INTRODUCTION

Historically slip in a pushbelt type CVT was regarded as destructive, because it was not controllable and resulted in damage to the variator. Recent publications suggest that limited amounts of slip can be allowed [7] without damage to the variator. This opens the door to other strategies for lowering the power consumption of CVT's. Not only can slip be used for optimizing variator efficiency [1], but also actuation efficiency can be improved. If the safety margin is eliminated, the clamping force can be reduced by more than 25%. This can be directly translated into a 25% decrease in actuation power. Shifting behaviour is also influenced by slip [3]. This can be used to greatly reduce the power needed for fast shifting during emergency stopping, tip-shifting and kickdown actions. Using this strategy the force needed for shifting is reduced. This has not only effects on the power consumed by the actuation system of the CVT, which is by itself a significant factor in the variator efficiency, but also has some implications on the design of the CVT. If actuation forces are smaller, the actuation system can be smaller and cheaper, and the CVT itself will be lighter. Furthermore, other actuation systems than hydraulics can be considered, for example electromechanical actuation, to further enhance the controllability and efficiency of the actuation system.

In this paper measurements are shown for shifting behavior of the CVT and a relation will be given with slip in the system. This data is then used to make a simple model of the transient behavior of the CVT, which includes the relationship between slip and shifting behavior. Although more sophisticated models exist, like described by Šrnik [6], these models often are very complex, thereby making them more computationally expensive, which limits their use for some applications where simulation time is important.

MEASUREMENTS

Measurements have been made on a variator test setup as shown in figure 3. This setup consists of a driving electromotor a variator belt-box and a braking elektromotor. Sensing devices include a torque sensor and a rotary incremental encoders on each axle and a linear encoder for pulleyposition measurements. Measured are primary torque $T_p$, secondary torque $T_s$, primary speed $\omega_p$, secondary speed $\omega_s$, and secondary pulley position $x_s$.

The governing equations of motion in the pushbelt CVT are shown by Kobayashi [4] and Asayama [2]. In these equations the friction coefficient of the contact between the belt and the pulley is distributed in the radial and tangential direction. In figure 2 the measured relationship between radial friction between the belt and the pulley is shown. From this figure it can be seen that the radial friction decreases with higher shifting speeds. This can be seen as slip-mode shifting according to Ide [3].

Since slip in tangential direction and slip in radial direction occur on the same surface to surface interface, there must be a relationship between traction in tangential and radial direction. Slip in tangential direction is shown in figure 4. The combined value of $\mu_t$ and $\mu_r$ is given by $\mu = \sqrt{\mu_t^2 + \mu_r^2}$. The result is shown in figure 5.

The shifting force is defined as the difference between
Figure 1: Forces acting on the belt

the actual primary clamping force and the primary clamping force for which no shifting occurs. Since the friction was assumed to be coulomb there should be no shifting until a certain threshold. This however is not what can be seen from the measurements in figure 2. This so-called creep-mode shifting[3] is not caused by slipping of the belt on the pulley, but by elastic deformation of the pulley. The belt crawls along the pulley. The resulting shift-speed is dependent on primary speed and the ratio of the pulley, as well as the stiffness of the pulleys as shown by Ide.

If the belt is slipping tangentially, slip in radial direction is almost instant, whereas where the belt is not slipping on the pulley tangentially the belt will only slip radially when the shifting force is sufficiently high. This can only be achieved if the clamping force is sufficiently low, because of limitations to the actuation system.

As stated in Bonsen [1], the belt will slip in tangential direction when torque is transmitted through the variator. Normally the CVT is operated in the microslip regime. In this regime only part of the blocks on the belt slip on the pulley. If all blocks are slipping on the pulley the CVT is in the macroslip regime. In the microslip regime the transmission will shift in creep-mode, when the CVT is in macroslip mode slip-mode shifting will occur and vice-versa.

From figure 2 can be seen that the radial friction coefficient decreases significantly for higher shift speeds. This is mainly due to the fact that high shift speeds cause large torques due to acceleration of the primary or secondary axis. The tangential friction causes a decrease in the radial friction.

MODELLING

For the purpose of modelling friction a coulomb friction model is used. This is expressed in:

$$\mu = \text{sign}(\nu)\mu_{\text{max}}$$  \hspace{1cm} (1)

This friction is divided in two directions. The friction in the tangential direction ($F_{wt}$) is dependent on the torque on the belt. The friction in the radial direction ($F_{wr}$) is dependent on the shifting force ($\Delta F$). The forces acting on the belt are shown in figure 1.

$$\Delta F = F_p - F_p^*$$ \hspace{1cm} (2)

$$F_p^* = \kappa(T, r) F_s$$ \hspace{1cm} (3)

$$F_{p,s}^* = \frac{F_{p,s}}{\cos \theta}$$ \hspace{1cm} (4)
where \( F_{p}^* \) is the primary clamping force for which the variator is not shifting. The value of \( F_{p}^* \) is dependent on the secondary clamping force and an experimentally obtained function \( \kappa \).

