Abstract

The control behaviour of an electro-dynamic motor, also known as a moving coil motor, is superior to other motor types. This is also true for the linear version of this motor type. The classic solution to produce an $x$-$y$ movement with a long-stroke is to apply a stack of at least two linear motors. A rising moving mass, combined with a limited mechanical stiffness, limits the bandwidth. An alternative is the planar motor, which generates $x$- and $y$-movements. This paper describes an electro-dynamic planar motor with moving coils, which levitates and propels itself over long $x$- and $y$-strokes under full control over six degrees of freedom (6-DOF).

Keywords: Planar motor; Magnetic levitation; Control; Modelling; Electro-dynamic; Precision engineering

1. Introduction

Magnetic levitation removes the need for the use of mechanical bearings in vacuum conditions where lubrication might pollute the vacuum. Magnetic bearings for rotating shafts are commercially available. Trumper [1] developed novel levitation linear motors and proposed their use in linear levitated stages for lithography. Electro-magnetically driven platforms with a long-stroke in the $x$- and $y$-direction based on the Sawyer linear stepper motor are described by Quaid and Rizzi [2], but here additional air-bearings are essential to levitate the platform. A levitated platform with a short-stroke is described by Kim et al. [3]. A long-stroke planar motor with a moving magnet provided with a mechanical bearing system is presented by Flores Filho et al. [4]. Spherical motors are studied at the RWTH-Aachen as reported by Weck and Reinartz [5].

Electro-dynamically levitated and propelled platforms are also described by several authors. The geometry of Cho and coworkers [6–8] analyses a planar motor with moving short coils with respect to the magnet pitch. Hazelton and Gery [9] patented many magnet and coil geometries including overlapping coils for a moving magnet design. Also the work of Jung and Baek [10] concentrates on the moving magnet geometry, with the advantage that there are no moving cables. No reference, however, is made to the parasitic pitch, roll or yaw torques, which can impact on systems with a millisecond settling time. Halbach [11] produced the first publication concerning the principle of Halbach magnet-arrays and Trumper et al. [12,13] recognized this as challenge for linear motors and analysed the force generation above a Halbach array.

The long-stroke planar motor concept described in this paper is based on the combination of a two-dimensional Halbach magnet array and relatively long coils, that becomes possible by rotating the coils $45^\circ$ with respect to the standard orientation as described by e.g. Cho and coworkers [6–8]. The advantage is that the length of the coils can be used to adapt the motor design to the application requirements. A further characteristic is that non-overlapping coils are used to enable ortho-cyclic winding technology [14], resulting in a high copper packing factor and an improved heat transfer.

The planar motor levitates and propels a platform, provided with non-overlapping coils, above a NdFeB-magnet plate. It is based on electro-dynamical principles and has a simple structure suitable for series production in combination with four three-phase amplifiers. After explanation of the principles, a description of the analytical equations of the force generation is given. The force and torque components are highly de-coupled to enable a relatively simple 6-DOF control. The exception is a pitch torque, the presence of which is demonstrated by a simple analytical model. The voltage...
Electromechanical performance and behaviour.

2. Electro-mechanics

The planar motor is an alternative to the well-known H-shaped gantry and can be used in vacuum conditions, where roller and air bearing are undesirable. The electro-dynamical motor principle is chosen to enable force generation in two directions. Zhu and Howe [15] gives an overview of special magnet geometries, called Halbach magnets, with the main objective of enhancing the motor performance. Fig. 1 shows a simple Halbach magnet structure with a schematic representation of the magnetic field lines.

