Discrete crack modelling of ductile fracture driven by non-local softening plasticity

J. Mediavilla¹,²,‡, R. H. J. Peerlings²,*,† and M. G. D. Geers²,§

¹Netherlands Institute for Metals Research, Rotterdamseweg 137, 2628 AL Delft, The Netherlands
²Department of Mechanical Engineering, Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands

SUMMARY

A combined approach towards ductile damage and fracture is presented, in the sense that a continuous material degradation is coupled with a discrete crack description for large deformations. Material degradation is modelled by a gradient enhanced damage-hyperelastoplasticity model. It is assumed that failure occurs solely due to plastic straining, which is particularly relevant for shear dominated problems, where the effect of the hydrostatic stress in triggering failure is less important. The gradient enhancement eliminates pathological localization effects which would normally result from the damage influence. Discrete cracks appear in the final stage of local material failure, when the damage has become critical. The rate and the direction of crack propagation depend on the evolution of the damage field variable, which in turn depends on the type of loading. In a large strain finite element framework, remeshing allows to incorporate the changing crack geometry and prevents severe element distortion. Attention is focused on the robustness of the computations, where the transfer of variables, which is needed after each remeshing, plays a crucial role. Numerical examples are shown and comparisons are made with published experimental results. Copyright © 2005 John Wiley & Sons, Ltd.

KEY WORDS: ductile damage; ductile fracture; non-local damage; fracture mechanics; finite element method; remeshing

1. INTRODUCTION

Ductile fracture is governed by microscopic processes such as void nucleation, void growth and coalescence, which ultimately lead to crack initiation and propagation. Macroscopically,
these processes become manifest in overall material softening, which leads to strain localization and eventually to the formation of discrete cracks. At this macroscopic scale, traditionally two different approaches have been followed in the modelling of fracture: continuous and discontinuous.

Continuous approaches describe the degradation processes which lead to the formation of new crack surface, either in a phenomenological way, as in continuum damage mechanics \[1, 2\], or in a micromechanical setting \[3–7\]. One or several variables—often called damage variables—are used to capture the essential features of the damage process. These variables generally have a weakening effect on the yield stress (and sometimes on the elastic properties) and therefore result in a strain-softening response.

Regularization techniques must be used to overcome the pathological localization and, in a finite element context, mesh dependence associated with softening materials, resulting in gradient models, integral-type non-local models, rate-dependent and micro-polar models (see for instance Reference \[8\] for a review). These enhanced formulations supply the continuum model with an internal length scale, for which partial physical interpretations have been suggested (e.g. average size of or distance between the largest inhomogeneities, voids, distributed cracks, etc.) and can be well adapted to a (numerical) finite strain setting \[9\]. Gradient models are especially interesting to model ductile failure, since they can account for non-local micro-void interactions \[10\]. Gradient models can be classified into implicit and explicit models, depending on whether the non-local variable is an explicit functional of its local counterpart or not \[11, 12\].

A discontinuous approach is typically used in fracture mechanics, where the focus is on modelling discrete cracks. The mechanical properties of the material next to the crack are assumed to remain intact, i.e. there is no material softening. Additional assumptions are made for initiation and propagation criteria (e.g. critical energy release rate, crack tip opening displacement) as well as for crack growth direction criteria (e.g. maximum circumferential stress). In a finite element framework the proper modelling of discrete cracks of any arbitrary geometry requires either remeshing \[23\] or embedding techniques such as XFEM \[24\], which allow to insert discontinuities in an existing mesh. Remeshing adapts the mesh topology to the crack geometry, whereas XFEM (based on the Partition of Unity concept) enriches the interpolation functions with displacement jumps.

To model the entire fracture process, from material degradation, via crack initiation, to crack propagation, both continuous softening and discontinuous crack growth should ideally be used in a combined continuous–discontinuous model. The transition between the continuous and discontinuous models is made at complete, local failure of the continuous material. The precise location of crack initiation is therefore the outcome of the damage evolution process and no initial crack needs to be defined. The interplay between the discrete crack(s) and the surrounding degrading material governs the crack growth behaviour and cracks can be seen as the limit result of strain localization. A finite sized process zone develops in a natural fashion ahead of the crack and the influence of initially present damage can be included in a straightforward manner. Since the continuous part of the description will generally exhibit strain...
softening, a regularization method must be used. A few attempts have been made to model cracks using gradient damage, either by employing a variable internal length parameter [25] or by the removal of failed elements [26], but these methods rely on a continuum description and result in a crude representation of the crack, respectively. More recently, XFEM has been applied to model discrete cracks in a regularized softening continuum [27,28]. Yet, XFEM loses part of its appeal in a large strain framework, since remeshing is nevertheless needed to avoid large element distortions.

The main motivation of this paper is to provide a combined continuous–discontinuous framework for the modelling of ductile fracture. Ductile damage is modelled by a strongly non-local gradient hyperelastoplasticity model [21], the main features of which are that: (i) it accounts for long range microstructural interactions (strong non-locality); (ii) it does not suffer from pathological mesh dependence; (iii) it can deal with large strains and rotations; (iv) it shows convergence to a discrete crack upon complete failure. It is emphasized that the damage field which is used to model the plastic degradation process influences the local yield stress and thus causes strain softening. This is unlike frameworks which use the damage field merely as an indicator for fracture, e.g. References [29,30]. Upon material failure, a discrete crack is introduced, for which a new mesh is created. Remeshing is simultaneously used to eliminate large element distortions. The combined approach proposed in this contribution is intended to be used whenever the size of the fracture process zone cannot be neglected and macroscopic cracks are initiated in locations which are not known a priori.

The structure of this paper is as follows. In Section 2 the material model is described. The numerical aspects are discussed in Section 3, where the emphasis lies on the robustness of the computations and in particular the correct treatment of transfer errors and the creation of new crack surface, which are crucial for the stability of the computations. In Section 4 a few applications are shown.

