in,i+1} - \Delta V_{out,i} + \frac{\partial L_{\theta,i}}{\partial \theta_i} \Delta \omega_i \right)$$  \hspace{1cm} (3.22)

with:

$$\omega = \text{Looper angular velocity} \quad [\text{rad} s]$$

3.6 Looper mechanics

The equations of motion for the looper is described by Newton’s second law

$$J \frac{d}{dt} \omega + D \omega = T_m - T_{\text{load}}$$  \hspace{1cm} (3.23)

with:

$$J = \text{Looper inertia} \quad [Kgf mms^2]$$
$$D = \text{Looper damping factor} \quad [Kgf mms]$$
$$T_m = \text{Motor torque} \quad [Kgf mm]$$
$$T_{\text{load}} = \text{Load torque} \quad [Kgf mm]$$

Hence, looper angular velocity deviation from a set-up point is described as follows by the linear approximation:

$$\Delta \omega_i = \frac{1}{sJ_i} \left( \Delta T_{m,i} \Delta \omega_i - D_i \Delta \omega_i - \frac{\partial T_{\text{load}}}{\partial \theta_i} \Delta \theta_i - \frac{\partial T_{\text{load}}}{\partial \sigma_i} \Delta \sigma_i \right)$$  \hspace{1cm} (3.24)

In the following subsections the expression for the load torque is given.
3.6.1 Load torque

The load torque \[2\] in 3.23 is described as:

\[
T_{\text{load}} = T_{\sigma} + T_s + T_b + T_{\text{LW}}
\]  
(3.25)

where:

\(T_{\sigma}\) is the torque due to the tension in the strip

\[
T_{\sigma} = \sigma \omega h[l \cos(\theta)f_1 + (l \sin(\theta) + r_1)f_2]
\]  
(3.26)

with:

\[
f_1(\theta) = (r_1 + l \sin \theta - y)(\frac{1}{l_1} + \frac{1}{l_2})
\]  
(3.27)

\[
f_2(\theta) = \left(\frac{L - a - l \cos \theta}{l_2} - \frac{a + l \cos \theta}{l_1}\right)
\]  
(3.28)

\(T_s\) is the torque to support the strip weight

\[
T_s = l g \rho \omega h(l_1 + l_2) \cos(\theta)
\]  
(3.29)

with:

\[
\rho = \text{Strip density } \left[\frac{Kg \text{ s}^2}{\text{mm}^4}\right]
\]

\(T_b\) is the torque to bend the strip over the looper roll

\[
T_b = l \cos(\theta) \frac{K \omega h^2}{4} \left(\frac{1}{l_1} + \frac{1}{l_2}\right)
\]  
(3.30)

\(T_{\text{LW}}\) is the torque to support the looper arm and roller weight. Due to the existence of a contra weight, the contribution of this part of the load torque can be neglected.

3.7 Actuators

The roll gap position regulators are approximated with first order lag elements.

\[
\Delta S_i = \frac{1}{1 + s \tau_{G,i}} \Delta S_{r,i}
\]  
(3.31)

and

\[
\Delta S_{i+1} = \frac{1}{1 + s \tau_{G,i+1}} \Delta S_{r,i+1}
\]  
(3.32)

The looper torque regulator is approximated in the same way with a first order lag element.
3.8 Linear multivariable dynamic model

In the previous sections the linear multivariable dynamic model describing two stands and the intermediary looper system of a hot strip finishing mill are given. This model is presented as a block diagram in figure 3.4.

The system inputs are all the inputs which can be controlled by an actuator which is part of the 2 stand-looper system, these actuators are: the roll gap position regulator of the i-th and the i+1-th stand, the looper torque regulator and the mill motor drive of the i-th stand. The other inputs work as disturbances on the system. The system inputs, disturbances and system outputs are shown in figure 3.5.

The model as given in 3.4 is implemented in Matlab Simulink to verify the behavior and to calculate the RGA given in 4.1.1. The conventional controllers as given in 4.1 are also implemented. The system is simulated with small permutation around the nominal values of the uncontrolled inputs (disturbances), the simulations showed the same behavior as the real close loop system.

\[ \Delta T_{m,i} = \frac{1}{1 + s \tau_{Ti}} \Delta T_{r,i} \]  
\[ \Delta V_{i} = \frac{1}{1 + s \tau_{Mi}} \Delta V_{r,i} \]
3.9 system parameters

Most of the nominal values of the system parameters described in this chapter have been directly obtained from data from Hylsa Steel company [3] and shown in Alma Obregón MSc thesis. Some had to be calculated. The strip Youngs modulus $E$ has been calculated by extrapolating the curve of the steel Youngs modulus at high temperature given in [5]. The value of the damping $D$ is estimated. The looper inertia $J$ wasn’t know in advance. The shape of the looper has been simplified, this simplified version is built up by some basic cylinder and plate shaped elements. First the moment of inertia of the basic elements has been calculated then the moment of inertia of the element with respect to the looper axle has been calculated with the parallel axis theorem (Steiner’s theorem).
Chapter 4

Hot strip finishing mill control

Hot strip finishing mill control is conventionally done by controlling the separate loops, this way ignoring the interaction between strip gauge, tension and looper angle. Due to this insufficient decoupling, the increasing demands for dimensional accuracy can not longer be reached. Therefore in the last decades there's a growing request for MIMO finishing mill controllers, which can better deal with the interactions.

In this chapter first the conventional control is discussed, then the selection of a suitable MIMO control strategy is explained.

4.1 Conventional control

In this section the conventional control of the hot strip finishing mill is described in detail.

The conventional way of controlling the strip thickness, tension and looper angle is shown in figure [4.1] The total control system exists of several separate SISO loops.