Coil A, shown in Fig. 1, generates a force in the vertical direction z whereas coil B generates a horizontal force. The electrical equation involved is represented by Eq. (1):

\[ u = iR + \frac{\partial \psi}{\partial t} = iR + \frac{\partial}{\partial t} \left( \frac{\partial \psi}{\partial x} \right) + \frac{\partial \psi}{\partial x} \frac{dx}{dt} . \]  

(1)

The symbols used are: \( u \) as voltage, \( i \) as current, \( R \) as coil resistance, \( L \) as self-inductance, which is assumed to be \( z \)-position independent, and \( \psi \) as the coupled flux generated by the magnets in the coil. Multiplication of Eq. (1) with the current \( i \) gives the power balance according to Eq. (2):

\[ P = ui = i^2 R + 0.5 \frac{dL^2}{dt} + i \frac{\partial \psi}{\partial x} \frac{dx}{dt} + i \frac{\partial \psi}{\partial z} \frac{dz}{dt} . \]  

(2)

The first term is the Ohmic loss, the second item is the change of the stored magnetic energy and the last terms represent the mechanical power in the \( x \) - and \( z \)-directions, respectively. The mechanical power in the \( x \) - and \( z \)-direction can also be expressed in mechanical terms. For example, Eq. (3) is valid for the \( x \)-direction:

\[ P_{\text{mech},x} = F_x \frac{dx}{dt} = i \frac{\partial \psi}{\partial x} \frac{dx}{dt} . \]  

(3)

This leads to the force constants in Eq. (4):

\[ K_x = \frac{F_x}{i} = \frac{\partial \psi}{\partial x} \quad \text{and} \quad K_z = \frac{F_z}{i} = \frac{\partial \psi}{\partial z} . \]  

(4)

A preferred function for the coupled flux \( \psi \) is the sinusoidal relationship with respect to the \( x \)-position, because this allows reuse of the algorithm applied for electronically commutated (EC) motors. An exponential dependency on \( z \) is most likely and a realistic function model for \( \psi \) is given in Eq. (5):

\[ \psi(\alpha x, \tau) = \psi_0 e^{-\tau/3} \sin \left( \frac{\pi x}{\tau} \right) . \]  

(5)

\( \alpha \) is a geometry-determined constant and \( \tau \) is the distance between the magnet poles N and S. The assumed sinusoidal relation between the flux and the \( x \)-position is reasonable, because higher order harmonics in the flux density above the Halbach array can be removed to a large extent by the integration over the coil plane when suitable coil dimensions are applied. The intended applications require a constant distance \( z \) between the coil and the magnets, so \( z \) is assumed a constant. The force constants can now be derived from Eqs. (4) and (5) as given in Eq. (6):

\[ K_{x,j} = -\frac{\psi_0}{\tau} \cos \left( \frac{\pi x}{\tau} \right) \]  

(6)

To create a 2-DOF long-stroke action we have to provide the actuator with three coils fed by a three-phase amplifier. The horizontal distance between the coils is chosen as \( 4\tau/3 \) to obtain a symmetrical three-phase motor, as given in Fig. 2.

For each of the three coils \((j = 1, 2, 3)\) the following Eqs. (7) and (8) are valid:

\[ K_{x,j} = \frac{\psi_0}{\tau} \cos \left( \frac{\pi x}{\tau} + \left( j - 1 \right) \frac{2\pi}{3} \right) = K_{x} \cos \left( \frac{\pi x}{\tau} + \phi \right) , \]  

(7)

\[ K_{z,j} = -\frac{\psi_0}{\tau} \sin \left( \frac{\pi x}{\tau} + \left( j - 1 \right) \frac{2\pi}{3} \right) = K_{z} \sin \left( \frac{\pi x}{\tau} + \phi \right) . \]  

(8)

The phase angle in Eqs. (7) and (8) is given by \( \phi \) as \( \phi = (j - 1)\pi/3 \) as usual for a three-phase system. The amplifier should generate the currents \( i_j \) according to Eq. (9):

\[ i_j = I \sin \left( \frac{\pi x}{\tau} + \phi + \theta \right) . \]  

(9)
The relation between the position and the current is usual in electronically commutated (EC) motors. The phase angle \( \phi \), however, is unusual. Its values determine the ratio between the force components. Combining Eqs. (7)–(9) gives

\[
\mathbf{F} = \begin{bmatrix} F_x \\ F_z \end{bmatrix} = \begin{bmatrix} K_{x,1} & K_{x,2} & K_{x,3} \\ K_{z,1} & K_{z,2} & K_{z,3} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}
\]

\[
= \frac{3}{2} \begin{bmatrix} \hat{K}_x \cos(\phi) \\ \hat{K}_z \sin(\phi) \end{bmatrix}.
\]  