2. CONSTITUTIVE MODELLING

2.1. Continuum model

Ductile damage is modelled here by means of a gradient enhanced hyperelastoplastic model proposed by Geers et al. [21], which extends an earlier small strain model [20] to a large strain setting. In this model, isotropic damage is introduced as a multiplicative degradation of the yield stress. It is based on the $J_2$ hyperelastoplasticity framework proposed by Simo and Miehe [31]. For the sake of completeness, the governing model equations are briefly discussed. The interested reader is referred to Reference [21] for a detailed description.

The deformation gradient $F$ is split into an elastic part $F_e$ and a plastic part $F_p$ according to

$$F = F_e \cdot F_p$$

The hyperelastic relationship between stress and elastic deformation reads

$$\tau = \frac{K}{2} (J^2 - 1) I + G \tilde{b}_e^d$$

where $\tau$ is the Kirchhoff stress tensor, $J$ is the volume change ratio ($J = \det(F)$) and $\tilde{b}_e^d$ is the deviatoric part of the isochoric elastic left Cauchy–Green tensor, i.e. $\tilde{b}_e^d = \tilde{b}_e - \frac{1}{3} \text{tr}(\tilde{b}_e) I$. 

\[ \bar{b}_e = J^{-2/3} F_e \cdot F_e^T, \]

where \( F_e \) is the elastic deformation gradient, \( J \) is the determinant of \( F_e \), \( K \) and \( G \) are the bulk modulus and the shear modulus, respectively. Plastic flow is assumed to be isochoric. The flow rule is given by

\[ \nabla \bar{b}_e = -3 \dot{\dot{\varepsilon}}_p \frac{\tau_d}{\tau_{eq}} \]

where \( \nabla \bar{b}_e \) is the objective Lie derivative of the elastic left Cauchy–Green tensor and the superscript \( (\cdot)^d \) denotes the deviatoric part of a second-order tensor; \( \dot{\dot{\varepsilon}}_p \) is the effective plastic strain rate.

\[ \dot{\dot{\varepsilon}}_p = \sqrt{\frac{1}{6} \left[ \nabla \bar{b}_e \right]^d : \left[ \nabla \bar{b}_e \right]^d} \]

and \( \tau_{eq} \) is the equivalent von Mises stress

\[ \tau_{eq} = \sqrt{\frac{2}{3} \tau_d : \tau_d} \]

An isotropic ductile damage variable \( \omega_p \) (\( 0 \leq \omega_p \leq 1 \)) is introduced as a softening factor of the virgin (effective) strain-hardening curve \( \hat{\tau}_y(\omega_p) \), leading to a combined hardening–softening yield stress which reads

\[ \tau_y = (1 - \omega_p) \hat{\tau}_y \]

or in rate form

\[ \dot{\tau}_y = (1 - \omega_p) \dot{\hat{\tau}}_y - \dot{\hat{\tau}}_y \dot{\omega}_p \]

In the computations of Section 4, linear hardening has been assumed for the undamaged material, i.e. \( \dot{\hat{\tau}}_y = h \dot{\varepsilon}_p \), with \( h \) a positive constant.

Plastic flow fulfills the standard Kuhn–Tucker loading–unloading conditions

\[ \dot{\varepsilon}_p \geq 0, \quad \phi \leq 0, \quad \dot{\varepsilon}_p \phi = 0 \]

where the yield function is defined as \( \phi = \tau_{eq} - \tau_y \).

The damage variable \( \omega_p \) is defined such that \( \omega_p = 0 \) represents the undamaged state, whereas \( \omega_p = 1 \) stands for complete failure, at which point the yield surface collapses to a singular point of zero yield stress. Upon loading, the material may undergo four different stages: (i) elastic behaviour, (ii) strain hardening, (iii) strain softening, (iv) failure and formation of a discrete crack (see Figure 1, in which these stages are indicated by E, H, S and D). This is a substantial difference with elastoplastic-fracture models, where the transition from strain-hardening to a discrete crack is introduced in an uncoupled, discrete manner [29, 30], without prior softening. The final hardening–softening response of the model is the combined effect of the usual hardening of the material and the softening influence of damage.

Damage evolution is described here by a phenomenological law, driven by a history variable \( \kappa \), which in general depends on some equivalent (scalar) measure of stress or strain. In rate form, the damage evolution equation reads

\[ \dot{\omega}_p = h_{\omega}(\omega_p) \dot{\kappa} \]
In the applications section of this paper (Section 4), a non-linear damage evolution law has been used (see Figure 2), which reads

$$ h_\omega(\omega_p) = \begin{cases} \frac{3}{\tanh(3)(\kappa_c - \kappa_i)}(1 - \tanh^2(3)(2\omega_p - 1)^2) & \text{if } \kappa_i \leq \kappa < \kappa_c \\ 0 & \text{otherwise} \end{cases} \quad (10) $$

where the parameters \(\kappa_i\) and \(\kappa_c\) are the initial and critical value of \(\kappa\), respectively. Equation (10) results in a slow initial and final damage growth, which can be computationally more robust, since it does not show any discontinuities in the damage growth (Figure 2).

To avoid pathological localization effects due to material softening, the damage driving variable \(\kappa\) is computed from a non-local variable \(\bar{\zeta}\) via a separate set of Kuhn–Tucker loading–unloading conditions

$$ \dot{\kappa} \geq 0, \quad \dot{\bar{\zeta}} \leq 0, \quad \dot{\kappa}(\bar{\zeta} - \kappa) = 0 \quad (11) $$
and the initial value $\kappa = \kappa_i$. The non-local variable $\bar{Z}$ is obtained from the local variable $Z$ by solving a Helmholtz type Partial Differential Equation (PDE)

$$\bar{Z} - \ell^2 \nabla^2 \bar{Z} = Z$$

(12)

In this equation $\ell$ is the internal length parameter, which—indirectly—sets the width of the localization band. The physical interpretation of $\ell$ can be related to void interactions. This second-order PDE is complemented by the Neumann boundary condition

$$\nabla \bar{Z} \cdot n = 0$$

(13)

on the boundary $\Gamma$ of the current domain, including the generated crack faces; $n$ is the normal to $\Gamma$.