The strip thickness $h_i$ is controlled by the Automatic Gauge Control. The AGC uses force feedback via the inverse of the mill modulus to adjust the hydraulic capsules. The exit strip thickness deviation is given by [3.6] from this equation it can be seen that the ideal gain to obtain a zero steady-state error is $K = \frac{1}{M}$. For a changing mill modulus the gain has to decrease in order to insure stability. Therefor the AGC gain is a trade-off between robustness and thickness performance. X-ray monitor AGC adjusts the roll gap to control the absolute strip thickness based on the measurements made by the X-ray thickness meter installed at the exit side of the finishing mill.

The looper angle is controlled using a PI controller which adjusts the upstream stand $i$ motor speed. The massflow control also contains a speed trim signal from the downstream stand $i+1$, the motor speed of stand $i$ is adjusted with the deviation of the motor speed of stand $i+1$ times a gain which is the proportion between the setup motor speeds of stand $i$ and $i+1$.

Strip tension is controlled using a static nonlinear model of the strip geometry and looper load to estimate the torque which should be applied to the looper arm to achieve the reference tension depending on the current looper angle.[1][7]
4.1.1 MIMO control

The first important factor in selecting the MIMO control strategy for a hot strip finishing mill is determining the amount of interaction in the system. This is done by calculating the relative gain array (RGA) at different frequencies. The RGA is a measure for the amount of interaction in the system [14].

\[
\text{RGA for } \omega = 1 \text{ [rad/s]} \\
\begin{array}{cccc}
in & out & h_i & h_{i+1} & \theta_i & \sigma_i \\
S_i & & 0.95 & -0.06 & 0.11 & 0.00 \\
S_{i+1} & & 0.00 & 0.94 & 0.06 & 0.00 \\
V_i & & 0.00 & 0.00 & 0.77 & 0.23 \\
T_i & & 0.05 & 0.12 & 0.06 & 0.77 \\
\end{array} \\
\text{RGA for } \omega = 10 \text{ [rad/s]} \\
\begin{array}{cccc}
in & out & h_i & h_{i+1} & \theta_i & \sigma_i \\
S_i & & 1.00 & 0.00 & 0.00 & 0.00 \\
S_{i+1} & & 0.00 & 1.00 & 0.00 & 0.00 \\
V_i & & 0.00 & 0.00 & 0.96 & 0.04 \\
T_i & & 0.00 & 0.00 & 0.04 & 0.96 \\
\end{array} \\
\text{RGA for } \omega = 50 \text{ [rad/s]} \\
\begin{array}{cccc}
in & out & h_i & h_{i+1} & \theta_i & \sigma_i \\
S_i & & 0.91 & 0.00 & 0.09 & 0.00 \\
S_{i+1} & & 0.00 & 0.55 & 0.45 & 0.00 \\
V_i & & 0.00 & 0.00 & 4.06 & 3.06 \\
T_i & & 0.09 & 0.45 & -3.60 & 4.06 \\
\end{array} \\
\text{RGA for } \omega = 100 \text{ [rad/s]} \\
\begin{array}{cccc}
in & out & h_i & h_{i+1} & \theta_i & \sigma_i \\
S_i & & 1.00 & 0.00 & 0.00 & 0.00 \\
S_{i+1} & & 0.00 & 1.00 & 0.00 & 0.00 \\
V_i & & 0.00 & 0.00 & -0.35 & 1.35 \\
T_i & & 0.050 & 0.01 & 1.34 & -0.35 \\
\end{array}
\]

In a hot strip finishing mill the most important disturbances lay in the low frequency range [11]. At these frequencies there is interaction between the different main channels of the controlled system as can be seen above. In other words the closed loop function \( T \) is not diagonal. Therefore one should look for a controller which can somehow deal with this interaction.
4.1 Conventional control

Another important matter is determining the MIMO control strategy is the amount of uncertainty in the system. The uncertainty in the hot strip finishing mill is high. There are high uncertain inputs (input disturbances) and the plant model used for designing the controller is uncertain, both from variation in the plant parameters itself and from inaccuracies in the modeling process. Because of these high uncertainty direct diagonalization of the open-loop transfer function is impossible.

A design technique which can deal with both the high uncertainty and the interaction is the Quantitative Feedback Theory (QFT). It is a robust design technique which works with sequential loop shaping. The interactions between the main channels are limited by the closed loop specifications [13]. An advantage of QFT compared to other robust techniques as $H_\infty$ is the fact that it’s a transparent technique, the trade-off between the complexity of the controller and the performance can easy be made [11]. Because of the characteristics mentioned above QFT is chosen to be the design technique for controlling the hot strip finishing mill.
Chapter 5

Quantitative feedback theory

Quantitative feedback theory (QFT) is a powerful robust control design technique developed by Isaac Horowitz in the 1960’s. The foundation of QFT is the fact that feedback is principally needed when the plant is uncertain and/or there are uncertain inputs (disturbances) acting on the plant. Uncertain plant does in this context not mean ‘unknown’ plant, the meaning of quantitative in QFT is that the uncertainties of the plant are known ‘quantitatively’. QFT is a transparent control design technique which uses the uncertainty upfront and reveals the trade-off between complexity of the controller and the performance specifications.

The basic objective of QFT is to design a SISO controller which will have robust stability and robust performance for parametric uncertainty and also will have a minimum complexity and a minimum bandwidth. Minimum bandwidth controllers are a natural requirement in practice in order to avoid problems with noise amplification, resonances and unmodeled high frequency dynamics.

In QFT the plant uncertainty is represented by a set of templates $\mathcal{P}$ on the Nichols chart, within each template all possible frequency responses $P(j\omega_k)$ for some frequency $\omega_k$ are enclosed. For each frequency $k$ the set $\mathcal{P}_\omega$ consist of a finite number of elements, therefore a discrete grid of uncertain parameters should be used for obtaining $\mathcal{P}_\omega$. The performance specifications consist of constraints $W(\omega)$ on the magnitude of a closed-loop frequency response $F(s)$. In QFT the main process is translating the frequency domain specifications $W$ on the uncertain feedback system into bounds in the Nichols chart within which the nominal loop transmission ($L_0 = P_0C : P_0\epsilon\mathcal{P}$) should lie.

