EXPERIMENTAL IDENTIFICATION OF A
DAMAGE EVOLUTION LAW FOR STEEL

M.W. Panis
MT04.05

Second internship

Coach (Philips CFT): dr.ir. O. van der Sluis
Coach (TU/e): dr.ir. R.H.J. Peerlings

Philips CFT
Product and Process Modelling Group

Eindhoven University of Technology
Faculty of Mechanical Engineering
Materials Technology Group

Eindhoven, February 2004
Abstract

The physical background of damage evolution in metals consists of initiation, growth and coalescence of voids. To include such a mechanical behaviour in a macroscopic sense, the concept of Continuum Damage Mechanics has been developed. An essential part of a continuum damage formulation is the damage evolution law, which is used to calculate the amount of damage during the deformation process. The damage evolves from 0 (no damage) to 1 (fully damaged material) as a function of the nonlocal equivalent plastic strain. To identify a damage evolution law from experiments, the amount of damage that has developed during deformation as well as the corresponding nonlocal equivalent plastic strain has been determined.

Micro-indentation appears to provide a way to determine the spatial distribution of damage, based on the decrease of hardness of the material during damage evolution. Tensile experiments are performed to produce damaged specimens in which the damage varies continuously from 0 (undeformed material) to 1 (fracture surface in the necking zone). Micro indentation experiments are performed on these specimens, namely a micro Berkovich hardness test and a micro Vickers hardness test, to obtain the spatial distribution of the hardness variation. The results of the micro Vickers hardness test did not show a variation in the hardness distribution. The decrease of the hardness and the Young’s modulus for the damaged material probably led to the same remaining indentation width after indentation as for the undamaged material. The micro Berkovich hardness experiments on the other hand did show a variation of the hardness over the domain of the specimen. For that reason, this technique has been used to determine the spatial distribution of the hardness of the specimen.

The spatial distribution of the nonlocal equivalent plastic strain is obtained by means of digital image correlation (DIC) software. During the tensile experiments, several images are recorded of the deforming tensile specimen. Afterwards these images are processed with the DIC software and the resulting local equivalent plastic strain is obtained. An analysis showed that for a physically motivated lengthscale of the material, the nonlocal and local equivalent plastic strain are almost similar. Therefore, the damage evolution law is written in terms of the local equivalent plastic strain instead of the nonlocal equivalent plastic strain.

Once the hardness distribution and the local equivalent plastic strain are determined, the damage evolution law has been determined according to the modified power law and an exponential damage evolution law, which described the measured damage evolution accurately. Unfortunately, these damage evolution laws do not incorporate triaxiality \((\frac{\sigma}{\sigma_{eq}})^\alpha\). Because damage also evolved during the indentation experiments, the obtained damage evolution law does not correspond to the tensile experiment only. The amount of damage that evolves by means of the indentation experiments has not been investigated. Furthermore, experiments showed a variation of the Young’s modulus over the domain of the specimen as well, which suggest that this material parameter also decreases during damage evolution. During the indentation experiments, a rapid decrease of the Young’s modulus as function of the indentation depth was measured. The reason for this behaviour has not been investigated.
Contents

1 Introduction 2

2 Continuum Damage Mechanics 3
   2.1 Constitutive framework 4
   2.2 Numerical examples 8
   2.3 Experimental techniques for damage analysis 10

3 Tensile experiment 12
   3.1 Experimental set-up 12
   3.2 ARAMIS 14
   3.3 Determination of the nonlocal strain distribution 16
   3.4 Nonlocal equivalent plastic strain of the tensile specimen 20

4 Indentation experiments 22
   4.1 Micro Vickers hardness measurement 22
   4.2 Micro Berkovich hardness measurement 24

5 Damage evolution law 32
   5.1 Determination of a damage evolution law 32

6 Discussion 37
   6.1 Conclusions 37
   6.2 Recommendations 38

A Elastic damage model 39

References 40

Acknowledgements 41
1 Introduction

Nowadays numerical tools, such as the finite element method, are indispensable for designing and optimising industrial processes, e.g., metal forming processes. These processes typically result in highly localised plastic deformation, often accompanied by the development of microstructural damage within the material. This damage development may lead to unintended fracture of the material during processing and, as a result, defines the process limits.