It has been demonstrated in Reference [11] that the above implicit definition of $\bar{Z}$ is truly non-local, in the sense that the behaviour at a point can be written as a weighted average over the entire problem domain. This is an important requirement when modelling cracks (see Reference [32]). Unlike non-local models of the integral type, however, the use of Equation (12) does not lead to integro-differential equations, but simply to an additional elliptic PDE which must be solved simultaneously with the equilibrium equation.

Both governing PDEs, i.e. the equilibrium equation

$$\nabla \cdot \sigma = 0$$

(14)

and the additional non-locality equation (12), are expressed with respect to the current (Eulerian) configuration. This is more convenient than a material description in view of the required remeshing and transfer, as will be seen later in this paper.

In what follows we assume the local variable $Z$, which ultimately drives the damage growth, to be the equivalent plastic strain:

$$Z = \varepsilon_p$$

(15)

This is a reasonable assumption for problems in which damage growth is mainly driven by shear strains. For problems with a high stress triaxiality, an influence of the hydrostatic stress may be expected, for which the present formulation can be extended (see Reference [33]). Here, however, our attention will be focused on the first class of problems.

2.2. Crack modelling

The material softening which is associated with the damage modelling as discussed in the previous section results in local material failure in a natural way. At some stage of the damage process (when $\omega_p = \omega_{\text{crit}} = 1$) the degraded yield stress $\tau_y$ vanishes and the material cannot sustain stress anymore. At this point a crack is initiated. From this moment on, the maximum damage will generally occur near the tip of this crack and therefore lead to growth of the crack when it becomes critical (i.e. equal to one). Crack growth thus is the natural consequence of the degrading constitutive response of the material and hence no separate criteria are needed to determine the direction and the rate of crack growth. This integrated approach towards damage and fracture reflects more closely the underlying physical process of ductile fracture (i.e. initiation, growth and coalescence of voids and microcracks) compared to, e.g. a fracture mechanics description. An essential requirement, however, is an accurate model for the damage process upon arbitrary loading paths.
The finite element implementation of the non-local damage-plasticity model used here has been described in detail by Geers et al. [21]. The governing PDEs of the continuum problem, the equilibrium equation (14) and the non-local averaging equation (12), are cast in a weak form and discretized by finite element shape functions. Note that these equations are coupled and must thus be solved simultaneously.

The numerical implementation presented by Geers et al. [21] uses an Updated Lagrangian description. Stresses are updated by a radial return mapping, based on an implicit backward-Euler rule. A consistent algorithmic tangent operator is obtained by linearization of the discrete time equations. The linear system is solved repeatedly within a standard Newton–Raphson procedure until convergence. Upon convergence, the time (and loads) can be incremented and the iterative procedure repeats itself. For details on the algorithms used, see Reference [21]. Here we detail only aspects of the present implementation which are not treated in Reference [21], i.e. the integration of the damage evolution law and hardening relation (Section 3.1), differences arising in the tangents (Section 3.2), the treatment of crack initiation and crack propagation (Section 3.3), as well as remeshing and state variable transfer (Section 3.4).

3.1. Integration of damage evolution and stress update

Unlike in Reference [21], the evolution of the yield stress $\tau_y$ and the damage variable $\omega_p$ are here written in a rate form. This form of the evolution equations is more convenient from a numerical perspective, as will be explained in Section 3.4.3.

A backward-Euler integration rule is employed for the integration of the damage evolution law Equation (9)

$$\omega_p = \omega_p^t + h\omega(\omega_p)\Delta k$$

where $\omega_p$ denotes the new damage variable at the end of the present time increment, and $\omega_p^t$ the known value at the beginning of the increment. The value of $\omega_p$ given by this rule must obviously be limited to $\omega_p = 1$, preventing it to exceed this critical level. Note that Equation (16) must be solved iteratively (at integration point level), for example by a Newton–Raphson algorithm.

The rate form of the combined hardening–softening evolution law, Equation (7), is also integrated using a backward-Euler rule, yielding

$$\tau_y = \tau_y^t + (1 - \omega_p)h\Delta\omega - \dot{\tau}_y\Delta\omega_p$$

where $\dot{\tau}_y$ denotes the time derivative of the yield stress.

Upon elimination of $\dot{\tau}_y$, this expression (17) can be rewritten as

$$\tau_y = \tau_y^t \frac{(1 - \omega_p)}{(1 - \omega_p^t)} + \frac{(1 - \omega_p)^2}{(1 - \omega_p^t)} h\Delta\omega_p$$

Enforcing the consistency condition now provides the increment of the effective plastic strain $\Delta\varepsilon_p = \Delta\varepsilon$ according to the radial return (see Reference [21])

$$\tau_y + 3GJ^{-2/3}\Delta\varepsilon_p = \tau_{eq}$$

where \( \tau_{\text{eq}} \) is the equivalent von-Mises stress corresponding to the trial Kirchhoff stress tensor \( \mathbf{\tau} \), for which the incremental deformation is entirely elastic. Substitution of Equation (18) in Equation (19) and reordering yields

\[
\Delta \epsilon_p = \frac{\phi}{(1 - \omega_p)^2 h + 3GJ^{-2/3}} \tag{20}
\]

where the trial value of the yield function \( \phi \) is defined as

\[
\phi = \tau_{\text{eq}} - \tau_y \frac{(1 - \omega_p)}{(1 - \omega_p')}. \tag{21}
\]

To prevent locking effects in the post-failure regime, i.e. when \( \omega_p \) reaches one (to be detected at a value close to one, e.g. \( \omega_{\text{crit}} = 0.99 \), in order to prevent a poorly-conditioned stiffness matrix), the yield stress is no longer updated, thus \( \tau_y = \tau_y' \). In this regime the material flows in an ideally plastic manner at a constant, negligible flow stress. This means that

\[
\Delta \epsilon_p = \frac{\phi}{3GJ^{-2/3}} \tag{22}
\]

Furthermore, the local damage driving variable is no longer updated, i.e. \( z = \tilde{z} \), and hence differs from \( \epsilon_p \), which can still increase. As a result, the convergence of the solution is not affected by the large variations of \( \epsilon_p \) which may occur during this post-failure regime. A beneficial feature of the radial-return algorithm used for the stress updated is that the return projection always leads to a solution on the updated yield surface, even close to failure when the yield surface is small.