A bound is obtained by determining all possible positions in the Nichols chart on which the uncertainty template of $\mathcal{P}(j\omega)$ can be translated without being rotated, such that the performance specifications $F(s)$ satisfies its magnitude bounds of $W(\omega)$. In the past this was a manual graphical task in Nichols chart which made it very difficult. With the QFT frequency domain control design toolbox for MATLAB calculating the bounds can now be done much faster and more accurate.[11] [12]
5.1 SISO QFT looper control

In this section the design of a QFT SISO looper controller is explained. This section can be seen as an example of QTF in a real control case. It is also a preparation on the yet to be designed QFT MIMO controller.

5.1.1 SISO looper modeling

First the linear dynamic model of the looper has been derived. This is done by isolating the looper dynamics from the rest of the total system model described in 3. The input of the SISO system is $\Delta V_{r,i}$, the output is $\Delta \theta_i$. This coupling is chosen because at low frequencies this input and output have the strongest coupling as can be seen in 4.1.1. It’s also the same coupling as in conventional SISO looper control. All other signals work as disturbances on the SISO looper system. The isolated SISO looper system $P_{tot}$ is shown in figure 5.1.

![Figure 5.1: SISO looper system](image)

In the SISO looper model the disturbances $D1$ and $D2$ don’t enter the system as input or output disturbances which is a problem because QFT can only work with disturbances on the input or output of a system. Therefore the system has been split up in two subsystems. The first subsystem is $P_1$ with input $V_{r,i}$ and output $V_{c,i}$ in this subsystem the plant uncertainty is neglected. In the second subsystem $P_2$ plant uncertainty is taken into account this part has input $V_{c,i}$ and output $\theta_i$. With this split up $D1$ becomes the input disturbance of subsystem $P_2$. Disturbance $D2$ would still enter $P_2$ in the middle therefore $D2$ has been moved to the input of subsystem $P_2$ by multiplying it with $Z$. $Z$ has been calculated with $Z \ast P_2$ equals the transfer function from $D2$ to $\theta_i$. The uncertainty of $Z$ has also been taken into account. The new system is shown in figure 5.2.

The total SISO feedback system is shown in figure 5.3. Reference disturbance and output disturbance in the conventional way are not being considered here, while high frequency sensor noise is being rejected according to the stability bounds in the QFT toolbox, therefore a more detailed characterization of the sensor noise is required for future work, furthermore sensor dynamics $H$ and pre-filter $F$ equal 1.
5.1 SISO QFT looper control

5.1.2 Plant Templates

As stated before a plant template $P_ω$ is the collection of all possible uncertain plant’s frequency responses $P(jω_k)$ for some frequency $ω_k$. In QFT, plant templates can be obtained from frequency response measurements or from specific parametric or non-parametric uncertainty models. In this case a parametric uncertainty model is used.

In QFT plant templates are represented in the Nichols chart. The Nichols chart represents complex numbers in terms of their magnitudes and phases, the coordinates of the Nichols chart are $(φ, 20\log(r))$ [12].

The transfer function $P_2$ from $V_{c,i}$ to $θ_i$ is determined, the transfer function coefficients are expressed in the system parameters. The transfer function $P_2$ is given by:

$$P_2(s) = \frac{-(B_2)E_i}{s^3 + \frac{D_1}{J_i}s^2 + \left(\frac{B_1}{E_i} + \frac{B_2L_1E_i}{J_iL_1}\right)s}$$

(5.1)
with:
\[ B_1 = \frac{\partial T_{\text{load}}}{\partial \theta_i} \]
\[ B_2 = \frac{\partial T_{\text{load}}}{\partial \sigma_i} \]
\[ L_1 = \frac{\partial L_{\theta,i}}{\partial \theta_i} \]

The uncertainty variations around the nominal values of the system parameters are estimated. Distance between stands \( L_i \pm 1\% \), partial derivatives \( \frac{\partial T_{\text{load}}}{\partial \theta_i}, \frac{\partial T_{\text{load}}}{\partial \sigma_i}, \frac{\partial L_{\theta,i}}{\partial \theta_i} \pm 3\% \), looper inertia \( J_i \pm 5\% \) and steel Young's modulus \( E_i \) and damping \( D_i \pm 10\% \).

With these uncertainties the minimum and maximum values of the transfer function coefficients are determined. A grid between these minimum and maximum values is formed. This leads to the following uncertain plant.

\[ P_2 = \left\{ P(s) = \frac{d_1}{s^3 + d_2 s^2 + d_3 s} \mid d_1 = [d_{1,\text{min}}, \ldots, d_{1,\text{max}}], d_2 = [d_{2,\text{min}}, \ldots, d_{2,\text{max}}], d_3 = [d_{3,\text{min}}, \ldots, d_{3,\text{max}}] \right\} \]

(5.2)

It is important to look at the shape of the template while determining the number of elements in the different grids. Too many elements will lead to a template which will be described by thousands of cases. Even with powerful computers, a solution may require unrealistic calculation time. Not enough elements will lead to a template which doesn’t cover the total response of the uncertainty plant. So, the process in determining the number of elements in each grid is, increasing the number of elements until there’s no change in the shape of the template anymore. The fact that for simply connected templates it is sufficient to only look at the shape, which is the boundary of the template is related to a celebrated result in complex variables, the maximum principle [12]. An example of determining the number of elements in \( d_3 \) is given in figure 5.4

The final template \( P_2 \) is described with 3 elements in \( d_1 \) and \( d_2 \) and 201 elements in \( d_3 \). The uncertain plant \( P_2(s) \) is given in Bode in figure 5.5, the final template \( P_2 \) is given in figure 5.6.

