Identification and control of a vehicle restraint system

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Abstract: To minimize occupant injuries, passive in-vehicle safety systems like the safety belt and the airbag restrain the occupant during a crash. This paper presents a design approach for a feedback controller for the belt force to reduce the maximum chest acceleration as a measure for the risk of occupant injuries. Only frontal crashes are considered. The available, experimentally validated numerical crash model is too complex to be used as a controller design model. Therefore, approximate linear models for the transfer from belt force to chest acceleration are derived by analysing the effect of stepwise perturbations of the belt force on the chest acceleration. Using these linear models, loop shaping is applied to arrive at a controller that satisfies a set of a priori defined criteria. The controller is implemented in and evaluated with the complex crash model, showing that a reduction of approximately 60 per cent in the adopted injury measure can be achieved. Furthermore, it is shown that this approach can be applied in different situations.

Keywords: adaptive/active restraint systems, safety belt, airbag, feedback control, loop shaping, identification, approximate realization

1 INTRODUCTION

Many passive in-vehicle safety facilities have been introduced to reduce occupant injuries in vehicle crashes. These facilities, with the airbag and the belt as widely known instances, attempt smoothly to absorb the kinetic energy of the occupant during a crash. According to the National Highway Traffic Safety Administration [1], safety belts have reduced the risk of a fatal injury for front-seat occupants by 45 per cent, whereas airbags have achieved a further reduction of 12 per cent.

Although passive safety systems have been shown to be effective, they themselves can cause injuries, e.g. when the occupant is in the path of a deploying airbag. Furthermore, it is known that occupant injuries are strongly influenced by characteristics of the occupant and the crash [2], whereas present-day restraint systems can rarely adapt to these characteristics [3].

These observations have stimulated the research on passive safety systems towards adaptable restraint systems. A way to influence the belt force by the ripping of stitches was implemented in 1970 [4]. In 1977, Hontschik et al. [5] introduced a hydraulic throttling element to regulate the belt force as a function of the initial velocity. In 2001, Clute [6] discussed a device to adapt the stiffness of the belt force limiter once during the crash. For the airbag, the point of time at which one or both inflators of the airbag system are inflated can be changed by state-of-the-art airbag systems [7].

For validation purposes of, among others, restraint systems, insight into the expected occupant injuries due to a crash is acquired by performing standardized tests with real vehicles and dummies instead of human occupants. During such a crash test, quantities such as accelerations of and forces on the dummy are measured. The translation of these measurements into injuries for human occupants, and vice versa, is not trivial, but standardized measures are available and are adopted as the injury measures throughout this paper.

Such real-world crash tests are very expensive and time consuming. For the design of (adaptive)
restraint systems, crash tests are therefore replaced as much as possible by simulations, using numerical packages such as MADYMO [8]. For the crash test, used as a working example throughout this paper, a MADYMO model is available. This model, depicted in Fig. 1, describes the US-NCAP frontal crash test [9] of a medium-class passenger car with a standard Hybrid III 50 percentile dummy as the driver. A simulation with this model with a time step of 1 µs takes approximately 1 h on Silicon Graphics with a 195 MHz processor. The relevant vehicle parts are represented by 49 rigid bodies. The model for the dummy is a commercially available model [8], consisting of 37 rigid bodies that are connected by joints and a total of 150 springs and dampers. The airbag is represented by a finite element model with 2800 finite elements. Contacts are modelled as force interactions by springs and dampers.

The belt is modelled as a line structure of elements, each consisting of a spring parallel with a damper. One end of the belt is attached to the vehicle floor. From there, the belt goes over the pelvis to the buckle, cross-diagonally over the chest, through the D-ring at the upperside of the B-pillar to the load limiter at the downside of that pillar. The Eytelwein friction model [8] is used to describe the interaction between the belt and the D-ring. The load limiter consists of a housing, rigidly attached to the floor, and a runner. The runner is connected at one side to the last belt element and at the other side to the housing via a translational joint with one degree of freedom, the so-called belt outlet \( x_1 \). The force, \( F \), of the load limiter on the belt is a highly non-linear function of \( x_1 \).

Typically, to determine the most appropriate characteristics of adaptive restraint systems, such as the deformation characteristics of the belt force limiter, numerical models like the one developed by the present authors are used. Characteristics of (adaptive) restraint systems are often determined using trial and error, e.g. reference [10], in combination with optimization, e.g. reference [11]. For this purpose, a cost function, representing one or more measures for the risk of injury, is minimized. Owing to the non-linear nature of the investigated system, optimization is often performed using response surfaces and stochastic methods [12].

Drawbacks of these approaches are the large number of simulations and the fact that the procedure has to be repeated for each new combination of crash test, vehicle, dummy, and injury measure. Drawbacks of adaptive restraint systems are that they have only a limited set of settings, meaning that the obtained characteristics are optimum for certain combinations only.