### 3.2. Consistent tangent operators

The tangent operators used in the global Newton–Raphson iterations differ somewhat from those obtained in Reference [21], as a consequence of the rate form of the hardening law and the damage evolution law. The modifications are reported below.

The variation of the damage variable according to (16) reads

\[
\delta \omega_p = \frac{\partial h_o}{\partial \omega_p} \delta \omega_p \Delta \kappa + h_o \frac{\partial \kappa}{\partial \tilde{z}} \delta \tilde{z} \tag{23}
\]

or, after rearranging,

\[
\delta \omega_p = c_{\omega} \delta \tilde{z} \quad \text{with} \quad c_{\omega} = \frac{h_o \frac{\partial \kappa}{\partial \tilde{z}}}{1 - \frac{\partial h_o}{\partial \omega_p} \Delta \kappa} \tag{24}
\]

The stress tensor has to be linearized in terms of the displacement variation \( \delta \mathbf{u} \) and the variation of the non-local field \( \delta \tilde{Z} \). The equation which relates the variation of stress \( \delta \mathbf{\tau} \) to \( \delta \mathbf{u} \) and \( \delta \tilde{Z} \), given in Reference [21], is retained and reads

\[
\delta \mathbf{\tau} = 4 \mathbf{C}_{\text{ep}} : \mathbf{L}^\dagger \delta \tilde{Z} - c_S \mathbf{r}^d \delta \tilde{Z} \tag{25}
\]
where \( \mathbf{L}_\delta^T = (\nabla \delta \mathbf{u}) \) and \( 4 \mathbf{C}_{\text{ep}} \) is the elastoplastic tangent operator as defined in Reference [21]. However, the constants \( c_1 \) and \( c_5 \) featuring in the definition of \( 4 \mathbf{C}_{\text{ep}} \) and in (25), respectively, must be reformulated as

\[
c_1 = 3 + \frac{(1 - \omega_p)^2}{(1 - \omega_p^k)} \frac{hJ^{2/3}}{G} \]

\[
c_5 = \frac{\varepsilon_{eq}^f + 2h\Delta \varepsilon_p(1 - \omega_p)}{c_1(1 - \omega_p^k)^{*\varepsilon_{eq}}} c_{\delta \omega} \tag{26}
\]

with \( c_{\delta \omega} \) given by Equation (24). The variation of the local variable \( z \) is given by [21]

\[
\delta z = \mathbf{H} : \mathbf{L}_\delta^T + c_6 \delta \mathbf{Z} \tag{27}
\]

with \( \mathbf{H} \) as defined in Reference [21] and \( c_6 \) computed using \( c_5 (26) \) as also indicated in Reference [21]. Upon insertion of definitions (25) and (27), the algorithm described in Reference [21] can also be used for the present modelling.

### 3.3. Prediction of crack initiation and propagation

The initiation and propagation of cracks is evaluated at the end of every time increment. In order to capture the exact moment when cracks initiate, the loading steps should not be too large. In practice, cracks usually initiate at the domain boundary. For this reason, the trigger for crack initiation is based on the critical nature of the damage values at the boundary nodes, which are obtained by extrapolation from the Gauss points. When the nodal damage value \( \omega_p \) becomes larger than the critical value \( \omega_{p \text{crit}} \), a crack is inserted in the geometry of the problem and the new domain is fully remeshed. Crack propagation is treated likewise, i.e. a new crack segment is added when the existing crack tip damage value reaches \( \omega_{p \text{crit}} \). This initiation–propagation method is quite reliable, because the damage gradients in the vicinity of the crack tip tend to be rather mild for the problems of interest.

The crack direction, defined by the angle \( \Theta_{\text{dir}} \), is given by the direction of the maximum damage in front of the crack tip. Numerically \( \Theta_{\text{dir}} \) is computed as the median of the angles \( \Theta_i \) for which the damage is maximum at different distances \( d_i \) from the crack tip, obtained by evaluating \( \omega_p \) at a number of discrete sample points (see Figure 3 and References [29, 30]). This method ensures that the crack is extended in the direction which is most affected by damage. On the other hand, it avoids abrupt changes in the crack orientation due to local (numerical) variations.

It should be emphasized that, contrary to what happens for elastoplastic models without a damage influence, stresses have already been reduced to almost zero at the crack tip and no large stress redistributions are therefore required upon crack growth. This does not imply, however, that the formation of a discrete crack can be neglected altogether, as is done in continuous approaches to fracture. Not introducing a crack when \( \omega_p = \omega_{p \text{crit}} \) leads to excessive straining, since the material across the damage zone remains kinematically connected (at almost zero stress levels), and consequently to an unrealistic lateral extension of the damage field. Whereas this was already noted and considered to be undesirable for small strains [25], it is even more troublesome for large strains, where large element distortions may result in poor convergence. This is illustrated in Figure 4, which shows a benchmark problem on a double-notched plain strain specimen, to be discussed later in Section 4.1, where crack growth has been prevented.
3.4. Robust crack propagation algorithm