The template of \( Z \) is determined in the same way as \( P_2 \)

**Frequencies**

In QFT templates are calculated for a discrete number of frequencies. The basic rule in determining these frequencies is that for the same specification, the bounds will only change with changes of the shape of the template, therefore, one should look for frequencies where the shape of the template shows significant variation compared to those of other frequencies. This will become clear when the calculation of the bounds is explained in subsection 5.1.3.

For example, at high frequencies the shape of the templates becomes fixed. Therefore the maximum frequency for calculating the bounds of a fixed specification is the first frequency for which this occurs. All bounds at higher frequencies will then be the same as this one [12].

In this design case the specifications are not fixed for all frequencies which means the bounds for frequencies with an equally shaped template can be different. Therefore the minimum and maximum frequencies for which templates are calculated equal the minimum and maximum frequencies defined in the specifications.
5.1 SISO QFT looper control

In order to compute bounds, one plant element from the uncertain set has to be chosen as the nominal plant. This is required to perform QFT design with a single nominal loop. The choice of the nominal plant is not important, from the way the bounds are calculated one can see that the choice has influence on the bounds but not on the resulting controller. In this case the first element of the uncertainty set in chosen as nominal plant, which is not the same as the plant with no uncertainty.

Figure 5.4: Plant templates with different number of elements for $d3$

Nominal plant

In order to compute bounds, one plant element from the uncertain set has to be chosen as the nominal plant. This is required to perform QFT design with a single nominal loop. The choice of the nominal plant is not important, from the way the bounds are calculated one can see that the choice has influence on the bounds but not on the resulting controller. In this case the first element of the uncertainty set in chosen as nominal plant, which is not the same as the plant with no uncertainty.
Figure 5.5: Bode plot of uncertain Plant $P_2(s)$

Figure 5.6: Final plant template $\mathcal{P}_2$
5.1.3 Bounds

In QFT with the given plant templates, closed loop magnitude specifications are converted into magnitude and phase constraints on the nominal open-loop function. These constraints are called QFT bounds.

So, the first task in calculating the bounds is determining the closed loop magnitude specifications. These specifications can come from all sorts of time-domain and frequency-domain criteria. In this case no hard criteria were known a priori.

To obtain the necessary closed loop magnitude specifications a low order controller is designed for the total no-uncertainty plant $P_{\text{tot.nu}}$, with $P_{\text{tot.nu}}$ is plant $P_{\text{tot}}$ with no uncertainty. This has been done with conventional loopshaping techniques. Criteria was maximum sensitivity $6\,\text{dB}$. The behavior of the closed loop was used to obtain the magnitude specifications for the QFT design case. In other words the performance of the robust QFT controller should be equal to the performance of the low order nominal loop controller.

**Specification on closed loop**

A specification on the magnitude of the close loop avoid problems with (sensor) noise amplification. This specification is defined in the high frequency range $[50-500\,\text{rad/s}]$. The closed loop for the nominal case and the obtained specification are shown in figure 5.7.

![Figure 5.7: Nominal case closed loop mag. and close loop mag. specification](image)

the specification on the closed loop is given by:

$$\left| \frac{y}{r} \right| = \left| \frac{P_{\text{tot}}C}{1 + P_{\text{tot}}C} \right| \leq \eta_t = 2 \left( \frac{1}{170^8 + 1} \right)^4 \omega \epsilon [50 - 500]$$  \hspace{1cm} (5.3)
**Input disturbance specifications**

Specifications on the input disturbance of a system ensure the rejection of the input disturbance on the output of the system. It is defined in the frequency range where input disturbance is expected, in this case the low frequency range $[0-10 \text{ rad/s}]$. The input disturbance to output transfer function is normally defined as the process sensitivity $(\frac{P_{\text{tot}}}{1+P_{\text{tot}}C})$. In this case because input disturbances $D_1$ and $D_2$ don’t enter the system on the input the transfer functions are different. The transfer function from $D_1$ to output $\theta_i$ is defined as $P_2/(1 + P_{\text{tot}}C)$ and the transfer function from $D_2$ to $\theta_i$ as $ZP_2/(1 + P_{\text{tot}}C)$. The magnitude of the transfer functions of the no-uncertainty case and the obtained input disturbance specifications are shown in fig 5.8 and 5.9.

![Figure 5.8: Mag. of TF $P_2/(1 + P_{\text{tot}}C)$ and input disturbance $D_1$ mag. specification](image)

![Figure 5.9: Mag. of TF $ZP_2/(1 + P_{\text{tot}}C)$ and input disturbance $D_2$ mag. specification](image)

The input disturbance specifications are given by:

\[
\left| \frac{y}{D_1} \right| = \left| \frac{P_2}{1 + P_{\text{tot}}C} \right| \leq \eta_{r.D1} = 8e^{-4} \frac{s}{s + 20} \quad \omega \epsilon [0 - 10] \tag{5.4}
\]

\[
\left| \frac{y}{D_2} \right| = \left| \frac{ZP_2}{1 + P_{\text{tot}}C} \right| \leq \eta_{r.D2} = 4e^{-8} \left( \frac{s}{s + 20} \right)^2 \quad \omega \epsilon [0 - 10] \tag{5.5}
\]
Sensitivity specifications

Sensitivity specifications guarantee stability and a low steady-state error. The specification on the sensitivity function is defined in the low and middle frequency range \([0-50 \text{ rad/s}]\). The magnitude of the sensitivity function for the nominal case and the obtained sensitivity specification are shown in figure 5.10.

\[
\left| \frac{e}{r} \right| = \left| \frac{1}{1 + P_{\text{tot}} C} \right| \leq \eta_s = 2 \left( \frac{s}{s + 20} \right)^2 \quad \omega \epsilon [0 - 50]
\]  \hspace{1cm} (5.6)

Figure 5.10: Nominal case sensitivity function mag. and sensitivity mag. specification

the sensitivity specification is given by:
Calculation of the bounds

With the given specifications the bounds on the open loop can be drawn in Nichols chart. This used to be a laborious and difficult manual graphical task. This process is explained in detail in [13]. In this case the bounds are drawn with the QFT frequency domain control design toolbox for MATLAB. With the given specifications, templates and nominal plants \((P_{2,nom} \text{ and } Z_{nom})\) the toolbox draws the bounds fast and accurate. A short explanation on how the bounds are calculated is given below.