In this paper, an innovative view on restraint systems is elaborated, avoiding these drawbacks. The problem of minimizing occupant injuries is formulated as a tracking problem. Basically, it is the introduction of ‘active’ restraint systems by adding actuators and sensors in order to apply feedback control [13–15].

1. One or more relevant variables for the risk of injury are chosen.
2. One or more relevant variables to manipulate the restraint system components are chosen.
3. A reference signal for the injury variables is defined, for which the risk of injuries is as low as possible.
4. A feedback controller is designed to force the controlled variables to follow the reference signal.
5. The controller is implemented and evaluated.

The focus of this paper is on the design of a feedback controller for the belt system. Henceforth, the MADYMO crash model with the state-of-the-art restraint system components will be called the passive model \( M_p \), and this restraint system will be referred to as the passive system. The numerical model with actuators instead of state-of-the-art components will be called the active model \( M_a \), whereas the corresponding restraint system will be called the active system.

The choice of the input(s) to manipulate the restraint system is fairly trivial here. This paper focuses on the reduction of chest injuries. Therefore, the airbag is not considered. Since the force of the belt on the occupant is believed to be directly influenced by the force at the load limiter, it is chosen to control the force at this point of the belt. This
is realized by replacing the load limiter with an actuator, exerting a force $F$ in the direction of the belt outlet $x_i$ in Fig. 1.

The choice of an appropriate injury measure is far from trivial [16, 17]. For the intended online control, a measure based on a continuously measurable signal is desired. Here, the maximum of the absolute value of the chest acceleration $\dot{e}$ is chosen as the injury measure to be minimized. Hence, this acceleration is the output of the system.

Implementation of the proposed concept in a real-world vehicle is not possible yet, mainly because the required sensing and actuating devices do not exist. However, developments towards necessary components have been initiated. Haß and Bertram [18] recently introduced an electric actuator for the belt force. In addition, a patent was granted for an hydraulic actuator [19]. In addition, van Poppel [20] has been granted a patent for an actuator for the belt force, based on piezoelements. Furthermore, sensors for the occupants’ position relative to the inner vehicle parts do exist [21]. Breed [22] discusses intelligent sensors to determine the severity of the crash.

The paper is organized as follows. The determination of the reference signal is described in section 2. Section 3 focuses on the derivation of simple models suitable for controller design. The actual controller design and evaluation are discussed in section 4. The design approach is elucidated in these sections only for one combination, i.e. the input $F$, the output $\dot{e}$, the US-NCAP frontal crash test, and a middle-class vehicle with an average male dummy as the driver, but it is applicable to other situations also. This is shown in section 5. More specifically, the airbag is taken into account and bounds on the belt force are introduced. It is shown that only the reference signal has to be modified to obtain a high performance of the controlled system. Section 6 gives conclusions and some recommendations for future research.

2 REFERENCE SIGNAL

An important requirement for the reference signal $r_c$ for the chest acceleration $\dot{e}$ is that, in case of perfect tracking, the maximum chest acceleration is minimized. Approaches to determine a suitable reference signal are discussed in references [23] and [24]. Unfortunately, for present purposes, the reference signal has to account for a number of additional constraints on the dummy motion, meaning that another approach is necessary.

Observations as reported in references [25] and [26] and confirmed by simulations with the crash models $M_p$ and $M_e$ show that, during frontal crashes, the head and the chest of the occupant move forwards along an almost straight line. These observations lead to the choice of the very simple crash model of Fig. 2 as the starting point for the derivation of the reference signal.

In this figure, $x_{veh}(t)$ and $c(t)$ denote the forward displacement of the vehicle and the chest at time $t$ with $x_{veh}(0) = c(0) = 0$ at the start of the crash at $t = 0$. For $t \leq 0$, the dummy is in contact with the back of the seat, so $x_{veh}(0) = \dot{c}(0) = v_0$, where $v_0$ is the initial vehicle velocity. The crash is considered to be finished when the forces on and the accelerations of the dummy have become negligible. A simulation with $M_p$ shows that the belt force is less than 1 kN and that the absolute chest acceleration is less than \(50 \text{ m/s}^2\) for $t \geq 100$ ms. Therefore, $t = t_e = 100$ ms is adopted as the end of the crash.

The reference signal $r_c$ has to guarantee that the maximum of the absolute value of the controlled chest acceleration is minimal if the tracking error $e = r_c - \dot{e}$ is 0 for all $t \in [0, t_e]$. This is the case if $r_c$ is equal to the average of the vehicle acceleration, $\dot{x}_{veh}$, over the time interval $[0, t_e]$. Unfortunately, this is not possible because the reference signal $r_c$ also has to account for the following constraints.

1. The chest cannot move through the back of the seat, so $c(t) - x_{veh}(t) \geq 0$ for all $t \geq 0$.
2. The chest velocity is equal to or lower than the vehicle velocity at time $t = t_e$, so $\dot{c}(t_e) - \dot{x}_{veh}(t_e) \leq 0$.
3. The chest may not collide with the steering wheel, so $c(t) - x_{veh}(t) < l_0$ for all $t \geq 0$, where $l_0$ is the initial distance between the chest and the steering wheel.