At each increment of crack growth, which has a fixed size, a new mesh is generated. The history dependence of the constitutive model requires information on the deformation and damage history of the material to be transferred to the new mesh before the simulation can continue. At this point the robustness of the simulation may be compromised by two factors: (i) inaccuracies and inconsistencies due to the transfer of state variables and (ii) unbalance due to the creation of new free surface and the resulting change of boundary conditions (vanishing surface tractions). If no special measures are taken, these disturbances occur at exactly the same time in the computation. As a consequence, computations easily break down, even for small crack increments and fine meshes. A similar lack of numerical stability was observed for the uncoupled damage approach which was followed in Reference [29]. There, it was effectively removed by uncoupling the two sources of unbalance—transfer and crack growth—and dealing with them separately. It should be mentioned that the present, fully coupled, damage framework is less sensitive to these sources of unbalance during the crack opening phase, because stresses across the crack have already been relaxed due to the damage influence. As a result, the stress redistributions which occur when a new crack increment is inserted are much smaller compared
to an uncoupled approach. Nevertheless, the strategy proposed in Reference [29] is adopted here as well, since it ensures a high level of robustness of the computations. The individual elements of the resulting algorithm are explained in detail below.

3.4.1. Remeshing. Full remeshing is done to accommodate cracks with arbitrary paths in a finite element mesh, and to simultaneously eliminate large element distortions. Although in principle only the region adjacent to the crack tip needs to be remeshed (local remeshing), in a large strain framework the use of full remeshing keeps the mesh in an overall better shape, resulting in a better convergence rate and higher accuracy. However, too frequent remeshing also leads to a loss of accuracy due to transfer errors. A powerful standard quadrilateral mesher is employed, which is capable to deal with complex domains that may result from the presence of cracks [34]. The mesher also allows to define regions with a required higher element density, e.g. at the crack tip.

The domain which is to be meshed is defined by the boundary segments of the existing mesh and the newly predicted crack segment. A mesh which conforms to the newly introduced crack segment is created, but the faces of the new crack segment initially remain connected. This ensures a proper transfer of state variables prior to the update of the geometry of the problem itself (i.e. opening of the new crack segment; see also Reference [29]).

3.4.2. Transfer of state variables. Since the material model at hand (gradient enhanced hyper- elastoplasticity) is history dependent, state variables must be transferred from the old to the new mesh. Most of these variables are available at the Gauss points of the previous mesh and must be mapped to the Gauss points of the newly created mesh. The transfer operator adopted has been described by Perić et al. [35, Section 3.1]. It uses subsequent interpolation and extrapolation operations to estimate the values of the relevant fields in the new Gauss points (see also Reference [29]).

Apart from Gauss-point data, data stored at the nodes of the finite element mesh generally need to be transferred as well. In the present situation, this would imply the transfer of the displacements and the non-local variable \( \bar{z} \). However, transfer of the displacements is not necessary in the present implementation because an updated Lagrangian formulation is used. This means that remeshing is done in the deformed configuration and only the incremental nodal displacements with respect to this configuration, \( \Delta \mathbf{u} \), are relevant for the sequel of the computations. These incremental displacements are—by definition—equal to zero in the remeshed state and thus do not need to be transferred, see also Reference [29].

Transfer of the nodal values of \( \bar{Z} \) can also be avoided by reconstructing them from the transferred local variables \( \bar{z}^{T} \) in the Gauss points, by solving the discretized form of the linear PDE which relates \( \bar{z} \) and \( \bar{z}' \), Equation (12). The solution of this global problem merely adds one iteration to the computations for every remeshing step. It ensures that the local and non-local effective plastic strain fields are consistent right from the start of the new increment and thus eliminates a possible source of numerical instabilities. With the nodal values of \( \bar{Z} \) now known, the integration point values of \( \bar{z} \), and hence the history variable \( \kappa \), can be updated using the element shape functions.

\(^{1}\)Transferred quantities are denoted in the following by a prime (').
The determination of new Gauss point data from discrete values at the old Gauss points inevitably introduces a transfer error. In particular, the transfer operator which we use here—and most other transfer operators proposed in literature—shows an artificial diffusion, particularly for relatively coarse meshes. As a consequence of these errors, a set of transferred variables generally no longer satisfies the constitutive equations which internally relate these variables. If not corrected, these violations of the constitutive equations, which are further denoted as inconsistencies, may easily induce a poor convergence or even divergence of subsequent loading increments [35–37]. Inconsistencies, which originate from the fact that non-linear relations among variables are not carried over by the—linear—transfer operator (see Reference [29]), may be avoided by transferring a minimal set of independent state variables and reconstructing the remaining (dependent) fields via the constitutive relations.

3.4.3. Restoring equilibrium. Apart from inconsistencies in the local state variables, transfer will generally also lead to loss of equilibrium, which can be regarded as a global inconsistency. To remove the unbalance introduced by the transfer, an equilibrium step is performed in which there is no change in the external loading, nor in the boundary conditions. In order to guarantee convergence in this step, an elastic response is assumed, i.e. \(\varepsilon_p\) and \(\omega_p\) are kept constant. This is justified since it is not a physical step, but merely a way to remove artificial, numerical unbalances.

The elastic iterations result in slight readjustments in the nodal positions and the non-local variable \(\bar{z}\). Note that even though the (weak form of) averaging equation (12) was initially satisfied, \(\bar{z}\) may vary slightly due to the displacement readjustments, which cause slight changes in the gradient operator. However, since (12) is solved simultaneously with equilibrium in the balancing step, it is still satisfied at the end of this step and the solution is consistent also in this respect.

Since stresses are not bounded by a yield surface during the elastic step, it may happen that upon convergence the stress state lies outside the yield surface given by the transferred yield stress, i.e. \(\tau_{eq} > \tau'_y\). The plastic state is restored by setting

\[
\tau_y = \max(\tau_{eq}, \tau'_y)
\]

(28)

Unlike the approach chosen in Reference [29] for the uncoupled damage modelling, \(z\) and \(\omega_p\) are not corrected for potential slight inconsistencies. In the present case, retrieving a completely consistent state would require solving a global problem, because the damage, which depends on \(\bar{z}\), and the plastic strain \(z = \varepsilon_p\) are coupled by the partial differential equation (12). In order to avoid solving such a global system, inconsistencies which may arise in the hardening/softening law are accepted, but their impact is greatly reduced by the fact that this law has been formulated in a rate form, for which the transferred state variables merely act as an initial condition. As a result, it can still be satisfied incrementally, even if the total forms from which the rate equations (7) and (9) have been derived may be violated.