If the torque load on the variator is known, which is usually the case in simulation environments, the value of \( \zeta \), the averaged angle the friction force makes with the tangential direction, can be calculated by substituting 6 in 7, which results in:

\[
\zeta_{p,s} = \arccos \frac{T_{p,s}}{R_{p,s} \mu F_{N}^s}
\]  

We can now calculate \( F_{wr_{max}} \). If the shifting force in the direction of the radial friction force is smaller than the maximum friction force, the variator will shift in sliding mode. If the shifting force is smaller creep-mode shifting will occur.

In creepmode, shifting (indicated by a translation of the secondary pulley) can be modelled by:

\[
\frac{dx}{dt} = c_1 \omega_p \Delta F
\]  

The constant \( c_1 \) is a measure for the spiral running of the belt when shifting. It can be thought of as a measure for the stiffness of the pulleys. Stiffer pulleys will give slower shifting. In creepmode the Ide model is followed, but is simplified to one parameter to facilitate faster estimation. In slip mode shifting can be modelled by:

\[
\frac{d^2x}{dt^2} = c_2 \left[ \Delta F - (F_{wr_{max}}^p + F_{wr_{max}}^s) \right] - c_3 \frac{dx}{dt}
\]  

In slip mode the spiral running is no longer leading for the shifting dynamics. Instead the belt is modelled as a damped single mass system not dependent on angular velocity of the primary pulley. This type of modelling follows the Shafai model.

**IDENTIFICATION**

The model parameters \( c_1, c_2 \) and \( c_3 \) are estimated using the least squares method using the measured data as input. The result is plotted in figure 6. The green line indicated with ‘tue’ is the result for the proposed model. The Shafai [5] and Ide model [3] were fitted using the same measurement data. The proposed model has a lower average error than the other two models.

The Shafai model uses only one parameter. The result, although the least accurate of the three, is an easy to implement model with little parameter estimation issues. Ide uses a variable number of parameters. In this comparison three parameters are used. With three parameters Ide’s model was not as accurate as the proposed model, which also uses three parameters.

From the parameter estimation follows that the mass term, which can be seen as \( m = 1/c_2 \), is not negligible. Moreover, it is an important factor.

The cost function used for parameter estimation is:

\[
J = a \sum (x_{mea} - x_{sim})^2 + \sum (\dot{x}_{mea} - \dot{x}_{sim})^2
\]  

with \( x \) the secondary pulley position. The values obtained for the constants are given in the table below.
The values obtained by the optimization routine are not what would be expected. The obtained inertia is much higher than expected. The mass of the oil in the pipes has an important influence on the results. The high value for the mass found can be accounted for by the oil in the pipes. The acceleration of the oil in the pipes is much higher than in the cylinders due to the high ratio between the piping cross-section and the cylinder surface area. Variations in the parameter-set are shown in figures 9, 10, 11 and 12. Clearly the model is most sensitive to variations in $\mu$, but sensitivity for all parameters is in the same order of magnitude.

In figure 7 the simulation results with respect to the pulley displacement speed. From this graph can be seen that all models differ from the measured data in the fast shifting range. Parameter fitting using different cost function should give more accuracy in this area.

**DISCUSSION**

The reduction of the required clamping force and shifting force reduce the power consumed by the actuation system. Moreover it reduces the maximum power output required from the actuation system. This would enable CVT designers to make the actuators smaller in size and therefore cheaper to build, while at the same time the losses are reduced. To fully utilize the possibilities of the described effect cheap ways to estimate slip should be investigated. More research on the robustness of pushbelts for slip is needed to gain insight in the reliability issues and durability of a CVT with slip control.

The proposed model accurately predicts the shift-speed characteristics of the pushbelt CVT. This model can predict the slip in the system by using a simple simulation algorithm. Parameter $c_1$ can be made dependent on the CVT ratio like in the Ide model. This would increase the accuracy of the simulation, but would add complexity and estimation effort. In essence the model uses the Ide description for creep-mode shifting and the Shafai description for slipmode shifting. Critical is the the switching between the two models.

The model should be verified using more data, but initial results are promising. With this model more efficient actuation systems can be designed that make use of the benefits of slip mode shifting.

**REFERENCES**


Figure 6: Simulation results with respect to ratio

Figure 7: Simulation results with respect to axial pulley speed

Figure 8: Switching between slip and creep mode (1: slipmode, -1: creepmode)

Figure 9: Parameter variation of parameter $c_1$

Figure 10: Parameter variation of parameter $c_2$
Figure 11: Parameter variation of parameter $c_3$

Figure 12: Parameter variation of parameter $\mu$