During the first phase of the crash, the vehicle deceleration is smaller than the average deceleration. Hence, it is impossible to make the controlled chest deceleration larger than the vehicle deceleration in this crash phase, since the dummy would move through the back of the seat. Therefore, from $t = 0$

![Fig. 2 Simple one-dimensional kinematic model](image-url)
until a yet to be determined time \( t_r \), it is desired that the dummy remains in contact with the back of the seat, so the reference \( r_c \) is chosen to be equal to \( \ddot{x}_{veh} \) for \( t \in [0, t_r] \).

With respect to the second and third constraints, it is noted that, for the remaining part of the crash, i.e. from \( t = t_r \) until \( t = t_e \), it is desired that \( r_c \) be constant and equal to its value at time \( t = t_r \), so \( r_c(t) = r_c(t_r) = \ddot{x}_{veh}(t_r) \) for \( t \in [t_r, t_e] \). Hence, the reference acceleration is given by

\[
\begin{cases}
\ddot{x}_{veh}(t) & 0 \leq t < t_r \\
\ddot{x}_{veh}(t_r) & t_r \leq t < t_e \\
0 & t_e \leq t
\end{cases}
\] (1)

The vehicle acceleration \( \ddot{x}_{veh}(t) \) is assumed to be known a priori for all \( t \in [0, t_e] \). Therefore, \( r_c(t) \) can be determined for all \( t \in [0, t_e] \) as soon as \( t_r \) is chosen.

With the initial conditions \( r_c(0) = c(0) = \ddot{x}_{veh}(0) = 0 \) and \( r_c(0) = c(0) = \ddot{x}_{veh}(0) = 0 \), it is possible for \( r_c \) and \( r_s \) also to be determined. For \( t \in [t_r, t_e] \), this results in

\[
r_c(t) = \ddot{x}_{veh}(t_r) + (t - t_r)\ddot{x}_{veh}(t_r)
\] (2)

\[
r_s(t) = \ddot{x}_{veh}(t_r) + (t - t_r)\ddot{x}_{veh}(t_r) + \frac{(t - t_r)^2}{2} \dddot{x}_{veh}(t_r)
\] (3)

Suppose that the controller does achieve perfect tracking, so \( c(t) = r_c(t) \) and \( c(t) = r_s(t) \) for all \( t \in [0, t_e] \).

The second constraint then implies that \( r_s(t_e) \leq \ddot{x}_{veh}(t_e) \), and, with equation (2), this yields one condition for \( t_r \)

\[
(t_e - t_r)\dot{x}_{veh}(t_r) \leq \ddot{x}_{veh}(t_e) - \ddot{x}_{veh}(t_r)
\] (4)

Likewise, the third constraint can be rewritten as

\[
(t_e - t_r)\dot{x}_{veh}(t_r) + \frac{(t_e - t_r)^2}{2} \dddot{x}_{veh}(t_r)
\leq \dot{\epsilon}_0 + \dddot{x}_{veh}(t_e) - \dddot{x}_{veh}(t_r)
\] (5)

In general, there is more than one value of \( t_r \) for which these constraints are satisfied. A suitable value is determined by trial and error, starting with \( t_r = 5 \) ms. For this value, it is checked whether the constraints are violated. If so, a larger value is chosen and the procedure is repeated. For the considered crash this results in \( t_r = 11 \) ms. Then all constraints are satisfied and, at the end of the crash, the chest is close to the steering wheel.

The reference signal from equation (1) is discontinuous for \( t = t_e \). This can lead to numerical problems in the simulations with the controlled system. Therefore, the given reference signal is smoothed by half a cosine in a small time interval \( 5 \) ms around \( t = t_e \). The result is shown in Fig. 3, together with the vehicle acceleration \( \ddot{x}_{veh}(t) \).

### 3 IDENTIFICATION FOR CONTROLLER DESIGN

The design of a feedback controller for the belt force requires a simple model that (roughly) describes the transfer from the input (belt force \( F \)) to the output (chest acceleration \( \dot{\epsilon} \)). For control design, the crash model \( M_c \) is far too complex. Commonly used approaches to arrive at a suitable controller design model are:

(a) use of first principles of physics and drastic simplifications, followed by the determination of the model parameters [27, 28];

(b) reduction of the complex model to a much simpler model with a small number of bodies, springs, and dampers [29];

(c) the black box approach, involving the choice of a set of mathematical relations between restraint system parameters and injury measures, followed by the determination of these parameters [12].

These time-consuming approaches are not attractive. Firstly they require a detailed insight into the dynamic behaviour, and secondly the whole procedure has to be repeated if another input–output combination is chosen or if another dummy, vehicle, crash, or restraint system is considered.

MADYMO can output the describing differential equations for simple multibody models [30] but not for complex models such as the model at hand. Therefore, the idea in this paper is to use ‘measurements’ of the input \( F \) and output \( \dot{\epsilon} \) of the model \( M_c \) to determine linear, low-order models suited for controller design. This approach can be very successful, as exemplified in, among others, reference [31] for a...

