Mesoscopic modeling of failure in brick masonry accounting for three-dimensional effects

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Received 21 July 2003; received in revised form 2 September 2004; accepted 9 September 2004

Abstract

The generalised plane state assumption is examined in order to assess its ability to represent in a simplified fashion three-dimensional effects in the behaviour of planar masonry structures subjected to in-plane loads. Realistic failure mode and load-bearing capacity predictions for various loading directions are verified using homogenisation techniques by building failure loci under proportional loading. When introduced in a homogenisation scheme, the resulting macroscopic continuum presents itself in a two-dimensional format, suggesting the use of generalised plane state in multiscale approaches where three-dimensional effects have to be taken into account.

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Keywords: Masonry; Damage; Generalised plane state; Unit cell; Homogenisation

1. Introduction

The design of repair operations for masonry structures requires the knowledge of the load-bearing capacity of this type of material. Due to its anisotropic and heterogeneous nature, costly procedures are required to characterise them experimentally. Numerical methods recently emerged as valuable tools for such a purpose, among which homogenisation procedures have drawn particular attention. Such methods may be useful if they account for the experimentally observed failure modes of the material. An extended experimental characterisation was performed by Page and co-workers [1–3] in order to identify a
macroscopic failure surface in relation to the observed failure modes. The failure mechanisms were found to substantially differ according to the loading case (see Fig. 1). The influence of the mortar joints is dominant when overall shear is applied (Fig. 1a), or when vertical tension prevails. Vertical cracking with failure of bricks is observed for vertical compression combined with horizontal tension (Fig. 1b). These failure

![Fig. 1. Typical failure modes in masonry corresponding to various applied macroscopic paths: (a) vertical compression compression combined with shear, (b) vertical compression combined with horizontal tension and (c) vertical compression combined with horizontal compression. The bold lines represent typical crack patterns.](image-url)
mechanisms can be captured by a mesoscopic model using a plane stress description. This assumption is usually motivated by the geometrical aspect ratio of the wall and neglects the out-of-plane stresses. It is often claimed to represent well the behaviour at the external faces of a masonry wall [4]. However, it is questionable in general, as the experiments show a transition from in-plane failure mechanisms towards out-of-plane failure for nearly equal compressive principal stresses (Fig. 1c). Due to the higher elastic stiffness of the brick material, a biaxial overall compressive loading causes the appearance of a triaxial stress state in the constituents in the mid-thickness plane. The mortar joints are subjected to a triaxial compressive stress state and the bricks to an in-plane compression/out-of-plane tension state (see Fig. 2). Since both materials have a much lower strength in tension than in compression, the out-of-plane tensile stress in the brick may cause mid-thickness plane splitting. This out-of-plane failure mode cannot be simulated with plane stress models [5], as three-dimensional failure criteria are needed to include the effects of out-of-plane stresses in the constituents. However, a fully three-dimensional analysis is often computationally too expensive for practical problems. These are sometimes avoided by the introduction of an artificially low limitation of the compressive stress states in two-dimensional failure criteria [6–8]. A generalised plane state assumption can include out-of-plane effects more rigorously, but still keeping a two-dimensional formalism. The idea of using the generalised plane state for masonry structures was originally suggested by Anthoine and co-workers [4,9], and illustrated for uniaxial compression. In [4], the plane stress assumption yielded a considerably lower overall masonry compressive strength than the generalised plane state assumption caused by different failure mechanisms. However, it is not known yet whether this conclusion may be extended to other loading paths or to other constitutive models.

The objective of this study is to compare masonry failure predictions obtained using the generalised plane state and plane stress assumptions, using homogenisation techniques as an extension to [4,5,9]. This comparison will be made on the basis of the predicted failure modes and of homogenised failure loci. The paper is organised as follows. First, homogenisation principles are briefly recalled in Section 2 for the case of plane masonry structures. A brief classification of plane state descriptions is given in Section 3, outlining their main simplifications with respect to a general three-dimensional representation. An enhanced two-dimensional finite element description is used in Section 4 to compare plane stress and generalised plane state assumptions, based on unit cell computations allowing failure predictions.

