HOW IMPORTANT IS THE FRICTION MODEL ON THE MODELING OF ENERGY DISSIPATION?

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Abstract

Frictional forces arising from the relative motion of two contacting surfaces are a well-known source of energy dissipation. Sometimes this is an unwanted effect of the design, but it can also be intentionally used to increase the damping of a certain system in a simple and cost-effective way. In an earlier work the energy dissipation of a 1-degree of freedom (DOF) system with Coulomb friction has been analytically studied for a friction law with equal dynamic and static friction forces and with a static friction force larger than the dynamic friction force. Closed-form expressions for the maximum energy dissipation per cycle and the optimal friction force were obtained. In the present work numerical simulations are performed with several different friction models currently used in the literature. For the stick phase smooth approximations like viscous damping or the arctan function are considered and the non-smooth switch friction model. For the slip phase several models of the Stribeck effect are used. The goal of this study is, for a given stable periodic solution, to determine the influence of the friction model on the predicted energy dissipation per cycle and especially on the maximum energy dissipation and the optimum friction force.

Key words

Friction, Damping, Energy dissipation

1 Introduction

Frictional forces arising from the relative motion of two contacting surfaces are a well-known source of energy dissipation. Sometimes this is an unwanted effect of the design, but it can also be intentionally used to increase the damping of a certain system in a simple and cost-effective way. In a recent publication [Lopez, Busturia and Nijmeijer 2004] have studied the energy dissipated through friction for a mass-spring-damper system subjected to frictional contact with an oscillating base plane, figure 1. In that work the existence and stability of a periodic solution is proved and subsequently analytical expressions for the energy dissipated per cycle are derived. Expressions for the optimum friction force and maximum energy dissipation are also obtained as a function of the system parameters.

![Mass-spring-damper on an oscillating base plane.](image)

In [Lopez, Busturia and Nijmeijer 2004] the friction force is modeled using the classical Coulomb friction model. In practice, this model is too simplified representation of frictional interaction. In the same work a Coulomb friction model with different static and dynamic friction forces \( F_s \neq F_d \) is used for the system with \( k = 0 \) and \( c = 0 \); that is, a mass on an oscillating base plane, figure 2. It is shown there that for \( F_d < 0.64 F_s \) the maximum energy dissipation and the corresponding optimum friction force change. This gives an indication that the choice of friction model can have a big influence on the predicted energy dissipation.

The goal of the work presented here is to answer the following two questions:

- Is it necessary to model the stick phase accurately when the goal is to model energy dissipation?
- How does the friction law for the slip phase influence the predicted energy dissipation?
In order to answer these questions a single degree of freedom model has been studied with a mass on an oscillating base. Different friction models have been considered, as will be shown in the following sections. The stable periodic solutions have been found using a single-point shooting method. Once the periodic solution is known, the energy dissipated per cycle has been determined.

The paper is organized as follows. First the most important definitions and results from [Lopez, Busturia and Nijmeijer 2004] are summarized in section 2. In section 3 the influence of the modeling of the stick phase is analyzed. The modeling of the slip phase is studied in section 4 and in section 5 the results are discussed. Finally some comments on future work are given section 6.

2 Mass on an oscillating base

In the system of figure 2 the base plane oscillates with a displacement \( x_0(t) = X_0 \sin(\omega_0 t) \). If the classical Coulomb friction model as defined in equation 1 is used, closed-form expressions for the motion of mass \( m \) can be derived and consequently, the energy dissipated per cycle can be determined.

\[
F_r = \begin{cases} 
F_d \text{sign}(v) & |v| > \eta, \\
[-F_d, F_d] & |v| \leq \eta
\end{cases}
\]  
(1)

where \( v \) is the relative velocity between mass and base.

Two non-dimensional parameters are defined:

Normalized friction force:

\[
f_r = F_r / mX_0\omega_0^2
\]  
(2)

Normalized energy dissipation:

\[
e_d = E_d / mX_0^2\omega_0^2
\]  
(3)

where \( F_r \) and \( E_d \) are the friction force and the energy dissipated per cycle.

The dissipated energy, \( E_d \), has been calculated according to the following expression:

\[
E_d = \int_0^T -F_r(\dot{x}_0(t) - \dot{x}_2(t)) \, dt
\]  
(4)

where \( T \) is the period length of the periodic solution.

An expression for the energy dissipation as a function of the friction force has been found in [Lopez, Busturia and Nijmeijer 2004] and is plotted in figure 3.

\[\text{Figure 3. Normalized energy dissipation vs. normalized friction force.}\]

It is also shown that the normalized optimum friction force and maximum energy dissipation are:

\[
f_r|_{\text{max}} = \sqrt{2} / \pi
\]  
(5)

\[
e_d|_{\text{max}} = 4 / \pi
\]  
(6)

In the following sections the influence of the friction model on the form of the curve in figure 3 and on the optimum values given by equations 5 and 6 will be analyzed.