3.4.4. Crack opening. Once a consistent equilibrium state has been restored on the geometry with the new crack segment still closed, this crack segment can be opened. This is done by duplicating the nodes along the new crack segment, attributing one node to each crack face [29].
The nodal forces which are acting on both sides of the new crack segment (which were self-equilibrating when the crack segment was closed) now become external forces. Likewise, a residue appears on the discrete set of averaging equations. In order to restore equilibrium with the new boundary conditions, these nodal forces must be eliminated. Note that these forces are very small since the new segment is introduced in a highly damaged region where only low residual stresses remain. As a result, their removal can usually be done in one step and a new equilibrium state is found after a few equilibrium iterations only. Nevertheless, if for some reason (e.g. a too large crack segment) this iteration process diverges, a nodal-relaxation procedure is automatically applied, whereby the initial unbalance is removed gradually in successive substeps [29].

Contrary to uncoupled damaging elastoplastic materials (e.g. Reference [29]), where the transition from a continuum to a discrete crack is abrupt, with large stress redistributions and a significant amount of energy release, the transition of damage to a crack is here much smoother. The stress redistribution and energy dissipation take place mainly during the damage evolution. This is illustrated in Figure 5, which shows results of the simulation of a shear test on an Arcan specimen, similar to those discussed further in Section 4.2. Figures 5(a) and (b) show the damage and stress state, respectively, before the opening of the newly inserted crack segment. Figures 5(c) and (d) represent the situation after the opening of this crack segment. Note that there is no significant change in either damage or stress fields. This contrasts with an uncoupled approach, as followed in Reference [29], where large stress redistributions are observed in a similar situation.
4. APPLICATIONS

In this section, a number of simulations are presented, and comparisons are made with published experimental data. The damage model which is used, i.e. that of Section 2, assumes that damage growth is governed solely by the effective plastic strain. This model typically focuses on shear failure. It is too restrictive for tensile fracture, which is governed by predominantly spherical void growth and is therefore significantly influenced by the presence of hydrostatic stresses. An extension of the damage theory which captures this effect is proposed in Reference [33]. Here we concentrate on situations which are dominated by shear straining, with little void growth, for which the present damage model provides meaningful results.

All examples shown are displacement-driven. Plane strain elements have been used. Bilinear interpolation functions have been used for both the displacement field \( u \) and the non-local strain field \( \bar{z} \) (see Figure 6). To prevent locking in the plastic regime, selectively reduced integration is used, i.e. full, four-point Gauss integration for the shear part of the stress and reduced, one-point integration for the volumetric part.

4.1. Tensile test on a double notched specimen

In order to assess the model’s performance, a number of simulations have been carried out of a tensile test on a double notched specimen. The geometry and boundary conditions are given in Figure 7 and were also used with the uncoupled damage approach presented in Reference [29]; the dimensions (in mm) are, \( r_1 = 2, r_2 = 2.5, r_c = 1, a = 10 \), with a thickness equal to 1 mm. The material properties have been summarized in Table I. These values are similar to those reported in Reference [38] for X30Cr13 steel; the internal length \( \ell \) is an estimate of the typical scale of the microstructure, e.g. the grain size. Unless otherwise specified, the default parameters of the computations shown are: Equation (10) for the damage evolution, a crack propagation increment of 0.4 mm and \( \omega_{\text{crit}} = 0.99 \). The crack direction was computed using a semi-circular fan (Figure 3), with intervals of 0.25, 0.5, 0.75 and 1 mm.

The tensile loading causes the development of a plastic shear band between the two notches and gives rise to damage initiation at the notches. Strains start to localize, culminating in the nucleation of two cracks which run towards the centre of the specimen. Figure 8 shows the evolution of the mesh, the damage variable \( \omega \) and the von-Mises stress \( \tau_{\text{eq}} \) at different stages of crack growth. Because of the material softening, stresses are not only low in the crack...
wake, for compatibility with the boundary conditions, but also at the crack tips. The maximum values are found away from the crack tips.

Figure 9 shows the force–displacement curve obtained in this analysis. The points at which the snapshots of Figure 8 were taken are indicated in the diagram. A smooth transition from the continuous damage phase to the cracked phase takes place. The global softening response is the combined effect of the material softening and the geometrical softening due to crack growth. The transfer of variables and the discrete geometric changes during crack propagation have no sharp discontinuous effect on the force–displacement curve. This is in contrast with an uncoupled approach, e.g. Reference [29], in which considerable force jumps can be observed at each increment of crack growth, particularly for coarse meshes, as a consequence of stress redistributions.

In order to examine the sensitivity of the crack growth algorithm to numerical parameters, the analysis has been repeated for several values of these parameters. Figure 10 shows the load–displacement curves (Figure 10(a)) and final crack shapes (Figure 10(b)) obtained with
different mesh densities, indicated by the number of elements in the initial mesh. These diagrams show a convergence of results as the finite element mesh is refined. This mesh objectivity is characteristic for the non-local constitutive model used. In the simulations cracks propagate by finite increments. Here again, the results obtained should converge as the size of these crack increments decreases. This can be seen in Figure 11, which illustrates the load–displacement curve and final crack shapes obtained for different sizes of crack increments. Note that the crack increment should be larger than the element size, to avoid a large influence of the numerical errors in the solution. The influence of the cut-off damage value \( \omega_{\text{crit}} \) in the model should also vanish for values close to 1. Figure 12 shows that there is almost no difference in the force–displacement curve and the crack path for slightly different values.
4.2. Arcan test

In order to be able to compare the behaviour of the damage-fracture modelling with experiments, simulations have been done of the experiments performed by Amstutz et al. [39, 40] using modified Arcan specimens made of Aluminium alloy 2024-T3. The Arcan test [41] allows, by means of a special fixture, to load a test specimen at different angles $\beta$, and thus to perform mixed mode fracture experiments. Figure 13 shows the geometry of the Arcan...
Figure 11. Sensitivity to crack increment length: (a) force–displacement; and (b) final crack paths.