2. Periodic homogenisation and failure predictions

2.1. Two-dimensional periodic homogenisation and failure detection

The mesostructure of masonry and the loading applied to it are assumed to be perfectly periodic in the plane of the wall. In a two-dimensional description, a homogeneous equivalent material may be identified if
the displacement field is also assumed to exhibit some form of local periodicity [10]. These assumptions allow to identify a representative volume element on which the equilibrium problem can be formulated and solved [10]. A displacement field is postulated with the form

$$\tilde{u} = \mathbf{E} \cdot \tilde{x} + \Delta \tilde{w}$$

where $\mathbf{E}$ is the macroscopic average strain tensor and $\tilde{x}$ is the position vector within the representative volume element. $\Delta \tilde{w}$ is a mesoscopic fluctuation field caused by the heterogeneity of the material and which is forced to be periodic at the boundary of the representative volume element. Using the periodicity assumption and the Hill–Mandel energy equivalence between the mesoscopic and macroscopic descriptions, the macroscopic stress tensor may be obtained as the volume average of the mesoscopic stress tensor

$$\Sigma = \frac{1}{V} \int_V \sigma dV$$

With mesoscopic equilibrium in the absence of body forces and the divergence theorem, the macroscopic stress is expressed in terms of representative volume element boundary contributions as

$$\Sigma = \frac{1}{V} \int_V \sigma dV = \frac{1}{V} \int_V \nabla \cdot (\sigma \tilde{x}) dV = \frac{1}{V} \int_{\partial V} (\tilde{p} \tilde{x}) dS$$

where $\tilde{p}$ is the traction vector at a point of the boundary and $\tilde{x}$ its position vector. The simplest representative volume element allowing to represent general two-dimensional loading conditions under the periodicity assumption is a single period unit cell as sketched in Fig. 3. It consists of one brick surrounded by half a joint on all sides. The periodicity of the displacement fluctuation $\Delta \tilde{w}$ forces corresponding boundary segments to exhibit the same shape in the deformed configuration [10]. It is imposed by means of linear constraint relations between couples of segments (A–D), (B–E) and (C–F):

$$\tilde{u}_D = \tilde{u}_A + \tilde{u}_2 - \tilde{u}_1 \quad \tilde{u}_E = \tilde{u}_B + \tilde{u}_3 - \tilde{u}_1 \quad \tilde{u}_F = \tilde{u}_C + \tilde{u}_3 - \tilde{u}_2$$

The macroscopic average quantities $\Sigma$ or $\mathbf{E}$ may then be imposed on the cell through three controlling points, numbered 1–3 in Fig. 3.

In Section 4, stress-controlled unit cell computations along different loading paths will be used in order to determine homogenised failure loci. A loss of uniqueness in the homogenised response is usually considered as an indication of failure in computational homogenisation procedures. In the present case, the loss of uniqueness may only appear as a result of material non-linearity since the description is geometrically linear. It is thus not dependent on the size of the chosen representative volume element and the load-bearing capacity can be estimated from unit cell computations. Additionally, it is assumed that the loss of uniqueness of the homogenised response occurs at the peak load [5,11,15]. The exact characterisation of the homogenised post-peak behaviour cannot be obtained from unit cell computations based on a strict periodicity assumption. As a result, only a qualitative value will be attributed to the unit cell failure patterns presented in Section 4. Their qualitative relevance is however strengthened by the presence of weaker constituents which lead to a limited set of preferential failure modes.

![Fig. 3. Two-dimensional unit cell definition for running bond masonry.](image-url)
2.2. Extension to incorporate three-dimensional effects

The ability to capture three-dimensional effects in the homogenisation procedure depends on the assumptions introduced at both the mesoscopic and the macroscopic scales (Fig. 4). Since no external load is applied along the thickness direction at the macroscopic scale, three options are possible, ranging from a fully two-dimensional approach without any out-of-plane effect (Fig. 4a) to a fully three-dimensional representation (Fig. 4c). The intermediate option (Fig. 4b) keeps a macroscopically two-dimensional formulation by constraining kinematically the behaviour in the out-of-plane direction, yet allowing to include simplified three-dimensional effects at the mesoscopic scale. The objective here is to assess the results obtained with the intermediate modeling option. A two-dimensional finite element formulation of the unit cell problem will be used for this purpose to investigate a wide range of loading directions.