(10)

So the current amplitude determines the force amplitude and the force direction is controlled by the phase angle \( \phi \). The forces have become independent of the \( x \)-position by the chosen \( x \)-position dependency of the current. The current amplitude and the phase angle \( \phi \) needed for the required values of the forces \( F_x \) and \( F_z \) are given in Eq. (11):

\[
\hat{I}(t) = \frac{2}{3} \sqrt{\left( \frac{F_z(t)}{\hat{K}_z} \right)^2 + \left( \frac{F_x(t)}{\hat{K}_x} \right)^2},
\]

\[
\phi(t) = \arctan \left( \frac{F_x(t)}{F_z(t)} \right) \hat{K}_z / \hat{K}_x.
\]  

(11)

Until now only an \( x \)-force is considered. It is our intention to use the magnet plate also for \( y \)-forces. This is achieved with the geometry given in Fig. 3, where the magnet borders are rotated \( 45^\circ \) with respect to the coils. The given coil enables \( y \)-forces, whereas a coil perpendicularly directed with respect to the one drawn, creates \( x \)-forces.

The behaviour of the coupled flux as a function of the position \( x \) determines the motor constants \( K_x \) (Eq. (4)). Two of the three line-to-line fluxes of a prototype are measured and their derivatives to the position \( x \) are given in Fig. 4. These curves confirm the sinusoidal behaviour of \( K_x \) in Eq. (7) as a consequence of Eq. (4). The sum of the enclosed fluxes in a symmetric three phase equals zero, so the third line-to-line flux can be reconstructed. With Eq. (4) and taking into account the conversion from line-to-line values to phase values, we get the force constant \( K = F/\hat{I} \) as shown in Fig. 5. The limited position dependency of the force constant is one of the conditions for good servo performance.

It is known that the air gap flux density of a segmented Halbach array exhibits significant spatial harmonics. These harmonics are minimized in the enclosed flux by the spatial integration of the flux density by the \( 45^\circ \) rotated coils.
The width of each coil equals 2L/3. For the sake of simplicity it is assumed that the coils have an infinite length in the y-direction and that we are considering a length l coil in this y-dimension only. A further simplification with respect to the flux-density is to omit its dependency on the z-direction. Eq. (12) gives the flux-density components \( B_1 \), \( B_2 \), and \( B_3 \) above the magnets in Fig. 2, including this last simplification:

\[
B(p,x) = \begin{bmatrix}
B_1 \\
B_2 \\
B_3
\end{bmatrix} = \hat{B} \begin{bmatrix}
-\cos \left( \frac{\pi x + p}{\tau} \right) \\
0 \\
\sin \left( \frac{\pi x + p}{\tau} \right)
\end{bmatrix} \tag{12}
\]

The current-density has a y-direction in the three coils (\( j = 1 \), 2 and 3) to the position according to the relationship (13):

\[
J(j, p, \phi) = J \sin \left( \frac{2\pi p}{\tau} + \phi + \frac{4\pi}{3}(j - 0.5) \right) \tag{13}
\]

This current-density has a y-direction only, so Eq. (14) holds:

\[
J(j, p, \phi) = J \sin \left( \frac{2\pi p}{\tau} + \phi + \frac{4\pi}{3}(j - 0.5) \right) \tag{14}
\]

The force and torque generated by the three coils (\( j = 1 \), 2 and 3) are based on Lorentz forces, as expressed by Eqs. (15) and (16):

\[
F = \sum_{j=1}^{3} \int \int \int_{\text{coil}_j} J(j, p, \phi) \times B(p, x) \, dV, \tag{15}
\]

\[
T = \sum_{j=1}^{3} \int \int \int_{\text{coil}_j} r \times J(j, p, \phi) \times B(p, x) \, dV. \tag{16}
\]