First the closed loop magnitude specifications are converted into magnitude and phase specifications on the open loop \((L = CP_{tot})\). As an example the converted sensitivity specification \(1/(1 + P_{tot}C) \leq 1.2\) is drawn in figure 5.11. From this the bounds for each specific frequency are drawn. It is important to note that since \(\ln L = \ln C + \ln P_{tot}\), the pattern of the template may be translated but not rotated on the Nichols chart. The amount of translation is given by the value of \(G(j\omega)\). In other words, the template can be moved vertically by \(|G(j\omega)|\)\ dB along the open-loop magnitude vertical axis, or be moved horizontally by \(\arg G(j\omega)\) along the open-loop phase horizontal axis, without rotation of the template.

With this in mind, a given open-loop specification and the template \(P_{2,\omega}\) and nominal plant \(P_{2,nom}\) for some frequency \(\omega_k\), the bound for this frequency can be drawn. This is done by translating (without rotating) the template around the open-loop specification without entering the restricted area by any part of the template. The path the nominal plant follows during this process is the bound at this frequency. This ensures that as long as the nominal open loop \(L_{nom}\) stays outside the bound, the template (and therefore the uncertain plant) stays outside the open-loop specification. This process is explained in figure 5.11.

Figure 5.11: Explanation of the calculation of a sensitivity bound
From this, one can see that the obtained bound $L(j\omega)$ at each frequency only depends on the specifications and on the shape (uncertainty character) of the template. For this reason subsystem $P_1$ has no influence on the bounds because it has no uncertainty and therefore no influence on the shape of the template $P_{tot}$. Also one can see why it is important to look for frequencies where the shape of the template differs from the rest, these different templates will, even for a constant specification, lead to totally different bounds \cite{12}.

The bounds on closed loop, sensitivity and input disturbance $D_1$ and $D_2$ are shown in figure \ref{fig:bounds}. In this representation the open-loop should lie above a hard line and under a dotted line.

![Figure 5.12: Bounds](image)

Now all specifications are transformed into bounds on the same open loop $L$. Therefore the bounds at each frequency can be combined and the worst-case bounds can be calculated. These worst-case bounds are shown in figure \ref{fig:worst_case_bounds}.
5.1.4 Loop shaping

In the previous section the bounds are derived. The next step involves the design of a stable nominal loop function \( L_{\text{nom}} \) that meets these bounds. During this process the trade-off between exactly meeting the bounds and the complexity of the controller has to be made. It is important to note that the nominal plant used in this process should be the same as the one used in deriving the bounds. The loop shaping is done in the Interactive Design Environment (IDE) of the QFT frequency domain control design toolbox. It involves adding poles and zeros until the bounds are met at all given frequencies. As an initial controller the nominal loop controller obtained in section 5.1.3 is used. The resulting open-loop is given in figure 5.13.

![Figure 5.13: Worst-case Bounds and final open-loop within IDE](image)

Ones the nominal open-loop \( L_{\text{nom}} \) has been derived, the controller \( C \) can easily be obtained from the relation

\[
C = \frac{L_{\text{nom}}}{P_{\text{nom}}}
\] (5.7)
5.1 SISO QFT looper control

5.1.5 Analysis

Once a QFT design has been completed, the closed loop response of all element of the uncertain plant should be analyzed. The fact that all bounds are met in the design phase does certainly not guarantee that this is the case for the magnitude specifications in the closed loop. There can be inaccuracies in the translation of the close loop magnitude specifications into QFT bounds and there can be frequencies in between those used for computing the bounds where the specifications are not met. Therefore the analysis should be done by different and many more frequency points than those used for computing the bounds. Also it is better to use a denser plant template [12].

The analysis is done within the Toolbox by calculating the worst-case of all closed loop frequency responses of the uncertainty set. The results are shown in figure 5.14.

As can be seen in figure 5.14 all closed loop magnitude specifications are met for the frequency range they were defined in.

![Graphs showing analysis results]

Figure 5.14: Analyses
5.2 MIMO QFT

In this section the extension to a Multiple Input Multiple Output QTF design problem is explained. The QFT method was originally developed for uncertain LTI SISO systems, but it can be extended to multi-loop systems using a sequential loop closure approach. In MIMO QFT the design problem is rigorously broken into an \( i \)-step sequential procedure. At each \( i \)th design step, the controller \( c_i \) is designed so that \( 1 + p_{ii}c_i \) is stable for each \( P \in \mathcal{P} \), and all performance specifications in this \( i \)th channel are met. \( p_{ii} \) describes the equivalent open-loop transfer function of the \( i \)th channel assuming the other channels have been closed.\[10\]. The problem with this approach is that, except for the last step, \( p_{ii} \) depends on yet to be designed controllers.

In the next subsection as an example of MIMO QFT, the \( 2 \times 2 \) MIMO looper control case is explained briefly.

5.3 MIMO QFT looper control

In this case QFT is used to control both the looper angle \( \theta_i \) and the strip tension \( \sigma_i \). The looper angle is controlled by the upstream work roll peripheral speed \( \Delta V_{r,i} \) and the strip tension is controlled by the motor torque \( T_{m,i} \). This is conventional pairing although it will be assumed that a direct measurement of tension is available. The MIMO looper system \( P \) is shown in figure 5.15.