3. Generalised plane state assumption

In this section, particular attention is given to the consequences of the generalised plane state assumption for the mesoscopic strain and stress fields. For clarity, this will be illustrated by the case of elastic compression of a laminate characterised by the mesostructure presented in Fig. 5. The laminate is subjected to a compression of 10 MPa perpendicular to the laminate direction (direction \( y \) in Fig. 5) and constrained in the \( x \) direction.

All deformations take place in the \( yz \) plane (\( z \) is the thickness direction) and a two-dimensional plane strain analysis is used. This analysis is performed on a mesh containing 300 eight-noded elements. The
mesoscopic out-of-plane stress components \( \sigma_{zz} \) and \( \tau_{yz} \) are plotted in Fig. 6 in the deformed configuration (with an amplification factor of 1200 for the horizontal displacement). The stiff material is subjected to tension while the soft phase undergoes out-of-plane compression. The shear stress vanishes at the mid-height of each phase, and is anti-symmetric with respect to the mid-thickness plane elsewhere. The out-of-plane tension generated in the stiff material is of the order of 10% of the applied average compression. Neglecting this component by assuming a plane stress state may therefore not always be justified, since the tensile strength of brick is most often less than 10% of its compressive strength [12,13].

The generalised plane state is defined by the following assumptions:

- the external faces normal to the out-of-plane direction \( z \) remain planar and vertical after deformation (see Fig. 4b),
- at the macroscopic scale, no external load is applied to the wall in the thickness direction. At the mesoscopic scale, surface tractions may however exist at the external faces as a result of the first assumption and of the interaction between the different phases,
- all mesoscopic quantities are such that the wall remains symmetric with respect to the mid-thickness plane,
- the out-of-plane component of the displacement at a given in-plane position \( (x, y) \) varies linearly through the thickness,
- the out-of-plane equilibrium is satisfied at the macroscopic scale only.

For the symmetric problem considered here, the above conditions respect the more general definition used in [4,14], where the generalised plane state is defined as a plane strain state superposed with three constant macroscopic strain components \( \varepsilon_{xz} \) being such that their dual stresses \( \Sigma_{xz} \) vanish. This may be shown using the averaging relation (3) for a three-dimensional masonry unit cell, taking into account the periodicity and anti-periodicity of the traction and position vectors respectively [15]. It results in constant mesoscopic strain and stress components along the thickness direction. The crucial point is however that mesoscopic out-of-plane normal strains and stresses can now exist.

Fig. 6. Normal (left) and shear (right) out-of-plane mesoscopic stresses in a laminate under compression (MPa).
In the fully three-dimensional representation of a masonry wall, the out-of-plane stress components evolve along the thickness. The out-of-plane displacement being an odd function of the out-of-plane coordinate, the out-of-plane normal strain and stress components are even functions and the out-of-plane shear components are odd functions of the thickness coordinate. The additional conditions introduced for the external faces to remain vertical planes constrains these quantities to be uniform through the thickness. For the mesoscopic out-of-plane shear components, this means that they are forced to vanish. The generalised plane state is therefore an approximation of the behaviour of a thin slice of material at its mid-thickness plane [4]. To illustrate this, the three dimensional variation of the mesoscopic stresses along the thickness direction under in-plane vertical compression of the laminate is compared to the generalised plane state solution along three cuts as indicated in Fig. 7. Since mid-thickness is precisely the position where the out-of-plane failure mechanism is likely to occur under strong overall biaxial compression, the generalised plane state assumption may be expected to represent correctly this failure mechanism. The phase-averaged stress components obtained by both assumptions are also compared to the average values delivered by a three-dimensional approach in Table 1. Clearly, the generalised plane state offers a better estimation of the in-plane averaged stress components in each phase than the plane stress assumption. This improvement is linked to the stiffness coupling terms between the out-of-plane and in-plane stress components which are taken into account in the generalised plane state description and neglected in plane stress.

