Simulation of ballistic impacts on armored civil vehicles
B. Adams
MT 06.03

University: Eindhoven University of Technology
Department: Mechanical Engineering
Professor: prof. dr. ir. M.G.D. Geers

University: Eindhoven University of Technology
Department: Mechanical Engineering
Contact teacher: dr. ir. J.A.W. van Dommelen

Company: PDE Automotive B.V.
Department: Computer Aided Engineering
Coach: ir. A.T.M.J.M. Huizinga
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Preface

Four interesting years of Mechanical Engineering at Eindhoven University of Technology have provided me enough knowledge and personal development to start the graduation. An external project had my preference as both the labour market and mechanical engineering in all day life could be discovered.

A vacancy at PDE Automotive in Helmond attracted my attention: "ballistic simulations on armored civil vehicles". During the first interview, PDE told me about a new service to be developed for the automotive market. In their vision, a student could bear the responsibility of setting up a new feature, the ballistic simulations. As such, I was recruited and to gain additional expertise, PDE arranged a study trip to the computational ballistic congress in Cordoba, Spain. The first great experience!

After a tough literature review, impressive ballistic experiments were defined and executed. These experiments can be seen as the verification of the ballistic simulation. To simulate these events, material models - dependent on high strain rates, high pressures, and high temperatures - were implemented in MSC.Dytran. The models are able to characterize the projectile and target material correctly. I would like to bring over my compliments to MSC-software for their support during my graduation and a special thanks to Erik Plugge and Wolter van de Veen for their hospitality and assistance during my stay at MSC.Software in Gouda.

After a satisfactory validation and final correlation with the ballistic experiments, the project was fulfilled. With the FE-modeling, a valuable tool arose for understanding and explaining impact (and penetration) of projectile on armored plates. The final results were published on the Virtual Product Development congress of MSC.Software in Munich which was informative and a true appreciation for the work delivered. Without my company coach, Freddy Huizinga, this project and all side activities would not have been realized. Therefore, I am very grateful. Moreover, I would like to thank the other colleagues at PDE Automotive for a pleasant working environment and their interest in bringing my research further.

The critical view and remarks of my academic coach, Hans van Dommelen were a significant added value to this final thesis. Another word of thanks to Marc v. Maris. He helped tremendously when performing research on the material composition of the bullets and his great pictures using the stereo and environmental scanning electron microscope.
Acknowledgements for Boudewijn van Schaik, Sander Molenaar and Jeroen Verhoef for their support and view on this final thesis. The last and most important feeling of gratitude goes out to my parents for facilitating this study.
Summary

PDE Automotive, a unit of the Benteler Automotive Company, foresees a new service to the automotive market, namely armored civil vehicles. Due to the increasing terrorism, the awareness of public safety raised enormously. Moreover, with a patent pending on hot formable armoring steel (BSEC180) of Benteler, PDE prospects to gain a share in this niche market. Having predictive capabilities using Computer Aided Engineering (CAE) could strengthen the chances for this share. In this thesis an approach is defined to obtain correlation between CAE simulation and the results of ballistic experiments.

From a review in literature it followed that bullet impacts are highly dependent on strain rate, temperature and pressure. Four models appeared to be appropriate to describe these phenomena, the Johnson-Cook and Steinberg-Guinan yield models, the Mie-Gruneisen equation of state, and the Johnson-Cook failure model. The last three models are implemented in the chosen finite element code MSC.Dytran and the first was already available.

The Taylor bar test was used to validate the implementations and good agreement between numerical, analytical, and experimental results were obtained. Further validation by comparison the Taylor bar test in MSC.Dytran with other FE-codes, LS-Dyna and Abaqus/Explicit, gave confidence in the correctness of the implementations.

The initiation and propagation of the shockwave through the material appeared to be of great influence on the failure behavior of the material. Since the Johnson-Cook failure model appeared to be sensitive to high local stress triaxialities caused by the shockwave, a simple 1D shockwave simulation was executed. This test verified that the shock propagated as a narrow transition zone through the material. The changes to density, pressure, and energy in this zone represent the shockwave. The order of magnitude of these variables agreed with the analytical model of Rankine and Hugoniot.

After this validation, several batches of experiments with three types of bullets - ss109 (5.56 × 45 mm), M80 (7.62 × 51 mm) and M193 (5.56 × 45 mm with lead filler) - impacting on different thicknesses of BSEC180 were performed. These experiments have been correctly reproduced by the upgraded FE-code MSC.Dytran.

A known phenomenon, called the body work effect (German: Karrosserie-effect), was recognized during the executed experiments. This phenomenon means that 5.0 mm of BSEC180 prevents bullet ss109 from penetrating. An additional sheet of body work of 1.0 mm thickness at a distance of 16 mm in front of 5.0 mm BSEC180 causes complete penetration of both plates. The K-effect is explained by the fact that the bullet deforms after penetrating the
body work and this new shape (flat nose) is able to initiate a larger pressure/shockwave that causes the armoring material to fail. This phenomenon proved that FE modeling is a valuable tool in understanding and explaining impact (and penetration) of projectiles on armored plates.
# List of symbols and abbreviations

- $a, b$: semi-axis length of holes
- $c_e$: elastic wave speed of flow field
- $c_s$: speed of sound
- $c_L$: linear AV constant
- $c_Q$: quadratic AV constant
- $d$: diameter
- $e$: specific internal energy
- $e_h$: Hugoniot energy
- $e_T$: thermal part of the energy
- $g$: gravitational constant
- $l$: length
- $l_{a,b}$: element length
- $l_f$: deformed length
- $l_0$: initial length
- $m$: thermal softening exponent
- $n$: strain hardening exponent
- $p$: pressure
- $p_h$: Hugoniot pressure
- $p_T$: thermal part of the pressure
- $q$: artificial viscosity
- $q_L$: linear artificial viscosity
- $q_Q$: quadratic artificial viscosity
- $r$: radius
- $s$: extent of plastic flow
- $t$: time
- $t_0$: initial time
- $v$: velocity
- $v_{p,pen}$: penetration velocity
- $v_{p,rear}$: rear projectile velocity
- $A$: JC initial yield stress
- $A_{Γ}$: Gruneisen parameter
- $B$: JC, SG strain hardening coef.
- $B_{Γ}$: Gruneisen parameter
- $C$: JC strain rate hardening coef.
- $C_v$: specific heat
- $C_0$: ZA initial yield stress
- $C_1$: ZA fit parameter
- $C_2$: ZA fit parameter
- $C_3$: ZA fit parameter
- $C_4$: ZA fit parameter
- $C_5$: ZA strain hardening coef.
- $D$: damage parameter
- $D_1$: JC fail void nucleation strain
- $D_2$: JC fail fit parameter
- $D_3$: JC fail stress triaxialty coef.
- $D_4$: JC fail strain rate coef.
- $D_5$: JC fail thermal coef.
- $E$: Young’s modulus
- $F$: yield function
- $F_{ij}, F_{a,b}$: hole-growth ratio
- $F_{a,b}^f$: fracture value for hole-growth
- $G$: shear modulus
- $H_1$: SG fit parameter
- $H_2$: SG fit parameter
- $HV$: Vicker’s hardness number
- $J$: volume ratio
- $K$: bulk modulus
- $K_T$: thermal bulk modulus
- $R_{impedance}$: % of ref. shockwave
- $R_t$: strength of target
- $S$: area/surface
- $S_D$: damaged surface
- $S_0$: initial surface
- $T$: temperature
- $T_0$: reference temperature
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_m)</td>
<td>melting temperature</td>
</tr>
<tr>
<td>(T_{impedance})</td>
<td>% of transmitted shockwave</td>
</tr>
<tr>
<td>(T')</td>
<td>dimensionless temperature</td>
</tr>
<tr>
<td>(U_p)</td>
<td>particle velocity</td>
</tr>
<tr>
<td>(V)</td>
<td>volume</td>
</tr>
<tr>
<td>(V_0)</td>
<td>initial volume</td>
</tr>
<tr>
<td>(Y_p)</td>
<td>strength of projectile</td>
</tr>
<tr>
<td>(Y_t)</td>
<td>strength of target</td>
</tr>
<tr>
<td>(Z)</td>
<td>acoustic impedance</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>thermal expansion coef.</td>
</tr>
<tr>
<td>(\alpha_t)</td>
<td>extent plastic flow field</td>
</tr>
<tr>
<td>(\beta)</td>
<td>volumetric expansion coef.</td>
</tr>
<tr>
<td>(\beta_T)</td>
<td>Taylor series constant</td>
</tr>
<tr>
<td>(\gamma_T)</td>
<td>Taylor series constant</td>
</tr>
<tr>
<td>(\varepsilon_i)</td>
<td>longitudinal strain wave</td>
</tr>
<tr>
<td>(\varepsilon_p)</td>
<td>effective plastic strain</td>
</tr>
<tr>
<td>(\varepsilon_{r,\theta})</td>
<td>radial and tangential strain</td>
</tr>
<tr>
<td>(\varepsilon_s)</td>
<td>strain of specimen in SHPB-test</td>
</tr>
<tr>
<td>(\varepsilon_t)</td>
<td>transmitted strain wave</td>
</tr>
<tr>
<td>(\varepsilon_v)</td>
<td>volumetric strain</td>
</tr>
<tr>
<td>(\varepsilon_\text{0})</td>
<td>initial strain</td>
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<tr>
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<td>strain rate</td>
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<td>equivalent strain</td>
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<tr>
<td>(\ddot{\varepsilon}_p)</td>
<td>equivalent plastic strain</td>
</tr>
<tr>
<td>(\ddot{\varepsilon}_p)</td>
<td>equivalent plastic strain rate</td>
</tr>
<tr>
<td>(\dot{\varepsilon}_p)</td>
<td>dimensionless plastic strain rate</td>
</tr>
<tr>
<td>(\varepsilon_\text{0})</td>
<td>reference strain rate</td>
</tr>
<tr>
<td>(\eta)</td>
<td>relative compression</td>
</tr>
<tr>
<td>(\mu)</td>
<td>reduced mass</td>
</tr>
<tr>
<td>(\nu)</td>
<td>specific volume</td>
</tr>
<tr>
<td>(\rho)</td>
<td>density</td>
</tr>
<tr>
<td>(\rho_p)</td>
<td>density of projectile</td>
</tr>
<tr>
<td>(\rho_t)</td>
<td>density of target</td>
</tr>
<tr>
<td>(\rho_0)</td>
<td>initial density</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>equivalent stress</td>
</tr>
<tr>
<td>(\sigma_{a,b})</td>
<td>stress in semi-axis direction</td>
</tr>
<tr>
<td>(\sigma_m)</td>
<td>mean stress</td>
</tr>
<tr>
<td>(\sigma_{r,\theta})</td>
<td>radial and tangential stress</td>
</tr>
<tr>
<td>(\sigma_s)</td>
<td>stress of specimen in SHPB-test</td>
</tr>
<tr>
<td>(\sigma_u)</td>
<td>ultimate strength</td>
</tr>
<tr>
<td>(\sigma_y)</td>
<td>yield stress</td>
</tr>
<tr>
<td>(\sigma_{y0})</td>
<td>initial yield stress</td>
</tr>
<tr>
<td>(\sigma_{1,2,3})</td>
<td>principal stresses</td>
</tr>
<tr>
<td>(\sigma^*)</td>
<td>stress triaxiality</td>
</tr>
<tr>
<td>(\sigma_{spall})</td>
<td>value for spallation</td>
</tr>
<tr>
<td>(\tau_y)</td>
<td>yield shear stress</td>
</tr>
<tr>
<td>(\tau_{y0})</td>
<td>initial yield shear stress</td>
</tr>
<tr>
<td>(\Delta t)</td>
<td>time step</td>
</tr>
<tr>
<td>(\Delta x)</td>
<td>grid spacing</td>
</tr>
<tr>
<td>(\Delta x_{\text{min}})</td>
<td>minimal element length</td>
</tr>
<tr>
<td>(\delta)</td>
<td>normal vector</td>
</tr>
<tr>
<td>(\hat{n})</td>
<td>surface vector field</td>
</tr>
<tr>
<td>(\hat{S})</td>
<td>nodal displacements</td>
</tr>
<tr>
<td>(\hat{\dot{u}})</td>
<td>nodal velocities</td>
</tr>
<tr>
<td>(\hat{\ddot{u}})</td>
<td>nodal accelerations</td>
</tr>
<tr>
<td>(\hat{\dddot{u}})</td>
<td>time derivative of the nodal acc.</td>
</tr>
<tr>
<td>(\hat{F}(t))</td>
<td>nodal forces</td>
</tr>
<tr>
<td>(\hat{\varepsilon}_p)</td>
<td>equivalent plastic strain</td>
</tr>
<tr>
<td>(\hat{\varepsilon}_p)</td>
<td>equivalent plastic strain rate</td>
</tr>
<tr>
<td>(\hat{\varepsilon}_p)</td>
<td>dimensionless plastic strain rate</td>
</tr>
<tr>
<td>(\hat{\varepsilon}_\text{0})</td>
<td>reference strain rate</td>
</tr>
<tr>
<td>(\varepsilon^d)</td>
<td>deviatoric strain tensor</td>
</tr>
<tr>
<td>(\varepsilon^v)</td>
<td>volumetric strain tensor</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>stress tensor</td>
</tr>
<tr>
<td>(\sigma^d)</td>
<td>deviatoric stress tensor</td>
</tr>
<tr>
<td>(\sigma^h)</td>
<td>hydrostatic stress tensor</td>
</tr>
<tr>
<td>(\mathbf{C})</td>
<td>right Cauchy-Green tensor</td>
</tr>
<tr>
<td>(\mathbf{D})</td>
<td>deformation rate tensor</td>
</tr>
<tr>
<td>(\mathbf{D}_p)</td>
<td>plastic deformation rate tensor</td>
</tr>
<tr>
<td>(\mathbf{I})</td>
<td>unit tensor</td>
</tr>
<tr>
<td>(\mathbf{L})</td>
<td>velocity gradient tensor</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

Recent violent incidents, such as the terrorist attacks on the World Trade Center in New York, trains in Madrid and the metro system in London, raised the awareness of public safety in the private sector as well as semi-government organization enormously. PDE Automotive, which is a unit of the Benteler Automotive Company, foresees a new service to the automotive market, namely armored civil vehicles. With a patent pending on hot formable armoring steel of Benteler, PDE prospects to gain a share in this niche market. Having predictive capabilities using Computer Aided Engineering (CAE) could strengthen the chances for this share. It is much cheaper than destructive experiments, gives better understanding of the dynamic behavior/phenomena and variables at high velocities. Since little information and knowledge about computational ballistics was present inside PDE Automotive this graduation project was initiated in order to gain expertise in this field.

<table>
<thead>
<tr>
<th>Rating</th>
<th>Type</th>
<th>Caliber</th>
<th>Mass [gram]</th>
<th>Velocity [m/s]</th>
<th>Cartridge type</th>
</tr>
</thead>
<tbody>
<tr>
<td>BR1</td>
<td>Rifle</td>
<td>0.22LR</td>
<td>2.592</td>
<td>320 - 340</td>
<td>Lead round nose</td>
</tr>
<tr>
<td>BR2</td>
<td>Handgun</td>
<td>9 mm Luger</td>
<td>8.035</td>
<td>390 - 410</td>
<td>Steel jacket, round nose, soft core</td>
</tr>
<tr>
<td>BR3</td>
<td>Handgun</td>
<td>0.357 Magnum</td>
<td>10.24</td>
<td>420 - 440</td>
<td>Steel jacket, coned, soft core</td>
</tr>
<tr>
<td>BR4</td>
<td>Handgun</td>
<td>0.357 Magnum</td>
<td>15.55</td>
<td>420 - 440</td>
<td>Copper jacket, flat nose, soft core</td>
</tr>
<tr>
<td>BR5</td>
<td>Rifle</td>
<td>0.223 REM (5.56x45)</td>
<td>4.02</td>
<td>940 - 960</td>
<td>Copper jacket, steel penetrator, soft core</td>
</tr>
<tr>
<td>BR6</td>
<td>Rifle</td>
<td>0.308 WIN (7.62x51)</td>
<td>9.53</td>
<td>820 - 840</td>
<td>Steel jacket, soft core</td>
</tr>
<tr>
<td>BR7</td>
<td>Rifle</td>
<td>0.308 WIN (7.62x51)</td>
<td>9.72</td>
<td>810 - 830</td>
<td>Copper jacket, steel hard core</td>
</tr>
</tbody>
</table>

Table 1.1: European standard EN 1063 [1].
The goal of PDE Automotive is to develop, certify and build armored civil vehicles up to the BR6 class. The armoring is achieved by inserting plates of the hot formable ballistic steel BSEC180 in the vehicle. For certification, bullets of a typical rating are not allowed to penetrate or fracture the armoring material. The ballistic ratings are described in table 1.1.

During the development of the armored vehicle and before executing the certification, which is highly expensive, one could benefit from predictive capabilities of finite element codes. Since such simulations (large deformations and parts are shattered) are a new field of operation for the CAE-department of PDE, a bottom start is required. An approach to start up computational ballistics is given by an organigram in figure 1.1 and is based on a literature study which will be discussed in more detail in chapter 2.

![Organigram for the field of ballistics.](image)

The final goal of this thesis is to validate the numerical approach and material models by correlating them to the results of the ballistic experiments. The block 'numerical simulations' will therefore be the central theme. The blocks 'experiments' and 'analytical models' are used for validation and material parameter characterization. As mentioned earlier, numerical simulation will be conducted in a finite element context. The impacting projectiles are often shattered and exhibit large deformation. Therefore an Euler approach was chosen. Because of the dynamic character of ballistics events, large meshes are used. A fast integration method was desirable and therefore an explicit formulation was chosen, more specific the explicit Euler package MSC.Dytran.

The numerical input in the organigram states that several ingredients are needed before the ballistic event can be modeled. These material models describe for example the yielding behavior, the hydrostatic response and the failure of the different materials.
Rate-, pressure- and temperature-dependent material models obtained from literature are implemented in the code and tested with the analytical 1D shockwave relations of Rankine-Hugoniot [2] and the theory of Taylor [3]. Moreover, a comparison with the explicit Lagrangian finite element codes Abaqus and LS-Dyna has been made.