Figure 12. Sensitivity to $\omega_{\text{crit}}$: (a) force–displacement; and (b) final crack paths.

specimen and clamps used by Amstutz et al., where the main dimensions (in mm) have been indicated with values $H = 38.1, w = 38.1, r = 15.34, l = 6.3$ mm (precrack); the thickness of the specimen equals 2.3 mm. A plane strain assumption is made in this direction for computational convenience. The material parameters are listed in Table II. These values correspond to those reported in Reference [42]. However, the hardening and damage evolution parameters have been determined in order to fit the experimentally determined force–crack length curves. Linear hardening and non-linear damage according to Equation (10) have been assumed. $\omega_{\text{crit}}$ has been taken equal to 0.99, the crack increment length is 1 mm. The crack growth direction is computed in a semi-circular fan of sample points (Figure 3), at distances of 0.5, 1, 1.5 and 2 mm.

Experiments on ductile metals have shown that predominantly mode-I fracture occurs for loading angles between $\beta = 90^\circ$ (pure tension) and $\beta = 30^\circ$, with crack paths either straight
Figure 13. Sketch of Arcan setup.

Table II. Material parameters used for the Arcan test simulations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear modulus $G$</td>
<td>27.5 GPa</td>
</tr>
<tr>
<td>Bulk modulus $K$</td>
<td>59.5 GPa</td>
</tr>
<tr>
<td>Initial flow stress $t_y^0$</td>
<td>345 MPa</td>
</tr>
<tr>
<td>Linear hardening parameter $h$</td>
<td>2 GPa</td>
</tr>
<tr>
<td>Damage parameter $\kappa_i$</td>
<td>0</td>
</tr>
<tr>
<td>Damage parameter $\kappa_c$</td>
<td>0.4</td>
</tr>
<tr>
<td>Internal length $\ell$</td>
<td>0.3 mm</td>
</tr>
</tbody>
</table>

($\beta = 90^\circ$) or bending upwards. For an even larger shear component, $\beta = 15^\circ$ and $0^\circ$ (simple shear), mode-II fracture occurs, and the cracks propagate downwards along a straight path. It is worth mentioning that this transition from mode-I to mode-II fracture only occurs in ductile fracture. Brittle fracture always happens in mode-I, even under pure shear loading. Since the present damage modelling is applicable only to shear dominated modes, i.e. mode-II fracture, attention is concentrated on the $\beta = 0^\circ$ and $15^\circ$ cases.

The initial mesh used in the simulation and the boundary conditions are shown in Figure 14. The mesh consists of a part which models the specimen, as well as a part which represents the loading clamps. Instead of the bolts used in the experiments, the two parts are here assumed to be perfectly bonded on the bolt line. Likewise, the external radius of the loading clamps is taken equal to the radius of the loading bolts (see Figure 14) and the loading force is modelled by a single nodal force at the appropriate position.

For the considered loading cases, the simulated and experimental crack paths show the same trend as in the experiments (Figure 15): propagation along a straight path. In the $\beta = 15^\circ$ case, this path has a downwards slope of $-5^\circ$, with no visible kinking. In the experiments, after a kink early in the growth process, this angle was approximately $-7^\circ$ [39, 40]. The kink near
the start of the crack is believed to be due to a transition from mode-I dominated growth to mode-II, which is not captured by the present model. In the pure shear case ($\beta = 0^\circ$), the simulation predicts crack propagation at an angle of $0^\circ$, which is consistent with the assumption of purely mode-II crack growth. The experimental cracks, nevertheless, deviate slightly from the horizontal line. These small discrepancies are probably due to the influence of the boundary conditions, which are not exactly the same in the experiments and simulations. In the experiments the load is transmitted from the plates to the Arcan specimen through bolts, hence allowing for some rotation of the plates; while in the simulations the plates and the specimen are perfectly bonded.

In the simulation of purely mode-II crack propagation ($\beta = 0^\circ$), the crack faces tend to touch each other or even to penetrate. Avoiding penetration requires a self-contact algorithm, which was not available in the present framework. However, computations done using a similar
model [43], in an operator-split implementation, showed that the influence of contact between the crack faces is only limited and hardly influences the crack growth direction. Therefore, the simulations shown here were done without self-contact, but the penetration is eliminated in each remeshing step.

Figure 16 shows the applied force versus crack length as measured by Amstutz et al. [39, 40] and as obtained in the two simulations. Figure 17 shows the evolution of the mesh, the damage variable $\omega_p$ and the equivalent stress $\tau_{eq}$ for the $\beta = 15^\circ$ case. It can be seen that the damage strongly localizes along the direction of the future crack.

A phenomenon that is observed in these simulations is that at the onset of damage growth the maximum damage is not found exactly at the notch tip, but at a little distance ahead of the notch tip, see Figure 18. Similar findings were reported by Simone et al. [28] in a brittle damage context, and were attributed to the adopted non-local formulation. Indeed, Figure 18 shows that the local variable $z$ does have its maximum at the tip, but the non-local averaging translates this maximum by some distance for $\bar{Z}$, resulting in maximum damage in the same location. For the computational algorithm this means that in principle a crack should first be initiated within the bulk of the material, rather than at the surface. Since in practice cracks are generally expected to be initiated at the notch edge and to grow into the material, no crack segment is introduced in the simulations until the damage has become critical also on the domain boundary (at the notch tip). This shift phenomenon disappears once the crack starts to propagate and is more pronounced for larger length scales, i.e. for stronger non-locality.