At Philips CFT, a dedicated finite element solver, called Crystal, has been developed to simulate forming processes in an accurate and robust way [1]. This solver is successfully applied at Philips DAP (Domestic Appliances and Personal care) to simulate the forming process of shaver heads. However, it was not possible to predict the process limits using this version of Crystal. For this reason, a recently developed gradient-enhanced ductile damage model [2] has been implemented by CFT [3].

An essential part in this model is the damage evolution law which is used to calculate the amount of damage in the material during the deformation process. The calculated damage variable evolves from 0 (undamaged material) to 1 (fully damaged material) as a function of the nonlocal equivalent plastic strain and influences the constitutive behaviour by reducing the yield stress. Currently, experimental quantification of the damage evolution law(s) is lacking. This report describes the experimental determination of damage evolution.

The experimental determination of a damage evolution law is performed as follows: first a tensile experiment is performed with the objective to produce damage in the material. Because the damage evolution laws are written in terms of the nonlocal equivalent plastic strain, the spatial distribution of the nonlocal equivalent plastic strain over the whole surface of the tensile specimen has to be obtained also. Therefore images of the deforming tensile specimen are recorded at several stages during the tensile experiment. Those images will be analysed afterwards with a digital image correlation package to calculate the equivalent strain, which will be used to obtain the nonlocal equivalent plastic strain. Because the nonlocal and local equivalent plastic strain are almost similar for a physically motivated lengthscale, it is assumed that the damage evolution law may be determined as a function of the local equivalent plastic strain instead of as a function of the nonlocal equivalent plastic strain. When the local equivalent plastic strain distribution of the tensile specimen is determined, micro-indentation experiments are performed, because micro-indentation appears to provide a way to determine the deterioration of the microstructure of the material based on the decrease of the hardness of the material during damage evolution. Finally, a damage evolution law can be determined when the variation of the hardness is coupled to the nonlocal equivalent plastic strain.
2 Continuum Damage Mechanics

The physical background for ductile fracture initiation in metals is well understood and mainly occurs in three stages. First microvoids initiate or are already present (i) in the interior of the material, for example by dislocation pile-ups at grain boundaries or by local interface decohesion of secondary phase particles in the matrix material. Those voids will grow (ii) as deformation continues and ultimately coalesce (iii) to form a microcrack that will grow further to a macrocrack. Finally, this macrocrack leads to total failure of the processing material. Stage i and ii can be observed in the left respectively the right part of Figure 1. This figure consist of two light microscope images of an axial cross section of a chromium steel tensile specimen, which was processed to the point of fracture. The secondary phase particles can be distinguished from the matrix material in the left part of Figure 1 (see arrow), the right part shows the grown voids near the crack surface and the ultimate coalescence.

![Figure 1: light microscope images of chromium steel: (a) undamaged and (b) voids near crack surface](image)

To include such a mechanical behaviour in a macroscopic sense, the concept of Continuum Damage Mechanics has been developed, see [4]. Research has been performed on numerical modelling of the characteristic softening response that ductile materials exhibit in forming processes. This characteristic softening is accompanied by the intense localisation of deformation into a narrow zone. The numerical solution that is obtained from finite element analysis employing standard continuum elasto-plasticity, where a local description of the softening material is used, reveals a pathological dependence on the direction and the fineness of the finite element mesh. Accordingly, upon mesh refinement no convergence to a physically meaningful solution exists. This problem is now well-known and stems from the continuum model rather than from the numerical implementation.