Finally, after validation of the model approach, ballistic simulations are executed. These are compared to ballistic experiments executed in a laboratory in Germany. The minimal thicknesses to prevent penetration through the ballistic steel is investigated.
Chapter 2

Literature review

2.1 Introduction

A literature study was conducted to gain knowledge about the field of ballistics and the scientists/research groups active in this field. The different parts of the organigram introduced in chapter 1 will be considered in the following sections.

Ballistics is defined as the science dealing with a great variety of phenomena that occur from the moment an object or projectile is fired until its effects are observed in a target. Ballistics in general can be divided in different sub-fields and is embedded in various sectors of industry. To demonstrate the size of the application area, a summation of the sectors is given.

- Academia and research institutes (TNO); *high velocity impact* (*0.05 < v < 1.5 km/s*), e.g. high strain rate measurement, analytical models, impact of projectiles;
- Aeronatics and astronautics (NASA, ESA); *hypervelocity impact* (*v > 1.5 km/s*), e.g. impact of meteorites on satellites;
- Automobile industry (BMW, Maserati); *low/moderate velocity impact* (*v < 0.05 km/s*), e.g. crashworthiness of vehicles;
- Aviation industry (Boeing, Airbus); *high velocity impacts*, e.g. bird strike in engine, cockpit safety;
- Military, Department of Defence; *high velocity impact*;
- Software industry (MSC-software, Autodyne, Abaqus/explicit, LS-Dyna); *low, moderate, high and hyper velocities*, e.g. bullet impact simulations, blast events, bird strikes;
- Weapons industry; *high velocity impact*.

All these sectors have their own interest in ballistics and the examples show that there are many angles of incidence. This report will focus on high velocity impacts and is found in various application areas stated above.
2.2 Experiments

2.2.1 Ballistic experiments

Ballistic experiments are crucial in the investigation of fundamental and applied problems in armor mechanics, armor applications and armor design. To ascertain the predictive capabilities of numerical simulations, validation experiments are required, despite the extreme high costs. Due to the high projectile velocities a careful instrumentation scheme is needed. In this section a brief summary of various contributions and methods is given.

Ballistic experiments are often executed by a company authorized by the government because of safety requirements. Standardized experimental set-ups are used to guarantee quality. Most often a universal launcher and the target material are fixed on a predefined position as depicted in figure 2.1.

![Experimental set-up for ballistic impacts](image)

Figure 2.1: Experimental set-up for ballistic impacts [4].

The different measuring devices are placed around the target material to acquire the specific data. Typically, the final depth of penetration provides one of the few links between experiments and the results of numerical simulations, although occasionally crater diameters are also compared. Preece and Berg [5] obtained a good comparison between these parameters.

Flash radiography and X-ray techniques can be used to obtain more information about other impact parameters. The positions of the projectile/target interface, projectile tail and the instantaneous length of the projectile as a function of time can be determined. Orphal et al. [6] used these data to calculate the speed of penetration into the target and the consumption of kinetic energy. To create sharp X-ray images the target dimensions are constrained since X-ray is absorbed by the target material. X-ray is mostly used in situations where a lot of dust is caused by the impact.

High-speed imaging of ballistic impacts is a technique of the last decade. The different events during ballistic penetration can be observed. Borvik et al [7] used a camera system that combines the advantages of an image converter camera and a CCD camera. The image
2.2. EXPERIMENTS

camera provides extremely fast shutter speeds, while the CCD camera provides
digital images. To obtain a full 3D-description of penetration, mirrors are placed above the
projectile path with an angle of 45 degrees. An advantage of this data-acquisition system is
that the impact and physical phenomena can be studied in time.

To measure velocities, Borvik et al. [7] used a photocell system having two identical light
barriers with LED-light sources on the upper side of the projectile path and detectors on the
lower side. When the projectile passes through the light, it is interrupted and signals are send
to a nanosecond counter. From the obtained data the velocities can be calculated. At the
interface six laser light sources are used to measure the residual velocities.

Various measurement techniques that have been used include:

- Photocell systems to measure velocities [8];
- Measurement of the penetration depth and crater radius [5];
- Flash X-ray system to measure the positions of the nose and tail at different times
during penetration [9, 6];
- High-speed imaging [10].

Two typical ballistic event can occur, (i) the projectile penetrates in the target material or
(ii) the object/projectile erodes or burst in pieces during impact. Four stages from initial to
zero projectile velocity are recognized by investigators, namely:

- Free flight from launcher (data acquisition for velocity of projectile);
- Impact moment (data acquisition for impact angle and speed);
- Penetration (data acquisition for deformation of target material and projectile);
- Resulting speed after impact (data acquisition for resulting speed).

2.2.2 Material characterization experiments

Standard mechanical testing procedures are well suited for material characterization in the
lower strain rate regime. Therefore the use of a servo hydraulic machine is common and
convenient [11]. However ballistic impacts are highly dynamic and strain rate dependent.
The strain rate’s involved can be divided into three categories/groups: lower \((\dot{\varepsilon} < 10^{-1} \text{ s}^{-1})\),
moderate \((10^{-1} \leq \dot{\varepsilon} \leq 10^{2} \text{ s}^{-1})\), and high strain rates \((\dot{\varepsilon} > 10^{2} \text{ s}^{-1})\). Experiments in the
whole range of rates are desired to characterize the materials properly. Different techniques
for moderate and high strain rate measurements are explained.

**Moderate strain rate**

Some more advanced techniques are needed for measurements of moderate strain rates. Wave
propagation can have some effect on the load measurement. The available techniques mainly
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originates from the automobile industry, for modeling their crashworthiness of civil vehicles. A commonly used technique is the falling weight impact experiment. A striker / impactor, which is guided by two guiding rods, is allowed to fall due to gravity (or it can be pneumatically pushed to achieve higher velocities) on the specimen in a fixture. An experimental set-up can be seen in figure 2.2.

![Figure 2.2: Schematic view of the falling weight impact test [11].](image)

The stress/strain behavior is calculated using the impact velocity and the data from an accelerometer fixed on the striker. The falling weight experiment is inexpensive, can accommodate different specimen geometries and allows easy variation of strain rate. The disadvantage is that it does not take wave propagation into account. The standard mechanical testing procedures can be adopted with a servo hydraulic frame with high capacity valve. This apparatus gives a more accurate relation between stress and strain. The only disadvantages are the high price and the fact that wave propagation is not adopted.

A high-speed system, the cam plastometer, was designed and built by Hockett and Lindsay [12] to acquire time, temperature and load data from experiments. The cam plastometer is a mechanical testing machine, which is capable to measure stress-strain relations up to a strain rate of 200 s\(^{-1}\). Deformation is achieved by inserting a cam follower between a rotating logarithmic cam to compress the specimen. The specimen is placed against a load cell and gives as output information about the load as a function of time. Measurement of the specimen before and after compression allows the calculation of the true strain rate and the strain is calculated from the known rotational speed of the cam and its profile.

**High strain rate**

Due to the short loading times and high deformation inherent to impact experiments, conventional measurements at high strain rates are not straightforward. Hopkinson introduced a technique for determining the pressure - time relation due to an impact produced by a bullet or explosive in 1913 [13]. With this technique a compressive wave is produced through an impact on a bar. This wave propagates down the bar, through a greased joint, into a billet and the wave would be reflected at the far end as a pulse of tension. Since the grease could not withstand any appreciable tensile loads, the billet would fly off with a definite momentum measured with a ballistic pendulum.
2.2. EXPERIMENTS

Kolsky improved the Hopkinson technique and developed the Split Hopkinson Pressure Bar (SHPB). The SHPB and all its variants [14, 15, 13] are one of the few known set-ups nowadays for studying material behavior at high strain rates. Impact and nuclear engineers are most interested and skilled with these techniques. This technique adopts a long input and output bar with a short specimen placed in between. The SHPB test set-up can be seen in figure 2.3. A compressive longitudinal wave $\varepsilon_i(t)$ is produced by a projectile impacting the input bar.

![Figure 2.3: Schematic view of the split Hopkinson pressure bar test [14].](image)

This wave is transported through the bar and once it reaches the specimen interface, a part of the waves is reflected, $\varepsilon_r(t)$, and a part is transmitted, $\varepsilon_t(t)$. These three waves $\varepsilon_i(t)$, $\varepsilon_r(t)$ and $\varepsilon_t(t)$ are recorded with strain gauges glued on the input and output bar. This data acquisition allows to calculate the strain rate ($d\varepsilon_s(t)/dt$), strain ($\varepsilon_s$) and stress ($\sigma_s$) respectively at the two faces of the specimen using the following equations:

\[
\frac{d\varepsilon_s(t)}{dt} = -\frac{2c_e}{l_0} \varepsilon_r(t) \quad (2.1)
\]

\[
\varepsilon_s(t) = -\frac{2c_e}{l_0} \int_0^t \varepsilon_r(t) dt \quad (2.2)
\]

\[
\sigma_s(t) = E S_0 S \varepsilon_t(t) \quad (2.3)
\]

where $c_e$ is the elastic wave speed defined as the speed of the input bar minus the speed of the output bar, $l_0$ instantaneous length, $E$ is the Young’s modulus of the input/output bar and $S_0$ and $S$ the initial and current surfaces. $\sigma_s$ and $\varepsilon_s$ are the stress and strain in the specimen respectively. A proper description of the force, strain and stress calculations and optimizations of this set-up can be found in [13, 14].

A smart variant to observe plastic deformation in a specimen was proposed by Verleijsen and Degrieck [15]. A line grid is attached to the specimen and the time evolution of the specimen deformation is recorded with a streak camera and a constant rotating drum. Figure 2.4 gives a schematical representation of the experimental set-up and a sample of a deformed line grid in time.

From the recorded deformed grating in figure 2.4, the specimen displacements are captured by means of an advanced numerical algorithm which is based on the interference between the specimen grating and a virtual reference grating. In [15, 16] a combination of geometric moiré and phase shifting is used to determine the specimen displacements.

Monitoring the deformation with a high-speed camera and measuring loading force from one-dimensional wave propagation using strain gauges was proposed by Kajberg et al. [17]. The
displacements are calculated by tracking the pixel position compared to a reference geometry from the monitored experiment.

The VNIIEF scientific research institute in Sarov, Russia, developed in cooperation with the Los Alamos National Laboratory [18], USA, an explosively driven pulsed power generator to load a sample for high strain and high strain rate measurements. Figure 2.5 shows a HEMG (helical explosive magnetic generator) and DEMG (multi-element disk explosive magnetic generator) which both use rapid magnetic flux compression to generate a very high short pulse of electrical current. This combination can deliver a peak current of $35 \text{MA}$ through a liner inside the physics package. This current causes a magnetically implosion of the aluminium liner which radially compresses the copper load sample via a polyethylene filler, see figure 2.5. A peak pressure of 160 kbar at a strain rate of $9 \cdot 10^5 \text{s}^{-1}$ can be achieved.

Figure 2.4: SHPB-experiment set-up with rotating drum and streak camera to record the line grid on the specimen [15].

Figure 2.5: HEMG (a) and DEMG (b) to generate a short pulse of electrical current and impose a material to high strain rate [18].
The volume inside the copper is equipped with a stainless steel central measuring unit (CMU). A VISAR interferometer is used to measure the velocity of the inside surface of the copper as a function of time via glass fibers in the CMU. X-ray radiographs are obtained to measure the perturbation growth. With the assumption that the yield strength of the polyethylene is small enough that it can be neglected, the dynamic yield strength of the copper can be determined directly from the growth rate of the perturbation amplitude.

2.3 Analytical models

Analytical models have been developed to predict various parameters such as the penetration depth of a projectile in a semi-infinite target, dynamics of cavity formation in the target, length of the projectile after impact, deceleration and many more. These parameters are derived for simplified models. In the following a brief explanation of various analytical models is given.

Alekseevskii-Tate model

Alekseevskii and Tate [19] independently developed the same model based on a semi-infinite target which is penetrated by a steel rod. Balancing the pressure at the material interfaces using Bernoulli’s equation gives

$$\frac{1}{2} \rho_p (v_{p,\text{rear}} - v_{p,\text{pen}})^2 = \frac{1}{2} \rho_t v_{p,\text{pen}}^2.$$  \hspace{1cm} (2.4)

In this equation $\rho_p$ and $\rho_t$ are the densities for the projectile and target respectively, $v_{p,\text{rear}}$ is the velocity of the rod and $v_{p,\text{pen}}$ is the penetration velocity. Experiments of Alekseevskii and Tate revealed that the penetration speed is much smaller than predicted by (2.4). The explanation is found by the hydrodynamic character of Bernoulli’s equation. Alekseevskii and Tate assumed that a material acts as a rigid body until a certain pressure, $Y_p$. Above this pressure, the material behaves hydrodynamically. Equation (2.4) therefore changes to

$$\frac{1}{2} \rho_p (v_{p,\text{rear}} - v_{p,\text{pen}})^2 + Y_p = \frac{1}{2} \rho_t v_{p,\text{pen}}^2 + R_t$$ \hspace{1cm} (2.5)

where $Y_p$ is often called the strength of the projectile and $R_t$ the target resistance. Two cases can arise, namely $Y_p < R_t$ where below a certain velocity penetration no longer occurs and $Y_p > R_t$ where below a certain velocity the projectile behaves like a rigid body.

Some candidate formulations for the target resistance $Y_p$ and $R_t$ are given in [20]. A study done by Anderson et al. [21] indicated that the constants vary with impact conditions and also change during the penetration process. Furthermore the predictive capability is limited due to the one-dimensional nature of the formulation [22]. However the theory demonstrates the essential physics of high velocity penetration. Despite these disadvantages, this model is the basis for analytical models for high velocity impacts and many authors derived their models based on this set of equations.
Walker-Anderson model

Walker and Anderson from the Southwest Research Institute (SWRI) reviewed the Alekseevskii-Tate model. Comparison with numerical simulations showed that the transient behavior at the beginning and near the end of the penetration was not properly described [23]. Therefore the z-component of the momentum equation of Euler was examined. To simplify the momentum equation three assumptions were made based on numerical results [23]:

- The velocity profile is taken along the centerline in axial direction ($u_x = u_y = 0$);
- The back end of the projectile is decelerated by elastic waves and the free surface gives $\sigma_{zz} = 0$;
- Shear behavior is specified in the target material assuming that the flow field is monotonically decreasing and has a hemispherical behavior.

The reduced $z$-component of the momentum equation becomes:

$$\rho \left( \frac{\delta u_z}{\delta t} + u_z \frac{\delta u_z}{\delta z} \right) - 2 \frac{\delta \sigma_{xz}}{\delta x} = 0. \tag{2.6}$$

The three assumptions above are reworked (for more detail see [23]) and the resulting terms are substituted in the Euler momentum equation in the axial direction, which yields:

$$\rho_p \dot{v}_{p,\text{rear}} (l_f - s) + \dot{v}_{p,\text{pen}} \left[ \rho_p s + \rho_t \left( \frac{\alpha_t - 1}{\alpha_t + 1} \right) \right] + \rho_p \left( \frac{\dot{v}_{p,\text{rear}} - \dot{v}_{p,\text{pen}}}{s} \right) s^2 \frac{\sigma_y}{2} + \rho_t \alpha_t \frac{2r v_{p,\text{pen}}}{(\alpha_t + 1)^2} =$$

$$\frac{1}{2} \rho_p (v_{p,\text{rear}} - v_{p,\text{pen}})^2 - \left[ \frac{1}{2} \rho_t v_{p,\text{pen}}^2 + \frac{7}{3} \ln(\alpha_t) Y_t \right], \tag{2.7}$$

$$\dot{v}_{p,\text{rear}} = - \frac{\sigma_y}{\rho_p (l_f - s)} \left[ 1 + \frac{v_{p,\text{rear}} - v_{p,\text{pen}}}{c_s} \right] + \frac{\dot{s}}{c_s}. \tag{2.8}$$

The different terms are visualized in figure 2.6a. If the crater radius $r$ and the extent of the plastic flow field in the projectile $s$ in this equation are allowed to go to zero and the Young’s modulus of the projectile is high ($c_s \to \infty$) than equation (2.7) is almost similar to Tate’s original model.

$$-\rho_p \dot{v}_{p,\text{rear}} l_f + \frac{1}{2} \rho_p (v_{p,\text{rear}} - v_{p,\text{pen}})^2 = \frac{1}{2} \rho_t v_{p,\text{pen}}^2 + \frac{7}{3} \ln(\alpha_t) Y_t. \tag{2.9}$$

The overall resistance of the target to penetration is derived as $R_t = \frac{5}{3} \ln(\alpha_t) Y_t$. This model satisfies for both rigid and eroding projectiles. Furthermore the transient effects at the beginning and in the end of the penetration are properly described. If a proper value for the
dimensionless extent of plastic flow field $\alpha_t$ in the target is chosen then the penetration depth as function of the time and the crater dimensions can be determined.

Walker and Anderson also developed a model for non-penetrating projectiles [24]. This model describes the radial outward flowing of the impacting projectile against a non-penetrable target (ceramics). During impact, the projectile loses kinetic energy due to mass loss (erosion) and deceleration. Using Tate’s model [19] and assuming a penetration velocity of zero, an equation for the kinetic energy is derived. To investigate the origin of these terms, this equation is split up in a part that decreases kinetic energy by means of deceleration (elastic waves traveling through the projectile) and a part due to erosion of the material. The loss of kinetic energy per unit of time can for example be predicted with the proposed equation.

**Ravid-Bodner model**

Ravid and Bodner [25] assumed flow fields for different regions during impacting, see figure 2.6b. A procedure is used to determine the longitudinal and radial flow of plastic deformation (combined with conditions between the various flow zones). On the basis of the assumed 2-D plastic flow fields in the penetrator and the target, the various work rate terms can be calculated and the overall work rate balance can be established. By defining correct boundary conditions for the different regions, some unknown terms in the work balance can be obtained. The overall work rate balance can then be reduced to three unknown terms namely interface velocity, velocity of undeformed section of the projectile and the crater radius. In [26] a rich selection of failure/exit modes defined by Ravid and Bodner suitable for finite-thick metallic targets are considered. These modes can describe the final shape of the crater.

![Figure 2.6: a. Variables of the Walker-Anderson model b. Flow field regions in the Ravid-Bodner model.](image)

**Taylor’s model**

In 1946, Taylor [3] developed a simple method to determine the average dynamical yield stress by measuring the initial length $l_0$ and the deformed length $l_f$ from an experiment, see figure 2.7. In the experiment, a cylindrical bar is fired straight against a rigid wall. After deformation a mushroom type cylinder must remain due to plastic work. Taylor derived a
simple equation relating the initial and final length to the dynamic yield stress, density and velocity.

\[ \sigma_y = -\frac{\rho v^2}{2 \ln(l_f/l_0)}. \]  

Figure 2.7: Taylor test.