The ‘measurements’ are obtained from simulations with $M_a$ during which a small perturbation $\delta F$ is added to a known belt force $F_p$, resulting in the perturbed input $\hat{F} = F_p + \delta F$. The perturbed output is given by $\hat{\varepsilon} = \hat{\varepsilon}_p + \hat{\varepsilon}$, where $\hat{\varepsilon}_p$ is the chest acceleration due to the force $F_p$. To find a suitable force $F_p$ as a function of time, the belt force is measured during a simulation with the passive model $M_p$. The obtained force in the belt at the load limiter is depicted in Fig. 4, whereas the obtained so-called passive chest acceleration $\hat{\varepsilon}_p$ is given in Fig. 5. This figure also gives the chest acceleration $\hat{\varepsilon}$ obtained for the case where the passive belt force $F_p$ is used as a prescribed actuator force in the active model $M_a$. As expected, this acceleration is equal to the passive chest acceleration $\hat{\varepsilon}_p$.

The preferred input perturbation, an impulse [27], cannot be used since numerical packages like MADYMO cannot deal with impulses. The alternative of harmonic perturbations is not attractive since it requires an extensive set of time-consuming simulations with $M_a$ to cover the frequency range of interest. Therefore, and to facilitate a quick and intuitive interpretation of the results, stepwise perturbations $\delta F_{i,j}(t) = \Delta F_{ij}(t - \tau_j)$ are used. Here, $\Delta F_{ij}(t - \tau_j)$ represents a unit step at the point of application or time $\tau_j$, and $\Delta F_i$ is the step size. The corresponding output perturbation $\hat{\delta} e_{i,j}(t)$ is scaled with the step size $\Delta F_j$, resulting in the normalized output perturbation $\hat{\delta} e_{i,j}(t) = \delta e_{i,j}(t)/\Delta F_j$. Figure 6 gives some simulation results for step sizes in the range from 10 N to 200 N, applied at $\tau_1 = 20$ ms.

Since the active model $M_a$ is non-linear, it is to be expected that the normalized output perturbations will not be the same for all sizes $\Delta F_i$, nor for all points of application $\tau_j$. From Fig. 6 it can be seen that nonlinearities become important for step sizes larger than 100 N, whereas the normalized output perturbations for sizes equal to or smaller than 50 N are very similar. Hence, it seems reasonable and possible to linearize $M_a$ locally at the point of application $\tau_1 = 20$ ms and to use the normalized output perturbations to determine a linear, simple controller design model.

The results in Fig. 6 show a striking oscillatory behaviour for $t > 37$ ms. A detailed analysis of simulation results shows that this behaviour is caused by a sign change of the friction in the joint between the left clavicle and the left upper arm. It is assumed that this friction does not significantly influence the controller design model to be determined, and therefore $M_a$ is modified by eliminating this friction. The normalized output perturbations $\hat{\delta} e_{i,1}$ for five input perturbations with step sizes $\Delta F_1 = 10$ N, $\Delta F_2 = 20$ N, $\Delta F_3 = 30$ N, $\Delta F_4 = 40$ N, and $\Delta F_5 = 50$ N, applied at $\tau_1 = 20$ ms and obtained with the modified model, are given in Fig. 7.

At $t \approx 39$ ms and $t \approx 42$ ms, indicated by A and B in Fig. 7, the response still shows a non-smooth behaviour. This implies that only the part of the output perturbations for $t \leq 39$ ms can be used to determine a linear controller design model.

To minimize the effect of computational noise, the obtained five normalized output perturbations are averaged. The averaged normalized perturbation
Fig. 7 Normalized output perturbation $\delta \tilde{c}_{i,1}$ for $\tau_1 = 20$ ms with the modified model

$\delta \tilde{c}_j(t)$ of the chest acceleration

$$
\delta \tilde{c}_j(t) = \frac{1}{5} \sum_{i=1}^{5} \frac{\delta \tilde{c}_{i,j}(t)}{\Delta F_i} = \frac{1}{5} \sum_{i=1}^{5} \delta \tilde{c}_{i,j}(t)
$$

(6)

is seen as the normalized output perturbation at $t = t_f$ and is used to determine a controller design model.

To investigate whether the simple model, obtained from the output perturbations at time $\tau_1$, reflects the relevant dynamic behaviour at other operating points, the outlined procedure is applied at different points of application $\tau_j$ in the interval 20–35 ms. The lower bound of this interval is based on the observation that, in the passive system, it takes nearly 20 ms into a continuous-time, state-space model (using the bound of this interval is based on the observation). Next, a discrete-time model is derived, transformed from the output perturbations at time $t = 20$ ms with the modified model and presented in Fig. 8.

