Anti-Windup: A Convergence-Based Problem Definition

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1 Introduction

In the field of dynamical systems, one of the main issues is stability. Most results in this area consider stability of one particular solution of the system. But wouldn’t it be nicer if we were able to guarantee the stability of all solutions of such a system? We therefore consider the concept of convergent systems, i.e., systems for which all solutions ‘forget’ their initial state and converge to each other, so that after a transient phase the system dynamics only depends on the input. An advantage of this property is that we can use simulation to analyse system performance for a given input, since we only need to consider one initial state. In this research we consider anti-windup design for the class of stable linear systems with input saturation. Whereas the performance of such a system is often expressed in $H_{\infty}$ or $H_2$ gain, we are interested in finding an anti-windup compensator which results in a convergent system, so that afterwards we can use simulation based techniques to optimize performance.

2 Convergent dynamics

Following Demidovich, see [1], a time-varying system, $\dot{x} = f(x, w(t))$, is called convergent if there exists a so-called limit solution $\bar{x}(t)$ that is defined and bounded for all $t \in (-\infty, \infty)$ and that is globally asymptotically stable. A result of this convergence property is that, given a bounded input, two arbitrary system trajectories $x_1(t)$ and $x_2(t)$ converge to each other in forward time independent of their initial condition, i.e.:

$$|x_1(t) - x_2(t)| \to 0 \quad \text{for} \quad t \to \infty.$$  

Consider a system (Figure 1), consisting of a linear plant and a (possibly discontinuous) scalar nonlinearity $\psi(y)$, which for some $\mu_1 < \mu_2$ satisfies the incremental sector condition:

$$\mu_1 \leq \frac{\psi(y_1) - \psi(y_2)}{y_1 - y_2} \leq \mu_2.$$  

In order to check whether this system is convergent, we can use LMI’s or the circle criterion, which for this case provide sufficient (but not necessary) conditions for convergence.

Figure 1: Linear plant with nonlinear feedback

Figure 2: Anti-windup closed-loop system

3 Anti-windup problem definition

In [2] the anti-windup problem for a linear plant with input saturation is roughly defined as follows: “design a linear anti-windup compensator such that the resulting closed-loop system (Figure 2) is quadratically stable for $w = 0$ and shows $L_2$ performance of level $\gamma$:  

$$||z||_2 \leq \gamma ||w||_2$$  

for a predefined output $z$. ” In our view, a more intuitive problem definition is to design the anti-windup compensator in such a way that the closed-loop system is convergent for any admissible input. In order to achieve this goal, the system in Figure 2 is transformed into the system in Figure 1 (the matrices $A$, $B$ and $C$ are then dependent on the design of the anti-windup compensator). For the resulting system the convergence property can be checked for all proposed anti-windup compensators. However, it has not been proven yet that there always exists an anti-windup compensator that leads to a convergent closed-loop system.

References