Eq. (15) gives the force components of the three phases of a former together as Eqs. (17) and (18):

\[
F_x = \frac{6\tau}{\pi} \hat{B} J_{hcoil} \sin \left( \frac{\pi b_h}{2\tau} \right) \sin \left( \frac{\pi b_h}{2\tau} + \frac{\pi}{3} \right) \sin(\phi), \tag{17}
\]

\[
F_y = \frac{6\tau}{\pi} \hat{B} J_{hcoil} \sin \left( \frac{\pi b_h}{2\tau} \right) \sin \left( \frac{\pi b_h}{2\tau} + \frac{\pi}{3} \right) \cos(\phi), \tag{18}
\]

As the reference centre for the torque we apply the centre of the former as indicated in Fig. 2: the arm can be written now as Eq. (19):

\[
r = \begin{bmatrix}
x - 2\tau \\
0 \\
0
\end{bmatrix}. \tag{19}
\]

The given geometry produces according to Eq. (16) a y-torque only and it consists of a position dependent and position independent term according to Eqs. (20) and (21):

\[
T_y = \frac{8\hat{B} J_{hcoil} \tau^2}{\pi \sqrt{3}} \sin \left( \frac{\pi b_h}{2\tau} \right) \sin \left( \frac{\pi b_h}{2\tau} + \frac{\pi}{3} \right) \sin(\phi) \times \sin \left( \frac{2\pi p}{\tau} + \phi \right), \tag{20}
\]

\[
\sin \left( \frac{\pi b_h}{2\tau} - \frac{2\pi}{3} \right) \left( \cos(\phi) + \frac{1}{3} \right) \sin(\phi). \tag{21}
\]

Substitution of the forces \( F_x \) and \( F_y \) in the first component \( T_y \) gives Eq. (22):

\[
T_y = \frac{4\tau}{\sqrt{3}} \left( F_x^2 + F_y^2 \right) \sin \left( \frac{2\pi p}{\tau} + \phi \right) = 0.77 \left| F \right| \tau \sin \left( \frac{2\pi p}{\tau} + \phi \right). \tag{22}
\]

This result shows that the position-dependent pitch effect behaves sinusoidally with a periodicity of \( \tau \), its amplitude is linearly related to the amplitude of the force vector and its phase is related to the phase angle \( \phi \). It is also important that the amplitude is linearly related to \( \tau \). This latter point suggests using a small pitch \( \tau \). However, it can be proven that the magnetic field density behaves according to \( |B| = |B_{co}| e^{-\omega \tau} \), so a smaller value of \( \tau \) reduces a suitable value of \( h_{coil} \). Finally, one will find higher losses at a certain force level. On the other hand a higher value of \( \tau \) leads to rising magnet assembly problems.

The best way to remove the torque \( T_y \) is to add of a second former shifted over the distance \( \pi \tau \), which is provided with currents with the same amplitudes and an appropriate phase-shift. Its torque is then directly opposing the torque of the first former.

The second component of the torque, \( T_x \), depends on geometrical variables as \( \tau, b_h \), and the controlled variable \( \phi \). The actual position \( p \) has no influence as can be seen in Eq. (21). The value of the coil side width \( b_h \) can be used to reduce this torque component, however this also influences the force produced. A point of interest is the ratio \( F^2/\phi \) as a function of \( b_h \) with \( P \) as the dissipated power in the coils. With a fixed current density \( J \) is the dissipation \( P \) in linear relation to the width of the coil sides \( b_h \), so for a comparison it is sufficient to determine the ratio \( C = F(b_h^2/b_h) \max(F(b_h^2/b_h^2)) \). Fig. 6 shows that \( b_h = 0.55\tau \) is a good choice as far as \( F^2/P \) is concerned.
The torque \( T^c \) depends on \( \phi \), \( \tau \) and \( b_s \). The value of \( \phi \) depends on the required values of \( F_x \) and \( F_z \), so \( \phi \) has to be considered as a time dependent variable. Fig. 7 shows the ratio \( 10^{4}T^c/[F_x] \) with \( \phi = 0, 2\pi \), \( \tau = 0.032 \) m and \( b_s/\tau = \{1/3, 2/3\} \).