The equation is derived from the energy balance and the following assumptions are made:

- a stationary yield front near the wall;
- a quasi-steady process.

For an homogeneously deforming, ideally plastic specimen, the energy-balance can now be rewritten in a yield stress formulation:

\[ \sigma_y = -\frac{\rho v^2}{2 \ln(l_f/l_0)}. \]  

This simple model is often used to validate a numerical experiment.

**Modified models**

A lot of effort is done in improving and modifying the Alekseevskii-Tate and Walker-Anderson models. In [27], a unified model of Walker-Anderson and Ravid-Bodner is composed. A combination of the advantages of both models is made, the momentum balance is used to calculate the semi-infinite penetration of the projectile and the Ravid-Bodner failure modes are used to determine projectile perforation and the shape of the crater.

An energy transfer mechanism to examine hypervelocity penetration is proposed by Walker [28]. The energy transfer from the projectile (which is only composed of kinetic energy) to the target (where a part is absorbed by elastic compression, kinetic energy and dissipation by means of plastic flow) is worked out in four examples. These examples clarified the analytic models and their terms in a physical sense. Walker also developed a model to describe the behavior of low aspect ratio projectiles \((l/d < 1)\) [29]. Velocities and penetration depth can be estimated with this model.
2.4 Numerical simulation

In the last decades, the performance and capability of computers has increased enormously. Due to this increase, a lot of effort is put in the development of codes based on the Finite Element Method (FEM). The advantage of this method is that a large quantity of information can be obtained for geometrically complex situations: full time-resolved displacements, strains, strain rates, moments, energies, forces, etc. In the aerospace society for example, numerical simulations are used to investigate impacts at velocities higher than 5 km/s. Since these velocities cannot be achieved in experiments, the only results can be obtained from FEM analysis. Furthermore material and configuration variations can be executed without the use of expensive experiments. First, the finite element formulations used in FEM-codes and integration methods are discussed. The codes frequently used in the ballistic society are discussed in the last part of this section.

2.4.1 Principles of formulations

In this subsection the principles of the different formulations used in computational ballistics are discussed. Both the advantages and disadvantages are discussed.

**Lagrangian formulation**

The Lagrangian formulation is the most common finite element solution technique for engineering applications. This formulation defines that the grid points on a body are unique, i.e. each grid point has different material coordinates [30]. Elements are created by connecting the grid points. As a body undergoes a deformation the grid points are forced to move and the element will be distorted.

Severe mesh distortions (material points which pass through each other) are a typical difficulty with a classical Lagrangian formulation for penetration problems. Remeshing is necessary to obtain a certain level of reliability. By erosion mechanisms distorted elements are removed to allow the calculation to continue, a method which lacks a physical basis. The great advantage is that small displacements/deformations and material interfaces are described with a high accuracy.

**Eulerian formulation**

The Eulerian formulation is most frequently used for analyses of fluids or materials undergoing very large deformations. The bullet and armoring material undergo such deformations and can reach the melting point which makes an Eulerian formulation suitable.

In an Eulerian formulation the grid points are fixed in space and elements are also created by connecting these points. An Eulerian mesh is a "fixed frame of reference" in which material moves from one element to the other and so its mass, momentum and energy. The Eulerian mesh must be large enough in order to model existing as well as future regions where material may flow.
Arbitrary Lagrange Euler Coupling (ALE-formulation)

The ALE-formulation is a coupling between the two formulations described before. This formulation allows the Eulerian material to move and coincide with the interface nodal points of the Lagrangian mesh by means of ALE coupling surfaces. The interface can be seen as a boundary condition for the Eulerian mesh on which it can exert a pressure for example. This pressure is then applied to the Lagrangian structure.

Smooth Particle Hydrodynamics (SPH)

SPH is a mesh free technique that can be applied for non-linear problems with large deformations. In [31] it is stated that SPH overcomes the disadvantages of the Lagrange and Euler approaches. In the SPH formulation free movable points with a fixed mass, called particles, have coherence by means of an interpolation function. A kernel estimate allows to describe the conservation of mass, momentum and energy in terms of interpolation sums [32]. A physical object is than defined by a field of SPH points instead of elements. The problem of this formulation are the large velocity oscillations in single particles.

2.4.2 Integration methods

It is critical that the basic of the solution techniques are understood to be able to analyse and explain unexpected results. Furthermore the integration method must agree well with the defined problem. The general dynamic equilibrium equations, (2.11), is used to advance in time and results after the spatial discretization.

\[ \ddot{u} + C \dot{u} + K u = F(t) \]  

(2.11)

where \( M \) is the mass matrix, \( C \) the damping matrix and \( K \) the stiffness matrix. The columns \( u, \dot{u}, \ddot{u} \) and \( F(t) \) are the nodal displacements, velocities, accelerations and forces, respectively. The implicit Newmark integration method and the explicit central difference methods to advance in time are explained as well as their (dis-)advantages.

Implicit integration

Implicit methods are characterized by the fact that unknown quantities at time \( t + \Delta t \) are required to solve the equilibrium [33]. The Newmark integration method is perhaps the most widely used implicit method. In [34] Newmark proposed to use a truncated version of a third order Taylor serie for the displacement and velocity at time \( t + \Delta t \).

\[ u_{t+\Delta t} = u_t + \Delta t \dot{u}_t + \frac{\Delta t^2}{2} \ddot{u}_t + \beta \Delta t^3 \dddot{u}_{t+\Delta t} \]  

(2.12)

\[ \dot{u}_{t+\Delta t} = \dot{u}_t + \Delta t \ddot{u}_t + \gamma \Delta t^2 \dddot{u}_{t+\Delta t} \]  

(2.13)
where $\beta_T$ and $\gamma_T$ are Taylor serie constants. An assumption is needed to determine a quantity for the unknown third order time derivative of the displacement at time $t + \Delta t$. Newmark assumed the acceleration linear within the time step:

$$\ddot{u}_{t+\Delta t} = \frac{(\ddot{u}_{t+\Delta t} - \ddot{u}_t)}{\Delta t}$$  \hspace{1cm} (2.14)$$

Substitution of (2.14) into (2.12) and (2.13) provides Newmark’s equations for the displacement and the velocities at time $t + \Delta t$:

$$u_{t+\Delta t} = u_t + \Delta t \ddot{u}_t + \frac{\Delta t^2}{2} (1 - 2\beta_T) \dddot{u}_t + \beta_T \Delta t^2 \dddot{u}_{t+\Delta t} = \dddot{u}_t^* + \beta_T \Delta t^2 \dddot{u}_{t+\Delta t}$$  \hspace{1cm} (2.15)$$

$$\ddot{u}_{t+\Delta t} = \dot{u}_t + (1 - \gamma_T) \Delta t \dddot{u}_t + \gamma_T \Delta t \dddot{u}_{t+\Delta t} = \dddot{u}_t^* + \gamma_T \Delta t \dddot{u}_{t+\Delta t}$$  \hspace{1cm} (2.16)$$

where $\Delta t$ is the time step and $u_t^*$, $\dddot{u}_t^*$ are used for convenience. The implicit character is recognized in equations (2.15) and (2.16) from the unknown quantities at time $t + \Delta t$ on the left and right side of the equal sign. If $\beta_T$, $\gamma_T$ are chosen zero, the quantities at time $t$ are deleted and an explicit formulations originates. Hughes [35] has proven that this integration method is unconditionally stable if $2\beta_T \geq \gamma_T \geq \frac{1}{2}$ and conditionally stable if $\gamma_T \geq \frac{1}{2}$ and $\beta_T < \frac{1}{2}\gamma_T$. Usually these constants are chosen as $\beta_T = \frac{1}{4}$ and $\gamma_T = \frac{1}{2}$ which is known as the average acceleration method. Substitution of (2.15), (2.16), and the constants in the discretized dynamic equilibrium equation (2.11) gives

$$\left[ M + \frac{1}{2} C \Delta t + \frac{1}{4} K \Delta t^2 \right] \dddot{u}_{t+\Delta t} = F_{ext}^{t+\Delta t} - C \dddot{u}_t^* - K \dddot{u}_t^*. \hspace{1cm} (2.17)$$

Rewriting (2.17) gives

$$M^* \dddot{u}_{t+\Delta t} = F_{ext}^{t+\Delta t} - \dddot{u}_t^* = F_{residual}^{t+\Delta t}. \hspace{1cm} (2.18)$$

The accelerations can be obtained by solving this set of equations. A widely used method is the iterative technique of Newton and Raphson. The displacement and velocity at time $t + \Delta t$ are then provided by Newmark’s equations ((2.15), (2.16)) and the increment is advanced in time. For each time step this set of equations must be solved, which makes it time consuming. The great advantage is that the method is unconditionally stable (for correctly chosen values of $\beta_T$ and $\gamma_T$).

**Explicit integration**

The central difference method is often called explicit integration. This method uses the general dynamical equilibrium equation of the current time $t$ to predict a solution at time $t + \Delta t$ [33]. The equilibrium determines the acceleration at the beginning of the increment
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(see 2.19) and it is assumed to be constant over the time step. To fulfil this assumption the step has to be chosen small. The acceleration at time $t + \Delta t$ becomes

$$\ddot{u}_{t+\Delta t} = M^{-1} F^\text{res}_t. \quad (2.19)$$

To proceed a step in time, the displacements and velocities are estimated by means of a central difference scheme. Examples from [30] are

$$\dot{u}_{t+\Delta t} = \dot{u}_t + \Delta t \ddot{u}_t \quad (2.20)$$

$$u_{t+\Delta t} = u_t + \Delta t \dot{u}_t + \Delta t \ddot{u}_{t+\Delta t} \quad (2.21)$$

To stabilize explicit codes, the time step must be chosen smaller than the smallest natural period in the mesh. In other words, the time step must be smaller than the time it takes for a stress wave to travel through the smallest element in the mesh. The time steps in explicit codes are therefore often 100 times smaller than those in implicit codes but unconditional stability is not ensured. However, since the values at the next time step are computed directly, costly formulation can be omitted. Explicit methods have greater advantage over implicit methods if the time step of the implicit solution has to be small for some reason and as the model size is large.

2.4.3 Commercial finite element codes

Different finite element codes are commercially available. In the next lines a brief summary is given of which codes are used in the field of computational ballistics.

Z. Rosenberg et al. [36] used the Eulerian processor of the PISCES 2-D ELK code (nowadays integrated in MSC.Software) to simulate impacts on brittle materials. A Lagrangian grid is attached to the outer surface of the target through a flow boundary to ensure the semi-infinite nature of the target.

In [9, 37, 38, 39] penetration and perforation of rods into steel targets has been examined for two-dimensional cylindrically symmetric and three-dimensional problems. All these authors used the Eulerian wave code CTH (from Sandia National Laboratory) for numerical simulations. CTH uses a 'van Leer'-algorithm for second-order accurate advection that has been generalized to account for a non-uniform and finite grid, and multiple materials [9]. All numerical simulations showed good correlation with experimental results.

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The non-linear, explicit, three-dimensional finite element code LS-Dyna-3D is used in [7, 40, 41] to simulate ballistic impacts on different types of armor. A cooperation between MSC.Software and LS-Dyna is established to increase their market share.

Autodyn, a branch of Ansys, is specialized in the development of software for the ballistic society. In association with the Department of Defense, Thalys, TNO-PML in the Netherlands for example and other major companies abroad they distinguish themselves from the
other software companies since it is their core business. All formulations are implemented in Autodyn and can directly be coupled with each other.

The Finite Element code MSC.Dytran is a non-linear dynamical formulation which will be used in this work. The Lagrange and Euler formulations and a coupling of these techniques are available in the code. The program is commonly applied for crash-simulation but knowledge on ballistics is available from past labor as well. Due to the large deformations and the fact that bullets are shattered, the Eulerian formulation is chosen.
Chapter 3

Material modeling

3.1 Introduction

The standard available material models in MSC.Dytran, do not suffice for simulating ballistic events. The literature review learned that ballistic impacts are complex and depend on a large number of variables. Therefore suitable material models for ballistic simulations are presented in this chapter to obtain agreement with ballistic experiments.

In section 3.2, the Mie-Gruneisen equation of state (pressure description) is treated and is a function of the new density and specific internal energy. The advantage for highly dynamical events is that the reference state of the pressure is described by the 1D-shockwave Rankine-Hugoniot relations. Section 3.3 describes the available yield criteria and the yield models obtained from literature. In section 3.4 the void growth model proposed by McClintock [42] and Rice and Tracey [43] and an extended version of Johnson and Cook [44] to account for strain rate, stress triaxiality and temperature is discussed for the failure of material.

These material models are implemented in MSC.Dytran, validated in chapter 4 and finally used for ballistic simulations.

3.2 Hydrostatic stress

An equation of state describes the hydrostatic stress or pressure as a function of the density and temperature. During a bullet impact, sufficiently high pressures \((p > 10 \text{ GPa})\) can arise in the loaded materials. Experimental data at these kind of pressures can be achieved using a shock transition experiment [45]. This leaves a so-called shock Hugoniot curve, an isothermal compressibility (pressure-volume) plot at 0K. The Mie-Gruneisen equation of state is related to this shock Hugoniot curve via the Rankine-Hugoniot relations and is therefore able to model the shock and its residual temperature more properly.

First, the Gruneisen parameter in physically measurable properties is derived. Then the 1D-shock wave theory of Rankine and Hugoniot is discussed and finally the Mie-Gruneisen equation of state is defined.
3.2.1 Gruneisen parameter

The Gruneisen parameter $\Gamma$ and the Mie-Gruneisen equation of state originate from statistical mechanics which treats energies of individual atoms [45]. The final expressions show similarities with terms in thermodynamics. The Gruneisen parameter can be considered as the measure of the change in pressure through an increase of specific internal energy at constant volume [46]. The basic definition of $\Gamma$ [47] is

$$\Gamma = \frac{1}{\rho} \left( \frac{\delta p}{\delta e} \right)_p. \quad (3.1)$$

In order to evaluate this parameter in physically measurable properties, $\left( \frac{\delta p}{\delta e} \right)_V$ is rewritten to

$$\left( \frac{\delta p}{\delta e} \right)_V = \left( \frac{\delta p}{\delta T} \right)_V \left( \frac{\delta e}{\delta T} \right)_V \quad (3.2)$$

where $\left( \frac{\delta e}{\delta T} \right)_V = C_v$ is the specific heat per unit mass. The term $\left( \frac{\delta p}{\delta T} \right)_V$ can be derived using Maxwell’s relation [48]:

$$\left( \frac{\delta p}{\delta T} \right)_V = \left( \frac{\delta p}{\delta V} \right)_T \left( \frac{\delta V}{\delta T} \right)_p \quad (3.3)$$

where $\left( \frac{\delta p}{\delta V} \right)_T = K_T$ is the isothermal bulk modulus and $\left( \frac{\delta V}{\delta T} \right)_p = \beta$ is the coefficient of volumetric expansion. For an isotropic solid the coefficient of volumetric expansion can be related to the coefficient of linear thermal expansion as $\beta = 3\alpha$. The substitution of the physical measurable parameters ($C_v, \alpha, K_T$) in equation (3.2) gives

$$\left( \frac{\delta p}{\delta e} \right)_V = \frac{3\alpha K_T}{C_V}. \quad (3.4)$$

Substitution of (3.4) in (3.1) gives the Gruneisen parameter in physically measurable parameters, defined as

$$\Gamma = \frac{3\alpha K_T}{\rho C_V} \quad (3.5)$$
3.2. HYDROSTATIC STRESS

3.2.2 Rankine-Hugoniot conditions and relation

Rankine and Hugoniot described the behavior of shock waves and their theory is generally accepted. The idea is to consider a one-dimensional, steady flow of a fluid described by the Euler equations where the shock wave conserves mass, momentum and energy.

The relation can be derived from the case of two plates which are impacted against each other [2]. This causes a shock front traveling through a compressible material in time $\delta t$ with a shock velocity $U_s$ from line A to line C, see figure 3.1. The shock wave creates a pressure $p_1$ (behind the shock front) which is suddenly applied to one face of the plate. This face was initially at pressure $p_0$ (in front of the shock front). This pressure compresses the material to a new density $\rho_1$ (compression from line A to line B) and at the same time the material is accelerated to a particle velocity $U_p$.

