Linear control strategies for the suppression of flame instabilities

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Abstract
The use of low NOx premixed burners with a large modulation range in modern central heating systems often leads to noise problems. These problems are often solved by passive techniques. In this study, the use of active model-based control strategies to interrupt the interaction between acoustic waves and unsteady heat-release is investigated in simulations and experiments. It is found that, though based on a linearized model, the LQG/LTR control and \( \mathcal{H}_\infty \) control are effective.

Introduction
Acoustic phenomena are almost inherent to any closed system with a flowing medium and can cause problematic behavior. In the case of condensing boilers two recent developments have given rise to a severe increase in acoustic problems. First, the trend to increase the modulation range, essential for the comfort offered by the boiler. Secondly the reduction of NO\(_x\) emission. To reduce the emission of NO\(_x\) a large class of boilers has been equipped with (fully premixed) surface burners. On these burners a premixed flat flame can be stabilized, leading to a heat transfer from the flame to the burner, and consequently a cooler flame and reduced NO\(_x\) emissions. A disadvantage is that these flames are sensitive to acoustic instabilities.

Often passive approaches are used to counter act these instabilities, such as the application of passive acoustic damping (baffles, Helmholtz resonators) or modifications of the burner geometry. The wish to operate under constantly changing operating conditions has led to the idea of exploring active control as a possible strategy to fight these instabilities.

In the last twenty years several experimental results have been reported for the active control of combustion driven oscillations. First a phase-shifter and filter were used as a controller by for instance Heckl [5], Lang et al. [6], and Poinset et al. [8]. The control strategies slowly developed into more sophisticated control design methods, for instance model-based control design, where the underlying physics is employed, Annaswamy et al. [1], and Campos-Delgado et al. [2], or input-output data together with system identification methods are used to derive the model, (Murray et al. [7]). Also adaptive control strategies are proposed (Evesque et al. [3]).

In this study, active controllers will be synthesized which use physically based modeling, in order to study the possibility of active suppression of combustion oscillations of flames stabilized on flat surface burners.

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Flame dynamics
In burner-stabilized flames a coupling exists between the heat loss of the flame to the burner and the flame velocity. The heat loss will change, whenever the flame moves. This influences the temperature of the flame and in consequence the flame velocity. If this heat loss gives positive feedback to the flame velocity, the flame response might display large amplitude oscillations on the burner.

A variation in flame temperature, \( T_b \), is assumed to result in an instantaneous change in the mass burning rate \( m = \rho_0 s_L \), with \( s_L \) the burning velocity and \( \rho_0 \) the density. It is assumed that the mass burning rate is a function of time only. Spatial dependence of the density is eliminated by introducing density-weighted coordinates (or von-Mises coordinates). The relation between \( T_b \) and \( m \) is then given by de Goey [4]:

\[
\frac{\partial m(\tau)}{\partial \tau} = \frac{Ze}{2} \frac{m}{T_b - T_a} \frac{\partial T_b}{\partial \tau} = \frac{Ze}{2} \frac{m}{c_p(T_b - T_a)} \frac{\partial J(\psi_f, \tau)}{\partial \tau}, \tag{1}
\]

with \( \tau \) the density weighted time, \( \psi \) the density weighted position, \( T_a \) the temperature of the burner, \( Ze \) the Zeldovich number, \( c_p \) specific heat capacity, and \( dJ \) a change in 'total enthalpy', given by:

\[
dJ = \Delta H dY + c_p dT,
\]

with \( \Delta H \) the combustion enthalpy and \( Y \) the fuel mass fraction. Note that \( J \) is not only a function of the time \( \tau \), but also of the space variable \( \psi \). Fluctuations in enthalpy will be traveling in space with a velocity equal to the gas velocity \( u_a \), so that heat losses to the burner at \( \psi = 0 \) arrive at the flame front \( \psi = \psi_f \) with a time lag. Furthermore, due to damping effects the shape of the enthalpy waves will change while they travel. In this study this will be neglected, so \( \frac{\partial J(\psi_f, \tau)}{\partial \tau} \) can then be replaced by:

\[
\frac{\partial J(\psi_f, \tau)}{\partial \tau} = \frac{\partial J(0, \tau - \psi_f/u_a)}{\partial \tau} = \frac{\Delta H}{c_p} \frac{\partial Y(0, \tau - \psi_f/u_a)}{\partial \tau}, \tag{2}
\]
where at $\psi = 0$ it is assumed that $T$ is constant and equal to the burner temperature. In order to solve the flame motion in terms of the fuel mass fraction $Y$, the following kinematic equation will be used.