From these figures we can conclude that \( T^c \) becomes equal to zero at \( b_s = 0.38r \) for all values of \( \phi \). Previously we found \( b_s = 0.55r \) as best value for a maximum steepness. In general it is clear that one should apply: \( 0.38r < b_s < 0.55r \). The value of \( [F_x]/[F_z] \) equals 0.77, so the position-dependent torque \( T^c \) is more than 10 times higher than the position-independent torque \( T^p \), even at \( b_s = 0.55r \). The penalty for choosing this value to limit thermal problems is clearly at a low level.

We have now obtained two torque components with a pitch effect. Only the first one is position dependent and its strength can be reduced to zero by applying \( b_s = 0.38r \), but the reduction of the ratio \( 10^{4}T^p/[F_z] \) compared to the optimum value at \( b_s = 0.55r \) is hardly acceptable. The value of this last torque is less than 10% of the first.

4. Electronics

The prediction of the voltage and current ranges to be covered by the amplifiers is not just an academic point, but is necessary for estimating the project costs. The amplifier specification is linked to the intended motion profile, which is of course a function of time. The four independent variables \( x(t), \dot{x}(t), \ddot{x}(t) \) and \( \phi(t) \) and their derivatives leads to a complex expression for a planar motor. The variables \( \dot{x}(t) \) and \( \phi(t) \) are linked to \( F_x(t) \) and \( F_z(t) \) by Eq. (11). A motion profile describes \( x(t), \dot{x}(t) \) and the forces \( F_x(t) \) and \( F_z(t) \) follows when the mechanics are taken into account. To obtain an expression for the required line-voltage the following conditions are assumed:

- a symmetrical three phase supply
- the motor phases are symmetrical
- the motor is star-point connected
- the displacement in the \( z \)-direction equals zero.

For a symmetrical three phase planar motor the following is valid:

\[
\left[\begin{array}{c}
\hat{u}_a \\
\hat{u}_b \\
\hat{u}_c
\end{array}\right] = \left[\begin{array}{ccc}
R & 0 & 0 \\
0 & R & 0 \\
0 & 0 & R
\end{array}\right] \left[\begin{array}{c}
\hat{i}_a \\
\hat{i}_b \\
\hat{i}_c
\end{array}\right] + \frac{d}{dt} \left[\begin{array}{ccc}
L & M & M \\
M & L & M \\
M & M & L
\end{array}\right] \left[\begin{array}{c}
\hat{i}_a \\
\hat{i}_b \\
\hat{i}_c
\end{array}\right] + \hat{\psi}_d(z) \left[\begin{array}{c}
\sin \left(\frac{\pi}{T} \right) \\
\sin \left(\frac{3\pi}{T} - 2\pi \right) \\
\sin \left(\frac{3\pi}{T} + 2\pi \right)
\end{array}\right]
\]

(23)

With:
- \( u_a, u_b, \) and \( u_c \) voltage over the coils a, b and c,
- \( i_a, i_b, \) and \( i_c \) current through the coils,
- \( R, L \) resistance and self-inductance of a coil,
- \( M \) mutual inductance between the coils,
- \( \psi_d \) the top value of the flux linkage induced by the permanent magnets.

The assumed symmetry of the phases allows the same value for all mutual inductances. However, the deviating spatial distances between the coils lead to slightly other values for \( M \). This effect is neglected to simplify the following expressions. In a shorter notation again Eq. (23):

\[
[U_{abc}] = [R][I_{abc}] + \frac{d}{dt} \left[\begin{array}{c}
|L| \frac{d}{dt}[I_{abc}] + [\psi_d(z)]
\end{array}\right]
\]

(24)

The Park-transformation [16] from an a, b, c to a, d, q, 0 system is applied to reduce the complexity of Eq. (24), giving the
The top-value of the phase voltage is given now as

$$|u_{\text{phase}}| = \sqrt{U_x^2 + U_y^2}$$  \hspace{1cm} (32)

Therefore, to obtain the amplifier line-to-line voltage we combine Eqs. (31) and (33) with the required motion profile and the mechanics, leading to $F(t)$ and $v_s(t)$ as will be described in Section 5. It is also necessary the temperature-dependent resistance $R$ of the motor phases, the connecting circuit and tolerances on the force constants.