![Figure 3.1: Schematic view of a shock front (line C) propagating through a compressible material [2].](image)

The consumption of mass, momentum, and energy in front and behind the shockwave results in three equations which are often called the Rankine-Hugoniot conditions:

$$\rho_0 U_s = \rho_1 (U_s - U_p),$$  \hspace{1cm} (3.6)

$$p_1 - p_0 = \rho_0 U_s U_p,$$  \hspace{1cm} (3.7)

$$p_1 U_p = \frac{1}{2} \rho_0 U_s U_p^2 + \rho_0 U_s (e_1 - e_0).$$  \hspace{1cm} (3.8)

The derivation of these equations can be found in appendix B. Here $e_0$ and $e_1$ are the specific internal energy in front and behind the shock front, respectively. Eliminating $U_s$ and $U_p$ from (3.8) gives the Rankine-Hugoniot relation, defined as

$$(e_1 - e_0) = \frac{1}{2} (p_1 - p_0) (\frac{1}{\rho_0} + \frac{1}{\rho_1})$$  \hspace{1cm} (3.9)

where $\frac{1}{\rho_0} = \nu_0$ and $\frac{1}{\rho_1} = \nu_1$ are the specific volumes. These four equations are often called the "jump conditions" which must be satisfied on both sides of the shock front. A relation for the shock velocity $U_s$ can be found by substitution of the particle velocity $U_p$ from equation...
(3.6) in (3.7) with the assumption that the pressure, density and internal energy are known at the initial state:

$$U_s^2 = \frac{1}{\rho_0} \frac{p_1 - p_0}{\nu_0 - \nu_1}. \quad (3.10)$$

The changes to the pressures and densities can also be calculated by measurement of only two parameters, the shock velocity $U_s$ and the particle velocity $U_p$. The shock Hugoniot curve provides such information. The experimental data often fits solids with a linear function of the following form,

$$U_s = A_\Gamma + B_\Gamma U_p \quad (3.11)$$

where $A_\Gamma$ represents the wave velocity in an extended medium, often chosen as the material speed of sound. The constant $B_\Gamma$ is related to the Gruneisen parameter, $B_\Gamma = \frac{1 + \Gamma}{2} [45]$. With all these equation, the well-known Hugoniot pressure and energy can be derived. This Hugoniot pressure $p_h$ (difference between the pressure in front and behind the shockwave) is a function of the density only. Equation (3.7) can be rewritten by assuming the relative compression as $\eta = 1 - \frac{\rho_0}{\rho} = 1 - J$, elimination of the velocities using $U_p = \eta U_s$ (derived from (3.6)), and $U_s = A_\Gamma \frac{1}{1 - B_\Gamma \eta}$ (derived from (3.11)). The Hugoniot pressure $p_h = p_1 - p_0$ is written as

$$p_h = \frac{\rho_0 A_\Gamma^2 \eta}{(1 - B_\Gamma \eta)^2}. \quad (3.12)$$

The Hugoniot energy $e_h = e_1 - e_0$ (difference between the energy in front and behind shockwave) is easily derived from equation (3.9) and is defined as

$$e_h = \frac{1}{2} p_h (\nu_0 - \nu_1) = \frac{p_h \eta}{2 \rho_0}. \quad (3.13)$$

### 3.2.3 Mie-Gruneisen equation of state

The Mie-Gruneisen equation of state also originates from statistical mechanics and can be expressed using the Gruneisen parameter (3.1):

$$\delta p = \Gamma \rho \delta e. \quad (3.14)$$

The difference in pressure and specific internal energy relates the current state with a reference state at 0 K. Equation (3.14) is then rewritten to

$$(p - p_{ref}) = \Gamma \rho (e - e_{ref}). \quad (3.15)$$
3.3. PLASTICITY

This reference state can easily be related to another. The most common known form of Mie-Gruneisen is related to the Hugoniot curve [45, 49, 50]. Substitution of the Hugoniot pressure (3.12) and energy (3.13) as the reference in (3.15) gives the full description of the Mie-Gruneisen equation of state:

\[ p = \frac{\rho_0 A_\Gamma^2 \eta}{(1 - B_\Gamma \eta)^2} (1 - \frac{\Gamma_0 \eta}{2}) + \Gamma_0 \rho_0 e. \tag{3.16} \]

3.3 Plasticity

The duration of ballistic impacts is less than 30 µs and associated velocities are above 600 m/s. This implies that a large amount of energy will be dissipated in the target and the projectile material. The deformation are very large and will therefore be dominated by plasticity.

A yield model is used to describe the stress in the material when the elastic limit is exceeded. This transition from elastic to elasto-plastic material behavior is given by a yield criterion. Here the Von Mises yield criterion is used which is defined as

\[ F = \bar{\sigma}^2 - \sigma_y^2 \tag{3.17} \]

where \( \sigma_y \) is the yield stress and the Von Mises equivalent stress \( \bar{\sigma} \) is given by:

\[ \bar{\sigma} = \sqrt{\frac{3}{2}} \sigma^d : \sigma^d = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \tag{3.18} \]

where \( \sigma_1, \sigma_2 \) and \( \sigma_3 \) are the principal stresses. The restrictions proposed by Kuhn-Tucker indicate whether the stress is inside or on the yield surface.

\[ (F < 0) \lor (F = 0 \land \dot{F} < 0) \rightarrow \text{elastic} \tag{3.19} \]

\[ (F = 0 \land \dot{F} = 0) \rightarrow \text{elasto} - \text{plastic} \tag{3.20} \]
The elastic contribution to the stress is defined as

\[ \sigma^e = 2G\varepsilon^e. \] (3.21)

The yield criterion in combination with a flow rule and yield model contribute for the plastic part. The materials will be subjected to high strain rate deformation and the process is essentially adiabatic. An adiabatic temperature rise is produced within the material and has a significant effect on the constitutive behavior. Typical temperatures range between 200 – 700°C and could exceed the melting temperature. Therefore rate and thermal effects need to be included into the yield models.

**Johnson-Cook**

Johnson and Cook [44] proposed a semi-experimentally based constitutive model for metals, which describes large strains, high strains rate, and high temperatures. For each phenomenon (strain hardening, strain rate hardening and thermal softening) an independent term is created. By multiplying these terms a flow stress as a function of the effective plastic strain (\(\tilde{\varepsilon}_p\)), effective plastic strain rate (\(\dot{\tilde{\varepsilon}}_p\)) and temperature (T) is obtained. The constitutive model is relatively easy to calibrate since it allows isolation of the various effects. Due to this property, the model is frequently used in the ballistic society. The yield stress is given by

\[
\sigma_y = [A + B\tilde{\varepsilon}_p^n] \left[ 1 + C \ln \left( \frac{\dot{\tilde{\varepsilon}}_p}{\dot{\varepsilon}_0} \right) \right] \left[ 1 - \left( \frac{T - T_r}{T_m - T_r} \right)^m \right]
\] (3.22)

where \(A\) is the initial yield stress, \(B\) the strain hardening coefficient and \(n\) the strain hardening exponent. \(\dot{\tilde{\varepsilon}}_p\) is the plastic strain rate, \(\dot{\varepsilon}_0\) the reference strain rate, and \(C\) is the strain rate coefficient. This equation shows that the yield model is valid from room temperature (\(T_r\)) to melting temperature (\(T_m\)). The thermal softening exponent is given by \(m\).

In [7] an onset to determine these fit parameters is given. This constitutive model is implemented in most common finite element software codes. In [51, 52] this material model is used to describe steel and tungsten, in [53, 54] for the gilding metal of the bullet and in [5] for the armored steel.

**Zerilli-Armstrong**

The Zerilli-Armstrong constitutive model [55] is based on dislocation mechanics. Two models are developed, one for face-centered cubic (fcc) materials and one for body-centered cubic (bcc) materials. The significant difference between the two crystal structure is found in the experimental dependency on strain of the thermal activation analysis parameter. For more details, see [55]. These constitutive material models are defined as

\[
\text{bcc : } \sigma_y = C_0 + C_1 \exp(-C_3 T + C_4 T \ln \left( \frac{\dot{\tilde{\varepsilon}}_p}{\dot{\varepsilon}_0} \right) + C_5\tilde{\varepsilon}^n_p),
\] (3.23)
3.4. FAILURE MODELS

\[ \text{fcc} : \sigma_y = C_0 + C_2 \sqrt{\dot{\varepsilon}_p} \exp(-C_3 T + C_4 T \ln \left( \frac{\dot{\varepsilon}_p}{\dot{\varepsilon}_0} \right)), \]  

(3.24)

where \( C_0 \) is the initial yield stress, \( C_1, C_2, C_3, C_4 \) are fit parameters, \( C_5 \) is the strain-hardening coefficient and \( n \) is the strain-hardening exponent. In [56] this material model is used for copper in the software of the CTH computer code developed at Sandia National Laboratory. In LS-Dyna this model can be applied for metals [57].

Materials which are sensitive for changes in temperature and strain rate are better described by the Zerilli-Armstrong model since these components are in the exponent. Due to the coupled dependency of temperature and strain rate, determination of the fit parameters is more difficult in comparison to the Johnson and Cook model.

**Steinberg-Guinan**

A constitutive model for metals applicable at high-strain rates was also proposed by Steinberg, Cochran and Guinan [58]. The yield stress increases with increasing strain rate \( (\dot{\varepsilon}) \) but according to the authors of [58] there must exist a limit. Experiments proved that at high pressure (> 5GPa), the rate dependency becomes insignificant. Since the yield stress increases with increasing pressure and decrease with increasing temperature this yield model is chosen as a function of effective strain, pressure and temperature. The Steinberg-Guinan model for the flow stress is written as:

\[ \sigma_y = (A + B (\dot{\varepsilon}_p + \varepsilon_0))^n \left( 1 + H_1 \frac{p}{J^{1/3}} - H_2 (T - 300) \right) \]  

(3.25)

where \( J = \frac{V}{V_0} \) is the volume ratio, \( A \) the initial yield stress and \( B, n \) are work-hardening parameters and \( \varepsilon_0 \) is the initial equivalent plastic strain, normally equal to zero. \( H_1 \) and \( H_2 \) are also material parameters.

In [56] this material model is also used for copper in the software of the CTH computer code developed at Sandia National Laboratory. In [53, 54] it is used for lead.

### 3.4 Failure models

Failure in ballistic events is complicated by the occurrence of high strains, high strain rates, high pressures and high temperatures. Therefore a failure description which takes into account these effects is desired. In [59], [60] and [61] failure models are proposed based on the concept of Continuum Damage Mechanics (CDM). Therefore a short introduction on CDM is given.

Damage of materials can be interpreted as the degradation of the material strength caused by loading, thermal, or chemical effects [62]. Microcracks and microvoids, which grow, coalesce and initiate cracks, are modeled with a continuous variable \( D \). In the majority of the models, this dimensionless damage variable is defined as a ratio between the damaged surface or volume and the total surface or volume. From the definition it follows that \( D \) is bounded between 0 (undamaged) and 1 (fully broken).
Previous experimental work [63] showed that failure occurs for $D < 1$ through a process of instability. Since damage starts with microcracks and microvoids, the void growth model of McClintock [42] will be introduced in subsection 3.4.1. The Johnson-Cook failure model, which will be used in the simulation, is an extended version on the results of McClintock and Hancock and Mackenzie and is treated in section 3.4.2.

3.4.1 Void growth

Corrosion, history loading or thermal treatments are possibilities which could cause void nucleation. Upon further loading, microscopic cavities are formed, called voids. These voids will grow and coalesce to a macroscopically observable crack. The condition of the material before the crack is initiated is often described with damage mechanics as introduced previously. The damage, which causes material degradation, is generally expressed in a state variable. This parameter can easily be implemented in a numerical method. Here a void growth model is used to describe damage.

The first studies to the contribution of void growth in ductile fracture have been done by McClintock [42], Rice and Tracey [43]. Both their models assume incompressibility grounded by the experiment of Bridgman [64] on many metals. McClintock proposed a cylindrical void with an elliptical cross section, under plane strain conditions. They assumed an element in which a circular or elliptical void is present, see figure 3.2. Fracture is assumed to initiate when voids coalesce at the boundaries of the element.

![Figure 3.2: Elliptical void inside an element [42].](image)

The element has lengths $l_a$, $l_b$ and $a$, $b$ are the corresponding semi-axes lengths of the holes. McClintock introduced two hole-growth factors in direction $a$ and $b$, $F_a = (a/l_a)/(a_0/l^0_a)$ and $F_b = (b/l_b)/(b_0/l^0_b)$. In the model, fracture occurs when there is complete loss of cross-section area, i.e. when $a = \frac{1}{2}l_a$ and/or $b = \frac{1}{2}l_b$. The failure criteria can be derived from these equations,

$$F_a = F_a^f = \frac{1}{2} (l^0_a/a_0), \quad (3.26)$$

$$F_b = F_b^f = \frac{1}{2} (l^0_b/b_0). \quad (3.27)$$

McClintock proposed a measure for damage which is additive and accumulates to unity at fracture.
3.4. FAILURE MODELS

The damage parameter is defined as

\[ \delta D_{a,b} = \delta \ln(F_{a,b}) / \ln(F_{a,b}^f). \]  

(3.28)

The motion of the void in the element is calculated in an infinite plane and the motion of each boundary is assumed zero. McClintock used the equilibrium equation, \( \nabla \cdot \sigma \), to derive a fracture strain assuming incompressibility and plane strain. A full derivation of McClintock’s fracture criterion is given in appendix C. The result \[42\] is formulated as

\[ \tilde{\varepsilon}_f = \frac{2}{\sqrt{3}} \ln \left( \frac{b_0}{2b_0} \right) \exp \left( -\frac{\sqrt{3}}{2} \frac{\sigma_a + \sigma_b}{\bar{\sigma}} \right). \]  

(3.29)

The examination of photomicrographs executed by Hancock and Mackenzie demonstrated that void coalescence could occur in any direction. Therefore it is appropriate to define an averaged growth factor for all directions. Hancock and Mackenzie showed that the average growth factor, \( (F_a F_b)^{\frac{1}{2}} \), for all the notches and for both specimens of their experiments were similar. If this value is considered for failure, it seems natural to take the mean stress ratio \( \frac{\sigma_m}{\bar{\sigma}} \) as the parameter to define the stress-state rather than \( \frac{\sigma_a + \sigma_b}{\bar{\sigma}} \) [65].

Rice and Tracey also developed a void growth model for spherical voids which gave good agreement with McClintock’s model. Here it was also found that the rate of growth is strongly dependent on the level of hydrostatic tension. In other words, fracture by coalescence of voids would be accelerated by a high level of triaxiality (\( \frac{\sigma_a + \sigma_b}{\bar{\sigma}} \)).

Hancock and Mackenzie generalized these two equation with two material parameters:

\[ \tilde{\varepsilon}_f = D_1 + D_2 \exp \left( D_3 \frac{\sigma_h}{\bar{\sigma}} \right) \]  

(3.30)

A constant \( D_1 \) was added to represent the appreciable plastic flow which occurs before voids nucleate. Equation (3.30) is then modified to

\[ \tilde{\varepsilon}_f = D_1 + D_2 \exp \left( D_3 \frac{\sigma_h}{\bar{\sigma}} \right) \]  

(3.31)

where \( D_1 \) is often taken as the void nucleation strain.
3.4.2 Johnson-Cook failure model

Johnson and Cook [59] proposed a failure criterion based on the fracture strain model of Hancock and Mackenzie as described in section 3.4.1. The fracture strain model is extended with a strain rate and a temperature term to allow their effect on the fracture strain. These terms are already present in the Johnson-Cook yield model. The failure criterion assumes a damage parameter $D$ given by

$$D = \frac{\varepsilon_p}{\varepsilon'_t}$$

(3.32)

with $\varepsilon_p = \int_{t=0}^{t} \dot{\varepsilon}_p d\tau$ and $\dot{\varepsilon}_p = \sqrt{\frac{2}{3}} D_p : D_p$. The fracture strain is defined as

$$\varepsilon^f = (D_1 + D_2 \exp(D_3 \sigma^*)) (1 + D_4 \ln(\dot{\varepsilon}_p))(1 + D_5 T')$$

(3.33)

where $\sigma^*$ is the stress triaxiality given by $\sigma^* = \frac{\sigma^h}{\bar{\sigma}}$. Here is $\sigma^h$ is the hydrostatic stress, $\bar{\sigma} = \sqrt{\frac{2}{3}} \sigma^d : \sigma^d$ is the effective stress and $D_1,...,D_5$ are empirically material parameters. An element is supposed to fail if the damage parameter equals unity. The damage parameter could also be defined to develop continuously with the stress to describe the weakening of the material.
Chapter 4

Computational aspects

4.1 Introduction

From the literature review, it was concluded that an Eulerian approach is most suitable for this application. Therefore a short description to advance the Euler domain (as defined in MSC.Dytran) with one time step is given. In the Eulerian approach, material is not attached to elements but can move from one Euler element to the other. The volume present inside an element is bounded by element faces and the polyhedron packets (which represent the boundary of a bottle for example in which a liquid is present). An algorithm in MSC.Dytran is then able to obtain the volume of the material. Conservation of mass, momentum and energy can be written as:

\[ \frac{d}{dt} \int_V \rho dV + \int_S \rho (\vec{v} \cdot \vec{n}) dS = 0, \]  
\[ \frac{d}{dt} \int_V \rho \vec{v} dV + \int_S \rho \vec{v} (\vec{v} \cdot \vec{n}) dS = - \int_S \rho \vec{v} dS - \rho g e_3 V, \]  
\[ \frac{d}{dt} \int_V \rho e dV + \int_S \rho e (\vec{v} \cdot \vec{n}) dS = - \int_S p (\vec{v} \cdot \vec{n}) dS. \]

The transported mass, momentum and energy through the Euler element faces are given by the surface integrals left of the equal sign in all three conservation equations. Equation (4.1) clearly states that the change of mass (volume integral) equals the summation of the donated and accepted mass through the element boundaries (surface integral). The integrals on the right of the equal sign in (4.2) and (4.3) take into account the impulses on the material. For example the compression work delivered by the body forces in equation (4.2).

During an increment, the new volume and mass of each Euler element are computed after transport and the density in an element can be obtained. The pressure is calculated using the equation of state as described in section 3.2. Since the internal energy is not known at the current time, an approximation for the internal energy and the speed of sound is required. These approximations are defined in subsection 4.2.1. The Taylor test is used
in subsection 4.2.2 to validate the implementation of Steinberg-Guinan yield model, Mie-Gruneisen equation of state and Johnson-Cook damage.

The strain rates are obtained by use of the divergence theorem. This theorem states that the total amount of volume which is transported and received in an element is equal to the net flow through the boundaries of the element. In integral form [30] this is written as

\[
\int_V \frac{dv_i}{dx_j} dV = \int_V (\vec{\nabla} \vec{v}) dV = \int_S v_i \vec{e}_j \cdot \vec{n} dS = \sum_{faces} v_i^{faces} S_j^{faces}.
\] (4.4)

The velocity gradient tensor \( \mathbf{L} \) follows from this equation. From this point elasticity and plasticity, as described in section 3, can be evaluated. The last step is to compute a new stable time step for the next increment based on the CFL stability criterion.

Shock waves have a rather discontinuous behavior and propagate through a material as a narrow transition zone in which density, pressure and many more quantities change rapidly. The shock waves manifest themselves mathematically (from the energy and momentum equilibrium) as a sudden change in density and pressure. Early investigators (1950) observed that the shock pressure was too high and elements or the time step collapsed. Furthermore oscillations were trailing the shock front. It was identified that the conversion of kinetic energy into heat could not be represented by equilibrium. Therefore a term is added in the balance laws to represent the dissipation into heat. This term is called artificial viscosity, which is capable to decrease the pressure and provide damping to discontinuities outside the narrow transition zone. These numerical shock waves are treated in section 4.3. Acoustic impedance is also introduced which describes the reflection or transmission of a part of a shock wave at material interfaces. These phenomena are validated using a 1D-shockwave experiment.

A new feature implemented in the code is damage, as introduced in section 3.4.2. Damage can be accounted for in a discrete or continuous manner. Subsection 4.4.1 describes both manners and the reason why the continuous implementation is chosen. From literature it is well-known that the continuous implementation could give localization and mesh dependencies. Therefore some attention will be payed to these phenomena in subsection 4.4.2. The Taylor bar test is here also used to demonstrate possible mesh-dependency.