Fig. 7. Out-of-plane stress distribution along thickness coordinate for laminate material. ‘Volume’ denotes the fully three-dimensional solution, ‘GPS’ means generalised plane state.
4. Comparison of failure predictions

4.1. Application

In this section, the effect of the generalised plane state assumption on the predicted failure behaviour of masonry is assessed. It is compared to the usual plane stress assumption for running bond masonry by comparing failure modes and failure envelopes obtained from both models in various stress directions. These envelopes are determined for proportional loading in the overall macroscopic stress space \([5]\). They are determined for the masonry mesostructure represented in Fig. 3. The considered brick has dimensions \(L \times h \times e\) of 165 \(\times\) 52 \(\times\) 100mm\(^3\) and mortar joints are 10mm thick. The mesh of this unit cell (which is represented in the results) consists of 396 eight-noded elements and 1440 nodes. Since in the generalised plane state assumption the faces normal to the thickness direction are assumed to remain vertical planes, the thickness variation is constant on the unit cell. A two-dimensional discretisation of the unit cell is used with a single degree of freedom for the thickness variation. The generalised plane state finite element formulation is elaborated in a standard way, taking into account the mesoscopic out-of-plane components.

4.2. Constitutive laws for constituents materials

The combined effect of mortar and brick–mortar interfaces in joints is represented with continuum elements, assuming a perfect bonding with the brick. This choice is made to correctly capture the generalised plane state effect. The interactions between the materials due to their different elastic properties—the main cause of the presence of out-of-plane effects—are then included in a natural fashion. Given its successful application in the description of the failure behaviour of quasi-brittle materials like concrete \([16]\), a strain-based implicit gradient damage model is used for both the brick and the mortar material \([17]\). This model uses a scalar damage variable entering the stress–strain relationship

\[
\sigma = (1 - D)^4 L : \varepsilon
\]

A damage criterion allows to determine whether a strain state change is accompanied by further damage

\[
f(\bar{\varepsilon}_{eq}, \kappa) = \bar{\varepsilon}_{eq} - \kappa \leq 0
\]

along with the set of Kuhn–Tucker relations

\[
f \leq 0 \quad \kappa \geq 0 \quad f \dot{\kappa} = 0 \quad \kappa(t = 0) = \kappa_i
\]

where \(\kappa\) represents the ultimate non-local equivalent strain state experienced so far by the material point (or the initial value \(\kappa_i\) if this threshold has not been exceeded yet); \(\bar{\varepsilon}_{eq}\) is a non-local (weighted averaged) equivalent strain introduced as the solution of the following partial differential equation incorporating a material intrinsic length scale in the constitutive setting \([18]\)

\[
\bar{\varepsilon}_{eq} - \bar{l}^2 \nabla^2 \bar{\varepsilon}_{eq} = \bar{\varepsilon}_{eq}
\]
This partial differential equation is complemented by a boundary condition, which is here of the Neumann type, i.e. a natural boundary condition for the gradient of the non-local strain field at the boundary
\[ \nabla e_{eq} \cdot \mathbf{n} = 0 \] (9)
The right hand side in (8) is the source term for the non-local averaging, i.e. \( e_{eq} \), which is a local equivalent (damage-sensitive) scalar measure of the tensorial strain state. A damage evolution law relates the value of the damage \( D \) to the most severe non-local strain experienced by the material \( \kappa \). Details related to the implementation of this implicit gradient damage model are available in [17] and the consequences of its use for masonry mesostructures are discussed in [5].