To control the forces we should adjust the current amplitude $i(t)$ and phase $\phi(t)$. The complex relationship (31) versus the simple relationship between the forces $F_x$, $F_z$ and $I$ indicates that a current controlled amplifier is the better choice to drive the motor. By referring to Eq. (31) amplifier voltage clipping can be avoided.

The amplifier specification concerning drift, offset and equal current gain per phase, has to be analysed thoroughly, because these factors will lead to a position-dependent control loop gain and to additional torques created by the planar motor. These disturbances can only be partly counteracted by the position control loop, given a finite loop gain, and positioning errors will occur. Experiments proved this statement for amplifiers with an offset of 1% of the full current range.

5. Mechanics

Until now we have only considered a single forcer with forces in the $x$- or $y$-directions. To create a planar motor we combine four of these forceps in a plane with two forceps perpendicularly directed with respect to the other two, as shown in Fig. 8. Further details are given in [17,18]. The length of
the coils is extendable without any theoretical limitation, so there is no need to increase the number of coils to increase the performance (we applied 8 coils in our set-up).

In general, the centre of mass does not coincide with the centre of the planar motor, because the load (e.g. a tool) is placed on the top of the motor. Let us assume a distance \( h_x \) between the centre of the motor and the centre of mass.

The four forcers, marked as 1 and 3 for \( x \)-forces and 2 and 4 for \( y \)-forces, have to counteract the gravitational force \( mg \) and to accelerate the motor and its load with \( a_x \) and \( a_y \) in the \( x \) and \( y \)-directions, respectively. The balance of forces leads to Eqs. (34)–(36):

\[
\begin{align*}
F_{x,1} + F_{x,3} &= ma_x, \\
F_{x,2} + F_{x,4} &= ma_y, \\
F_{x,1} + F_{x,2} + F_{y,3} + F_{y,4} &= mg.
\end{align*}
\]

The balance of torques gives Eqs. (37)–(39):

\[
\begin{align*}
-l_yF_{x,1} + l_yF_{x,2} + l_yF_{y,3} - l_yF_{y,4} &= mha_x, \\
-l_yF_{x,1} - l_yF_{x,2} + l_yF_{y,3} + l_yF_{y,4} &= mha_y, \\
(-F_{x,1} + F_{x,2} - F_{y,3} + F_{y,4})h_y &= 0.
\end{align*}
\]

We have eight independent forces, but only six equations, based on the 6-DOF, are given. Two additional constraints must be added to obtain a solution. There are a number of possible options, e.g.

\[
F_{x,1} = F_{x,3} \quad \text{and} \quad F_{x,2} + F_{x,4} = 0.5mg.
\]

Rearrangement of the equations leads to the force-vectors of the individual forcers. Eqs. (41)–(44) are the force components required per force-vector:

\[
F_1 = \begin{bmatrix} F_{x,1} \\ F_{x,2} \end{bmatrix} = \begin{bmatrix} a_x \\ \frac{g}{2} + c(a_xl_x + a_yl_y) \end{bmatrix} \frac{m}{2},
\]

\[
F_2 = \begin{bmatrix} F_{y,1} \\ F_{y,2} \end{bmatrix} = \begin{bmatrix} 0 \\ a_y \frac{g}{2} + c(a_xl_x - a_yl_y) \end{bmatrix} \frac{m}{2}.
\]

These equations clearly show the impact of \( h_x \). At a high value of \( h_x \) we should modify the values of \( l_x \) and \( l_y \) to reduce the force levels. It also indicates that a planar motor stage should be made as low-profile as possible (of course still satisfying thermal, mechanical and control constraints).