In the literature, several models are proposed to overcome this deficiency. Among them the higher-order continuum-based models, which are best motivated from both the physical and computational point of view. A common feature of such formulations is the incorporation of an intrinsic length scale, which is related to the microstructure and the failure mechanisms during plastic slip. These higher-order continuum-based models often incorporate a nonlocal quantity that is defined as the spatial weighted average of the corresponding local field quantity over a finite volume surrounding the point under consideration. Commonly, the nonlocal quantity is computed with an integral format, in which the associated intrinsic length scale
influences the weight amplitude in the vicinity of a material point. The gradient-enhanced formulations avoid this integral format by approximating the nonlocal kernel with a Taylor series expansion, which yields a differential format. A first subclass of the gradient-enhanced formulations are based on the direct use of the higher-order gradient terms of a local quantity in the constitutive frame-work. They are therefore referred as ‘explicit’. Although considerable progress has been made in restoring the well-posedness of the boundary value problem (BVP), several problems persist for this type of gradient-enhanced formulations. A second subclass of the gradient-enhanced methods consist of the so-called ‘implicit’ approaches [5]. These formulations adopt a differential approximation of a nonlocal variable, which involves the higher-order derivatives of a nonlocal variable rather than a local field variable.

The implicit gradient-enhanced damage model will be employed in this report. Therefore, the next subsection describes the damage formulation as well as its incorporation in the plasticity framework. Some features of this gradient-enhanced damage-plasticity formulation will be described according to several numerical examples. At last, several experimental techniques for damage evolution are discussed in the last subsection.

2.1 Constitutive framework

The gradient-enhanced ductile damage constitutive model is based on the hypo-elastic relation between the objective Jaumann rate of the Cauchy stress tensor \( \dot{\sigma} \) and the elastic deformation rate tensor \( D^e \) is given by Hooke’s law:

\[
\dot{\sigma} = ^4 C : D^e, \tag{1}
\]

in which the isotropic elastic fourth order stiffness tensor is given by:

\[
^4 C = \frac{\nu E}{(1 + \nu)(1 - 2\nu)} \left( \mathbf{II} + \frac{1 - 2\nu}{\nu} 4 \mathbf{I} \right), \tag{2}
\]

with \( E \) and \( \nu \) are Young’s modulus and Poisson’s ratio, respectively.

The additive decomposition of the total deformation rate tensor into an elastic and a plastic part \( (\mathbf{D} = \mathbf{D}^e + \mathbf{D}^p) \) gives

\[
\dot{\sigma} = ^4 C : (\mathbf{D} - \mathbf{D}^p). \tag{3}
\]

The plastic deformation rate tensor \( \mathbf{D}^p \) is determined by assuming an associative flow rule, in which the direction of plastic flow is defined by the normal on the yield surface

\[
\mathbf{D}^p = \dot{\lambda} \frac{\partial F}{\partial \sigma} = \dot{\lambda} \mathbf{n}, \tag{4}
\]

in which \( \dot{\lambda} \) is the plastic multiplier, which determines the magnitude of the plastic strain rate tensor, and \( \mathbf{n} \) is the normal on the yield surface \( F \), which for von Mises plasticity is given by:

\[
F(\sigma, \varepsilon_p) = \sigma_{eq}(\sigma) - K(\varepsilon_p) = \sqrt{\frac{3}{2} \sigma^d : \sigma^d - K(\varepsilon_p)}, \tag{5}
\]
with the deviatoric stress $\sigma^d$ and $K$ the hardening law as a function of the effective plastic strain $\varepsilon_p$, which is defined as

$$\varepsilon_p = \int_0^t \dot{\varepsilon}_p(\tau) \, d\tau,$$

(6)

with

$$\dot{\varepsilon}_p = \sqrt{\frac{2}{3} \mathbf{D} : \mathbf{D}^p}.$$  

(7)

Now, the gradient-enhanced damage-plasticity formulation will be incorporated by including the softening behaviour into the post-yield function. To this end, the local yield stress is multiplied by a factor which depends on a nonlocal ductile damage variable [2] (see also Figure 2):

$$F(\sigma, \varepsilon_p, \bar{\kappa}) = \sigma_{eq}(\sigma) - [1 - \omega_p(\bar{\kappa})] K(\varepsilon_p).$$  