$$\rho_u \frac{\partial Y}{\partial \tau}(\psi, \tau) = (m(\tau) - \phi_u(\tau)) \frac{\partial Y}{\partial \psi}(\psi, \tau)$$  \hspace{1cm} (3)

Substituting (2) and (3) in (1) yields:

$$\frac{\partial m(\tau)}{\partial \tau} = \frac{Z e}{2} \pi_a \delta \frac{\partial Y(0, \tau - \psi_t/u_a)}{\partial \psi}$$

$$\times (m(\tau - \psi_t/u_a) - \phi_u(\tau - \psi_t/u_a))$$  \hspace{1cm} (4)

It is possible to express the gradient $\partial Y(0)/\partial \psi$ in terms of stationary flame heat losses, because small acoustic variations in $\frac{\partial Y}{\partial \psi}(0, \tau - \psi_t/u_a)$ are neglected since they will be of higher-order in the acoustic distortion:

$$\Delta H \frac{\partial Y}{\partial \psi}(0, \tau - \psi_t/u_a) = - \frac{c_p(T_{ad} - T_b)}{\delta}.$$  \hspace{1cm} (5)

where $\delta = \frac{T_{ad} / (\pi u_c p)}{\pi a}$ is the flame thickness and $T_{ad}$ is the adiabatic temperature.

Substituting this in equation (4) gives:

$$\frac{\partial m(\tau)}{\partial \tau} = (\phi_u(\tau') - m(\tau')) \frac{Z e}{2} \pi_a \frac{\Delta H}{\delta} \frac{T_{ad} - T_b}{T_b - T_a}.$$  \hspace{1cm} (6)

with $\tau' = \tau - \psi_t/u_a$, the delay time. The previous equation can be written as a first order differential equation:

$$\frac{\partial m_u(\tau)}{\partial \tau} + \frac{Z}{\tau_t} m_u(\tau') = \frac{Z}{\tau_t} \phi_u(\tau'),$$  \hspace{1cm} (7)

where the ‘flame time’ is defined by $\tau_t = \delta / \pi a$ and the flame ‘feedback’ coefficient given by:

$$Z = \frac{Z e}{2} \frac{T_{ad} - T_b}{T_b - T_a}.$$  \hspace{1cm} (8)

A relation between the fluctuating (total) heat-release, $Q_{rel}$, and the fluctuating mass burning rate is given by:

$$Q'_{rel}(\tau) = c_p(T_b - T_u) m'(\tau)$$  \hspace{1cm} (9)

Substituting equation (7) in equation (5) and using $\phi_u(\tau) = \rho u_c u(\tau')$ yields:

$$\frac{\partial Q'_{rel}(\tau)}{\partial \tau} + \frac{Z}{\tau_t} Q'_{rel}(\tau') = \frac{Z}{\tau_t} \rho u_c (T_b - T_u) u'(\tau')$$  \hspace{1cm} (10)

The burner system was simulated, results are shown in Fig. 1, with the following parameters: $\pi_a = 14$ cm/s, $\phi = 0.8$, $\rho_u = 1.131$ kg/m$^3$, $\lambda = 0.0275$ J/Ksm, $c_p = 1060$ J/kgK, $T_a = 300$ K, $T_b = 1836$ K, $T_{ad} = 2016$ K and $Ze = 13.2$. 

**Control**

The goal is the active control of (forced) velocity oscillations using the model developed in the previous section. This is a disturbance attenuation problem. Fig. 2 gives the general feedback representation for this problem. With $r$ is the reference input (0 in this case), $d$ is the disturbance ((forced) velocity fluctuations), $u$ is the control signal and $y$ is the measured output (the heat-release signal). $P(s)$ represents the model and $C(s)$ the controller both in Laplace notation.

Two commonly used control methodologies are presented in this section for the design of the output feedback law $u = -C(s)y$, namely LQG/LQ control and $H_\infty$ control.

The LQG/LTR control procedure consists of a combined estimator-state feedback design, with the former assuming a fictitious Gaussian noise and a quadratic cost in the estimation error and the latter based on a quadratic cost in the system response as well as the control effort. The Controller $C(t)$ (the time equivalent of $C(s)$) is determined using the cost function:

$$J = \int_{0}^{\infty} (y^T Q y + u^T R u) dt,$$  \hspace{1cm} (11)

where $Q$ and $R$ are suitably chosen design parameters, respectively the control error and the control effort. When a loop transfer recovery (LTR) is applied, $Q = I$ and $R = \rho I$.