The applied orientation of the coils ensures a low rig height and allows for a cooling system to be mounted on top of the coils.

6. Losses

The motor performance is directly related to the permitted temperature rise. Forced air flow or water-cooling and the introduction of strong rare-earth permanent magnets, as NdFeB, will reduce the temperature rise. Before starting the thermal analysis we have to predict the Ohmic losses in the planar motor, which are in general given by

\[
P = \frac{3}{2} IR = \frac{3}{2} \left( \frac{F_x(t)}{K_x} \right)^2 + \left( \frac{F_y(t)}{K_y} \right)^2.
\]

Introduction of \( S_{\text{lax}} = 1.5 K_{\text{x}}^2/R \) with Eq. (46) leads to the simple form represented by Eq. (47):

\[
P = \frac{F_x(t)^2}{S_{\text{ax}}} + \frac{F_y(t)^2}{S_{\text{ay}}}.
\]

Now all the information is collected to give the full losses of a planar motor levitating and accelerating a mass \( m \). Substitution of Eqs. (41)–(44) subsequently into Eq. (47), assuming that \( S_{\text{ax}} = S_{\text{ay}} \) is valid, leads to the total power loss in Eq. (48):

\[
P = m^2 \left( \frac{g^2}{4S_{\text{ax}}} + (a_x^2 + a_y^2) \frac{1}{2S_{\text{ax}}} + \frac{h_x^2}{20(l_x^2 + l_y^2)S_{\text{ax}}} \right).
\]

This Eq. (48) shows that loss reduction can be obtained by limiting the ratio between the position of the centre of mass \( h_x \) and the footprint, determined by \( l_x \) and \( l_y \).
7. Electro-magnetics

We have already given a simplified description of the field above the magnet array. A more detailed analysis has been made of the position dependency of all force and torque components and the losses with a three-dimensional finite element package (three-dimensional FEM). However, these time-consuming calculations are less suited as a design tool to investigate the influence of the geometrical dimensions (the coil and magnet dimensions) on the losses, forces and torques. In addition to the FEM tools a numerical program has been used, based on the equations given by Akoun and Yonnet [19], describing the field of a cuboidal permanent magnet in free air. Some results will be presented to demonstrate the behaviour of the torques and forces of one forcer.

The field of a permanent magnet in free air is given by the equations under the assumption that the magnet permeability equals 1 and the magnet behaves linearly in the second quadrant of the \( B-H \) curve. This is true for modern NdFeB-magnets. The fixing of magnets to a iron plate can be modelled by mirroring the magnets in the iron, as described by Binns and Lawrenson [20] assuming that the dimensions of the iron plate in the \( x \)- and \( y \)-direction can be considered as infinite and that magnetic saturation does not occur in the iron. (Fig. 9 shows the principle used.) Summation of the fields of all magnets gives the local values of the flux density, so \( B_{x,y,z}(x,y,z) \). This approach is followed and tested by Compter et al. [21] for a linear moving coil actuator.

The field components \( B_x \) and \( B_z \) over an \( xy \)-plane 4 mm above the magnets are given in Fig. 10 as an example.

\[
\begin{align*}
F &= \sum_{j=1}^{3} \int \int \int_{V_{coil,j}} J(x,y,z,p,\phi) \times B(x,y,z) \, dV, \quad (49) \\
T &= \sum_{j=1}^{3} \int \int \int_{V_{coil,j}} r \times J(x,y,z,p,\phi) \times B(x,y,z) \, dV. \quad (50)
\end{align*}
\]

These equations are applied to a calculation program and the results used to determine the geometrical dependencies and to minimize unwanted force and torque components. For example, a forcer for the \( x \)-direction should give no \( y \)-force, \( z \)- and \( x \)-torque. Changing the phase angle \( \phi \) should only change the direction of the force in the \( x-z \) plane and not the amplitude of the force. What remains is the \( y \)-torque given by Eqs. (21) and (22), which has to be accepted as belonging to the chosen geometry in Fig. 2. The deviation of the spatially averaged force \( F_z \) (being 152.5 N) as a function of the position is given in Fig. 11 and demonstrates the nearly perfect behaviour.