(8)

![Figure 2: incorporation of damage into post-yield function](image)

Here, $\omega_p(\bar{\kappa})$ denotes the ductile damage variable, defined as a function of $\bar{\kappa}$; $\bar{\kappa}$ is the nonlocal plastic multiplier:

$$\bar{\kappa} = \max\{\bar{\varepsilon}_p(\tau), 0 \leq \tau \leq t\},$$

(9)

with $\bar{\varepsilon}_p$ the nonlocal equivalent plastic strain. The choice for $\bar{\kappa}$ is motivated by the fact that during equilibrium iterations, the nonlocal equivalent plastic strain may decrease. To avoid numerical problems, the nonlocal plastic multiplier $\bar{\kappa}$ is introduced. In (8), the damage parameter $\omega_p$ evolves from 0 (undamaged material) to a value of 1 (fully damaged material). The damage formulation given in Equation (8) is a typical plastic damage formulation. An elastic damage formulation is presented in Appendix A.
The nonlocal equivalent plastic strain $\bar{\varepsilon}_p$ in (9) can be written as a weighted volume average of the local equivalent plastic strain $\varepsilon_p$ in the entire problem domain $\Omega$:

$$\bar{\varepsilon}_p(\vec{x}) = \frac{1}{\Psi(\vec{x})} \int_{\Omega} \psi(\vec{y}; \vec{x}) \varepsilon_p(\vec{y}) \, d\Omega$$  \hspace{1cm} (10)

in which $\vec{y}$ is the position vector of the infinitesimally small volume $d\Omega$, $\psi(\vec{y}; \vec{x})$ is the weight function and $\Psi(\vec{x}) = \int_{\Omega} \psi(\vec{y}; \vec{x}) \, d\Omega$ is the normalising factor. Consequently, for a homogeneous strain state, it holds that $\bar{\varepsilon}_p = \varepsilon_p$. A Taylor series approximation is used to reformulate (10) \[5\]

$$\varepsilon_p(\vec{y}) = \varepsilon_p(\vec{x}) + \frac{\partial \varepsilon_p}{\partial x_i} (y_i - x_i) + \frac{1}{2!} \frac{\partial^2 \varepsilon_p}{\partial x_i \partial x_j} (y_i - x_i)(y_j - x_j) + \frac{1}{3!} \frac{\partial^3 \varepsilon_p}{\partial x_i \partial x_j \partial x_k} (y_i - x_i)(y_j - x_j)(y_k - x_k) + \frac{1}{4!} \frac{\partial^4 \varepsilon_p}{\partial x_i \partial x_j \partial x_k \partial x_l} (y_i - x_i)(y_j - x_j)(y_k - x_k)(y_l - x_l) + ...$$  \hspace{1cm} (11)

Substitution of this expansion into equation (10) and evaluation of the integral yields:

$$\bar{\varepsilon}_p(\vec{x}) = \varepsilon_p(\vec{x}) + c(\ell) \nabla^2 \varepsilon_p(\vec{x}) + d(\ell) \nabla^4 \varepsilon_p(\vec{x}) + ...$$  \hspace{1cm} (12)

where the Laplacian operator $\nabla^2 = \sum_i \frac{\partial^2}{\partial x_i^2}$ and $\nabla^2 n = (\nabla^2)^n$. For $\Omega = \mathbb{R}^n$, the odd derivatives in equation (11) vanish. This relation is called the explicit gradient approximation. The implicit gradient formulation is obtained in the following way. First, the Laplacian of equation (12) is taken:

$$\nabla^2 \bar{\varepsilon}_p(\vec{x}) = \nabla^2 \varepsilon_p(\vec{x}) + c(\ell) \nabla^4 \varepsilon_p(\vec{x}) + ...$$  \hspace{1cm} (13)