The Mie-Gruneisen model contains an elastic part and should somehow correspond to a constant bulk modulus. The comparison of both is given in section 4.6.

### 4.2 Material model adaptions and validations

The Steinberg-Guinan yield model and the Mie-Gruneisen equation of state have been implemented in the FE code MSC.Dytran. The implementation of the latter model will be discussed in more detail.
4.2. MATERIAL MODEL ADAPTIONS AND VALIDATIONS

4.2.1 Mie-Gruneisen implementation

The Mie-Gruneisen equation of state (equation (3.16)) is a function of the density and the internal energy. Each increment the new internal energy needs to be calculated. However it is dependent on the new pressure which is then still unknown. Therefore an approximation for the internal energy is derived. In the theory manual of MSC.Dytran [30], section "the standard Euler solver", an approximation for the new internal energy is given. This equation is derived from the energy balance (4.3) with the main assumption that the pressure is constant across the element and is chosen from a trapezium rule. The new internal energy $e_i$ can be written as

$$ e_{i}^{n+1} - e_{i}^{n} = -\frac{1}{2} \left[ p\left(\rho_{i}^{n+1}, e_{i}^{n+1}\right) + p\left(\rho_{i}^{n}, e_{i}^{n}\right) \right] \frac{\varepsilon^v}{\rho^{n+1}}. \quad (4.5) $$

Substitution of the equation of state (equation (3.16)), $p = \xi(\rho) + \Gamma_0 \rho_0 e_i$, in (4.5) gives:

$$ e_{i}^{n+1} - e_{i}^{n} = -\frac{1}{2} \left[ \xi(\rho^{n+1}) + \Gamma_0 \rho_0 e_i^{n+1} + p^n \right] \frac{\varepsilon^v}{\rho^{n+1}}. \quad (4.6) $$

In this function there is only one unknown parameter, $e_i^{n+1}$. Rewriting equation (4.6) gives

$$ e_{i}^{n+1} = e_{i}^{n} - \frac{1}{2} \left[ \xi(\rho^{n+1}) + p^n \right] \frac{\varepsilon^v}{\rho^{n+1}} \left[ 1 + \frac{1}{2} \Gamma_0 \rho_0 \varepsilon^v/\rho^{n+1} \right]. \quad (4.7) $$

The new pressure $p^{n+1}$ can now be calculated using the new internal energy $e_i^{n+1}$. The new velocity of sound is calculated using

$$ c^2 = \frac{dp}{d\rho} = \frac{dp}{d\eta} \frac{d\eta}{d\rho} = \frac{dp}{d\eta} \frac{\rho_0}{\rho^2}. \quad (4.8) $$

Furthermore the denominator of Mie-Gruneisen, equation (3.16), could give an infinite pressure if $\eta \to \frac{1}{B^2}$ i.e. if $\rho \to \frac{B}{B^2-1} \rho_0$. To prevent this, a maximum density of $(\frac{B}{B^2-1} - \frac{1}{2}) \rho_0$ is defined.
CHAPTER 4. COMPUTATIONAL ASPECTS

Figure 4.1: (a) Taylor test results for a steel AISI 4340 cylinder with an initial length of 8.1 mm and a diameter of 7.82 mm having an initial velocity of 343 m/s, experimental result in insert is taken from [44] and (b) the euler mesh, 5 degree wedge.

4.2.2 Validation tests

The Taylor (bar) test was introduced in the literature study (chapter 2) and is an important validation test in the science of ballistics. Experimental data for various materials are largely available in literature due to the simplicity of the test. The final shape and length are often compared with numerical simulations. Here the Taylor test is used to validate the implemented material models. An example for the Johnson-Cook yield model is given in figure 4.1(a), where numerical results are compared with experimental data for steel AISI 4340.

The computational specifications for this particular test are given in table 4.1. Axi-symmetry, which is not available in MSC.Dytran, is approach by a 3D model of a wedge shape of 5 degrees as depicted in figure 4.1(b) to decrease the calculation time. The simulation compares quite well with the experimental result (picture in insert). For robustness, three simulations (different initial lengths and velocities) are executed and agreed well with experimental data. The length ratio \( \frac{l_f}{l_0} \) and the theoretical equation of Taylor (equation (2.10)) are also compared and show agreement, see figure 4.2.

Another valuable validation is comparison with other commercial FE-codes (LS-Dyna and Abaqus/Explicit, both Lagrangian and explicit codes). In figure 4.3 the simulations are the same as specified for figure 4.1. In figure 4.4 and table 4.1, the equation of state is replaced by the Mie-Gruneisen model. Both figures show similarities between the different FE-codes. The errors (in the order of maximum 10%) can be accountable to the different formulations used by the FE-codes.
4.2. MATERIAL MODEL ADAPTIONS AND VALIDATIONS

Table 4.1: Computational specifications for the Taylor bar test.

<table>
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<th>element type</th>
<th>element size</th>
<th># element</th>
<th>material</th>
<th>references</th>
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<td>0.1 mm</td>
<td>5854</td>
<td>steel AISI 4340</td>
<td>section 5.3.1</td>
</tr>
<tr>
<td>initial velocity</td>
<td>[m/s]</td>
<td>shear model</td>
<td>equation of state</td>
<td>yield model</td>
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<td>shear modulus</td>
<td>polynomial (bulk modulus)</td>
<td>Johnson-Cook</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 4.2: Comparison between Taylor’s equation and the numerical results obtained.

Figure 4.3: Validation of the implemented Johnson-Cook yield model using 3 FE-codes, MSC.Dytran, Abaqus/Explicit and LS-Dyna. The effective plastic strain is shown.
Figure 4.4: Validation of the implemented Mie-Gruneisen equation of state model using 3 FE-codes, MSC.Dytran, Abaqus/Explicit and LS-Dyna. The pressure [MPa] is shown.

4.3 Shockwaves

A shock wave can be represented as a mechanical wave of finite amplitude and is initiated when the material undergoes a fast compression. A forced motion in a deformable medium creates these waves, often called a disturbance. This medium is comprised of material points, which are forced away from their equilibrium position as the disturbance propagates through the material. The propagation carries amounts of energy in the forms of kinetic and potential energy. This energy is transmitted from material point to material point. A mechanical wave can be characterized by the transport of energy through oscillatory motions of material point about an equilibrium position.

Figure 4.5: Motion of particles about their equilibrium.

The medium provides resistance against the motion and eventually it will subside until a state of static deformation will be reached due to frictional losses and spreading of waves.
4.3. SHOCKWAVES

4.3.1 Numerical shockwaves

A shockwave is not a real physical discontinuity but a very narrow transition zone in which density, velocity, temperature, and the like rapidly change. In most cases a finite element model cannot correctly solve this discontinuity (shockwave) since the transition zone is too small to be solved by the grid size. Moreover, the necessary boundary conditions are non-linear and the shock surfaces are in motion compared to the network of points [66].

Solving the discontinuity in an explicit central difference integration method will cause oscillation in amplitude which trail the shock front [30]. These oscillations behind the shock front could cause singularities and the critical time step could be reached. The critical time step can be influenced by a sudden change in density, often defined by the CFL-stability [30] condition:

\[
\Delta t = \frac{\Delta x_{\text{min}}}{c_s} \quad \text{or} \quad c_s = \sqrt{\frac{E}{\rho}} \quad \text{or} \quad c_s^2 = \frac{\delta p}{\delta \rho} \quad \text{(for equation of state)} \quad (4.9)
\]

These oscillations and instabilities come into existence during the conservation of mass, momentum and energy. Von Neumann and Richtmyer [66] recognized that a natural dissipation mechanism was absent in the Euler equation. A certain amount of energy in a natural shockwave is converted irreversibly into heat and such a term was absent. The idea of Von Neumann and Richtmyer [66] was to introduce viscous dissipation (to account for the heat), later on called artificial viscosity (q), in the balance laws and use this for the entire calculation. The dissipation was introduced for purely mathematical reasons. The artificial viscosity could damp the oscillations trailing the shock front and/or give a resisting pressure to an element to prevent collapsing or reaching the critical time step. The momentum and energy equations then change to

\[
\frac{d}{dt} \int_V \rho \vec{v} dV + \int_S \rho \vec{v}(\vec{v} \cdot \vec{n}) dS = - \int_S (p + q) \vec{n} dS - \rho g e_3 V, \quad (4.10)
\]

\[
\frac{d}{dt} \int_V \rho e dV + \int_S \rho e (\vec{v} \cdot \vec{n}) dS = - \int_S (p + q)(\vec{v} \cdot \vec{n}) dS. \quad (4.11)
\]

The artificial viscosity term q of Neumann and Richtmyer originate from the inelastic collision of two masses. After conservation of momentum, a decrease in kinetic energy of \(\mu(\Delta v)^2/2\) is found, where \(\mu\) is the reduced mass and \(\Delta v\) is the difference in the velocities of the two masses before the collision. The reduced mass is replaced by the density and for a one-dimensional description, q can be written as,

\[
q_Q = \rho (c_Q \Delta x)^2 \left(\frac{\dot{V}}{V}\right)^2, \quad (4.12)
\]

where \(\rho\) is the density, \(c_Q\) a constant and \((c_Q \Delta x)\) determines the amount of reduction. The gradient term in equation (4.12) assures that the dissipation and pressure decrease occurs in
the region of the shock layer and is the strain rate in one-dimension. $\Delta x$ is the characteristic length or element dimension.

The artificial viscosity above was found adequate to capture strong shocks although oscillations behind the shock front were still observed [67]. To overcome this problem Landshoff [68] introduced an additional term, which vanishes less rapidly, as

$$q_L = \rho (c_L \Delta x) c_s \left( \frac{\dot{V}}{V} \right).$$

where $c_L$ is a constant and $c_s$ the speed of sound. Since this viscosity provides spreading over too much elements (physically not representative), the factor $c_Q$ is usually set a magnitude higher than $c_L$. In MSC.Dytran, a combination of these two terms is used, $q = q_l + q_{nl}$. The requirement is that the balance laws give satisfying shock jumps and oscillations are negligible outside the thin shockwave layer.

$$x \to \infty: v = v_{inf}, p = p_{inf}, e = e_{inf}, q = 0,$$

$$x \to -\infty: v = v_{inf}, p = p_{inf}, e = e_{inf}, q = 0.$$ (4.14) (4.15)

The desired result is to spread the shock over the minimum number of grid spacings (to represent the narrow transition zone) while damping of the oscillations behind the shock front is solved by the numerical method itself. For the finite element equations and the mathematical stability of these functions see [66].

### 4.3.2 Acoustic impedance

In a multi-material configuration, shock waves will interact at the material interface in a way comparable to sound waves. Due to differences in material properties, a part of the shockwave will be reflected and a part transmitted by the other material. A key parameter called acoustic impedance describes reflection and transmission and is defined as

$$Z = \rho c_s$$

where $\rho$ is the material density and $c_s$ the material sound of speed. Material interfaces with the same acoustic impedance value transmit the energy of a shockwave without reflection. This is often called impedance matching. When there is a difference in acoustic impedance, mechanical waves at the interface can be reflected which is called impedance mismatch. The fraction of the reflected wave is given by

$$R_{impedance} = \left( \frac{(Z_1 - Z_2)}{(Z_1 + Z_2)} \right)^2$$

and the fraction that is transmitted as
4.3. SHOCKWAVES

\[ T_{\text{impedance}} = 1 - R = \frac{(4Z_1Z_2)}{(Z_1 + Z_2)^2}. \] (4.18)

4.3.3 Validation tests

To verify the triggering and propagation of shockwaves a simple test is executed. A row of two hundred elements, with an element size of 0.1x0.1 mm and having the material properties of bronze is considered. The element at the right-hand side has an initial velocity of 200 m/s during one increment to initiate a shockwave. The input for this simulation is given in table 4.2.

![Figure 4.6: Set up to verify the initiation and propagation of a shockwave through bronze.](image)

Table 4.2: Computational specifications for the 1D shockwave test.

<table>
<thead>
<tr>
<th>element type</th>
<th>element size</th>
<th># element</th>
<th>material</th>
<th>references</th>
</tr>
</thead>
<tbody>
<tr>
<td>initial velocity</td>
<td>shear model</td>
<td>equation of state</td>
<td>(c_L)</td>
<td>(c_Q)</td>
</tr>
<tr>
<td>(200, \text{m/s})</td>
<td>shear modulus</td>
<td>polynomial (bulk modulus)</td>
<td>0.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The result of this simulation is displayed in figure 4.7. The outcome is satisfying, a narrow transition zone in which density, pressure and specific internal energy change rapidly. To validate the amounts of these variables, a comparison with the RH-relations (equations (3.6), (3.7) and (3.8)) is made. The results of these relations are displayed with circles in the figure and are close to the numerical results.

Artificial viscosity

The shockwave validation test introduced above is slightly changed to be able to study the function of the artificial viscosity. All elements except the most left are given an initial velocity of 200 m/s during the first increment. A high shock pressure will arise and load this single element. Figure 4.8a represents the results of this simulation with artificial viscosity not active \((c_L = c_Q = 0)\).

The oscillations behind the shock front, which travels to the right, are clearly visible. Artificial viscosity could give a solution and the problem was executed with \(c_L = 0.5\) and \(c_Q = 1.5\) (the constants of the artificial viscosity). Figure 4.8b shows the results. The oscillations are
Figure 4.7: Comparison between 1D-shockwave simulation and Rankine-Hugoniot relations at \( t=2.3 \ \mu s \).

Figure 4.8: Effect of artificial viscosity, (a) no artificial viscosity, (b) artificial viscosity.
damped, the time step was more stable and the peak pressure is decreased (or in physical sense, heat is dissipated). The peak in figure 4.8b can be damped by increasing \( c_Q \).

To acquire more feeling for the effects of these constants, the test introduced at the beginning of subsection 4.3.3 is executed with different values. The results are given in figure 4.9 and the pressure is monitored in time at \( x = 10 \text{ mm} \). Figure 4.9b shows that the shock is reflected by the air at the end and travels back with the correct speed. The decrease in pressure by the second time the shock passes is also logical since heat is dissipated for example.

![Figure 4.9: Time history of element 151 (x = 10 mm) to study the sensitivity of artificial viscosity. (a) Sensitivity of linear viscosity, \( c_Q = 0 \), (b) verifying the travel time of the shock wave, (c) sensitivity of quadratic viscosity, \( c_L \).](image)

To create more damping for the oscillations, the linear constant \( c_L \) smears out the transition zone over more elements as seen in figure 4.9a (\( c_Q = 0 \)). The quadratic constant on the other hand retains the same width but the value of the shock pressure is reduced by increasing \( c_Q \), see figure 4.9c (which is a magnification of the left peak in figure 4.9b and \( c_L = 0 \)). In a physical sense this means that more heat is dissipated.

Investigation of the artificial viscosity (AV) also learned that internally in MSC.Dytran, linear AV applies only for positive strain rate and quadratic AV for negative strain rate. Linear AV is designed to damp oscillations behind the shock front and a positive strain rate is typically found here. The material in front of the shockwave is compressed and has a negative strain rate. In this region \( q_Q \) needs to dissipate heat. This restriction makes sure that \( q_L \) and \( q_Q \) are applied in the region where the viscosity should have effect.

**Acoustic impedance**

Subsection 4.3.1 described acoustic impedance as a property of shockwaves. Here the shockwave simulation is modeled using two materials (see table 4.3 and figure 4.10), steel AISI 4340 and zinc. Again an initial velocity is applied to the most right element during the first increment.
Table 4.3: Material properties of AISI 4340 and zinc used in the simulation.

<table>
<thead>
<tr>
<th></th>
<th>density [kg/m³]</th>
<th>Bulk modulus [GPa]</th>
<th>Acoustic impedance [1e⁷·kg/m²·s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>AISI 4340</td>
<td>172.5</td>
<td>7850</td>
<td>3.68</td>
</tr>
<tr>
<td>Zinc</td>
<td>69.7</td>
<td>7140</td>
<td>2.231</td>
</tr>
</tbody>
</table>

Figure 4.10: Set up to verify the acoustic impedance between steel AISI 4340 (right) and zinc (left).

Figure 4.11 shows a reflection of 8.56 % produced by MSC.Dytran which is calculated using the mean of the peak pressures. Equation (4.18) learns that theoretically 6.01% of the shockwave should be reflected. No explanation is found for this difference.

Figure 4.11: Verifying the acoustic impedance and the percentage reflected and transmitted. The interface between both materials is found at \( x = 15 \) mm and the shockwave propagates to the left.

Also the behavior of acoustic impedance on the interface air-metal is studied. Since air has a very low impedance value the wave should almost fully reflect, see table 4.4 for the chosen properties. A simulation in which the zinc is replaced by air showed full reflection of the shockwave. This reflection was already noticed in figure 4.9b where the shockwave reflected at the far end of the bar and passed the observation element twice.

Default the surrounding area in the Euler mesh is defined as void material. Void material is not considered during a simulation and reduces the calculation time significant. Therefore
4.4. DAMAGE IMPLEMENTATION

Table 4.4: Material properties of air used in the simulation.

<table>
<thead>
<tr>
<th></th>
<th>density [kg/m$^3$]</th>
<th>Bulk modulus [MPa]</th>
<th>Acoustic impedance [kg/m$^2$s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>1.29</td>
<td>0.152</td>
<td>4.425e$^2$</td>
</tr>
</tbody>
</table>

the effect of the reflected shockwave by air on the failure behavior of the materials will be investigated in section 4.5.

4.4 Damage implementation

In section 3.3, three appropriate yield models were introduced to account for material hardening upon increase of pressure and plastic strain rate and material (thermal) softening upon increase of temperature. Damage evolution could also cause softening (a reduction of strength is found in damaged materials). In the following, two methods to implement damage are explained.

4.4.1 Discrete versus operator split technique

Damage is used to describe the failure of material. Two ways of taking damage into account are considered, the discrete and the operator split technique. The discrete manner calculates at the end of each increment the damage parameter and checks if unity has been reached. If so, the material fails and the deviatoric stress is set to zero.

Damaged materials will have less resistance against applied stresses. To account for the weakening/softening of the material during loading, Boers et al. [69] proposed to multiply the yield stress with $(1 - D)$, called the operator split technique. The damage parameter is known from previous increment. The material hardening will decrease upon damage evolution and even soften upon large deformation. Finally the material reaches zero yield stress and the material has failed.