3 Influence of stick phase

The stick phase has been modeled using three different friction laws: viscous damping, arctan function and switch model (see [Leine, 2000] [Leine and Nijmeijer 2004]). In all three cases a constant friction force is used in the slip phase and \( F_s = F_d \). The corresponding equations are:

Viscous damping:

\[
F_r = \begin{cases} 
F_d \text{sign}(v) & |v| > \eta, \\
F_d / \eta v & |v| \leq \eta
\end{cases}
\]  
(7)
Arctan function:

\[ F_r = F_d \frac{2}{\pi} \arctan(\epsilon v) \]  

Switch model:

\[ F_r = \begin{cases} 
F_d \text{sign}(v) & |v| > \eta, \\
\min \{|F_{\text{ex}}|, F_d\} \text{sign}(F_{\text{ex}}) + \alpha v & |v| \leq \eta 
\end{cases} \]  

The zero velocity interval \( \eta \) is normally chosen such that \( 1 >> \eta > T_0 \), where \( T_0 \) is the tolerance of the integration method. The parameter \( \epsilon \) in the arctan function is the steepness parameter and \( \alpha > 0 \).

The switch model with \( \eta = 0.001 \) and \( \alpha = 10 \) gives a result which is almost identical to the analytical solution and the energy dissipated per cycle as a function of the friction force is very much the same as the analytical result shown in figure 3. This is the analytical curve plotted in figures 4 and 5.

Figure 4. Normalized energy dissipation vs normalized friction force. Viscous damping during stick-phase.

The normalized energy dissipation predicted with viscous damping for the stick phase is shown in figure 4 for several different values of the parameter \( \eta \), which is given as a function of the amplitude of the base velocity, \( \omega_0 X_0 \). As expected, for \( \eta = \omega_0 X_0 \) the predicted energy dissipation is very different from the analytical result for Coulomb friction and increases as the friction force increases because the viscous damping coefficient increases as well. But as \( \eta \) decreases the energy dissipation converges to the analytical result and for \( \eta = 0.001 \omega_0 X_0 \) the two curves are nearly the same. For the case where the arctan function is used to model the stick phase, similar results are obtained as can be seen in figure 5. In this case the steepness parameter \( \epsilon \) has been varied from 1 to 1000. Again the predicted dissipated energy converges rapidly to the analytical result.

\[ g(v) = F_s \left[ \gamma + (1 - \gamma) e^{-|v/v_s|^\delta} \right] \]  

Stribeck1:

\[ \gamma = \frac{F_s}{F_d} \]  

Stribeck2:

\[ g(v) = F_s \left[ \gamma + (1 - \gamma) \frac{1}{1 + |v/v_s|^\delta} \right] \]  

where \( v_s > 0 \) is called the Stribeck velocity, \( \delta \) is the shaping parameter of the Stribeck curve and \( \gamma = \frac{F_s}{F_d} \) with \( F_s \) and \( F_d \) the static and dynamic friction force.

4 Influence of slip phase

The slip phase has been modeled using two different parametrizations of the Stribeck effect discussed in [Putra, 2004] [Putra and Nijmeijer, 2004]. In the following we will refer to these approximations as Stribeck1 and Stribeck2. In both cases the stick phase has been modeled using the switch model. The equations used are:

It can be concluded that using a viscous damping model or the arctan function to model the stick-phase will not influence the predicted energy dissipation if a sufficiently small \( \eta \) or a sufficiently large \( \epsilon \) are used. But it does not seem interesting to do this because the switch model is computationally more efficient and predicts an energy dissipation curve almost identical to the analytical result.

Figure 5. Normalized energy dissipation vs normalized friction force. Arctan function for stick-phase.
1 has been chosen and \( v_s \) has been varied between 1 and 0.001. \( v_s = 0.001 \) gives a very steep decay of the friction force and \( v_s = 1 \) gives a very slow decay.

Two different values of \( \gamma \), 0.7 and 0.5, have been considered as was done in [Lopez, Busturia and Nijmeijer 2004]. Relationships between energy dissipation and static friction force similar to the one shown in 3 were derived for \( \gamma = 0.7 \) and \( \gamma = 0.5 \). Those are the analytical curves shown in figures 6 and 8 and figures 7 and 9.

The results of the simulations with the two Stribeck models and with the two values of \( \gamma \) are shown in figures 6 to 9. It should be noted that all four plots show the normalized energy dissipation as a function of the normalized static friction force.

In all four cases the dissipated energy curve for \( v_s = 0.001 \) is very similar to the analytical prediction. This result validates the analytical derivations presented in [Lopez, Busturia and Nijmeijer 2004], since it is clear that the numerical prediction converges to the analytical result for very small values of \( v_s \).