To investigate the effect of damage as the precursor to fracture, the stress and damage field in the wake of the crack and in the fracture process zone ahead of the tip are further scrutinized, where a comparison is made with an uncoupled variant, in which there is no softening prior to crack initiation and crack growth. The uncoupled model has been obtained from the present combined model by eliminating the damage influence from the yield stress, i.e. by setting $\tau_y = \hat{\tau}_y$. However, the damage and non-local variables are still computed, by solving the equilibrium and averaging equations in an uncoupled manner. A similar uncoupled approach, albeit local, was followed in Reference [29]. In this uncoupled approach, damage serves only as a discrete crack initiation and propagation criterion. Figure 19 shows the damage and stress
state in the vicinity of the crack for the coupled damage model (left) and for the uncoupled model (right). It can be observed that the damage fields are similar, although the distribution in the coupled model is somewhat more localized. Differences are more clear in the stress field, in the process zone and in the crack wake. In the coupled model, the crack wake and process zone are highly damaged, and the equivalent stress is therefore low. In the uncoupled model, which does not experience the damage influence in the continuum elastoplastic phase, the low equivalent stress in the crack wake results from the presence of the traction-free crack faces. On the contrary, the equivalent stress reaches its maximum in the fracture process zone, at the crack tip. This is better illustrated by means of the stress profiles at a certain stage of the crack propagation, along the crack axis (Figure 20) and across it (Figure 21), which correspond to the cross-sections indicated in Figure 19. Along the crack axis ('a–a') (Figure 20), the stresses predicted by the coupled model grow from zero at the crack tip to a maximum at a distance of 5 mm, where the damage vanishes, after which it reaches the same stress level as in the
uncoupled model. In the uncoupled model, the yield stress at the crack tip is not degraded by the damage variable and the stress level therefore remains high. Adjacent to the crack, Figure 21, the largest differences in stresses between the coupled and uncoupled model are found in an area of 1 mm at each side of the crack. In front of the crack tip the influence of the damage in the degradation of the yield stress is already visible (Figure 21 ‘b–b’ left). In the section at the crack tip (‘c–c’) the maximum stress level appears at the crack tip in the uncoupled model, whereas a minimum value can be found in the coupled model. In the section in the crack wake (‘d–d’) the same effects occur.

The effect of damage on the constitutive response of the continuum, which distinguishes the coupled from the uncoupled approach, becomes apparent in the force–clamp displacement curves, depicted in Figure 22. In the coupled model, the force decrease is caused by two factors: geometric softening, due to the loss of load-carrying area and material softening, due to the damage growth. In the uncoupled model, there is only geometric softening. The pronounced jumps in the uncoupled model are due to the build-up and release of stresses at the discreetly
Figure 21. Coupled (left) versus uncoupled approach (right). Damage ($\omega_p$) and von-Mises stress ($\tau_{eq}$) fields.
Figure 22. Force versus displacement ($\beta = 15^\circ$). Uncoupled, coupled and continuous (only damage, no cracks) approaches.

moving crack tip. In the coupled approach, stresses are nearly completely relaxed by the damage influence before a new crack segment is inserted and no such stress jumps are therefore observed. The force–displacement curve of the continuous model without any crack growth has also been plotted. This model is obtained by removing the crack modelling from the numerical framework. As a result, stresses can only decrease because of damage. However, part of the curve is meaningless, since the damaged area extends over an unrealistically wide zone towards the right of the specimen.

5. CONCLUSIONS AND FUTURE WORK

A combined continuous damage–discontinuous crack model for the simulation of ductile fracture has been presented, which enables to simulate the entire fracture process, from crack initiation to propagation. The gradient damage model which is thereby used acts as localization limiter and can be seen as a bridge between the microscopic and the continuum level, accounting for the microscopical non-local interactions.

The following conclusions can be drawn:

- Macroscopic softening is accompanied with localizing strains, where further plastic straining decreases the finite width of the localization zone until a discrete crack is approached in the limit. Therefore, the transition from continuous damage to a discrete crack is smooth, which for ductile fracture is more realistic than the sudden changes assumed in elastoplastic fracture mechanics or when using an uncoupled damage model and the widespread damage obtained with a continuous damage model.
- Since $J_2$ (deviatoric) plasticity and an effective plastic strain driven damage evolution law have been used, the model is mainly suitable for shear failure, and can therefore be applied to metal forming processes such as blanking in which shear strains are dominant. For more general applications a dependence on the hydrostatic stress should be included in the failure mechanism.
• Although ductile damage affects mainly the inelastic material properties in the present modelling, there is evidence that it also affects the elastic properties, which is disregarded in this model. Its relative importance remains to be seen.
• The crack direction and rate of crack propagation depend on the evolution of the localization in space and time, respectively. Therefore, the validity of the damage evolution law should be assessed with the experimentally measured crack paths. Although the trend of the crack propagation has been captured, more experiments are needed in order to have more reliable data to compare with.
• The benefits of remeshing in this ductile fracture context are twofold. First, it allows to trace the crack geometry, and secondly it keeps elements from distorting too much. The disadvantage is that transfer of state variables from one mesh to the other introduces errors, which cause inconsistencies among state variables and have a detrimental effect on the convergence of the computations. The robustness of the crack propagation algorithm can be considerably improved by separating the numerical transfer unbalance from the unbalance due to the creation of new traction-free surface. Additional improvements in the robustness could be achieved with a better transfer operator, which would minimize the diffusion errors.
• During crack propagation, localization takes place mainly in front of the crack tip (in the FPZ), therefore a finer mesh is used there. However, the location of crack initiation is not known a priori and in the present framework one needs an ‘a priori’ knowledge on where this happens in order to have finer mesh there. A more reliable approach would be to use an adaptive remeshing technique, either based on an error norm or a phenomenological criterion.

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