In the MIMO looper model 5.15 the disturbance \( D_1 \) doesn’t enter the system as input or output disturbances, therefore \( D_1 \) has been moved to the input of the system \( P \) by multiplying it with \( Z \). \( Z \) has been calculated with \( Z \ast p_{11} \) equals the transfer function from \( D_1 \) to \( \theta_i \). The total MIMO feedback system is shown in figure 5.16. In this control design case again reference disturbance, output disturbance and sensor noise are neglected, also sensor dynamics \( H \) and pre-filter \( F \) equal I.

In MIMO QFT the design problem is broken into a \( i \)th step sequential procedure. At each step single-loop QFT problems are solved in accordance with the one explained in section 5.1.

\[\text{Figure 5.15: MIMO looper system}\]
5.3 MIMO QFT looper control

Figure 5.16: MIMO feedback looper system

For this reason no attention will be given here to the plant templates and the loop shaping procedure. Only the procedure of deriving the bounds will be given.

5.3.1 Bounds on 1st loop

As stated, the controllers will be designed sequentially with the looper angel controller first.

When the MIMO feedback system is treated, we are interested in the same transfer functions as defined for SISO systems. However, in MIMO systems, the open-loop transfer matrix $L$ is defined differently, according to the location where the loop is opened. For instance, if the loop is opened at the plant output, then $L_o = PC$, but if the loop is opened at the plant input $L_i = CP$, in MIMO cases this gives different matrices.

Specifications on closed loop

The closed loop function is defined as:

$$T = PC (I + PC)^{-1}$$  \hspace{1cm} (5.8)

with:

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix}$$

for the first loop this given:

$$t_{11} = \frac{p_{11}c_1 + det(P)c_1c_2}{1 + p_{22}c_2 + p_{11}c_1 + det(P)c_1c_2}$$  \hspace{1cm} (5.9)

$$t_{12} = \frac{p_{12}c_2}{1 + p_{22}c_2 + p_{11}c_1 + det(P)c_1c_2}$$  \hspace{1cm} (5.10)
The problem here is that the closed loop functions $t_{11}$ and $t_{12}$ depend on the, yet to be designed, controller $c_2$. To facilitate a solution, it is assumed that there exist solutions to the total MIMO robust performance problem, and hence that:

$$|t_{21}| \leq \eta_{k,21} \quad (5.11)$$
$$|t_{22}| \leq \eta_{k,22} \quad (5.12)$$

with $\eta_{k,21}$ and $\eta_{k,22}$ specifications on the closed loop \cite{10}.

With \cite{5.8}, expressions for $c_2$ can be derived in terms of $t_{21}$ and $t_{22}$ respectively and with \cite{5.11} and \cite{5.12} in terms of the closed loop specifications $\eta_{cl,21}$ and $\eta_{cl,22}$. Substituting these expressions for $c_2$ in \cite{5.9} and \cite{5.10} gives the specifications on the closed loop:

$$|t_{11}| = \frac{|det(P)c_1 + |p_{12}t_{21}|}{|p_{22} + det(P)c_1|} \leq \frac{|det(P)c_1| + |p_{12}\eta_{cl,21}|}{|p_{22} + det(P)c_1|} \leq \eta_{cl,11} \quad (5.13)$$
$$|t_{12}| = \frac{|p_{12}t_{22}|}{|p_{22} + det(P)c_1|} \leq \frac{|p_{12}\eta_{cl,22}|}{|p_{22} + det(P)c_1|} \leq \eta_{cl,12} \quad (5.14)$$

**Sensitivity specifications**

The expressions for the sensitivity function is given by:

$$S = (I + PC)^{-1} \quad (5.15)$$

the sensitivity functions for the first loop are rewritten in the same way as above. The specifications are given by:

$$|s_{11}| = \frac{|p_{22}| + |p_{12}s_{21}|}{|p_{22} + det(P)c_1|} \leq \frac{|p_{22}| + |p_{12}\eta_{s,21}|}{|p_{22} + det(P)c_1|} \leq \eta_{s,11} \quad (5.16)$$
$$|s_{12}| = \frac{|-p_{12}| + |p_{12}s_{22}|}{|p_{22} + det(P)c_1|} \leq \frac{|-p_{12}| + |p_{12}\eta_{s,22}|}{|p_{22} + det(P)c_1|} \leq \eta_{s,12} \quad (5.17)$$

**Input disturbance specifications**

The expression for the input disturbance to output function is here defined as the process sensitivity it is given by:

$$R = P (I + PC)^{-1} \quad (5.18)$$

The input disturbance $D_1$ is moved to the input of the system by multiplying it by $Z$, the transfer function from input disturbance $D_1$ to the outputs is given by:

$$r_{11} = \frac{Zp_{11} + det(P)Zc_1c_2}{1 + p_{22}c_2 + Zp_{11}c_1 + det(P)Zc_1c_2} \quad (5.19)$$
5.3 MIMO QFT looper control

\[ r_{12} = \frac{Zp_{12}}{1 + p_{22}c_2 + Zp_{11}c_1 + \text{det}(P)Zc_1c_2} \] (5.20)

In this case, specifications are only defined for the input disturbance \( D_1 \) and not for \( D_2 \). For this reason the specifications on the 2nd loop cannot be used in an expression for \( c_2 \). Instead the extreme values of \( c_2 \) are used:

\[ |r_{11}| = \frac{|Zp_{11}|}{|1 + Zp_{11}c_1|} \leq \eta_{r.11} \quad (c_2 \to 0) \] (5.21)

\[ |r_{11}| = \frac{|Z\text{det}(P)|}{|p_{22} + Z\text{det}(P)c_1|} \leq \eta_{r.11} \quad (c_2 \to \infty) \] (5.22)

\[ |r_{12}| = \frac{|Zp_{12}|}{|1 + Zp_{11}c_1|} \leq \eta_{r.12} \quad (c_2 \to 0) \] (5.23)

\[ |r_{12}| = \frac{|Zp_{12}|}{|p_{22} + Z\text{det}(P)c_1|} \leq \eta_{r.12} \quad (c_2 \to \infty) \] (5.24)

With these specifications, the templates of \( P \) and the nominal open loop \( L_{11,n} = p_{11}c_1 \) the bounds can be calculated.

