OPTIMIZATION OF CARDIAC FIBER ORIENTATION FOR HOMOGENEOUS FIBER STRAIN AT BEGINNING OF EJECTION

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Abstract—Mathematical models of left ventricular (LV) wall mechanics show that fiber stress depends heavily on the choice of muscle fiber orientation in the wall. This finding brought us to the hypothesis that fiber orientation may be such that mechanical load in the wall is homogeneous. Aim of this study was to use the hypothesis to compute a distribution of fiber orientation within the wall.

In a finite element model of LV wall mechanics, fiber stresses and strains were calculated at beginning of ejection (BE). Local fiber orientation was quantified by helix (HA) and transverse (TA) fiber angles using a coordinate system with local r-, c-, and θ-directions perpendicular to the wall, along the circumference and along the meridian, respectively. The angle between the c-direction and the projection of the fiber direction on the cr-plane (HA) varied linearly with transmural position in the wall. The angle between the c-direction and the projection of the fiber direction on the cr-plane (TA) was zero at the epicardial and endocardial surfaces. Midwall TA increased with distance from the equator. Fiber orientation was optimized so that fiber strains at BE were as homogeneous as possible.

By optimization with TA = 0°, HA was found to vary from 81.0° at the endocardium to −35.8° at the epicardium. Inclusion of TA in the optimization changed these angles to respectively 90.1° and −48.2° while maximum TA was 15.3°. Then the standard deviation of fiber strain (σ_d) at BE decreased from ±12.5% of mean σ_d to ±9.5%. The root mean square (RMS) difference between computed HA and experimental data reported in literature was 15.0° compared to an RMS difference of 11.6° for a linear regression line through the latter data.

Keywords: Left ventricle; Fibers; Optimization; Finite element analysis

INTRODUCTION

Within the cardiac wall, stress and strain are important determinants of the distributions over the cardiac wall of blood flow, oxygen consumption (Delhaas et al., 1994) and tissue adaptation effects (Arts et al., 1994). Experimental assessment of the stress and strain distributions over the wall is difficult. Strain can be accurately measured at only a limited number of sites in the wall (Prinzen et al., 1986; Waldman et al., 1988). The reliability of the measurement of wall stress is limited, because insertion of a force transducer damages the tissue at the site of measurement (Huisman et al., 1980). In the assessment of cardiac function, stress and strain expressed with respect to muscle fiber orientation is most relevant. However, accurate knowledge of regional muscle fiber orientation is often not available.

Because of these experimental limitations, mathematical models have been developed to facilitate assessment of the distributions of stress and strain. Advances in computer hardware and numerical methods have enabled an increasing number of aspects of cardiac mechanics to be simulated. The more recent models generally consider the fibrous nature of cardiac tissue, increasingly accurate descriptions of wall shape, nonlinear stress-strain relationships and finite deformations (Arts and Reneman, 1989; Bovendeerd et al., 1992; Huyghe et al., 1992; Guccione et al., 1995). Unfortunately the predicted distributions of fiber stress and strain disagree. Guccione et al. (1995) used a measured wall geometry and distribution of fiber orientation and calculated an inhomogeneous distribution of fiber stress and strain. Bovendeerd et al. (1992) and Huyghe et al. (1992) adapted fiber orientation heuristically so that the distribution of fiber stress was more even. The thus determined fiber orientation appeared not to be significantly different from what has been measured in experiments (Nielsen et al., 1991; Streeter, 1979). The latter models indicate that the distribution of fiber stress is sensitive to the distribution of fiber orientation, even within the limits of biological variance. Combining this finding with the fact that muscle fiber contraction is most efficient for a particular combination of fiber shortening and fiber stress, we came to the hypothesis that mechanical load may be distributed evenly over the wall by proper adjustment of local fiber orientation. Various experimental findings are in compliance with homogeneous mechanical loading of the LV wall. If the distribution of mechanical load is disturbed, adaptation effects, such as growth (Cooper IV et al., 1985; Grossman et al., 1975; Prinzen et al., 1995) or changes in fiber orientation (Carew and Covell, 1979; Pearlman et al., 1982) result in at least partial recovery to the original loading level.

In this study we predicted the orientation of muscle fibers using the hypothesis that mechanical load in the cardiac wall is homogeneous. In a finite element model of LV wall mechanics the distribution of fiber stress and
strain over the wall was calculated at a single moment during the cardiac cycle for given fiber orientation and given stiffness of the fibers. In the reference state of deformation left ventricular cavity pressure was zero and wall stress was zero everywhere. The stress state at beginning of ejection was obtained by stiffening of the fibers while pressurizing the cavity. In an optimization procedure inhomogeneity of mechanical load, defined as the variance of fiber strain at the beginning of ejection, was minimized by proper adjustment of the fiber orientation. The thus predicted fiber orientation was compared with anatomical findings.