The energy dissipation curves predicted with the two Stribeck models are very similar, although the maximum energy dissipation predicted with Stribeck1 seems to be a little more sensitive to changes in \( v_s \).

For the two values of \( \gamma \) considered the maximum energy dissipation goes through a minimum and the optimum friction force decreases as \( v_s \) increases. Mainly the predicted optimum friction force varies significantly when compared to the value predicted by the
classical Coulomb friction model. As one would expect, for very large values of $v_s$ the energy dissipation curve tends to be equal to the analytical result for $F_s = F_d$.

In practice, viscous effects are also observed as the relative velocity increases. In order to model these effects a viscous damping term has been added to the Stribeck curve as shown below.

$$F_r = \begin{cases} g(v) \text{sign}(v) + dv & |v| > \eta, \\ \min \{|F_{ex}|, F_s\} \text{sign}(F_{ex}) + \alpha v & |v| \leq \eta \end{cases}$$

(13)

where $d$ is the damping coefficient.

The results in the following figures correspond to the parametrization Strubeck 1. The parametrization named Stribeck 2 has not been further considered due to the small differences in the results obtained with both parametrizations.

Figure 10. Normalized energy dissipation vs normalized friction force. Strubeck1 and viscous damping for $v \neq 0$ with $\gamma = 0.7$, $\delta = 1$ and $v_s = 0.001$.

The corresponding curves of energy dissipation versus static friction force can be seen in figures 10 to 13 for $\gamma = 0.7, 0.5$, $\delta = 1$ and $v_s = 0.001, 0.01$. The damping coefficient $d$ has been varied between 0.001 and 1 Ns/m.

The main effect of including viscous damping is that the energy dissipated in the continuous sliding region (low static friction force) increases and decreases in the stick-slip region (high static friction force) as the viscous damping coefficient increases. For low values of the static friction force and large values of the viscous damping coefficient ($d = 1$) the viscous term is dominant, which explains why the dissipated energy increases as the static friction force decreases.

The values of the damping coefficient $d$ found in practice [Putra, 2004] [Putra and Nijmeijer, 2004] are between 0.01 and 0.1 Ns/m. For this range of values of $d$ the maximum energy dissipation increases and the optimum friction force decreases as $d$ increases, although these effects are less pronounced than when the Strubeck velocity $v_s$ is varied.

Figure 11. Normalized energy dissipation vs normalized friction force. Strubeck1 and viscous damping for $v \neq 0$ with $\gamma = 0.7$, $\delta = 1$ and $v_s = 0.01$.

Figure 12. Normalized energy dissipation vs normalized friction force. Strubeck1 and viscous damping for $v \neq 0$ with $\gamma = 0.5$, $\delta = 1$ and $v_s = 0.001$.

Summarizing, the choice of friction model for the slip phase has a significant influence on the predicted curve of dissipated energy versus friction force and also on the optimum values of friction force and energy dissipation. Both the Strubeck effect and including viscous damping have an influence on the results.

5 Conclusions

In the light of the results presented in the preceding sections the following conclusions can be derived. The use of continuous approximations to model the stick phase (viscous damping, arctan function) has little
influence on the predicted energy dissipation when realistic values of the parameters are chosen. It does not seem interesting to use these approximations since the switch model is computationally cheaper and a more realistic model of the stick phase.

Taking the Stribeck effect into account has a significant influence on the predicted energy dissipation. The predicted maximum energy dissipation and optimum friction force depend on the Stribeck velocity $v_s$. The optimum friction force is more sensitive to variations of $v_s$ than the maximum energy dissipation.

Including viscous damping for $v \neq 0$ has also a significant effect on the energy dissipation. Again the optimum friction force is more sensitive to changes in the viscous damping factor $d$ than the maximum energy dissipation, provided one stays in the range where friction is dominant. It is important to know what the realistic values for the damping coefficient are.

In consequence, the values of maximum energy dissipation and optimal friction force derived from the analytical study must be regarded as upper-bounds. This is specially important for the optimum friction force, which strongly depends on $v_s$ and $d$. This means that, when designing a friction damper, the friction characteristic has to be accurately known in order to determine the right optimum for the friction force.

The optimum stays in the continuous sliding region (or just on the border between sliding and stick-slip). This fact together with the conclusions from the analysis of the stick phase indicates that the exact behavior during the stick-phase will not significantly influence the prediction of the maximum energy dissipation and the optimal friction force. These can be an indication that modeling the pre-sliding behavior with a LuGre type model, for example, might not be necessary for this type of analysis. However, further research is needed to clarify this point.

6 Future Work
The results obtained here have been obtained for a mass on an oscillating base plane. The next step is to perform similar calculations for the mass-spring-damper system of figure 1.

In addition to this, experiments will be carried out to validate the theoretical results presented here.

References