The ductile damage parameter proposed by Johnson-Cook will be implemented using the operator split technique since it gives more realistic material behavior. The material is weakened by decreasing the yield stress according to $(1 - D)\sigma_y$ in the yield criterion. In this way the ductile damage has a direct effect on the stress level since the yield criterion is given by

$$F = \bar{\sigma}^2 - ((1 - D)\sigma_y(\varepsilon_p))^2.$$  \hspace{1cm} (4.19)

Damage is based on void nucleation, growth and coalescence which is in reality local behavior. However the average behavior can be described isotropically, which grounds the use of a scalar damage variable [69]. Since in reality damage is not initiated directly when strains reach the plastic zone, a damage threshold value should be introduced.
A disadvantage of the operator split technique could be that the formulation gives localization and mesh dependency. In the next subsection, the cause of localization and the results of the Taylor test with different mesh sizes is discussed. An option to overcome this dependency is the discrete manner.

### 4.4.2 Convergence and localization

Continuum damage mechanics approaches for the prediction of failure in FE-codes gained a lot of interest in the last decades. A topic often discussed is the localization and mesh-dependency of continuous damage. The problem is often described by using a bar with an imperfection of one element, see [62], [70]. During a tensile test, the deformation tends to localize in the weakest cross-section. Mesh-refinement decreases the localization zone and so its volume in which energy should be dissipated. An infinite number of elements will lead to an infinitely small volume for the imperfection. This element is initiating fracture with a minimal amount of energy. This amount of energy is certainly not equal to the fracture initiation energy and is therefore physically unacceptable.

Since highly dynamical events are investigated, a tensile test is unappropriate. The Taylor test will be used and is modeled axi-symmetrically. The simulation specifications are the same as defined in section 4.2.2. This test is first executed without damage for four different element sizes (0.03, 0.05, 0.1 and 0.2 mm quads) to observe convergence to one solution. The results on the centreline in \( x \)-direction are evaluated at \( t = 1.05 \, \mu s \) and oscillations trailing the shock front were noticed in the equivalent stress in all four simulations. In figure 4.12 only the results of 0.1 mm are given.

![Figure 4.12: Notified oscillations behind the shock front in a Taylor bar test for a mesh-size of 0.1 mm. Results on the centerline of the cylinder at \( t = 1.05 \, \mu s \) are given.](image)

The oscillations are not dependent on the element size and setting the thermal component of the Johnson-Cook yield to zero proved that there isn’t thermal softening behind the shock front. A plausible explanation was found in [71], which states that the (velocity) oscillations
can be interpreted as heat energy (fluctuating molecular energy) instead of kinetic energy. Subsection 4.3.3 proposed to raise the constant $c_L$ in the artificial viscosity to overcome these oscillations and to be able to dissipate heat. The results are satisfying ($c_L = 2.0$) and can be seen in figure 4.13.

![Figure 4.13: Damped oscillation and convergence testing. Results on the centerline of the cylinder at $t = 1.05 \mu s$ are given.](image)

Figure 4.13 shows that convergence is found with an element size of 0.05 mm. The Taylor test with damage active is executed to investigate the mesh dependency of the operator split technique. The results are given in figure 4.14.

![Figure 4.14: (a) Damage parameter and (b) equivalent stress on the centerline in a Taylor bar test with operator split technique at $t = 1.0 \mu s$.](image)

Figure 4.14 (a) Damage parameter and (b) equivalent stress on the centerline in a Taylor bar test with operator split technique at $t = 1.0 \mu s$.

The weakening of the stress through the damage is clearly visible in figure 4.14(b) when it is compared to figure 4.13. Also the same convergence to mesh refinement is observed. No additional effect of damage to the mesh dependency is noticed. The damage contours of the
CHAPTER 4. COMPUTATIONAL ASPECTS

Simulation with damage active are given in figure 4.15 and no localization is noticed. Only the damage is spread wider in the coarse mesh in comparison to the fine.

![Figure 4.15: Analyzing the mesh dependency using element sizes of 0.05, 0.1 and 0.2 mm.](image)

The damage parameter, given in figure 4.14(a), shows a sharp and fast damage evolution from one to zero directly behind the shock front. The shock pressure results in a high local triaxiality and causes a low fracture strain, see equation (3.33) and therefore unity of the damage parameter can be reached quickly. Since the failure takes place in such a short period of time, the mesh dependency and localization have little effect on the results and are assumed to be negligible.

The discrete manner is comparable to the operator split technique since unity of the damage parameter is reached quickly. For this reason the discrete manner is left out of consideration.

4.5 Effects of air

In section 4.3.3 is explained that a shockwave is reflected by air and it could load the material for the second time. To investigate this effect of air on the failure of the material, a Taylor test is modeled with void and air to fill the Eulerian mesh (surroundings), see figure 4.16 (a) with void, (b) with air. Considering the previous section, the first shockwave already failed the material so the reflected shockwave is expected to have no additional effect.

Examination of figure 4.16 (c),(d) shows that the effective plastic strain contours are almost identical for both simulations. Only the calculation time for the Taylor test with air was twice as high.
Because the air is compressed enormously, the calculation becomes instable. To run the calculation, several numerical adaptations have been made. The time step is limited, a maximum velocity is prescribed and the artificial viscosity is increased. Due to all these adaptations the calculation time has increased. Since the effect of the air on the final result is small but the influence on the calculation time is large, the bullet impact simulations will be executed using void material to fill the surrounding space.

4.6 Benefits of Mie-Gruneisen

As described in section 3.2, an accurate description of the shock wave propagation is important in impact calculations. The benefit of the Mie-Gruneisen equation of state is that the shock Hugoniot curve is taken as a reference. During simulations this experimental fit will be followed and should describe the propagation more precise.

Another benefit is that the pressure description is dependent on the temperature via the specific internal energy. As explained in subsection 4.4.2, the energy which is not dissipated by the equilibrium equations will be interpreted as kinetic energy and causes (velocity) oscillation behind the shock front. Due to the consumption of the specific internal energy in the Mie-Gruneisen model, the amount of energy not dissipated is reduced. Less oscillation behind the shockwave are found and the amount of artificial viscosity is decreased.

The reduction of the oscillations is clearly visible in figure 4.17, comparing a constant bulk modulus (a) with the Mie-Gruneisen equation of state (b). Larger and more oscillations are present behind the shockwave using a constant bulk modulus.
Figure 4.17: Pressure and temperature result of the Taylor test using a constant bulk modulus (a,c) and the Mie-Gruneisen equation of state (b,d).

A consequence of these oscillations is a local temperature rise (see figures 4.17c, d), which can affect the yield and failure behavior of the materials. Due to these facts, the Mie-Gruneisen equation of state is preferred above a constant bulk modulus.
Chapter 5

Geometry, material analyses and characterization

The manufacturer of the projectiles used in this study has not published the shape and material properties of the bullet. Therefore, its shape is determined by means of an image analysis in section 5.1 after being parted using a electrical discharge machine. An environmental scanning electronic microscope (ESEM) was used to obtain the material composition. The hardness of the material was obtained with Vickers hardness test. This information as well as information from literature allows to define the bullet materials in section 5.2. To execute numerical simulations, material models to characterize the material behavior are needed. In chapter 3, various appropriate models were proposed and in section 5.3 for each material it is investigated which model is the most appropriate. The material properties and model parameters are also given.

5.1 Geometry analysis

The accuracy of a comparison between a ballistic impact and a FEM simulation depends for a part on the geometry of the bullet. The size and shape determine for example the mass and volume. To acquire a correct mesh description, digital stereo microscopic photos are made after the microscope was calibrated. Lines to indicate the diameter and length were created with the available software. Since protection is required up to the BR6 class, the bullets of this class are analyzed and used in the simulations. The photos for the ss109 (5.56 × 45 mm) and M80 (7.62 × 51 mm) bullet can be seen in figure 5.1(a) and (b) respectively.

The geometry and size of the bullet were obtained using the reference length present in the image. A scale factor can be calculated to converse the pixel lengths/coordinates to millimeters. In an enlarged image of the bullet, the coordinates of the pixels at the interfaces were extracted. Multiplying these coordinates with the scale factor gave a geometrical description of the bullet. With a least square routine, circles and lines were fitted through these point and a mesh is then created in Altair Hypermesh, see figure 5.2.
5.2 Material analysis

To acquire more information about the materials present in the bullet, electron microscopy is executed. The second column in table 5.1 gives the measured composition of each material. A Vickers hardness test was conducted to investigate effects of the thermo-mechanical history (heat treatments). In literature agreement between these compositions and hardness and materials were obtained. A summary is given in table 5.1 and more details about the analysis can be found in appendix A.

The most likely reason for the choice for lead as bullet filler is its high density. More mass provides more kinetic energy. Good formability and low price could be reasons for the choice of the bullet jacket materials.

5.3 Material characterization

The composition and state materials associated with the different parts of both bullets were obtained in the previous section. In this section, the physical, mechanical and thermal properties of the bullet (projectile) and target material are given. The next step is to define
5.3. MATERIAL CHARACTERIZATION

Table 5.1: Results of material analysis, see appendix A

<table>
<thead>
<tr>
<th>Name</th>
<th>Composition [wt%]</th>
<th>Vickers hardness</th>
<th>Material</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bullet jacket 5.56 mm</td>
<td>89.5% Cu ; 10.5% Zn</td>
<td>104.11</td>
<td>Bronze</td>
<td>[72]</td>
</tr>
<tr>
<td>Bullet jacket 7.61 mm</td>
<td>99.8% Fe</td>
<td>137.5</td>
<td>Steel 1006</td>
<td>[73]</td>
</tr>
<tr>
<td>Bullet penetrator</td>
<td>97.25% Fe ; 0.91% Mn 0.87% Si ; 0.72% Mo 0.25% Cr</td>
<td>394.4</td>
<td>Steel 4340</td>
<td>[73]</td>
</tr>
<tr>
<td>Bullet filler</td>
<td>99.9% Pb</td>
<td>&lt;10</td>
<td>Pure Lead</td>
<td></td>
</tr>
</tbody>
</table>

constitutive models of chapter 3 for the different materials. These models should fit the experimental data to obtain reliable results from the FE-code. These experimental data are often available in the literature and frequently along with the model parameters.

At large strains, significant heat is generated from the plastic work, and at high strain rates there is no time for the heat to be conducted away from the test specimen. Therefore the impact process becomes adiabatic and it is crucial to use adiabatic sets of experimental data.

Finally, appropriate failure parameters for each material are determined.

5.3.1 Projectile material

The correct definition of the material properties are crucial to obtain reliable results out of the simulation. The physical properties of the bullet materials are given in table 5.2. The densities are taken from [74], the velocity of sound for lead from [49] and for the other materials from [75].

Table 5.2: Physical properties of projectile materials.

<table>
<thead>
<tr>
<th></th>
<th>Steel AISI 1006</th>
<th>Steel AISI 4340</th>
<th>Commercial bronze</th>
<th>Lead</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density [kg/m³]</td>
<td>7850</td>
<td>7850</td>
<td>8800</td>
<td>11340</td>
</tr>
<tr>
<td>Speed of sound [m/s]</td>
<td>3075</td>
<td>3850</td>
<td>3720</td>
<td>2028</td>
</tr>
</tbody>
</table>

Table 5.3: Mechanical properties of projectile materials.

<table>
<thead>
<tr>
<th></th>
<th>Steel AISI 1006</th>
<th>Steel AISI 4340</th>
<th>Commercial bronze</th>
<th>Lead</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulk modulus [GPa]</td>
<td>172.5</td>
<td>172.5</td>
<td>99.3</td>
<td>37.5</td>
</tr>
<tr>
<td>Shear modulus [GPa]</td>
<td>79.6</td>
<td>79.6</td>
<td>44</td>
<td>4.7</td>
</tr>
<tr>
<td>Elongation at break [%]</td>
<td>20</td>
<td>7</td>
<td>45</td>
<td>30</td>
</tr>
</tbody>
</table>
Some mechanical properties are given in table 5.3 and are taken from various literature [74, 75, 76]. The elongation at break is determined from stress-strain curves in [76].

The thermal properties are obtained from [74, 75] and the melting temperatures from [44, 58]. These properties are summarized in table 5.4.

<table>
<thead>
<tr>
<th></th>
<th>Steel AISI 1006</th>
<th>Steel AISI 4340</th>
<th>Commercial bronze</th>
<th>Lead</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat capacity [J/kgK]</td>
<td>486</td>
<td>475</td>
<td>376</td>
<td>129</td>
</tr>
<tr>
<td>Coefficient of thermal expansion [10^-6/K]</td>
<td>11.7</td>
<td>12.3</td>
<td>18.4</td>
<td>29.3</td>
</tr>
<tr>
<td>Melting point [°C]</td>
<td>1538</td>
<td>1520</td>
<td>1030</td>
<td>327.5</td>
</tr>
</tbody>
</table>

With the properties above, the Gruneisen parameter, $\Gamma = \frac{3\alpha K}{\rho C_V}$, the parameter $A_\Gamma$ and the parameter $B_\Gamma = \frac{1+\Gamma}{2}$ can be calculated and are summed in table 5.5.

<table>
<thead>
<tr>
<th></th>
<th>Steel AISI 1006</th>
<th>Steel AISI 4340</th>
<th>Commercial bronze</th>
<th>Lead</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_\Gamma$ [m/s]</td>
<td>3075</td>
<td>3850</td>
<td>3720</td>
<td>2028</td>
</tr>
<tr>
<td>$B_\Gamma$ [-]</td>
<td>1.294</td>
<td>1.354</td>
<td>1.328</td>
<td>1.627</td>
</tr>
<tr>
<td>$\Gamma$ [-]</td>
<td>1.587</td>
<td>1.707</td>
<td>1.657</td>
<td>2.253</td>
</tr>
</tbody>
</table>

Three optional yield models are available from the literature study, the Johnson-Cook, Zerilli-Armstrong and Steinberg-Guinan yield model (see chapter 3). The Johnson-Cook model is frequently used in literature since it gives a good fit of experimental results. In their paper [44] good agreement between the fit parameters and the experiments for steel AISI 1006 and steel AISI 4340 was found. Therefore, this model will be used for these materials, the parameters used are given in table 5.6.

There is a lack of high strain rate experimental data for bronze in literature. Since experiments at the strain rates of interest are too expensive to conduct in this study, the Johnson-Cook parameters for yield of cartridge brass, which are known from literature, are taken. The physical, mechanical and thermal properties of these two materials are in close agreement and their yield and failure behavior should be comparable. The parameters are given in table 5.6.

Lead is a low strength metal with a melting temperature which could be exceeded during a ballistic impact. A pressure driven yield description is more appropriate than a strain rate driven one since the material could be in a liquid state. Therefore the pressure-dependent yield model of Steinberg-Guinan is used to describe plastic deformation in lead. The model parameters are given in table 5.7 and correspond well with experimental data [58].
5.3. MATERIAL CHARACTERIZATION

Table 5.6: Johnson-Cook parameters.

<table>
<thead>
<tr>
<th></th>
<th>Steel AISI 1006</th>
<th>Steel AISI 4340</th>
<th>Cartridge brass</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$ [MPa]</td>
<td>350</td>
<td>792</td>
<td>112</td>
</tr>
<tr>
<td>$B$ [MPa]</td>
<td>275</td>
<td>510</td>
<td>505</td>
</tr>
<tr>
<td>$n$ [-]</td>
<td>0.36</td>
<td>0.26</td>
<td>0.42</td>
</tr>
<tr>
<td>$C$ [-]</td>
<td>0.022</td>
<td>0.014</td>
<td>0.009</td>
</tr>
<tr>
<td>$m$ [-]</td>
<td>1.00</td>
<td>1.03</td>
<td>1.68</td>
</tr>
</tbody>
</table>

Table 5.7: Steinberg-Guinan parameters.

<table>
<thead>
<tr>
<th></th>
<th>Lead</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{y0}$ [GPa]</td>
<td>0.008</td>
</tr>
<tr>
<td>$\beta$ [-]</td>
<td>110</td>
</tr>
<tr>
<td>$B = \beta \cdot \sigma_{y0}$ [GPa]</td>
<td>0.88</td>
</tr>
<tr>
<td>$n$ [-]</td>
<td>0.52</td>
</tr>
<tr>
<td>$H_1 \left[10^{-12}\text{Pa}^{-1}\right]$</td>
<td>116</td>
</tr>
<tr>
<td>$H_2 \left[10^{-3}\text{K}^{-1}\right]$</td>
<td>1.16</td>
</tr>
</tbody>
</table>

Failure of steel AISI 4340 and AISI 1006 is modeled with the Johnson-Cook failure model (subsection 3.4.2). The model takes the dependencies of stress triaxiality, strain rate, and temperature on the fracture strain into account. The damage parameters $D_1...D_5$ are adopted from [59, 77]. The parameters are given in table 5.8.

Table 5.8: Johnson-Cook damage parameters.

<table>
<thead>
<tr>
<th></th>
<th>Steel AISI 1006</th>
<th>Steel AISI 4340</th>
<th>OFHC Copper</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_1$ [-]</td>
<td>−0.8</td>
<td>0.05</td>
<td>0.54</td>
</tr>
<tr>
<td>$D_2$ [-]</td>
<td>2.1</td>
<td>3.44</td>
<td>4.89</td>
</tr>
<tr>
<td>$D_3$ [-]</td>
<td>0.5</td>
<td>2.12</td>
<td>3.03</td>
</tr>
<tr>
<td>$D_4$ [-]</td>
<td>0.0002</td>
<td>0.002</td>
<td>0.014</td>
</tr>
<tr>
<td>$D_5$ [-]</td>
<td>0.61</td>
<td>0.61</td>
<td>1.12</td>
</tr>
</tbody>
</table>

The failure properties of bronze are replaced by those OFHC Copper since the parameters are known and are most close to those of bronze. Failure of the lead was simulated using a maximum equivalent plastic strain failure model with a value of $\varepsilon_{p,max} = 0.30$. 

5.3.2 Target material

Benteler, the owner of PDE Automotive, has developed an armored material (called BSEC180) which acquires the armoring properties after the armoring plates are hot-formed and cooled down. A patent is pending for the ballistic material and the forming process. Integration of various parts into one single part becomes possible. Figure 5.3 shows the current non-formable ballistic steel and BSEC180. The advantages are that complex shapes are possible, weak spot (welds) are prevented and it requires less handling.