METHODS

Finite element model of left ventricular wall mechanics

Reference state. To calculate fiber stresses and strains with a finite element model a reference state must be defined. We used as a reference the state in which transmural pressure across the LV wall and the stresses in the wall were 0 kPa.

Geometry. In the reference state the LV wall is considered thick-walled and rotationally symmetric around the long axis. The center of the equatorial plane is a point of symmetry. Consequently, only the mechanics of the region between the apex and equator was considered. The shape of the LV wall in the reference state was defined by a prolate spheroidal midwall surface with a wall thickness perpendicular to the midwall. The wall shape was defined by five parameters (Appendix) whose values were chosen so that the LV had given values for: cavity volume, ratio of cavity-to-wall volume, ratio of equatorial-to-apical-wall thickness, and ratio of midwall-long-to-short-axes. The geometry parameters were also such that wall thickness decreased smoothly from equator to apex. For the evaluation of cavity and wall volumes it was assumed that the base extended vertically above the equatorial plane by a distance equal to half the semi-major axis of the midwall surface (Streeter and Hanna, 1973). The choice of parameter values of the finite element model, including LV wall geometry, are described in a separate section below.

Fiber orientation. Fiber orientation in the reference state was quantified by the helix and transverse fiber angles (Streeter, 1979) measured with respect to the local transmural, circumferential and longitudinal directions (Fig. 1). To be able to define the local transmural, circumferential and longitudinal directions, we first define a wall-bound coordinate system \((u, v)\) (Fig. 2); the coordinate \(u\) decreases from 0 at the equator to \(-1\) at the apex in direct proportion with distance along the midwall surface in the equator-to-apex direction; the coordinate \(v\) increases from \(-1\) at the endocardium to \(+1\) at the epicardium in direct proportion with distance in the direction perpendicular to the midwall surface. The local transmural direction was then defined as the outward normal to a surface of constant \(u\). The local longitudinal direction is orthogonal to both the transmural and circumferential directions. The helix fiber angle was defined as the angle between the local circumferential direction and the projection of the fiber direction on the plane perpendicular to the local transmural direction. The transverse fiber angle determines the degree to which fibers cross over between inner and outer wall surfaces. It was defined as the angle between the local circumferential direction and the projection of the fiber direction on the plane perpendicular to the local longitudinal direction. The spatial distributions of the helix and transverse fiber angles are specified with respect to the wall-bound coordinate system \((u, v)\). The helix fiber angle, \(\alpha_h\), varies with \(v\) according to (Fig. 3):

\[
\alpha_h(v) = p_1 + p_2v,
\]

where \(p_1\) and \(p_2\) are parameters whose optimal values are to be determined. The transverse fiber angle, \(\alpha_t\), is zero at the wall boundaries, i.e. fibers are assumed not to end here. Because the equatorial plane is a plane of symmetry, \(\alpha_t\) is an odd function in coordinate \(u\) (Streeter et al., 1978). To satisfy the latter conditions, we used the following equation (Fig. 3):

\[
\alpha_t(u, v) = p_3(1 - v^2) \sin(\pi u/2),
\]

where parameter \(p_3\) has to be optimized.
and Prothero, 1992). Sarcomere length, $l$, is related to strain in the fiber direction, $\varepsilon_{ll}$, by

$$\varepsilon_{ll} = \varepsilon_{l0} (I_{l}^{1/2} + 1) ^{1/2} - 1,$$

(5)

where $l_{0}$ is sarcomere length in the reference state. The total second Piola–Kirchhoff stress, $S$, in the passive tissue, $S_{p}$, is obtained by differentiation of equation (7) with respect to $E$:

$$S_{p} = \partial W(E)/\partial E.$$

(8)

The passive second Piola–Kirchhoff stress, $S_{p}$, is zero in the unstrained state and increases exponentially with strain.