The formulation of the damage model requires the definition of the scalar equivalent strain which is chosen differently here for each of the constituents. Given the lack of experimental knowledge for some of the compressive material parameters, simple damage criteria are used. For the brick material, the equivalent strain is defined in terms of the principal effective stresses by
\[ e_{eq} = \max_i \left( \frac{\langle \hat{\sigma}_i \rangle}{E}, \frac{\langle -\hat{\sigma}_i \rangle}{kE} \right) \] (10)
where \( \hat{\sigma}_i \) are the principal values of the effective stress tensor \( \hat{\sigma} = 4L : \varepsilon \) and \( k \) represents the ratio compressive strength/tensile strength for the brick material. The McAuley brackets \( \langle \cdot \rangle \) are defined as \( \langle x \rangle = \frac{1}{2} (x + | x |) \) and \( 4L \) is the undamaged stiffness tensor of the material. For the combined effect of mortar and brick–mortar interface, a Drucker–Prager criterion, modified with a cap in the compressive regime has been used. The equivalent strain is defined in terms of the strain tensor invariants as (summation on repeated indices)
\[ I_1 = e_{ii}, \quad J_2 = \frac{1}{6} \hat{f}_i^2 - \frac{1}{2} e_{ij} e_{ij} \] (11)
\[ e_{eq} = \begin{cases} 
A \left( 1 - 2v \right) I_1 + \frac{B}{(1 + v)} \sqrt{J_2} & \text{if } \frac{\sqrt{J_2}}{(1 + v)} \geq \frac{A - C}{D - B} \left( 1 - 2v \right) \\
C \left( 1 - 2v \right) I_1 + \frac{D}{(1 + v)} \sqrt{J_2} & \text{if } \frac{\sqrt{J_2}}{(1 + v)} \leq \frac{A - C}{D - B} \left( 1 - 2v \right) 
\end{cases} \] (12)
where the coefficients \( A, B, C \) and \( D \) controlling the shape of the damage criterion are defined in terms of the strength parameters
\[ A = \frac{f_c - f_t}{2f_c}, \quad B = \sqrt{3} \frac{f_c + f_t}{2f_c} \]
\[ C = -\frac{f_t}{3f_h}, \quad D = 2\sqrt{3} \frac{f_c}{f_b} - 4 \frac{f_t}{\sqrt{3} f_h} \] (13)
\( f_t, f_c, f_h, f_b \) representing respectively the mortar strengths in uniaxial tension, uniaxial compression, hydrostatic triaxial compression and a special case of triaxial compression (with two of the principal stress values halved with respect to the major one), see [5] for details. The shape of these criteria in the principal stress space is illustrated in Fig. 8 for the material parameters used in the computations (see Table 2) and for the case of plane stress. Note that the shape of the damage criterion used for joints reflects a high compressive strength under multi-axial compression. In the three-dimensional space of principal stress components, the shape of the mortar criterion presents a symmetry of revolution as a double conical surface centered on the hydrostatic axis. Note also that other criteria might be used if additional experimental evidence is available, especially for the combined effect of the mortar material with the brick–mortar interface.
An exponential damage evolution law is used to quantify damage growth

$$D = 1 - \frac{\kappa_i}{\kappa} e^{-\beta(\kappa - \kappa_i)}$$

(14)

The equivalent strain expressions (10) and (12) are defined such that damage is initiated when they reach the initial threshold value $\kappa_i = f_t/E$. The parameter $\beta$ allows to control the softening tail of the stress–strain relation and is essentially related to the tensile fracture energy of the material.

In order to allow a quantitative comparison with experimental results, the constitutive parameters used in this study were taken from [1–3] for as far as available. The bulk tensile properties of the mortar are adjusted in order to represent the tensile strength and fracture energy of joints in mode I as obtained in experiments [1–3]. The uniaxial compressive properties of mortar and bricks are taken without modification from the same sources. The material parameters used in the simulations are summarised in Table 2. Only the average values reported experimentally for the constituents were considered, ignoring the experimental scatter. For parameters not reported in [1–3], typical values were extracted from [12,13]. The Young’s modulus of mortar was obtained from [12] based on the mortar composition reported in [3] (cement:lime:sand volumetric ratio of 1:1:6). The damage evolution law parameter $\beta$ is related to the tensile fracture energy. The tensile fracture energy density is expressed in terms of $\beta$ and integrated over the full joint thickness (damage is allowed to spread over the whole thickness of joints). The result is required to represent the observed tensile fracture energy. The values for the tensile fracture energy are 9 Nm/m² and 16.2 Nm/m² for the mortar and brick materials respectively, the same order as the values reported in [13] for the considered mortar composition and similar tensile bond strengths.