![Figure 5.3: Current non-formable ballistic steel (left) and BSEC180 (right)](image)

The physical properties of BSEC180 are equal to normal steel. From static testing the mechanical properties of the material are acquired and the melt temperature is known. No data is available of the mechanical response at various strain rates and temperatures due to the limited budget.

The Johnson-Cook yield and failure models are used since they allow isolation of the different dependencies and easily fit experimental results. With the static data, the parameters in the first brackets of the Johnson-Cook yield model (equation 3.22) are determined. To acquire the remaining parameters for the yield and damage model, they are adjusted in order to obtain agreement between numerical simulations and the ballistic experiments.

A well-known phenomenon recognized in the experiments, called the body work effect (German: Karrosserie effect), is used to obtain the parameters. This phenomenon means that 5.0 mm of BSEC180 prevents bullet ss109 from penetrating. An additional sheet of body work of 1.0 mm thickness in front of 5.0 mm BSEC180 causes complete penetration of both plates. The K-effect is clarified in figure 5.4. Robustness of the parameters is tested by simulating two other bullets on BSEC180. Exactly the same model parameters are used and experimental results are available as well.

A first guess for the strain rate hardening and thermal softening parameters is obtained from steel AISI4340. For failure, the Johnson-Cook damage model is chosen. None of these parameters are known and also a first guess is taken from AISI4340. The final model parameters are confidential and are given in appendix ??.
Figure 5.4: K-effect, no penetration (left) and penetration (right) with extra body work sheet on the same plate of BSEC180 (5.0 mm)
Chapter 6

Computational results

6.1 Introduction

In chapter 4, the Taylor and the 1D-shockwave test were simulated in order to validate the numerical approach. In this chapter, ballistic impacts are simulated using the bullets and target materials and associated material models as described in the previous chapter.

The axi-symmetric character\(^1\) is again represented by a 3D model of a five degree wedge shape. The wedge is given in figure 6.1. 6-node pentahedral elements are used on the centerline and 8-node hexahedral elements for the remaining parts of the mesh. The contour of the mesh, consisting of 64060 elements with a size of 0.1 mm in the \(x\)- and \(y\)-direction and one element over the \(z\)-direction, is given in figure 6.2a. The axi-symmetric \((x,y)\)-planes of the Euler mesh are by default impenetrable (boundary conditions) and all elements are defined as structural

\(^1\)axi-symmetric elements are not available in MSC.Dytran
multi-material with shear strength. The \((x,z)\)- and \((y,z)\)-planes are open so material can flow axially and radially out of the mesh. The two bulges in the mesh are used to fix the plates to the real world and allow springback.

Triangular and quadrilateral shell elements are used to describe the initial position and material interfaces of the bullet and the target plate, see figure 6.2b. These shells also define a multifaceted surface which accounts for the initial contacts and coupling interfaces. Furthermore with a transient initial condition, the regions (elements) inside these surfaces are filled with material and given an initial velocity and material properties.

For the bullet impact simulations, the following models are used:

- Mie-Gruneisen equation of state;
- Constant elastic shear modulus;
- Steinberg-Guinan (lead) and Johnson-Cook (remainder materials) yield models;
- Johnson-Cook failure model.

![Figure 6.2: (a) Front view of the contour of the Euler mesh. The Euler mesh elements are too small to be displayed. (b) The shell elements are visible and define the initial position of the materials.](image)

During first testing, a significant increase of pressure, equivalent stress and plastic strain during free flight of the bullet was noticed. To discover the origin of the growth of these variables, the free flight simulation was executed with different material models. Figure 6.3 shows that the pressure is present in all simulations and therefore it was excluded that the material models caused the increase.
6.1. INTRODUCTION

Figure 6.3: Free flight simulation of the ss109 bullet with different material models. Bulk = constant elastic bulk modulus, Shear = constant shear bulk modulus, MG = Mie-Gruneisen equation of state, YLDJC = Johnson-Cook yield model, YLDSG = Steinberg-Guinan yield model and FAILJC = Johnson-Cook damage model.

Another source for the high pressures could be the Euler mesh, which was created in a $(x,y,z)$-coordinate system. Rotation of the $xy$-plane with 5 degrees to create a 3D mesh could be inaccurate. Small errors in the normals of the planes can be present and could cause small strains. Therefore the Euler mesh was modeled in a $(r,\phi,z)$-coordinate system but the results did not change. The initialization was checked by simulating the free flight of the bullet with an initial velocity of zero during 5 increment, no pressures were visible.

Most probably the pressure arises during incorrect transport of the different materials. The strains could grow during transport through the mesh or rounding can be inaccurate. This increase in pressure was also noticed in LS-Dyna and solved by calculating with double precision. This was not the case for MSC.Dytran. However, it appeared from the simulation that the pressure decreased in time during free flight, see figure 6.4. The incorrect transport probably works the other way around.

Figure 6.4: Notified decrease of pressure in time during free flight of the ss109 bullet.
CHAPTER 6. COMPUTATIONAL RESULTS

The damage caused by the pressure increase at \( t = 4 \mu s \), see figure 6.5, is negligible. Therefore, to overcome the problem, the bullet is placed 3.8 mm in front of the target and will arrive almost stress free (the pressure and equivalent stress are in the order of 2 MPa).

![Figure 6.5: Damage present at \( t=4.0 \mu s \) in the ss109 bullet during free flight.](image)

To execute the calculation correctly the velocity is bounded (maximum is 5000 m/s) and the artificial viscosity constants are \( c_L = 0.1 \) and \( c_Q = 1.0 \).

The material model parameter values for the target plate are obtained by matching simulations with the K-effect experiments. The results are given in section 6.2. To prove robustness of these values, a number of ballistic simulations with different type of bullets and plate thicknesses are executed and compared with experimental results in section 6.3. Finally, an explanation for the K-effect is found in section 6.4.

## 6.2 Characterization of BSEC180

In section 5.3.2, the lack of dynamical material data for the target material BSEC180 was indicated. Only a few ballistic experiments are available, the BR5 class ss109 bullet (5.56×45 mm) against 5 mm of BSEC180 and the same experiment with a 1 mm sheet of body work metal in front are two of them. These experiments showed the K-effect, i.e. the bullet penetrates with an extra sheet of metal in front of it and is stopped without it. The experiments are executed by Benteler Automotive Company according to the standard EN1063.

This K-effect is used to obtain the missing values for the model parameters (tuning to the experimental results) of the target material BSEC180. The final results and the correlation with the ballistic experiments are given in subsection 6.2.1 and subsection 6.2.2. Most probably, the K-effect is caused by the shape change of this type of bullet while penetrating the 1 mm sheet. Hereafter, the bullet acquires better penetration capabilities with this new shape. Results in section 6.4 confirm this conclusion.

### 6.2.1 ss109 bullet (5.56×45 mm) on 5.0 mm BSEC180

In this numerical simulation, the ss109 bullet is fired against 5.0 mm of BSEC180 with an initial velocity of 950 m/s. By comparing with experimental data, the unknown parameter values for BSEC180 are obtained. The parameter values for the equation of state, elastic shear behavior and \( A, B \) and \( n \) of the Johnson-Cook yield model of the ballistic steel are known...
from static and thermal testing. A first guess for the remaining values of the Johnson-Cook model (yield: \( C \) and \( m \) and failure: \( D_1 - D_5 \)) are taken from steel AISI4340. The sensitivity of both models is investigated and it is demonstrated that the stress triaxiality parameter \( D_3 \) of the Johnson-Cook failure model has the greatest influence.

Figure 6.6: Experimental result of the ss109 bullet on 5.0 mm BSEC180. The crater has a radius of \( r = 3.08 \) mm.

Figure 6.7: (a) ss109 bullet on 5.0 mm BSEC180 at \( t = 0 \) \( \mu \)s and (b) the density, (c) equivalent stress, and (d) material fraction of BSEC180 at \( t = 60 \) \( \mu \)s after the start of the impact. The crater indicated in (d) has a radius of \( r = 3.16 \) mm.

After executing various calculations with different parameter values, agreement with the experimental result in figure 6.6 is found. The simulation shows realistic results, the crater dimensions (figure 6.7(d)) are similar to the experimental result in figure 6.6. The peak stress (figure 6.7(c)) is found behind the bullet impact area and furthermore the bullet is completely shattered as in the experiment, see figure 6.7(b).
One has to keep in mind, that in this numerical simulation the model parameters are obtained by means of tuning to the experimental result and that it is logical that the numerical results in figure 6.7 correlate with the ballistic experiment in figure 6.6. The same results were obtained using air instead of void to fill the surrounding area which validates the choice for void material.

### 6.2.2 ss109 bullet on 5.0 mm BSEC180 and 1.0 mm body work sheet

The K-effect is used to validate the chosen material parameters. The model parameters for BSEC180, which were obtained in subsection 6.2.1, are used in this simulation. The simulation is executed with the same plate thickness and bullet velocity. The body work is modeled as steel AISI 1006, is 1.0 mm thick, and the distance between the two plates is 16.0 mm.

Agreement is found between the experimental results in figure 6.8 and numerical result in figure 6.9. The bullet falls apart but a part penetrates the armored plate. The crater radius in figure 6.8 ($r = 2.69$ mm) agrees with figure 6.9(d) ($r = 2.56$ mm). Also a fractured fragment of the armored plate is visible in figure 6.9(d) and has a velocity over 400 m/s which makes this fragment lethal. This result confirms that the BSEC180 is characterized correctly.

![Figure 6.8: Experimental result of the ss109 bullet on 5.0 mm BSEC180 with steel body work sheet in front. The crater has a radius of $r = 2.69$ mm.](image)

### 6.3 Robustness

From section 6.2, it can be concluded that with the material model parameter values used, both simulations correlate with the experimental results. To further prove the validity of the material properties of BSEC180, other simulations with different types of bullets and plate thicknesses are executed. The value of the material parameters remains unchanged.
6.3. ROBUSTNESS

Figure 6.9: (a) ss109 bullet on 5.0 mm BSEC180 at \( t = 0 \) \( \mu \)s with extra body work sheet of 1.0 mm and (b) the density, (c) equivalent stress, and (d) material fraction of BSEC180 and body work sheet at \( t = 70 \) \( \mu \)s after the start of the impact. The crater indicated in (d) has a radius of \( r = 2.56 \) mm.

6.3.1 ss109 bullet on 5.1 mm BSEC180 with strain grid

To check the order of magnitude of the plastic strain, a grid consisting of circles with a radius of 1 mm was etched on the back side of the target plate. The ss109 bullet impacted the plate with a velocity of 938 m/s. An estimate of the plastic strain was obtained by measuring the area \( S \) of the deformed circles in figure 6.10 and dividing them by the initial area. The experimental equivalent plastic strains, \( \bar{\varepsilon}_p = \ln \left( \frac{S}{S_0} \right) \), indicated in figure 6.10(a) are given in table 6.1.

<table>
<thead>
<tr>
<th>( y ) [mm]</th>
<th>Experiment</th>
<th>Numerical</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon_{p,1} )</td>
<td>0.0</td>
<td>0.48</td>
</tr>
<tr>
<td>( \varepsilon_{p,2} )</td>
<td>3.1</td>
<td>0.32</td>
</tr>
<tr>
<td>( \varepsilon_{p,3} )</td>
<td>6.3</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Figure 6.10(b) shows a zoom of the cross-section of the target plate at \( t = 60 \) \( \mu \)s where the bullet impacted. The squares indicate the center of the measured circles on the strain grid and the equivalent plastic strain of the simulation is also given in table 6.1. A deviation of approximately 25 percent was found. This deviation can be explained by the coarse grid and difficulty of digital imaging of the impacted plate.

6.3.2 M80 bullet (7.62×51 mm) on 4.9 mm BSEC180

The M80 bullet is launched with a velocity of 833 m/s corresponding to the classification of the BR6 class. The bullet weight and hence its kinetic energy is almost twice as high as the
Figure 6.10: (a) Experimental result of the ss109 bullet on 5.1 mm BSEC180 with a strain grid on the backside. The length of the line is 6.3 mm. (b) Zoom on the impact area from the simulation and the markers indicate the place of measurement in the experimental result.

ss109 (\(e_{ss109} = \frac{1}{2} \cdot 3.95 \cdot 10^{-3} \cdot 950^2 = 1780\) J and \(e_{M80} = \frac{1}{2} \cdot 9.33 \cdot 10^{-3} \cdot 830^2 = 3214\) J). Figure 6.11 clearly shows that the ballistic steel failed to stop the bullet.

Due to the higher kinetic energy of this bullet, the maximum equivalent stress in figure 6.12(c) (\(\sigma_{\text{max}}=2.109\times10^4\) MPa) is seven times higher than in figure 6.7(c) (\(\sigma_{\text{max}}=2.815\times10^3\) MPa). The crater dimension are also comparable (\(r_{\text{experimental}} = 1.42\) mm and \(r_{\text{simulation}} = 1.203\) mm).

Figure 6.11: Experimental result of the M80 bullet on 4.9 mm BSEC180 and notice that the crater is not axi-symmetric. The hole is elliptical (a=3.3 mm and b=2.46 mm) and a mean radius \(r\) is derived from \(\frac{1}{2} \pi ab = \pi r^2\). The radius becomes \(r = 1.42\) mm.
The material fraction in figure 6.12(d) shows good agreement with the fractured crater in figure 6.11, a fragment is broken and accelerated. The melted lead around the impact area in figure 6.11 is also visible in figure 6.12(b).

Figure 6.12: (a) M80 bullet on 4.9 mm BSEC180 at $t = 0 \mu s$ and (b) the density, (c) equivalent stress, and (d) material fraction of BSEC180 at $t = 65 \mu s$ after the start of the impact. The hole indicated in (d) has a radius of $r = 1.203$ mm.

6.3.3 M193 bullet (5.56×45 mm full lead) on 5.0, 7.0, and 8.0 mm BSEC180

This type of bullet is not ranked in the European standard EN 1063 but is less expensive than the bullets in the BR5 and BR6 class and has better penetration capabilities. From experimental results\(^2\), it is known that 7.5 to 8.0 mm of armored steel is needed to stop this bullet.

This test is used for further robustness testing. The M193 is fired against 5.0, 7.0 and 8.0 mm BSEC180 with an initial velocity of 950 m/s. The results given in figure 6.13 show good correlation with the experimental data. This bullet has the same shape as the ss109 bullet but is filled with lead (the mass of the bullet is increased). This proves that the kinetic energy is an important parameter in bullet impacts.

6.4 Origin of the K-effect

Since the velocity is decreased after penetrating the body work steel, one should expect that the bullet is stopped by BSEC180. This is not the case and the armored plate, which is mounted behind the body work, is penetrated. The hypothesis is that the deformed shape of the ss109 bullet acquires better penetration capabilities with this new shape. Therefore the ss109 bullet is modeled as a flat nose and impacted on 5.0mm BSEC180 with the residual velocity (920 m/s). Figure 6.14 shows the result of this simulation and confirms the hypothesis.

\(^2\)no photographic results available for these experiments
CHAPTER 6. COMPUTATIONAL RESULTS

Figure 6.13: M193 on different thicknesses of BSEC180, 5.0 and 7.0 mm failed to stop the bullet and 8.0 mm prevents penetration. The bullet jacket is bronze and lead is used as filler.

Due to the flat nose of the deformed bullet, the first contact area is increased. Figure 6.15 shows that the flat nosed ss109 is able to exert a larger pressure (more kinetic energy is transferred during first contact) on the armored steel in comparison to the ss109. The increased pressure (3.4 times larger) initiates a larger shockwave which loads BSEC180 even though the kinetic energy of the bullet is less.

This pressure and initiated shockwave give an increased stress triaxiality $\frac{\sigma_h}{\sigma}$ which reduces the failure strain in the Johnson-Cook damage model, see equation (3.33). Through this the material fails more easily, see equation (3.32). The larger exerted pressure or larger initiated shockwave produced by the deformed or flat nosed bullet is therefore a reasonable explanation for the K-effect.
6.4. ORIGIN OF THE K-EFFECT

Figure 6.14: Numerical results of the ss109 bullet \((v_{\text{ini}} = 950 \text{ m/s})\) and ss109 flat nosed bullet \((v_{\text{ini}} = 920 \text{ m/s})\) against 5.0 mm BSEC180. (a), (b) give the density at \(t = 0 \mu\text{s}\), (c), (d) give the density at \(t = 60 \mu\text{s}\).

Figure 6.15: Numerical results of the ss109 bullet and ss109 flat nosed bullet against 5.0 mm BSEC180. (a), (b) show the density at \(t = 2.5 \mu\text{s}\), (c), (d) give the pressure at \(t = 2.5 \mu\text{s}\).
Chapter 7

Conclusion and recommendations

7.1 Conclusion

To gain knowledge and experience with computational ballistics, which was lacking at the start of the project, a thorough literature study was executed. From the review in the literature, a bottom approach to simulate ballistic events is defined which was summarized in the organigram of figure 1.1. It followed that these phenomena are highly dependent on strain rate, temperature and pressure.

Candidate material models, which describes these phenomena, were investigated thoroughly in literature and four models appeared to be appropriate, the Johnson-Cook and Steinberg-Guinan yield models, the Mie-Gruneisen equation of state, and the Johnson-Cook failure model. The last three models are implemented in the chosen FE-code MSC.Dytran and the first was already available.

The Taylor bar test was used to validate the implementations and good agreement between numerical, analytical, and experimental results were obtained. Further validation by comparison the Taylor bar test in MSC.Dytran with other FE-codes, namely LS-Dyna and Abaqus/Explicit, gave confidence in the correctness of the implementations.

The initiation and propagation of the shockwave through the material appeared to be of great influence on the failure behavior of the material. Therefore the pressure, energy, and density values followed from a 1D-shockwave test were compared to the Rankine-Hugoniot relations and gave a deviation of approximately 8 percent. The narrow transition zone (shockwave) in which density, pressure and energy change rapidly was correctly produced by MSC.Dytran.

Since the reference pressure and energy of the Mie-Gruneisen equation of state are the Rankine-Hugoniot relations, this model was able to describe the propagation of the shockwave/pressure more accurately than a constant bulk modulus. The little oscillations left were damped using artificial viscosity, reducing the pressure in the equilibrium equation to represent heat dissipation.

