Partial synchronization of bidirectionally coupled Chua systems

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1 Introduction

Synchronization of coupled dynamical systems receives much attention in literature. One of the reasons for this is that synchronization can be found in several fields such as nature, brain dynamics and robotics. Also, the potential use of synchronization in communication and coordination forms a reason for this interest. Recently partial synchronization in networks of identical systems is receiving particular interest.

Although there are many published results on global synchronization of a network of coupled systems, less attention sofar has been devoted to experimental results for bidirectional coupled systems.

In this presentation attention will be drawn to synchronization of diffusively coupled Chua circuits and in particular to partial synchronization.

2 Partial synchronization

A networked system consist of coupled identical systems that cannot be decomposed into disconnected smaller networks. Besides full synchronization of networked systems, there can exist a number of stable linear invariant manifolds, depending on the size and topology of the network, corresponding to a partial synchronized situation [2]. Partial synchronization is defined here as the situation where some circuits synchronize with each other, while others do not.

Consider a single Chua circuit [1] given by

\[ \begin{align*}
C_1 \dot{v}_1 &= G(v_2 - v_1) - f(v_1) \\
C_2 \dot{v}_2 &= G(v_1 - v_2) + i \\
L \dot{i} &= -v_2 - R_0 i
\end{align*} \]

with \( f(v_1) = G_a v_1 + \frac{1}{2}(G_a - G_b)(|v_1 + B_p| - |v_1 - B_p|) \). The variables \( v_1 \) and \( v_2 \) are the voltages across the capacitors, \( C_1 \) and \( C_2 \), \( i \) is the current flowing through the inductor \( L \), which has an internal resistance \( R_0 \). \( G_a \) and \( G_b \) are the conductances of the piecewise characteristic. \( B_p \) is voltage of the breakpoint.

An experimental setup consisting of four diffusively coupled circuits, using the voltages across the capacitors \( C_1 \), is build to show the possibility of experimental partial synchronization. One of the drawbacks of an experimental setup is that it is impossible to achieve a zero synchronization error due to tolerances in presumably identical electrical components of the circuits. Therefore a form of practical synchronization is introduced to be able to specify synchronization.

3 Results

In the case where four systems are symmetrically coupled, see figure 1, there exist three linear invariant manifolds independently of the particular individual cell and are given by

\[ \begin{align*}
A_1 &= x \in \mathbb{R}^4: x_1 = x_2, x_3 = x_4 \\
A_2 &= x \in \mathbb{R}^4: x_1 = x_3, x_2 = x_4 \\
A_3 &= x \in \mathbb{R}^4: x_1 = x_4, x_2 = x_3
\end{align*} \]

where \( x_i \) is the state of the \( i \)th system. The intersection of any two of these manifolds describes the full synchronization manifold \( (x_1 = x_2 = x_3 = x_4) \).

The two possible ways to global synchronization for the given layout depend on the ratio \( K_0 \) and \( K_1 \).

For the experimental setup the manifolds (2) and (4) are locally stable if the trajectories remain bounded by the double scroll attractors. Global stability is not obtained because a Chua circuit is not (semi)passive [3], which can result in unbounded trajectories. Semipassivity is a requirement for global stability [2].

However on the experimental setup the solutions are normally bounded by the attractor. Therefore by altering the coupling strength \( K_0 \) or \( K_1 \) either manifold (2) or (4) can be made locally stable and as a result partial synchronization is experimentally observed.

References