5.3.2 Bounds on 2nd loop

During the 2nd step in the sequential procedure the controller \( c_1 \) is already known. Therefore defining the closed loop functions for the 2nd loop is more straight forward. The specifications for the 2nd loop are defined as:

\[ |t_{21}| = \frac{|p_{21}c_1|}{|1 + p_{11}c_1 + p_{22}c_2 + \text{det}(P)c_1c_2|} \leq \eta_{t.21} \] (5.25)

\[ |t_{22}| = \frac{|p_{22}c_2| + |\text{det}(P)c_1c_2|}{|1 + p_{11}c_1 + p_{22}c_2 + \text{det}(P)c_1c_2|} \leq \eta_{t.21} \] (5.26)

\[ |s_{21}| = \frac{|-p_{21}c_1|}{|1 + p_{11}c_1 + p_{22}c_2 + \text{det}(P)c_1c_2|} \leq \eta_{s.21} \] (5.27)

\[ |s_{22}| = \frac{1 + |p_{11}c_1|}{|1 + p_{11}c_1 + p_{22}c_2 + \text{det}(P)c_1c_2|} \leq \eta_{s.21} \] (5.28)

With these specifications, the templates of \( P \) and the nominal open loop \( L_{22,n} = p_{22}c_2 \) the bounds can be calculated. \( p_{22}^e \) is the equivalent open-loop transfer function of the 2nd channel assuming the 1st has been closed, \( p_{22}^e = p_{22} - \frac{p_{22}p_{21}c_1}{1 + p_{11}c_1} \).
Chapter 6

Conclusions

A linear model has been derived that describes the behavior of the 2 stands and the intermediary looper system of a hot strip finishing mill around a set-up point. Despite the fact that a lot of simplifications have been done while deriving the model, the model, with the conventional controllers implemented, responds in the same way as the real closed loop system. A start is made in developing a robust MIMO controller for the hot strip finishing mill under parametric uncertainties using quantitative feedback theory (QFT). A robust QFT controller is derived for the SISO looper system. It was shown that the closed loop transfer functions remain within tolerance. Quantitative feedback theory is proven to be a suitable design technique for the uncertain SISO looper system. The extension to a MIMO uncertain looper system is described. The extension is based on sequential loop closing techniques, so the basic process is the same as in the SISO case. Therefore quantitative feedback theory is also expected to be suitable for uncertain MIMO systems and one should be able to design a robust controller with the quantitative feedback theory for a MIMO hot strip finishing mill system.
Chapter 7

Recommendations

A lot of recommendations can be done to improve the accuracy of the model. Also the model has to be validated with the real system data, this has to be done in different set-up points. Some parameters have been estimated or calculated very rough, further research on the values of the looper inertia $E$, mill modulus $M$, moment of inertia of the looper $J$ and damping factor $D$ can lead to more accurate results. Furthermore no explicit expression has been found for the strip tension effect factors $m_1$ and $m_2$. They have been calculated with $P_\sigma = \frac{1}{100} P_{tot}$ and $m_1 = m_2$. Finding an explicit expression for $m_1$ and $m_2$ can improve the accuracy of this parameters very much. Also the yield stress $K$ is calculated with the expression for $P$ itself instead of with his own expression, finding an explicit expression for $K$ can improve the accuracy.

Also the partial derivatives can be calculated more accurate. During the calculation of the partial derivatives of $P$, the deformed roll radius $R'$ and the yield stress $K$ are kept constant while they actually depend on $H$, $h$, $\sigma_{in}$ and $\sigma_{o}$. Involving expressions for $R'$ and $K$ in the expression for force $P$ can improve the accuracy of the partial derivatives. A simplified expression for the geometric factor $Q$ is used in $P$, introducing a more accurate expression, which also can be found in [6] can lead to more accurate results for the partial derivatives. During the calculation of the partial derivatives of forward slip $f_i$ and backward slip $b_{i+1}$, $R'$ is kept constant. Because of this $f_i$ and $b_{i+1}$ only depend on $H_i$, $h_i$ and $H_{i+1}$, $h_{i+1}$ the partial derivatives $\frac{df_i}{\sigma_{in}}$, $\frac{df_i}{\sigma_{o}}$, $\frac{df_i}{\sigma_{i}}$, $\frac{df_{i+1}}{\sigma_{in}}$, $\frac{df_{i+1}}{\sigma_{o}}$, $\frac{df_{i+1}}{\sigma_{i}}$ are all zero. Involving the expression for $R'$ in $f$ and $b$ will result in non zero term for the given partial derivatives and therefore in a more accurate model. Involving an explicit expression for the yield stress $K$ while calculating the partial derivative $\frac{\partial P_{load}}{\partial \sigma_{i}}$ also leads to a more accurate result.

The model has to be validated in more detail, by comparing the simulation results, of the model with conventional controllers implemented, with the real closed loop system.