Due to the shockwave, high local stress triaxialities were present. Since the Johnson-Cook failure model is sensitive for changes in stress triaxialities, variations caused rapid evolution
of the damage parameter from zero to one. Since failure takes place in such a short period of time, mesh dependency and localization were assumed to be negligible.

The presence of air (to fill the Euler mesh) in the simulation increased the calculation time with a factor two but did not have a significant effect on the failure of the material. Therefore, so called void (empty) material was used to fill the surrounding space.

After satisfying validation results had been obtained, bullet impact simulations have been performed. During first testing, a significant increase of pressure during free flight of the bullet was noticed. It appeared that the transport of multi-material through the Euler mesh was not accurate enough. To overcome the problem, the bullet is placed 3.8 mm in front of the target and will arrive almost stress free.

First, the unknown material model parameters of BSEC180 (armored steel) are characterized. Through variation of parameter values the numerical result was correlated with the experimental result of the ss109 bullet (5.56×45 mm) against 5.0 mm BSEC180 (no penetration case). The K-effect simulation also showed agreement (penetration case) with the same parameter values and proved that the material parameters were well defined.

To further validate the model, simulations with two types of bullets - M80 (7.62×51 mm) and M193 (5.56×45 mm with lead filler) - impacting different thicknesses of BSEC180 were executed. Good correlation between the calculation and experiments was found.

The K-effect is explained by the fact that the bullet deforms after penetrating the body work and this new shape (flat nose) is able to initiate a larger pressure/shockwave which fails the armoring material. This phenomenon proved that FE modeling is a valuable tool in understanding and explaining impact (and penetration) of projectiles on armored plates.

7.2 Recommendations

The improved FE-code can now be used to optimize the armoring of vehicles and to investigate other phenomena. Some recommendation are:

- Obtain an optimum distance (to prevent penetration of the ss109 bullet) between the ballistic steel and body work. Already executed simulations proved that a distance of 3.5 mm is able to stop the bullet and 7.5 mm gives penetration;
- Investigate the effect of a filler between both plates;
- Simulations of oblique impacts (under an angle);
- The simulation of the M193 bullet proved that the influence of the kinetic energy on penetration is large. Perhaps a small layer of ceramics can absorb a large amount of kinetic energy;
- Analytical models can be derived and investigated using the numerical code.
Appendix A

Material analyses

The manufacturer of the projectiles used in this study has not published the shape and material properties of the bullet. An analysis with an environmental scanning electronic microscope (ESEM) was used to obtain the material composition. The hardness of the materials was tested with Vickers hardness test.

A.1 Environmental Scanning Electronic Microscope

In this part of the appendix, each material is analysed using scanning electron microscopy. A figure is given to indicate the area which was analysed. Furthermore, the weight percentage of the materials are shown.

Jacket of the ss109 bullet

![Figure A.1: Analysis position for the ss109 bullet jacket.](image-url)
APPENDIX A. MATERIAL ANALYSES

<table>
<thead>
<tr>
<th>Elem</th>
<th>Wt %</th>
<th>At %</th>
<th>K-Ratio</th>
<th>Z</th>
<th>A</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>CuK</td>
<td>89.48</td>
<td>89.75</td>
<td>0.8944</td>
<td>0.9998</td>
<td>0.9998</td>
<td>1.0000</td>
</tr>
<tr>
<td>ZnK</td>
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<td>10.25</td>
<td>0.1055</td>
<td>1.0019</td>
<td>1.0016</td>
<td>1.0000</td>
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<td>100.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

kV: 20.00  Tilt: -0.30  Take-off: 34.79  Tc: 100.0
Det Type: UTW, Sapphire  Res: 132.60  Lsec: 44

The weight percentages for Cu and Zn agree with the properties of commercial bronze [75].

Penetrator of the ss109 bullet

The analysis shows a steel which probably was subjected to a heat treatment. From the results of the Vickers hardness test, explained in the next paragraph, one can conclude that
the steel is austenized until approximately 800°C and then rapidly cooled down. In [73] good agreement is found with steel AISI 4337 and AISI 4340. Only the percentage of nickel is not present in the bullet penetrator.

**Filler of the ss109 bullet**

![Analysis position for the ss109 bullet filler.](image)

<table>
<thead>
<tr>
<th>Elem</th>
<th>Wt %</th>
<th>At %</th>
<th>K-Ratio</th>
<th>Z</th>
<th>A</th>
<th>F</th>
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<tbody>
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<td>AlK</td>
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<td>0.0007</td>
<td>1.2583</td>
<td>0.3935</td>
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<tr>
<td>CuK</td>
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<td>0.9357</td>
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<td>PbL</td>
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<td>64.17</td>
<td>0.8416</td>
<td>0.9502</td>
<td>0.9996</td>
<td>1.0000</td>
</tr>
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<td><strong>Total</strong></td>
<td><strong>100.00</strong></td>
<td><strong>100.00</strong></td>
<td></td>
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</tbody>
</table>

kV: 20.00   Tilt: -0.30   Take-off: 34.93   Tc: 100.0
Det Type:UTW, Sapphire   Res: 132.60   Lsec: 66

The results produced by the analysis are not unambiguous. It looks like that there are particles of the polishing process present in the soft lead (silicium from the polishing paper, Fe and Cu from the bullet jacket and penetrator). Therefore another analysis is performed on the bullet filler and the results of this analysis are given in figure A.4.

In figure A.4, the different colors mark the materials present in the lead. The colors clearly show that silicium, copper, and aluminium are present in the lead. An alloy composed of these materials was not found in literature. Since lead is very soft, a slice of the bullet filler is cut off and analyzed. From these results it can be concluded that the bullet filler consists predominantly of lead (> 95 wt%).
APPENDIX A. MATERIAL ANALYSES

Filler of the M80 bullet

The results of this material are the same as for the bullet filler of the ss109.

Jacket of the M80 bullet

```
<table>
<thead>
<tr>
<th>Elem</th>
<th>Wt %</th>
<th>At %</th>
<th>K-Ratio</th>
<th>Z</th>
<th>A</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>C K</td>
<td>7.15</td>
<td>40.59</td>
<td>0.0274</td>
<td>1.2481</td>
<td>0.3074</td>
<td>1.0000</td>
</tr>
<tr>
<td>SnL</td>
<td>80.68</td>
<td>46.35</td>
<td>0.7797</td>
<td>0.9547</td>
<td>1.0123</td>
<td>1.0000</td>
</tr>
<tr>
<td>CuK</td>
<td>12.17</td>
<td>13.06</td>
<td>0.1127</td>
<td>1.0596</td>
<td>0.8738</td>
<td>1.0000</td>
</tr>
<tr>
<td>Total</td>
<td>100.00</td>
<td>100.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

kV: 30.00  Tilt: 0.00  Take-off: 35.00  Tc: 100.0
Det Type:UTW, Sapphire  Res: 132.60  Lsec: 118

```
<table>
<thead>
<tr>
<th>Elem</th>
<th>Wt %</th>
<th>At %</th>
<th>K-Ratio</th>
<th>Z</th>
<th>A</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>C K</td>
<td>5.76</td>
<td>22.12</td>
<td>0.0115</td>
<td>1.1498</td>
<td>0.1744</td>
<td>1.0007</td>
</tr>
<tr>
<td>FeK</td>
<td>94.24</td>
<td>77.88</td>
<td>0.9351</td>
<td>0.9888</td>
<td>1.0035</td>
<td>1.0000</td>
</tr>
<tr>
<td>Total</td>
<td>100.00</td>
<td>100.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

kV: 30.00  Tilt: 0.00  Take-off: 35.00  Tc: 100.0
Det Type:UTW, Sapphire  Res: 132.60  Lsec: 19

Analysis on the inside, outside, and cross-section of the M80 bullet showed that it mainly consists of normal steel (AISI 1006/1008) [73]. The presence of copper and tin on the outside of the bullet is probably due to thin layers to protect the steel against corrosion and to reduce friction in the barrel.
A.2 Vickers hardness test

The Vickers hardness test gives a measure of the hardness of a material, calculated from an impression produced by a pyramid shaped diamond indenter under load. The indenter employed in the Vickers test is a square-based pyramid whose opposite sides meet an angle of $136^\circ$. The diamond is pressed into the surface of the material at different loads. The Vickers number ($HV$) is given by

$$HV = 1.8544 \frac{F}{D^2}$$  \hspace{1cm} (A.1)

where $F$ is the load and $D$ denotes the length of the diagonal of the square. Table A.1 gives the mean value and standard deviation of the different materials in the bullet.

<table>
<thead>
<tr>
<th>Material</th>
<th>mean(HV)</th>
<th>std(HV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>bullet jacket 5.56 mm</td>
<td>104.11</td>
<td>9.1266</td>
</tr>
<tr>
<td>bullet jacket 7.61 mm</td>
<td>137.5</td>
<td>18.858</td>
</tr>
<tr>
<td>bullet penetrator</td>
<td>394.4</td>
<td>26.747</td>
</tr>
<tr>
<td>bullet filler</td>
<td>&lt;10</td>
<td>-</td>
</tr>
</tbody>
</table>
Appendix B

Rankine-Hugoniot relation

In this appendix, the Rankine-Hugoniot conditions and relation are derived. The basis is formed by the one-dimensional Euler equations (mass, momentum and energy) with the assumption of steady flow.

\[
\frac{\delta \rho}{\delta t} + \rho \frac{\delta v}{\delta x} = 0 \quad (B.1)
\]
\[
\frac{\delta \rho v}{\delta t} + \rho v \frac{\delta v}{\delta x} + \frac{\delta p}{\delta x} = 0 \quad (B.2)
\]
\[
\frac{\delta e}{\delta t} + \rho v \frac{\delta e}{\delta x} + \rho v \frac{\delta v^2}{\delta x} + v \frac{\delta p}{\delta x} = 0 \quad (B.3)
\]

The starting point is figure B.1 where a shock wave propagates through a material.

Figure B.1: Schematic view of a shock front (line C) propagating through a compressible material. \( U_p \) and \( U_s \) are the particle and shock velocity respectively.

**Mass conservation**

Balancing the mass in front (state of the material on the right side of line C in figure B.1) and behind (state of the material on the left side of line A in figure B.1) the shock wave gives:
\[ \rho_0 v_0 = \rho_1 v_1. \quad (B.4) \]

From figure B.1 can be concluded that \( v_0 = U_s \) (line C) and \( v_1 = U_s - U_p \) (line B). Substitution of these terms give the first Rankine-Hugoniot condition,

\[ \rho_0 U_s = \rho_1 (U_s - U_p). \quad (B.5) \]

**Momentum conservation**

The momentum conservation equation can be defined in 1-D and steady state as:

\[ \frac{\delta p}{\delta x} + \rho v \frac{\delta v}{\delta x} = 0. \quad (B.6) \]

Balancing this equation on the left and right side yields:

\[ p_0 + \rho_0 U_s^2 = p_1 + \rho_1 (U_s - U_p)^2. \quad (B.7) \]

Rewriting equation (B.7) gives

\[ p_1 - p_0 = \rho_0 U_s^2 - \rho_1 (U_s - U_p)^2. \quad (B.8) \]

Substitution of equation (B.5) in (B.8), gives

\[ p_1 - p_0 = \rho_0 U_s U_p. \quad (B.9) \]

**Energy conservation**

The energy conservation equation can be defined in 1-D and steady state as:

\[ \rho v \frac{\delta e}{\delta x} + \rho v \frac{\delta v^2}{\delta x^2} + v \frac{\delta p}{\delta x} = 0. \quad (B.10) \]

Balancing on the left and right side of the shock yields:

\[ (U_s - U_p)[p_1 + \rho_1 e_1 + \rho_1 (U_s - U_p)^2/2] = U_s(p_0 + \rho_0 e_0 + \rho_0 U_s^2/2). \quad (B.11) \]

Reordering and substitution of equation (B.5) and (B.9) in (B.11) gives

\[ p_1 U_p = \frac{1}{2} \rho_0 U_s U_p^2 + \rho_0 U_s (e_1 - e_0). \quad (B.12) \]
**Rankine-Hugoniot relation**

The Rankine-Hugoniot relation is found by eliminating $U_s$ and $U_p$ from the energy equation. Equation (B.11) is first rewritten to:

$$\frac{p_1}{\rho_1} + e_1 + \frac{1}{2}(U_s - U_p)^2 = \frac{p_0}{\rho_0} + e_0 + \frac{1}{2}U_s^2$$

(B.13)

Eliminating $U_s$ and $U_p$ using (B.5) and (B.9) gives the Rankine-Hugoniot relation:

$$(e_1 + \frac{p_1}{\rho_1}) - (e_0 + \frac{p_0}{\rho_0}) = \frac{1}{2}(p_1 - p_0)(\frac{1}{\rho_0} + \frac{1}{\rho_1}).$$

(B.14)

At high strain rates, as in bullet impacts, there is no time for heat to be conducted away from the target. Therefore the impact process becomes adiabatic in which $pV^\gamma = c$ applies. The Rankine-Hugoniot relation can therefore be reduced to:

$$(e_1 - e_0) = \frac{1}{2}(p_1 - p_0)(\frac{1}{\rho_0} + \frac{1}{\rho_1}).$$

(B.15)
Appendix C

McClintock void growth model

In this appendix, the derivation of McClintock for growth of elliptical voids with plane strain conditions and incompressibility is given. The results define a fracture strain.

The derivation starts with the definition of a cylinder in a infinite medium as depicted in figure C.1. The equilibrium equation, $\nabla \cdot \sigma = 0$, reduces to:

\[
\frac{\delta \sigma_r}{\delta r} + \frac{1}{r}(\sigma_r - \sigma_\theta) = 0
\]  

(C.1)

with the assumption of plain strain and incompressibility. McClintock solved the equilibrium equation in [42] for a non-hardening Von Mises material as

\[
d \ln \left( \frac{r}{r_0} \right) = \sqrt{3}d\varepsilon_{r,\infty} \sinh \left[ \sqrt{3} \frac{\sigma_{r,\infty}}{\sigma_{r,\infty} - \sigma_{z,\infty}} \right] + d\varepsilon_{r,\infty}.
\]  

(C.2)

This circular equation can be transformed into an elliptical by substitution of the following terms in equation (C.2):
APPENDIX C. MCCLINTOCK VOID GROWTH MODEL

\[ \varepsilon_{r,\infty} \to \tilde{\varepsilon} = \frac{(\varepsilon_a + \varepsilon_b)}{2}, \quad \sigma_{r,\infty} \to \frac{(\sigma_a + \sigma_b)}{2}, \quad r \to b \quad \text{and} \quad (\sigma_{r,\infty} - \sigma_{z,\infty}) \to \tilde{\sigma} \]  

(C.3)

which yields

\[ d \ln \left( \frac{b}{b_0} \right) = \frac{\sqrt{3} \tilde{\varepsilon}^2}{2} \sinh \left[ \frac{\sqrt{3}}{2} \frac{\sigma_a + \sigma_b}{\tilde{\sigma}} \right] + \frac{d(\varepsilon_a + \varepsilon_b)}{2}. \]  

(C.4)

Replacing the strains in the second terms by \( \varepsilon_a = \left[ \sigma_a - 1/2(\sigma_b - \sigma_z) \right] \tilde{\varepsilon}/\tilde{\sigma} \) and \( \varepsilon_b = \left[ \sigma_b - 1/2(\sigma_a - \sigma_z) \right] \tilde{\varepsilon}/\tilde{\sigma} \) gives

\[ d \ln \left( \frac{b}{b_0} \right) = \frac{\sqrt{3} \tilde{\varepsilon}^2}{2} \sinh \left[ \frac{\sqrt{3}}{2} \frac{\sigma_a + \sigma_b}{\tilde{\sigma}} \right] + \frac{3 \sigma_a + \sigma_b}{4 \tilde{\sigma}} d\tilde{\varepsilon}. \]  

(C.5)

McClintock assumed an element in which an elliptical void is present, see figure C.2. Two hole-growth factors in direction \( a \) and \( b \), \( F_a = (a/l_a)/(a_0/l_0^0) \) and \( F_b = (b/l_b)/(b_0/l_0^0) \), are introduced. Fracture is assumed to initiate when voids coalesce at the boundaries of the element \( (a = \frac{1}{2}l_a \text{ and } b = \frac{1}{2}l_b) \).

![Figure C.2: Elliptical void proposed by McClintock.](image)

In the derivation is assumed that the growth of the element boundaries is zero \( (l_a = l_0^0) \) which gives \( F_b = \frac{b}{b_0} \). Then equation (C.5) becomes

\[ d \ln (F_b) = \frac{\sqrt{3} \tilde{\varepsilon}^2}{2} \sinh \left[ \frac{\sqrt{3}}{2} \frac{\sigma_a + \sigma_b}{\tilde{\sigma}} \right] + \frac{3 \sigma_a + \sigma_b}{4 \tilde{\sigma}} d\tilde{\varepsilon}. \]  

(C.6)

McClintock desired a simpler expression and dropped the last term of equation (C.6). Hancock and Mackenzie [65] showed with their experiment to various metals that the last term is small compared to the first and validated McClintock’s decision. The equation is rewritten to

\[ \frac{2}{\sqrt{3}} d \ln (F_b) = \sinh \left[ \frac{\sqrt{3}}{2} \frac{(\sigma_a + \sigma_b)}{\tilde{\sigma}} \right] d\tilde{\varepsilon}. \]  

(C.7)
Equation (C.7) can be integrated to obtain the fracture strain in $b$-direction.

$$
\frac{2}{\sqrt{3}} \int_0^{l_b/2} d \ln(F_b) = \int_0^{\tilde{\varepsilon}} \left[ \sinh \left( \frac{\sqrt{3} (\sigma_a + \sigma_b)}{2} \right) \right] d\tilde{\varepsilon}.
$$

(C.8)

Hancock and Mackenzie [65] also showed that the sinus hyperbolic function is approximated well by half an exponential function. Rewriting equation (C.8) after integration gives an expression for the fracture strain:

$$
\tilde{\varepsilon}^f = \frac{2}{\sqrt{3}} \ln \left( \frac{t^0_b}{2b_0} \right) \exp \left( -\frac{\sqrt{3} \sigma_a + \sigma_b}{2} \right).
$$

(C.9)
Bibliography


[72] Material properties of copper, [http://www.copper.org/resources/properties](http://www.copper.org/resources/properties)


[77] LS-Dyna user guide, Appendix B, Material model examples, B.2.21.