Muscle fiber contraction in the real LV depends on sarcomere length, sarcomere velocity of shortening, time and extracellular calcium concentration (de Tombe and ter Keurs, 1991). In the finite element simulations the first Piola–Kirchhoff fiber stress, $T_{f}$ (kPa) depended linearly on sarcomere length, $l_{s}$, and active stiffness, $K$ (kPa $\mu m^{-1}$):

$$T_{f} = K (l_{s} - l_{s0}).$$

(9)

For physiological circumstances such a linear relationship is quite an accurate description throughout the cardiac cycle (Sagawa, 1978). In the reference state sarcomeres have a uniform length, $l_{s0}$ ($\mu m$), while cavity pressure is zero and cavity-to-wall-volume ratio corresponds to mid-diastole. In this state the left ventricular wall is stress-free, regardless of active stiffness, $K$. To obtain the stress state at beginning of ejection, the stiffness, $K$, was estimated such that at the cavity pressure of beginning of ejection a physiologically realistic cavity-to-wall-volume ratio resulted.

Equations solved. Calculations of fiber stresses and strains in the LV wall were based on the law of conservation of momentum (Malvern, 1969). Neglecting inertial effects (Moskowitz, 1981; Peskin, 1989) and gravitational effects, conservation of momentum expresses the static equilibrium of forces in the wall due to blood pressure in the cavity and internal stresses in the wall:

$$\nabla \cdot (S \cdot F^T) = 0.$$

(10)

Solution method. In principle the solution to the equations can be expressed in terms of the displacements of all points in the wall. A Galerkin-based finite element method, implemented in the package DIANA-5.1 (Diana Analysis B.V., Delft, The Netherlands), was used to calculate the displacements of a finite number of points, so-called nodes. Quadratic interpolation was used to determine the displacements of points in between the nodes. Nodes are grouped into elements which constitute the building blocks of the LV wall. To reduce the computational effort, we used the rotational symmetry of the LV wall in the model. The LV wall mesh comprised only the 1/8th-section of the LV in the region $x > 0, y > 0, z \leq 0$ (Fig. 2). The mesh consisted of 27 twenty-node brick elements with 3 element layers in the transmural...
direction and 5 in the equator-to-apex direction. Kinematic boundary conditions on the through-wall faces of the mesh allowed cavity volume changes and torsion to occur. The LV inner wall surface was loaded perpendicularly by cavity pressure while the outer wall surface experienced no external forces.

**Design of finite element simulations.** Finite element simulations started in the reference state of deformation, corresponding to approximately mid-diastole. The state of deformation and stress at the beginning of ejection was obtained by applying a cavity pressure of 10.64 kPa (80 mm Hg). The stiffness, K, was chosen such that after application of the cavity pressure loading, the cavity-to-wall-volume ratio was about 0.6. This volume ratio and pressure were considered representative for the beginning of ejection (Douglas et al., 1991).

**Quantification of mechanical load.** Regional mechanical load was quantified as fiber strain at the beginning of ejection. The LV wall mesh was divided into 729 regions with similar volumes. Sarcomere length at the central point of a region was considered representative for that region. For region i, fiber strain, $e_{f,i}$, is given by

$$e_{f,i} = \frac{l_{s,i} - l_{s,0}}{l_{s,0}}. \quad (11)$$

where $l_{s,i}$ is the instantaneous sarcomere length in the region and $l_{s,0}$ is the sarcomere length in the reference state.

**Optimization procedure.**

Fiber orientation was optimized to make regional differences in mechanical load as small as possible. The optimization consists of minimizing an objective function, $G$, defined as the variance of fiber strain at beginning of ejection normalized to the average fiber strain at beginning of ejection:

$$G(p) = \sum_i w_i \left( \frac{e_{f,i} - e_{f,av}}{e_{f,av}} \right)^2. \quad (12)$$

$G$ depends on the fiber orientation parameters $p_1$, $p_2$, and $p_3$ which are stored in the vector $p$. Region $i$, representing a fraction $w_i$ of the total wall volume, has a representative fiber strain at beginning of ejection, $e_{f,i}$. The average fiber strain at beginning of ejection, $e_{f,av}$, is the wall volume-weighted sum of the regional fiber strains.

Most optimization methods require many evaluations of the objective function. Since evaluation of $G$ with the finite element model is computationally expensive, the concept of sequential approximate optimization was used (Barthelemy and Haftka, 1993). For a given set of fiber orientation parameters, $p_k$, a finite element analysis was performed to evaluate regional fiber strains, $e_{f,k}$, at beginning of ejection. By perturbing the fiber orientation parameters one at a time by $\Delta p$, and using the already available state of deformation and stress as a first estimate, the regional fiber strains for the perturbed set of fiber orientation parameters were computed, with little additional computational effort. Hence, finite difference derivatives of regional fiber strains with respect to $p_k$ were obtained. Regional fiber strains, $e_{f,k}$, were estimated for arbitrary $p$ by performing a first order Taylor series expansion on $e_{f,i}$ around the value $p_k$:

$$e_{f,i}(p) = e_{f,i}(p_k) + \sum_{j=1}^n \left( \frac{p_j - p_{k,j}}{\Delta p_j} \right) \frac{\partial e_{f,i}}{\partial p_j}(p_k). \quad (13)$$

where $p_{k,j}$ and $p_j$ are the jth components of $p_k$ and $p$ respectively. The partial derivatives $\partial e_{f,i}/\partial p_j$ were calculated as finite difference derivatives. The numbers $n$ and $N$ refer to the number of fiber orientation parameters, and to the number of regions in the LV wall mesh ($N = 729$) respectively. By substitution of equation (13) in equation (12), an approximation model, $\tilde{G}$ is obtained which gives an estimate of the objective function, $G$, for an arbitrary set of fiber orientation parameters:

$$\tilde{G}(p) = \sum_i w_i \left( \frac{e_{f,i} - \tilde{e}_{f,av}}{\tilde{e}_{f,av}} \right)^2. \quad (14)$$

Repeated evaluation of $\tilde{G}$ is very efficient due to the explicit dependence on $p$. The function, $\tilde{G}$, is minimized by a sequential quadratic programming algorithm (Schittkowski, 1986). This algorithm requires limits on the parameter values. The parameters $p_1$, $p_2$, and $p_3$ were allowed to vary in the range $(-90^\circ, +90^\circ)$, $(-90^\circ, 0^\circ)$ and $(0^\circ, +90^\circ)$ respectively. The result of the optimization of the approximation model is a new set of fiber orientation parameters, $p_{k+1}$. Two checks are performed to evaluate whether convergence has occurred. The finite element evaluations of the objective function of the current and previous iterations, $G(p_k)$ and $G(p_{k-1})$, should agree to within a tolerance of $\delta$:

$$\|G(p_k) - G(p_{k-1})\| / G(p_k) \leq \delta, \quad (15)$$

where $\delta = 0.00001$. Furthermore, the optimum of the approximation model based on parameters $p_k$, $\tilde{G}_{opt,p_k}$ should coincide with the finite element evaluation of the objective function at $p_k$, to the same tolerance of $\delta$:

$$\|G(p_k) - \tilde{G}_{opt,p_k}\| / G(p_k) \leq \delta. \quad (16)$$

If convergence has not occurred a new approximation model is set up around the parameters $p_{k+1}$ and the process is repeated. Because the optimization results may depend on the starting values, it is expedient to perform optimizations with several starting values.

**Applied parameter values in finite element model.**

Wall geometry in reference state. In potassium-arrested canine left ventricles cavity volume and wall mass were reported to be 40 ± 9 ml and 145 ± 19 g (McCulloch et al., 1989). Assuming a tissue density of 1050 g cm$^{-3}$ measured wall volume is 138 ml. In the model we have used a cavity volume of 42 ml, and a cavity-to-wall volume ratio of 0.3. The ratio of midwall long-to-short axes was set to 2.08 (Streeter and Hanna, 1973). From preliminary calculations of fiber stress and strain we found that fiber stress was most homogeneous if we chose the ratio of equatorial-to-apical wall thickness to be 3.0. Parameter values are summarized in Table 1.

**Sarcomere length in reference state.** In rat LVs that were formalin-fixed at zero transmural pressure, sarcomere lengths have been measured as 2.04 ± 0.02 μm.
Table 1. Parameter values for model of left ventricular (LV) wall mechanics

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>LV wall geometry</td>
<td></td>
</tr>
<tr>
<td>$V_c$</td>
<td>40 ml</td>
</tr>
<tr>
<td>VR</td>
<td>0.3</td>
</tr>
<tr>
<td>WTR</td>
<td>3.0</td>
</tr>
<tr>
<td>MAR</td>
<td>2.08</td>
</tr>
<tr>
<td>Passive material behavior</td>
<td></td>
</tr>
<tr>
<td>$a_0$</td>
<td>0.5 kPa</td>
</tr>
<tr>
<td>$a_1$</td>
<td>3</td>
</tr>
<tr>
<td>$a_2$</td>
<td>6</td>
</tr>
<tr>
<td>$a_3$</td>
<td>3</td>
</tr>
<tr>
<td>$a_4$</td>
<td>500 kPa</td>
</tr>
<tr>
<td>Contractile material behavior</td>
<td></td>
</tr>
<tr>
<td>$l_{iso}$</td>
<td>1.95 µm</td>
</tr>
<tr>
<td>$K$</td>
<td>77.0 kPa/µm</td>
</tr>
</tbody>
</table>

Wall geometry in the reference state was chosen so that the following parameters had given values: cavity volume ($V_c$), ratio of cavity-to-wall-volume (VR), ratio of equatorial-to-apical-wall thickness (WTR), and ratio of midwall-long-to-short-axis (MAR). Passive material properties are described by parameters $a_0$, $a_1$, $a_2$, $a_3$, $a_4$ (equation (7)). Sarcomere length in the reference state is denoted by $l_{iso}$. Active fiber stiffness at beginning of ejection is denoted by $K$ (equation (9)).