![Fig. 8. Shape of constituents damage loading surface in plane stress space.](image-url)

Table 2
Material parameters (values with a * are typical values retrieved from various sources in the literature)

<table>
<thead>
<tr>
<th>Material</th>
<th>$E$ (MPa)</th>
<th>$v$</th>
<th>$l_c$</th>
<th>$f_t$ (MPa)</th>
<th>$\beta$</th>
<th>$f_c$ (MPa)</th>
<th>$f_b$ (MPa)</th>
<th>$f_h$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brick</td>
<td>16,700*</td>
<td>0.15*</td>
<td>1.73</td>
<td>0.75</td>
<td>800*</td>
<td>15*</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Mortar</td>
<td>3900*</td>
<td>0.20*</td>
<td>1.73</td>
<td>0.13*</td>
<td>140*</td>
<td>5.6*</td>
<td>8.72</td>
<td>7.45</td>
</tr>
</tbody>
</table>
4.3. Failure patterns for non-shearing overall stress states

The type of failure mode obtained computationally is an important result that allows to assess the validity of the description. Therefore, the damage distributions at the end of the unit cell computations obtained for both assumptions are depicted in Fig. 9. These patterns were obtained only well beyond the peak loads needed to construct the failure envelopes, using advanced path following techniques [19]. Similar failure modes have been found for most of the loading paths for both assumptions. A distributed vertical cracking of bricks with head joints failure is captured for both assumptions under vertical compression of masonry (case A) as reported in experiments [1–3]. Note however that for the plane stress final failure pattern, the bed joints are also mostly degraded, while this is not the case for generalised plane state because of the tri-axial compressive stress state. For tension dominated loading paths (cases B and C), the failure modes are consistent with the reported experiments. As expected, the results delivered by the plane stress and generalised plane state assumptions differ in the biaxial compressive range (cases D, E and F) in terms of identified failure modes. The experimentally observed transition towards out-of-plane failure is captured in the generalised plane state description as a result of its ability to capture mesoscopic out-of-plane stresses, while it is not captured by the plane stress description. Case D represents a transition from an in-plane to an out-of-plane failure. In case E, the failure mode detected in the generalised plane state description indicates an almost complete failure of both the brick and the mortar. For the corresponding plane stress computation, an in-plane failure mode is found with vertical brick cracking and partial bed joint failure, which is not consistent with the experimentally observed failure mode.

4.4. Failure patterns for vertical compression combined with shear

The failure patterns obtained along loading paths combining vertical compression and shear are illustrated in Fig. 10. They are similar, except under pure shear (case A). For this case, the head joints fail for the generalised plane state description, while only bed joints fail in the plane stress description. When the shear stress is of the same order of magnitude as the compressive stress, the anisotropy development is prescribed by the geometrical arrangement as damage only appears in a part of the bed joints. For these two first stress paths illustrated in Fig. 10, the overall induced anisotropy strongly alters the initial orthotropic symmetry of the material. For moderate shear, brick cracking appears. In such loading conditions, the loss of orthotropy is still present, but is less pronounced and is mainly driven by the damage growth in the brick.

4.5. Envelopes for non-shearing overall stress states

The failure envelopes obtained for both assumptions are next compared for non-shearing overall stress states with experimental results reported in [1–3]. The computational envelopes are built based on the maximum load-bearing capacity reached for each proportional stress direction, and are represented in Fig. 11, where each dot represents a finite element computation (the capital letters refer to the corresponding failure patterns reported in Fig. 9).