To ensure that the small, but possibly significant, differences between these responses are accounted for during controller design, a linear model is derived for $\delta \tilde{c}_{1}$ and $\delta \tilde{c}_{2}$. As mentioned earlier, the usable part of these responses is limited owing to the non-smooth behaviour of $M_a$ at $t \approx 39$ ms. For $\delta \tilde{c}_3$, the usable time span is only 9 ms. It is expected that this is too short, so the response for $\tau_3 = 30$ ms is left out of consideration in the sequel.

Given the step response of a linear model, a variety of techniques exists to determine the order and the model parameters, e.g. reference [27]. For the present purposes, a technique is desired that can deal with step responses, measured over a time span shorter than the time required to attain the steady state. This is possible, for instance, with the modified approximate realization method [33, 34]. This method is attractive since it supplies information to choose the order of the model and it can directly estimate the model parameters. Here, the modified approximate realization algorithm, as outlined in reference [35], is applied.

The transfer functions of the linear models, obtained with this method applied to $\delta \tilde{c}_1$ and $\delta \tilde{c}_2$, are denoted by $H_1(s)$ and $H_2(s)$. Singular value decomposition is performed on the transformed Hankel matrices, constructed from $\delta \tilde{c}_j(t)$ with $t \in (\tau_j, 39)$ ms, $j = 1, 2$. The five largest singular values are given in Table 1. Based on these results, the linear models to be constructed are established as second order.

Next, a discrete-time model is derived, transformed into a continuous-time, state-space model (using the matched pole-zero method [36]) and written in transfer function form

$$
H_j(s) = K_{st} \frac{1 - s/2\pi z}{(s/2\pi f_p)^2 + 2\zeta s/2\pi f_p + 1}
$$

(7)

where $K_{st}$ is the static gain, $f_p$ is the undamped eigen-frequency, $\zeta$ is the damping factor, and $2\pi z$ is system zero. The obtained model parameters are given in Table 2.

| $\sigma_1$ | 1.4620 | 1.0945 |
| $\sigma_2$ | 0.2955 | 0.1206 |
| $\sigma_3$ | 0.0350 | 0.0117 |
| $\sigma_4$ | 0.0118 | 0.0054 |
| $\sigma_5$ | 0.0100 | 0.0047 |

Table 1: Singular values for $\tau_1$ and $\tau_2$

<table>
<thead>
<tr>
<th>$K_{st}$ (m/N s$^2$)</th>
<th>$f_p$ (Hz)</th>
<th>$\zeta$</th>
<th>$z$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_1(s)$</td>
<td>0.0188</td>
<td>40.0</td>
<td>0.630</td>
</tr>
<tr>
<td>$H_2(s)$</td>
<td>0.0139</td>
<td>39.1</td>
<td>0.721</td>
</tr>
</tbody>
</table>

Table 2: Parameters of $H_1(s)$ and $H_2(s)$
From the Bode diagrams in Fig. 9 it can be concluded that, in the frequency range of interest, i.e. from 5 Hz to 5 kHz, the differences between the obtained linear models are small.

4 CONTROLLER DESIGN

In this paper, only feedback control is considered. Feedforward control is not attractive or even possible at the moment, because the crash model $M_r$ is too complex as a basis for inverse dynamics and no sufficiently accurate inverse crash model is yet available. Besides, the controller to be designed has to be robust and applicable without major changes for a wide variety of crashes and occupants. This requires some kind of feedback.

From the wide variety of feedback controller design techniques, e.g. reference [37], the classical loop-shaping technique, e.g. references [38] and [39], is chosen. This technique is widely accepted as a starting point for controller design, the controller output can intuitively be understood, and the structure of the controller is well known. The transfer function of the controller will be denoted by $C(s)$.

The controller has to satisfy a number of design criteria concerning, among others, stability and performance. The desired stability is formulated here in terms of the gain margin (GM) and phase margin (PM). To account for the lack of accurate knowledge about model errors, relatively high values GM = 3 and PM = 45° are aimed for. Since the differences in the local dynamics, as characterized by the transfer functions $H_1(s)$ and $H_2(s)$, are fairly small and because the Nyquist plots are very smooth, extra measures to ensure additional robustness are omitted.

The desired performance is formulated in terms of a maximum allowable tracking error $e_{\text{max}}$ and a desired 5 per cent settling time $t_{\text{sek}}$. Based on the maximum of the vehicle deceleration, and to avoid unrealistic requirements for the actuator, rather arbitrarily $e_{\text{max}} = 20 \text{ m/s}^2$ and $t_{\text{sek}} = 11 \text{ ms}$ are chosen.

The bandwidth $f_{\text{bw}}$, defined as the 0 dB crossover frequency of the open-loop transfer function $H(s)C(s)$, is related to the settling time. The desired bandwidth is lower bounded by the duration of the crash. The aim is a bandwidth as low as acceptable, because then the actuator specifications will be less demanding than for a high bandwidth. For a standard, linear, second-order system with damping factor $\zeta$, a simple, approximate relation between the bandwidth and the settling time is given by [38]

$$f_{\text{bw}} \approx \frac{3}{2 \pi t_{\text{sek}} \zeta} \quad (8)$$

Using this relation with $t_{\text{sek}} = 11 \text{ ms}$, the minimum bandwidth is then 69 Hz and 60 Hz for the systems with transfer function $H_1(s)$ and $H_2(s)$ respectively.