A lot of work still has to be done in developing the full MIMO controller. The extension to the full $4 \times 4$ MIMO controller will increase the complexity of the controller design process, but the basic structure should stay the same as in the $2 \times 2$ looper system. It is recommendable to obtain more specific performance specification. The specification which are now used are derived from a nominal loop, it’s better to obtain specification from the people at the plant in Monterrey. Obtaining a more accurate model as discussed above will lead to a smaller uncertainty, this will lead to less tight bounds and therefore to a less complex controller.
Bibliography


[3] Hylsa steel company, Monterrey, Mexico, internal documents


Appendix A

SISO QFT looper control m-file

clear all

%

%% a continuous-time, siso feedback system
%%
%%
%% ---- ---- | D1(s) | D2(s)
%% ---- ---- ---- ---- ---- ----
%% ------->|G(s)|--Vr--|A(s)|--V--|P1(s)|--T-->|P2(s)|--->theta--->
%% % % R(s) | ---- ---- ----- ---- ----| Y(s)
%% % % | --- --- --- |
%% % % -------------------------- -1 -------------------------
%% % % ---
%% % % PROBLEM DATA
%%

% % % % % % % % % % % % % %
% % system parameters % % %
% % % % % % % % % % % % % % %

run('gains_MTYHSM')

% % uncertainty in +/- procent

uB1 = 3;
uB2 = 3;
uT6 = 5;
uT7 = 10;
uT8 = 1;
ul1 = 3;
uK7 = 10;

% % min , max values op system parameters
B1min = B1*(100-uB1)/100;
B1max = B1*(100+uB1)/100;
B2min = B2*(100-uB2)/100;
B2max = B2*(100+uB2)/100;
T6min = T6*(100-uT6)/100;
T6max = T6*(100+uT6)/100;
T7min = T7*(100-uT7)/100;
T7max = T7*(100+uT7)/100;
T8min = T8*(100-uT8)/100;
T8max = T8*(100+uT8)/100;
L1min = L1*(100-uL1)/100;
L1max = L1*(100+uL1)/100;
K7min = K7*(100-uL1)/100;
K7max = K7*(100+uL1)/100;

%% min , max TF parameters
b1max = (-T8max)/(B2min*T7min);
b1min = (-T8min)/(B2max*T7max);

d1max = (-B2max*T7max)/(T6min*T8min);
d1min = (-B2min*T7min)/(T6max*T8max);
d2max = K7max/T6min;
d2min = K7min/T6max;
d3max = B1max/T6min+(B2max*L1max*T7max)/(T6min*T8min);
d3min = B1min/T6max+(B2min*L1min*T7min)/(T6max*T8max);

%% Templates

%% compute the boundary of the plant template P12
n1=3;
n2=3;
n3=201;
c = 1;
for d1 = linspace(d1min,d1max,n1);
    for d2 = linspace(d2min,d2max,n2);
        for d3 = linspace(d3min,d3max,n3),
            P12(1,1,c) = tf([d1],[1,d2,d3,0]);  c = c + 1;
        end
    end
end
% compute the boundary of Z

c = 1;
for b1 = linspace(b1min, b1max, n1*n2*n3), % number of cases
    Z(1,1,c) = tf([b1 0],[1]); c = c + 1;
end

% actuator

A = tf(-K3,[T3,1]);

w = [.1,1,10,50,100,500]; % working frequencies

nomZ = 1; % define nominal plant case
nomP12 = 1; % define nominal plant case

Znom = Z(:,:,nomZ); % nominal plant
P12nom = P12(:,:,nomP12); % nominal plant

plottmpl(w,Z,nomZ), title('Z Templates') % Plot Z Templates
plottmpl(w,P12,nomP12), title('Plant Templates') % Plot Plant P12 Templates

% BOUNDS

wbd1 = [50,100,500]; % compute bounds at all frequencies in wbd1
W1 = tf([2],[1.197e-9 8.142e-7 0.0002076 0.02535 1]); % specification
bdb1 = sisobnds(1,wbd1,W1,P12,[],nomP12,A);
plotbnds(bdb1),title('Closed loop Bounds'); % plot Robust Margins Bounds

wbd2 = [0.1,1,10,50]; % compute bounds at all frequencies in wbd2
W2 = tf([2 0 0],[1 40 400]); % specification
bdb2 = sisobnds(2,wbd2,W2,P12,[],nomP12,A);
plotbnds(bdb2),title('Sensitivity Bounds'); % plot Sensitivity Bounds

wbd3 = [.1,1,10]; % compute bounds at all frequencies in wbd3
W3 = tf([0.00008 0],[1 20]); % specification
bdb3 = sisobnds(3,wbd3,W3,P12,[],nomP12,A);
plotbnds(bdb3),title('Robust Input Disturbance D1 Rejection Bounds'); % plot Input D1 rejection Bounds

wbd4 = [.1,1,10]; % compute bounds at all frequencies in wbd4
W4 = tf([4e-8 0 0],[1 40 400]); % specification
V = [nomP12,nomZ];
\texttt{bdb4 = sisobnds(3,wbd4,W4,Z*P12,[],V,A/Z);}
\texttt{plotbnds(bdb4),title('Robust Input Disturbance D2 Rejection Bounds');}
% plot Input D2 rejection Bounds

% % All bounds

\texttt{bdb=grpbnds(bdb1,bdb2,bdb3,bdb4);}
\texttt{plotbnds(bdb),title('All Bounds');}

% % worse case bounds

\texttt{ubdb=sectbnds(bdb);}
\texttt{plotbnds(ubdb),title('Intersection of Bounds');}

% % % % % Controller design % % %
% % % % % % % % % % % % % % %

\texttt{Cin=tf([0.5599 8.443 837.3 9067 2.513e4],[8.063e-8 3.546e-5 0.0114 1 0])}
% initial controller
\texttt{w1 = logspace(-2,4,100); % define a frequency array for loop shaping}
\texttt{lpshape(w1,ubdb,A*P12nom,Cin); % With open loop so, A*P12nom!!!!!!!}

% % % Analyse % % %
% % % % % % % % % % % % % % %

\texttt{wa=[logspace(-2,4)];}
\texttt{chksiso(1,wa,W1,P12,[]),Cfinal,A); % analysis with final controller}
\texttt{chksiso(2,wa,W2,P12,[]),Cfinal,A);}
\texttt{chksiso(3,wa,W3,P12,[]),Cfinal,A);}
\texttt{chksiso(3,wa,W4,Z*P12,[]),Cfinal,A/Z);}