The forces \( F_x \) and \( F_y \) are less than 0.5% of \( F_z \) over this area. The same percentage holds for \( F_y \) and \( F_z \) with respect to the...
high force $F_x$ (spatially averaged force level again 152.5 N) when the phase angle $\phi$ rises $\pi/2$. The torque $T_y$ is calculated over the same $x$–$y$ range and is given by Fig. 12. Its sinusoidal behaviour as predicted by Eqs. (20) and (21) is clearly seen. Division of this torque by the force found earlier reveals that the line of operations of the force $F_z$ moves within the range $x = -23$ to $+23$ mm or expressed in the pole pitch $\tau$ of $32$ mm: $x = -0.72\tau$ to $+0.72\tau$. Eq. (22) predicts $3.7$ N m, whereas $3.6$ N m is given by this method.

The values of $T_x$ and $T_z$ are less than 10% of $T_y$ over this area applying an appropriate length of the coils.

This analysis shows that the assumed nearly perfect behaviour is in agreement with a more detailed analysis. The only remaining imperfection is predicted by Eqs. (20)–(22).

8. Status

A prototype of this planar motor design has been built and a controller fitted for all six degrees of freedom has been built, tested and approved. The tests were done with the set-up given in Fig. 13. The planar motor is initially mounted under a gantry consisting of two linear motors driving the gantry in the $y$-direction and one linear motor for the $x$-direction. Three force sensors, each one sensing $x$, $y$ and $z$-forces, are mounted between the planar motor and the gantry. This set-up enables the verification of the predicted forces and torques and ensures that correct gain factors will be available for the 6-DOF control loops. The mechanical fixing for each of the degrees of freedom of the planar motor is removed and the 6-DOF control loops are subsequently implemented.

Inductive position sensors are sensing the relative $x$- and $y$-position of the planar motor with respect to the gantry. These signals are added to the linear encoder signals of the gantry to obtain the position of the planar motor with respect to the magnets, as needed for the electronic commutation of the planar motor currents. Sensors are also used to measure the $z$-position of the planar motor with respect to the magnet plate.

The applied position sensing is possible by the presence of the gantry. A recent position sensor development is described by Frissen and Compter [22]. This position sensor is based on sensing the magnet field with Hall sensor arrays as indicated in Fig. 14. Two arrays with $N$ Hall sensors each, with a distance $2\tau(N - 1)/N$ between the Hall elements, give a classic S0–S90 position signal when the arrays are shifted over $\tau/2$. The accuracy is mainly determined by the offsets and gain errors of the Hall elements and a comparison with encoders indicated that an absolute accuracy of 1% of the pitch $\tau$ is realistic.
The experiments indicate the presence of a pitch torque higher than expected. The cause of this is found to be the amplifier offsets and deviating gains of the phases of the three-phase amplifier. This, however, is compensated for, by gain adjustment and an improvement of the current sensing circuits. Further activities are mainly directed to more advanced controllers to increase the bandwidth of the control loops and to eliminate bandwidth limiting eigen-frequencies and cross-talk. The final target is the integration of this planar motor in an application linked with the semiconductor industry.

9. Summary

An electro-dynamical planar motor development is described starting with an idealized behaviour for a Halbach-based magnet geometry and a three-phase coil system, called a forcer. The introduction of a phase angle $\phi$ in the electronically commutated three-phase system allows an independent control of two orthogonal force components. A more detailed analysis reveals the presence of a pitch torque which has to be considered as a design related disturbance, which can be compensated for by a second forcer with an appropriate spatial shift. The voltage equation is obtained to give the specification for an amplifier.

Four forcers together can control a load in all six degrees of freedom and the electrical loss related to the levitation and acceleration of the load is predicted.

An analytical/numerical model is used to verify the assumed ideal behaviour, proving that a nearly position-independent behaviour is obtained with only the pitch-torque as the exception. Two different methods give virtually the same value for the ratio between the pitch torque and the force of a forcer.

References