 Constitutive behavior. The values of passive material parameters $a_0$, $a_1$, $a_2$, and $a_3$ were taken from Bovendeerd et al. (1992) (Table 1). Equibiaxial stretching experiments on sheets of passive canine myocardium (Yin et al., 1987) indicate that the ratio of fiber to cross-fiber stress ranges from 1.10 to 2.95. In the present model the values of parameters $a_1$, $a_2$ and $a_3$ were set such that the ratio is 2.0 under equibiaxial loading. The absolute values of $a_0$ and $a_1$ were chosen so that calculated passive pressure-volume curves agreed with experiments on canine hearts (Nikolic et al., 1988). The volume of the myocardium may change by a few percent as a result of changes in coronary inflow and outflow during the cardiac cycle (Judd et al., 1991; Yin et al., 1996). For simplicity, we have assumed that the LV wall is nearly-incompressible. The value of $a_4$ was chosen large enough so that in the simulations numerical stability was maintained and LV wall volume changed by less than 2%.

In experiments (ter Keurs et al., 1980), the sarcomere length at which no contractile force can be generated has been measured as about 1.6 µm. For simplicity of calculation we have chosen a zero-force length equal to $l_{iso}$ so that the reference state is stress free, regardless of the instantaneous active stiffness, $K$. Using the finite element model, the stiffness of the fibers was chosen so that fiber stress at beginning of ejection was physiological. This resulted in a value of 77.0 kPa µm$^{-1}$ for $K$.

Finite element mesh. The size of the elements was not changed to investigate their influence on the computed stress and strain distributions. However, for a given distribution of fiber orientation the predicted stress and strain distributions were similar to those predicted by an independently developed finite element model of LV wall mechanics by Huyghe et al. (1992).

Performed optimizations

Two sets of optimizations, optim1 and optim2, were carried out. In optim1 regional differences in mechanical loading were minimized by optimizing only the helix fiber angle parameters $p_1$ and $p_2$ for the case without any fiber cross-over ($p_3 = 0$). Three optimizations were performed in which the starting values in degrees for ($p_1$, $p_2$) were (0, 0), (0, $-90$), and (90, 0). In optim2 the fiber cross-over parameter $p_3$ was included. Seven different sets of starting values were tried for ($p_1$, $p_2$, $p_3$): (0, 0, 0), (0, $-90$, 0), (90, 0, 0), (0, 0, 90), (0, $-90$, 90), (90, 0, 90), and (0, 0, 90). The optimizations required between 3 and 16 finite element evaluations of mechanical load. Each finite element analysis required about one CPU-hour on a Sun SPARC ELC workstation with 24 MB RAM.

RESULTS

In optim1 a single minimum in inhomogeneity of mechanical load was found, with optimized values of $p_1$ and $p_2$ of respectively 22.57° and $-58.41°$. For a wide range of starting values a fiber orientation resulted in which subendocardial fiber paths resembled a right-handed and subepicardial fiber paths a left-handed helix. At the optimum, fiber strain, $e_t$, at the beginning of ejection was 0.103 ± 0.013 (mean ± sd), if 6% of the LV wall volume near the apex was excluded (information from the three elements in the mesh adjoining the apex). The coefficient of variation of fiber strain at beginning of ejection, defined as the standard deviation divided by the mean, was 12.5%. For the whole LV wall, fiber strain at the beginning of ejection was 0.102 ± 0.020 and the coefficient of variation 20.0%.

In optim2 a single optimum was also found: $p_1 = 20.96°$, $p_2 = -69.21°$ and $p_3 = 15.33°$. For the computed optimum, fiber strain at the beginning of ejection was 0.111 ± 0.001 and the coefficient of variation 9.5%, if 6% of the wall volume near the apex was excluded. For the whole LV the fiber strain at the beginning of ejection (Fig. 4) was 0.111 ± 0.016 and the coefficient of variation 14.4%.