The requirement $|e| \leq e_{\text{max}}$ for the tracking error is translated into a requirement for the sensitivity function $S(s)$, describing the closed-loop transfer from reference $r_c$ to tracking error $e = r_c - \hat{c}$

$$S_j(s) = \frac{1}{1 + H(s)C(s)} \quad (9)$$

The translation is not trivial. First of all it is noted that, for $t < t_c$, it is no problem if the tracking error is larger than $e_{\text{max}}$, since then the absolute chest accelerations are moderate or small and exceeding $e_{\text{max}}$ will in general not increase the maximum absolute chest acceleration. For $t_c \leq t \leq t_e$, the reference acceleration is constant and, at least for the considered crash, approximately equal to $-200 \text{ m/s}^2$. This means that the allowable relative tracking error is then 10 per cent or $-20 \text{ dB}$. Using the method of Welch’s averaged, modified periodogram [27, 36] on the chest acceleration $\ddot{c}$, it turns out that almost 95 per cent of the energy is stored in components with frequencies lower than 100 Hz. It is therefore required that $|S_j(2\pi f)| \leq -20 \text{ dB}$ for $f \leq 100 \text{ Hz}$.

From the Bode diagram in Fig. 9 it can be concluded that a properly tuned PI controller in combination with a second-order low-pass filter may be sufficient [39]

$$C(s) = P \left(1 + \frac{1}{\tau_1 s} \right) \frac{\omega_\zeta^2}{s^2 + 2\zeta\omega_1 s + \omega_1^2} \quad \text{with } \omega_1 = 2\pi f_t \quad (10)$$

where $P$ and $\tau_1$ are the parameters of the PI controller and $\zeta = 0.7$ and $f_t = 1 \text{ kHz}$ are the damping factor and
the undamped eigenfrequency of the low-pass filter. This filter effectively suppresses the (computational) noise in the output of the MADYMO model.

A first choice for the controller parameters, using the transfer function $H_1(s)$ and aiming at a bandwidth of 69 Hz, shows that the desired disturbance reduction is not achieved. Next, the bandwidth is increased until a controller is found that satisfies all design criteria. This turns out to be the case for a bandwidth of 400 Hz for $H_1(s)$ and 468 Hz for $H_2(s)$. The controller parameters are $P = 100$ Ns²m and $\tau_1 = 160$ Hz. The Bode diagram of the resulting sensitivity functions $S_1(s)$ and $S_2(s)$ and the interesting part of the Nyquist plot of $H_1(s)C(s)$ and of $H_2(s)C(s)$ are shown in Figs 10 and 11 respectively.

The obtained controller is combined with the active system $M_a$. Some results of the simulation with this closed-loop system are given in Fig. 12 for the controlled chest acceleration $\ddot{c}$ and in Fig. 13 for the required belt force $F$. It can be observed that the maximum of the absolute chest acceleration has decreased from 50 g for the passive system to 21 g for the active system.

The results indicate a stable and sufficiently robust closed loop. Interpreting the tracking error during the first phase of the crash for $t \leq 11$ ms, it seems that the settling time $t_{sp}$ is achieved. The error at $t = 11$ ms is approximately 4 per cent.

At $t \approx 35$ ms, the maximum allowable tracking error $e_{max}$ is slightly violated, indicated by A in Fig. 12. A detailed investigation of the forces in the belt system shows that, at $t \approx 35$ ms, the velocity of the belt over the D-ring changes sign. As a result of this, the friction force between the belt and the D-ring starts to contribute to restrain the dummy instead of counteracting it. This phenomenon can be observed in Fig. 13, where the force $F_{eff}$ in the belt between the D-ring and the left clavicle of the dummy is shown by the dash-dotted line.

Until $t = 25$ ms, high belt forces are needed to keep the dummy in contact with the back of the seat. In this period, the dummy is even slightly pushed back. After $t = 25$ ms, the dummy starts to move forward relative to the vehicle. Then, the velocity of the belt over the D-ring changes sign. These two phenomena cause the belt force to decrease significantly around $t \approx 35$ ms.

Figure 13 also clearly shows that the exerted belt force is significantly higher in the controlled case. This may be due to the fact that the airbag is not inflated in this case. For modern cars involved in a serious frontal crash, this is unrealistic. In section 2, the airbag will be accounted for.

The results in the preceding part of this section indicate that the transfer functions $H_1(s)$ and $H_2(s)$ are sufficiently accurate for controller design. More
than that, these results suggest that the linear models do make sense even for operating points of the closed-loop system. To evaluate these observations, closed-loop identification is performed (see Fig. 14). Stepwise perturbations \( \delta F(t) = \Delta F_{\text{c}}(t - \tau) \) are added to the controller output \( F \) at two points of application \( \tau = \tau_1 \) and \( \tau = \tau_2 \).