The idea behind $H_\infty$ is to minimize the peak value of the transfer function between the disturbance signal and the output. It is possible to add a weighting filter on
the output to be able to optimize worst-case performance. More information on LQG/LTR control and \( \mathcal{H}_\infty \) control can be found in Skogestad et al. [9].

For the control design the \( \mu \)-toolbox of Matlab will be used. Therefore the system has to be presented in a state-space notation. The heat-release signal and here derivatives are chosen to be the state variables. This can only be done if the system is linearized at \( u_0 = 14 \text{ cm/s} \) and \( \phi = 0.8 \), and an approximation is made for the time-delay by placing six first order Padé approximations in series. A comparison of the model and the approximation is given in Fig. 1. The state-space notation of the system is given by:

\[
A = \begin{pmatrix}
1.7 \cdot 10^4 & 1 & 0 & 0 & 0 & 0 \\
-1.1 \cdot 10^8 & 0 & 1 & 0 & 0 & 0 \\
-6.3 \cdot 10^{11} & 0 & 0 & 1 & 0 & 0 \\
-1.2 \cdot 10^{15} & 0 & 0 & 0 & 1 & 0 \\
-3.63 \cdot 10^{18} & 0 & 0 & 0 & 0 & 1 \\
-2.5 \cdot 10^{20} & 0 & 0 & 0 & 0 & 0 \\
1.3 \cdot 10^{24} & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}^T
\]

\[
B = \begin{pmatrix}
1.2 \cdot 10^2 \\
-1.9 \cdot 10^6 \\
1.5 \cdot 10^{10} \\
-6.67 \cdot 10^{13} \\
1.8 \cdot 10^{17} \\
-3.1 \cdot 10^{20} \\
2.4 \cdot 10^{23} \\
\end{pmatrix}^T
\]

\[
C = \begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{pmatrix}
\]

\[
D = \begin{pmatrix}
0 \\
\end{pmatrix}
\]

Results and Discussion

In this section the performance of the controllers proposed in the previous section is evaluated. Fig. 3 and 4 give the results of simulations, where a disturbance input, \( u_0 = 0.2 \sin(320 \pi t) \) is used.

It can be seen that both control strategies are able to suppress the disturbance effectively. \( \mathcal{H}_\infty \) control is a little bit faster then \( LQG/LTR \) control, but that can be caused by the choice for the performance weighting function (notch filter) and the choice for the required control effort (\( \rho = 0.1 \)).

The setup used to judge the performance of the controllers during experiments is a burner system consisting of a 0.5m long tube with a diameter of 0.05m, open at the downstream end. In the closed upstream end a small hole serves as inlet for the premixed methane/air mixture. Approximately 0.07m from the open end a burner plate, a perforated brass disc with a great deal of holes, is mounted. The flat methane/air flame will stabilize on top of this disc. To be able to measure the velocity upstream of the burner deck two (calibrated) capacitance microphones are mounted in the side of the tube.
This allows the use of the two-microphone method. A (real-time) measurement of the heat released by the flame is performed using chemiluminescence. Chemiluminescence is the emission of light as a result of a chemical reaction. Here the emitted light (ultraviolet) of excited OH molecules is measured, because the concentration of excited OH molecules is considered to be a quantitative measure for the heat-release. For this measurement a photomultiplier is used.

We only focus on the attenuation of forced oscillations, therefore the flow of premixed methane/air is perturbed by a loudspeaker (100W horn speaker). The chemiluminescence signal is used as the input for the controller. The output of the controller is connected to a second loudspeaker. This loudspeaker introduces a second oscillation in the system used to attenuate the first one. Experiments were conducted with a gas velocity of 14 cm/s and an equivalence ratio of 0.8, which corresponds to a linear burning regime.

Fig. 5 and 6 give the results of experiments, where a disturbance input $d = 0.2 \sin(320\pi t)$ is fed to the first loudspeaker.

The only conclusion that can be drawn from these Figures is that both control strategies are able to suppress the forced oscillation. Nothing can be said over how strong their suppression is, because the uncontrolled and controlled signal for the heat-release are measured during different measurements. That is also the reason that in Fig. 6 both signals are out of phase.

**Conclusion**

The results described in this paper show the potential of active model-based control of flame instabilities. Two different approaches for controller design are used: LQG/LTR control, and $\mathcal{H}_\infty$ control. Their performance is validated during simulations and experiments. The results show that both control designs are effective despite the fact that they are based on a linearized model. The choice of the weighting functions in $\mathcal{H}_\infty$ control, and the matrices $Q$ and $R$ in LQG/LTR control have influence on the performance. More study is needed on that topic.

**References**