DISCUSSION

A finite element model of LV wall mechanics has been developed in which regional mechanical load can be calculated for given fiber orientation, LV cavity pressure, wall geometry and material properties. Regional mechanical load was quantified as fiber strain at the beginning of ejection. Regional differences in mechanical load have been successfully minimized by proper adjustment of fiber orientation. A helical arrangement of fibers in the LV wall formed automatically for a wide range of starting values of the parameters describing the distribution.
of fiber orientation. Inclusion of fiber cross-over between inner and outer wall surfaces in the optimization further reduced load inhomogeneity.

For simplicity, residual load has not been incorporated in the present finite element model of LV wall mechanics. Residual circumferential strain in the unloaded LV wall (zero transmural pressure) has been revealed in the rat heart by making a longitudinal-transmural cut in the LV wall, upon which the wall opens (Omens and Fung, 1989). Several studies have emphasized the importance of residual load for accurate prediction of passive mechanical behavior (Kang and Yin, 1996), as a potential mediator during cardiac morphogenesis (Taber et al., 1993), and for its potential effect on systolic wall stress (Taber, 1991). Experiments in the rat heart indicate that circumferential residual strain leads to a transmural gradient in sarcomere length in the unloaded LV of the rat, with sarcomeres in the subepicardium being longer than those in the subendocardium (Rodriguez et al., 1993). At positive filling pressures, sarcomere length becomes more uniformly distributed over the LV wall of the rat (Grimm et al., 1980), indicating that residual strain may contribute to homogeneity of fiber strain at beginning of ejection.

Solving optimization problems requiring stress analyses by the finite element method easily leads to unacceptably large computational efforts. Computational efficiency can be manipulated by the number of elements in the finite element mesh and by the number of time steps during the cardiac cycle included in the evaluation of the objective function. We have tried to limit the computational effort in three main ways. First, the number of finite element analyses was reduced by using a suitable optimization strategy, namely sequential approximate optimization (Barthelemy and Haftka, 1993). Second, we quantified mechanical load as regional fiber strain at the beginning of ejection relative to the diastolic reference state. Hence, evaluation of the objective function by the finite element method required only strain information at the beginning of ejection, not throughout the whole cardiac cycle. Other finite element simulations of LV wall mechanics (Bovendeerd et al., 1992) show that the distribution of fiber stress at beginning of ejection gives a good approximation of the stress distribution during the entire ejection phase. In our model of LV wall mechanics, cardiac tissue is elastic and the stiffness uniform so that homogeneity of fiber stress automatically implies homogeneity of fiber strain. Therefore, by making fiber strain at the beginning of ejection as homogeneous as possible, we expect, to a good approximation, to have made fiber strain homogeneous during the entire ejection phase. Third, we chose a simplified constitutive law for contracting cardiac tissue in which the zero-force sarcomere length equals the sarcomere length in the reference state. As a direct computational gain, the reference state is then stress-free regardless of the active stiffness so that the state of beginning of ejection can be reached by merely loading the endocardial surface with cavity pressure. To make fiber stresses at beginning of ejection physiological the active fiber stiffness was chosen such that cavity pressure and volume had physiological values (Arts et al., 1991). Following optimization of fiber orientation, fiber stress at beginning of ejection is both physiological and homogeneous (with a coefficient of variation less than 10%). In conclusion, the measures we have taken to restrict computational effort have made the optimization problem more manageable.

Fiber strains near the apex remain inhomogeneous (Fig. 4), despite optimization of fiber orientation. Others (Peskin, 1989) who have calculated left ventricular fiber orientation in a mathematical model have ignored the apex on the basis that apical fibers are an integral part of the right ventricle (Thomas, 1957). We have included the apex in our model, but recognize that the description of its mechanics is probably erroneous due to an inadequate description of fiber orientation, wall geometry and possibly also material properties. After all, the apex itself is a singular point in the mathematical description of the distribution of fiber orientation. As a consequence we have presented the results of optimizing fiber orientation in the whole LV wall both with and without information from the three elements (6% of wall volume) adjoining the apex.

The choice of the number and nature of the fiber orientation parameters in the model is somewhat arbitrary. Our aim was to use as few parameters as possible that still allowed a reasonable description of measured fiber angles. Within limits, more parameters would result in more homogeneous loading of the wall as indicated in this study by the improvement in load homogeneity when adding transverse fiber angle parameter, $p_3$, to the two helix angle parameters. Additional helix fiber angle parameters can be chosen on the basis of anatomical measurements which reflect that (1) helix fiber angle is a nonlinear function of transmural position, especially close to the wall boundaries (Nielsen et al., 1991; Streeter, 1979; Streeter et al., 1969) and (2) helix fiber angle increases towards the apex (Streeter and Hannah, 1973). Inclusion of fiber angle parameters in the optimization that allow such behavior are likely to further improve load homogeneity.