An analysis of the perturbed belt force \( F \) and perturbed chest acceleration \( \ddot{c} \) suggests a locally linear behaviour. Next, the transfer functions from \( \delta F \) to \( \ddot{c} \) are determined, both for \( t = 15 \) ms. For this point of time, the closed system is in almost the same state as the passive model at point of time \( t = 20 \) ms. From these functions, the transfer functions \( H(s) \) of the local model at the operation points of the controlled system at time \( t = 15 \) ms can be derived, e.g. reference [40]. The Bode diagrams of the resulting \( H(s) \) and the Bode diagram of \( H_1(s) \) and \( H_2(s) \) are shown in Fig. 15.

For low frequencies and around the bandwidth \( f_{\text{BW}} = 400 \) Hz these models are quite similar to those for the passive system, whereas the operating points are quite different. This suggests that the non-linearity of the system is fairly weak. The bandwidth \( f_{\text{BW}} \) of the open-loop system of the controller \( C(s) \) combined with \( H(s) \) is 280 Hz, suggesting that the actual bandwidth for an actuator may be even lower than the 468 Hz earlier determined.

5 TOWARDS A MORE REALISTIC RESTRAINT SYSTEM

In the preceding sections, the restraint system was not very realistic. First of all, the dummy was only restrained by the belt. Secondly, the belt forces were relatively high, probably leading to undesired injuries, not accounted for by the adopted measure for the risk of injury. For example, a belt force of 15 kN (see Fig. 13) probably leads to a broken clavicle or rib. Besides that, actuators that are able to apply a force increasing from 0 to 8 kN within 2 ms do not (yet) exist.

In this section, these aspects are taken into account.

1. A finite element model of the airbag is included in the active model \( M_a \). This airbag is passive, meaning that the gas flow into and out of the bag behaves as in the passive system.

2. Bounds on the maximum belt force and its time derivative are implemented by clamping the controller output.

Load limiters are designed with the objective of limiting the force applied to the dummy by the belt. These limits vary from 2 to 7 kN [41–43], depending on dummy mass or crash test characteristics. Here, a bound of \( F_{\text{max}} = 6 \) kN is adopted. Bounds on the time derivative of the belt force \( |\dot{F}(t)| \) are rarely discussed in the literature. Therefore, the time history \( F_p(t) \) of the belt force for the passive belt system is analysed. The maximum of the absolute rate per millisecond, i.e. max \( |\dot{F}_p(t)| \), is approximately 1.5 kN/ms. Hence, a bound of \( |\dot{F}_{\text{max}}| = 1.5 \) kN/ms seems reasonable.

Owing to the bounds on the belt force, it will not be possible to track the reference signal with sufficient accuracy. In addition, the constraints on the dummy motion will be violated if the reference signal of the previous sections is used. Hence, the reference signal has to be modified. By trial and error, a constant reference signal, \( r_c = \dot{\ddot{c}}_{\text{des}} \), is used, where \( \dot{\ddot{c}}_{\text{des}} \) is the acceleration to be determined. Again, the reference signal is smoothed around \( t = t_e \). To prevent integral wind-up problems, the integral controller is reset when the tracking error passes zero for the first time [44].

Starting with an initial value of \( -200 \) m/s\(^2\) for \( \ddot{c}_{\text{des}} \), a simulation with the closed-loop system, including the bounds on the controller output, is performed. The simulation results show that the constraints on the dummy motion are violated. Therefore, the value of \( \ddot{c}_{\text{des}} \) is decreased until the motion constraints are satisfied. After a few simulations, a value of \( \ddot{c}_{\text{des}} = -230 \) m/s\(^2\) is found. The controlled chest acceleration and the required belt force are shown in Figs 16 and 17 respectively. Until \( t \approx 35 \) ms, the required belt force and the controlled chest acceleration are fully dominated by the imposed constraints.
The problem, normally solved by optimization, is translated here into a tracking problem. The reference signal has to reflect the lowest possible occupant injuries and account for a number of constraints on the dummy motion and belt force. The design approach embraces three main steps: the determination of the reference signal, the derivation of linear controller design models, and the actual controller design.

The local dynamic behaviour, derived from the response in the chest acceleration on small perturbations in the belt force, is linear and second order and weakly depends on the operating point.

Fig. 16  Chest acceleration $\ddot{c}(t)$ for the controlled system with a passive airbag

Fig. 17  Belt force $F(t)$ for the controlled system with a passive airbag

In comparison with the case without the airbag, the chest acceleration and the belt force are more noisy, especially for $t > 50$ ms. This is caused by the interaction of the chest with the finite element model of the airbag. The time history of the effective restraint force $F_{\text{eff}}$ in the belt between the D-ring and the left clavicle is shown by the dash-dotted line in Fig. 13. It can be seen that, for the active system with the passive airbag and extra bounds on the belt force, the effect of the friction in the D-ring on $F_{\text{eff}}$ is much smaller. A very important result of the simulations is that the required force in the first phase of the crash and the maximum of this force are significantly smaller than for the controlled system without the airbag, whereas the maximum of the absolute chest acceleration is only slightly increased.