The shaded area in this figure represents the experimentally obtained envelope with its scatter [1–3]. For the considered damage laws and constitutive parameters, the failure envelopes determined for plane stress and generalised plane state nearly coincide. A maximum difference of 5% is found, which is much lower than the experimental scatter. From the load-carrying capacity point of view (i.e. without considering failure pattern prediction), the benefit of using the generalised plane state description seems negligible at least for the considered failure criteria and material parameters. The shape of both computational failure envelopes is qualitatively similar to the experimental envelope, given the limited knowledge of geometrical data from [1–3]. The quantitative agreement is fairly good in the tension related quadrants. The difference in the
Fig. 9. Failure (damage distribution) for loading directions defined in Fig. 11 for generalised plane state (left) and plane stress (right) assumptions.
horizontal compressive strength is linked to the definition of failure used for this loading direction in [1–3], which was there identified through the failure of bed joints and not on the basis of the load-bearing capacity as done here. In the biaxial compressive range, the non-symmetric shape of the envelope with respect to the equibiaxial compression axis is captured. This anisotropy is slightly stronger for the computational envelopes than for the experiments. Differences in the aspect ratio of the bricks (not reported for the experimental case) and the rather simple brick failure criterion in the biaxial compressive range may explain this deviation.

In the results reported in Fig. 11, the plane stress assumption consistently yields a higher load-bearing capacity than the generalised plane state. The lower strength values obtained here for the generalised plane state are in contradiction with the observation reported in [4] for vertical compression. A modified Mazars damage evolution setting with distinct tensile and compressive damage parameters was used in [4], with the appearance of different failure mechanisms under vertical compression for both descriptions. A lower load-bearing capacity is thus obtained here for the generalised plane state assumption (for the same overall

Fig. 10. Failure (damage distribution) for compression combined with shear for generalised plane state (left) and plane stress (right) assumptions.
failure mechanism) due to the acceleration of tensile failure of the brick. Under vertical compression, and upon almost complete failure of the head joints, the bed joints are submitted to a biaxial compressive stress state in the plane stress description, or triaxial compression for the generalised plane state case. In-plane tensile stresses also appear in the brick. In our simulations, tensile failure in the brick is found to be predominant for both assumptions as illustrated in Fig. 9(A). In the generalised plane state description, out-of-plane tensile stresses are causing the stress state in the brick to be more critical than for the plane stress case. The ultimate failure is governed by a competition between triaxial compressive failure of mortar and compressive—biaxial tension of bricks in the generalised plane state case. For plane stress, a competition appears between biaxial compressive failure of mortar and biaxial compressive-tensile failure of bricks.

4.6. Envelopes for overall compression combined with shear

The envelope corresponding to vertical compression combined with shear is next identified. This type of overall stress state is usually encountered in masonry structures under normal loading conditions. Both the plane stress and the generalised plane state envelopes are represented in Fig. 12. The obtained failure
envelopes are nearly coincident when shear is dominant while the difference is more pronounced for low values of shear. For high shear stress values, the plane stress assumption leads to lower peak stresses than the generalised plane state assumption. The generalised plane state becomes more critical for lower shear stresses. In addition, the envelope obtained with the generalised plane state description is (slightly) non-convex for low values of shear stresses combined with strong compression.

5. Conclusions

A two-dimensional implementation of the generalised plane state formulation has been used in an extension of mesoscopic analyses of masonry failure performed in earlier works [4,5]. An implicit gradient damage model was used to simulate the quasi-brittle behaviour of mortar and brick materials. Based on simple degradation criteria, a comparison with experimental evidence was achieved using homogenisation techniques. It shows that the generalised plane state assumption allows to capture correctly the failure mechanisms for various stress paths, while the plane stress assumption is not able to detect the correct failure modes under biaxial compressive states. However, for the considered failure criteria a good qualitative approximation may be obtained for the load-carrying capacity with both descriptions. This effect may to be attributed to the shape of the applied damage criteria in the compressive range, for which the experimental data is scarce. More substantial differences may however be found for other criteria, parameters or evolution laws. Further investigations would be required to assess the influence of various modelling choices in this sense. Coupled to the overall two-dimensional format of the homogenised behaviour, the generalised plane state assumption could be used fruitfully in multi-scale structural analyses of planar masonry walls.

Acknowledgement

Fruitful discussions with Dr. A. Anthoine are gratefully acknowledged. The first author was supported financially by the Région Wallonne (Belgium) under grant 215089 (HOMERE).

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