To assess the physiological relevance of the computed transmural course of helix fiber angle, comparisons were made with reported measurements (Nielsen et al., 1991; Streeter, 1979) and model predictions (Arts et al., 1994; Bovendeerd et al., 1992; Peskin, 1989). The anatomical measurements were performed in the equatorial and

![Fig. 4. Distribution of fiber strain at the beginning of ejection after optimization of fiber orientation parameters $p_1$, $p_2$ and $p_3$. Fiber strain over whole LV wall is $0.111 \pm 0.016$ (mean $\pm$ sd).](image-url)
adjacent more apical regions of human and canine left
ventricles (Nielsen et al., 1991; Streeter, 1979) (Fig. 5). For
a quantitative comparison, the root mean square (RMS)
difference between the computed transmural course of
helix fiber angle and a measured data set was determined
(Table 2). Since the ability to describe the measured data
with a straight line is limited by inherent scatter, the
RMS differences of linear regression lines to the measure-
ments were also determined. Table 2 shows that the RMS
differences of the computed transmural course of helix
angle are only slightly larger (range: 11.9°–18.4°) than
those of the linear regression lines (11.4°–12.1°). Notably,
the optimized helix fiber angle distribution of optim2
agrees better with the measurements than that of optim1.
Also shown in Table 2 are RMS differences for predic-
tions of helix fiber angle distributions of several mathe-
matical models. In a finite element model of LV wall
mechanics by Bovendeerd et al. (1992) equatorial fiber
stress, averaged over the ejection period, was made as
homogeneous as possible by making trial and error cha-
mechanics by Guccione et
studies. Results of a finite element model of LV wall
mechanics to calculate left ventricular fiber orienta-
tion (Peskin, 1989), transverse fiber angle was zero at the
inner and outer wall surfaces and greatest in the middle
of the wall. The midwall transverse fiber angle was 0°
at the equator position and increased towards the apex to
about 18°. In conclusion, there is too little experimental
data available to make a quantitative evaluation of the
computed transverse fiber angle. Qualitatively, our pre-
dictions do not conflict with either available measure-
ments or other model predictions.

Comparison of computed transverse fiber angle with
measurements is more difficult due to the scarcity of
experimental data. The transverse fiber angle is positive
above the equator and negative below but more accurate
latitude-dependence could not be detected (Streeter et al.,
1978). Mean through-wall values have been measured as
-4.6 ± 0.8° (mean ± SEM) near the equator and
-3.5 ± 0.6° near the apex (Streeter, 1979). In our model
the spatial average of α0 below the equator is -6.5°
which compares reasonably well with these measure-
ments. Reported predictions of fiber orientation by mathe-
matical models are another available source to compare
the computed transverse fiber angle field with. In a mathe-
matical model to calculate left ventricular fiber orienta-
tion (Peskin, 1989), transverse fiber angle was zero at the
inner and outer wall surfaces and greatest in the middle
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data available to make a quantitative evaluation of the
computed transverse fiber angle. Qualitatively, our pre-
dictions do not conflict with either available measure-
ments or other model predictions.

The finding that an anatomically realistic distribution
of fiber orientation results in rather homogeneous me-
chanical loading of the wall at the beginning of ejection is in
agreement with other (Arts and Reneman, 1989; Boven-
deer et al., 1992; Huyghe et al., 1992), but not all, model
studies. Results of a finite element model of LV wall
mechanics by Guccione et al. (1995) showed there was
considerable regional heterogeneity of end systolic fiber
stress. The latter model incorporated measured wall

![Fig. 5. Comparison of measured and computed helix fiber angle, α0, as
a function of transmural position in the wall, v. Measurements: (●)
Streeter (1979), from equatorial region of human LV. (△) Nielsen et al.
(1991), from equatorial region of canine LV. (▲) Nielsen et al. (1991),
from adjacent more apical region of canine LV. (x) Computations: (---)
optimization of parameters p1 and p2, (——) optimization of parameters
p1, p2, and p3.