6 CONCLUDING REMARKS

In this paper, an approach is presented for the design of a belt force controller to reduce the maximum absolute chest acceleration as much as possible. The problem, normally solved by optimization, is translated here into a tracking problem. The reference signal has to reflect the lowest possible occupant injuries and account for a number of constraints on the dummy motion and belt force. The design approach embraces three main steps: the determination of the reference signal, the derivation of linear controller design models, and the actual controller design.

The local dynamic behaviour, derived from the response in the chest acceleration on small perturbations in the belt force, is linear and second order and weakly depends on the operating point. ‘Measurements’ over a short time span are shown to be sufficient for a suitable identification of the required linear models. Control design based on these models resulted in a PI controller in combination with a second-order low-pass filter. The required bandwidth of the controller is in the range 200–400 Hz. Simulations with the controlled system show that the occupant injuries could be reduced significantly if appropriate sensors and actuators were available.

The presented approach for restraint controller design is powerful because of its simplicity and because of its widely known and accepted technique for controller design. The approach is very versatile, as long as a suitable reference signal can be determined. It has been shown that the approach can be seen as a design expedient, i.e. as a basis for the design of controllers for other than the considered restraint systems.

Research has been initiated to investigate the possibility of using the same approach to manipulate the belt, aiming at reduction in the chest deflection. Predictive control is being implemented to deal with constraints that cannot simply be translated into bounds on the reference signal. More insight has to be obtained into the relation between the injury measures, injury mechanisms, and the behaviour of the airbag and the belt. This insight is necessary for the design of controllers simultaneously to manipulate the airbag and belt with the purpose of minimizing more than one injury measure.

For several reasons, the designed controller cannot yet be implemented in a real vehicle. First of all, no appropriate hardware exists as yet. Secondly, the formulation of the control problem as a tracking problem implies that the reference signal has to be known at the very beginning of the crash. In the current approach, this means that the vehicle deceleration during the whole crash has to be available a priori. It is a topic of future research how to
determine, in the first few milliseconds of a crash, a suitable reference signal, using measurements of, for instance, the initial velocity, \(v_0\), the initial distance, \(d_0\), between the occupant and the steering wheel, and the vehicle decelerations in these first few milliseconds.

A further factor is that the present-day actuators cannot realize the belt forces that are prescribed by the controller, especially because their bandwidth is too low. Besides this, the prescribed forces are very high. This is a problem not only for the actuator but also for the occupant since these forces may result in injuries such as a broken clavicle. This again underlines the need for future research on a more realistic reference signal that not only reflects the mentioned constraints on the occupant speed and displacement at the end of the crash and on the maximum of the absolute chest acceleration but also accounts for requirements with respect to the maximum of the belt force.

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APPENDIX

Notation

- $c$, $\dot{c}$, $\ddot{c}$: chest acceleration, velocity, and displacement
- $\delta c$, $\delta \dot{c}$, $\delta \ddot{c}$: normalized perturbed chest acceleration, averaged normalized perturbed chest acceleration
- $e$: tracking error
- $f$: frequency
- $f_u$: undamped eigenfrequency
- $F$: belt force
- $F_{\text{eff}}$: effective belt force
- $GM$, $PM$: gain margin and phase margin
- $H(s)$: transfer function
- $K_s$: static gain
- $\zeta_0$: initial distance between the front of the chest and the steering wheel
- $M$: numerical crash model
- $P$, $\zeta_1$: controller parameters
- $r_c$, $r_f$, $r_e$: reference signal for the chest acceleration, velocity, and displacement
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>( s )</td>
<td>Laplace variable</td>
</tr>
<tr>
<td>( t )</td>
<td>time</td>
</tr>
<tr>
<td>( t_e )</td>
<td>point of time at which the crash is considered to be ended</td>
</tr>
<tr>
<td>( t_{5%} )</td>
<td>5 per cent settling time</td>
</tr>
<tr>
<td>( v_0 )</td>
<td>initial vehicle velocity</td>
</tr>
<tr>
<td>( x_1 )</td>
<td>belt outlet</td>
</tr>
<tr>
<td>( x_{\text{veh}}, \dot{x}<em>{\text{veh}}, \ddot{x}</em>{\text{veh}} )</td>
<td>vehicle acceleration, velocity, and displacement</td>
</tr>
<tr>
<td>( z )</td>
<td>damping factor</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>( \zeta )</td>
<td>system zero</td>
</tr>
<tr>
<td>( \tau )</td>
<td>point of time at which a perturbation is applied</td>
</tr>
<tr>
<td>( \omega_f, \zeta_f )</td>
<td>low-pass filter parameters</td>
</tr>
</tbody>
</table>

**Subscripts**

- \( a \): active restraint system
- \( i \): step size
- \( j \): point of application
- \( p \): passive restraint system