Table 2. Root mean square (RMS) differences of computed transmural course of helix fiber angle with respect to measurements

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Root Mean Square Difference (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Streeter, 1979</td>
<td>11.4</td>
</tr>
<tr>
<td>Nielsen, 1991 A</td>
<td>12.1</td>
</tr>
<tr>
<td>Nielsen, 1991 B</td>
<td>11.4</td>
</tr>
</tbody>
</table>

Sources of measurements: Streeter, (1979), from equatorial region of human LV; Nielsen et al. (1991), A and B, from equatorial and adjacent more apical regions of canine LV respectively. Other columns show RMS differences in degrees for computed transmural course of helix fiber angle. (1) best linear fit to measurements; (2) optim1: when optimizing parameters p1 and p2 only; (3) optim2: when additionally optimizing p3; (4) Bovendeerd et al. (1992), and (5) Peskin (1989), and (6) Arts et al. (1994).
geometry and fiber orientation. Unfortunately, in their study, Guccione et al. (1995) did not present a sensitivity analysis of the model parameters. It may well be that small changes in fiber orientation within the anatomical range lead to more homogeneous fiber stress over the wall.

CONCLUSIONS

A finite element model of LV wall mechanics has been set up. Fiber orientation in the model, quantified by the helix and transverse fiber angles, was described by up to three parameters. Fiber orientation was optimized to make regional differences in fiber strain at the beginning of ejection as small as possible. A single optimum in fiber orientation was found when helix fiber angle was optimized for a wide range of starting values of the fiber orientation parameters. The standard deviation of the fiber strain at beginning of ejection, divided by the mean fiber strain was 12.5%. Inclusion of the transverse fiber angle in the optimization reduced this measure of inhomogeneity to less than 10%. The computed transverse fiber strain was 12.5%. Inclusion of the transverse fiber orientation parameters. The standard deviation of the orientation was found when helix fiber angle was optimized to make regional differences in fiber strain at the beginning of ejection.

Acknowledgements—The authors thank Prof. K. Schittkowski for providing an implementation of the sequential quadratic programming algorithm, NLPLQF.

REFERENCES


Homogeneous fiber strain at beginning of ejection


APPENDIX: WALL GEOMETRY

To specify wall geometry in the reference state a spherical coordinate system as well as the rectangular coordinate system of Fig. 2 were used. Given the coordinates of a point with respect to the spherical coordinate system, (r, θ, φ), its coordinates in the rectangular system, (x, y, z) are

\[
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix} = r \begin{pmatrix}
\sin(\theta) \cos(\phi) \\
\sin(\theta) \sin(\phi) \\
\cos(\theta)
\end{pmatrix}.
\]  

The LV wall was assumed to be rotationally symmetric. The shape of the wall was defined by specifying a midwall surface and a wall thickness perpendicular to the midwall. A point on the midwall surface at a distance \( r_m \) from the origin has position vector \( \mathbf{r}_m \) given by

\[
\mathbf{r}_m(\theta, \phi) = r_m(\theta) \begin{pmatrix}
\sin(\theta) \cos(\phi) \\
\sin(\theta) \sin(\phi) \\
\cos(\theta)
\end{pmatrix}.
\]  

where \( \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3 \) are unit vectors along the Cartesian coordinate axes. The shape of the midwall surface is described by the function \( r_m(\theta) \) which depends on two parameters, \( a_m \) and \( b_m \):

\[
r_m(\theta) = a_m + b_m \cos(2\theta)^{-1}. \]

The wall thickness perpendicular to the midwall is specified by the function \( t(\theta) \), which depends on three parameters, \( a_t, b_t, \) and \( c_t \):

\[
t(\theta) = a_t + b_t \cos(2\theta) + c_t \cos(4\theta). \]

The unit outward normal vector to the midwall surface is given by \( \mathbf{n}_m \):

\[
\mathbf{n}_m(\theta, \phi) = \left[ \frac{\mathbf{r}_m}{|\mathbf{r}_m|} \right] - \mathbf{r}_m \frac{\partial}{\partial \theta} \left( \frac{\mathbf{r}_m}{|\mathbf{r}_m|} \right) \sin(\phi) \mathbf{e}_1 + \sin(\phi) \mathbf{e}_2 + \cos(\phi) \mathbf{e}_3.
\]  

Hence the position vector \( \mathbf{r} \) of a point in the LV wall is given by

\[
\mathbf{r}(\theta, \phi) = \mathbf{r}_m(\theta, \phi) + t(\theta) \mathbf{n}_m(\theta, \phi). \]  

The parameter \( r \) varies linearly with transmural position from \(-1\) at the endocardium to \(+1\) at the epicardium. The used values of \( a_m, b_m \), and \( c_t \) were 0.109 and \(-0.0680\) cm\(^{-1}\), respectively, and those of \( a_t, b_t, \) and \( c_t \) were 0.643, \(-0.289\) and \(-0.0643\) cm, respectively.