Then, the infinite series of higher-order derivatives of $\varepsilon_p(\vec{x})$ are transferred to the left-hand side of equation (12) after multiplying (13) with $c(\ell)$:

$$\bar{\varepsilon}_p(\vec{x}) - c(\ell) \nabla^2 \varepsilon_p(\vec{x}) = \varepsilon_p(\vec{x}) + [d(\ell) - c(\ell)^2] \nabla^4 \varepsilon_p(\vec{x})$$  \hspace{1cm} (14)

If the weight function is chosen to be Green’s function of the operator on the left-hand side, it can be shown that $d(\ell) - c(\ell)^2 = 0$ \cite{10}. In addition, using Green’s function results in $c(l) = l^2$. As a result, the following partial differential equation (PDE) for the nonlocal equivalent plastic strain is obtained:

$$\bar{\varepsilon}_p(\vec{x}) - l^2 \nabla^2 \varepsilon_p(\vec{x}) = \varepsilon_p(\vec{x})$$  \hspace{1cm} (15)

For this additional PDE, also additional boundary conditions should be prescribed. The following homogeneous Neumann boundary condition has been chosen:

$$\nabla \bar{\varepsilon}_p \cdot \vec{n} = 0 \quad \text{on } \Gamma,$$  \hspace{1cm} (16)

in which $\Gamma$ is the external boundary of $\Omega$, and $\vec{n}$ is the unit outward normal on $\Gamma$. 
Several phenomenological evolution laws $\omega_p(\bar{\kappa})$ exist [6]. A damage evolution law for linear softening can be formulated as

$$\omega_p = 1 - \left( \frac{\kappa_c - \bar{\kappa}}{\kappa_c - \kappa_i} \right),$$

where $\kappa_i$ is defined as an initial threshold value and $\kappa_c$ as a critical value of $\bar{\kappa}$. The value of $\omega_p$ is a linear function of $\bar{\kappa}$; if $\bar{\kappa} < \kappa_i$ no damage is introduced and if $\bar{\kappa} > \kappa_c$ the damage has reached a value of 1.

The power-law model is defined as

$$\omega_p = 1 - \frac{\kappa_i}{\bar{\kappa}} \left( \frac{\kappa_c - \bar{\kappa}}{\kappa_c - \kappa_i} \right)^\alpha,$$

(18)

The exponent $\alpha$ mainly influences the slope of the stress decrease. The introduction of a second exponent $\beta$ leads to a modified power-law

$$\omega_p = 1 - \left( \frac{\kappa_i}{\bar{\kappa}} \right)^\beta \left( \frac{\kappa_c - \bar{\kappa}}{\kappa_c - \kappa_i} \right)^\alpha,$$

(19)

which provides a more flexible formulation.

An exponential damage evolution law can be formulated as

$$\omega_p = 1 - \left( \frac{\kappa_i}{\bar{\kappa}} \right) \left( (1 - \delta) + \delta \exp^{-\gamma(\bar{\kappa} - \kappa_i)} \right),$$

(20)

where the coefficients $\delta$ and $\gamma$ are material parameters.

The four different damage evolution laws are presented in Figure 3 with values for $\kappa_i = 1$, $\kappa_c = 3$, $\alpha = 2$, $\beta = 5$, $\gamma = 1$ and $\delta = 0.5$.

Figure 3: several damage evolution laws
Another damage evolution law can be formulated according to a fracture initiation criterium $C_G$ as proposed by [7]. The criterium is formulated as an integral of a function $f(\sigma, \varepsilon_p)$ over the equivalent plastic strain:

$$C_G = \int_{\varepsilon_p} \left( 1 + 3.9 \frac{\sigma^h}{\sigma_{eq}} \right) \varepsilon_p^{0.63} d\varepsilon_p. \quad (21)$$

The values 3.9 and 0.63 have been fitted on tensile experiments on several metals. The brackets $\langle \cdot \rangle$ are defined as $\langle \varphi \rangle = \frac{1}{2}(|\varphi| + \varphi)$. If during a simulation the integral value is larger than a threshold value $C_G$, which is a material parameter, fracture initiation will occur. Equation (21) can be rewritten to a damage evolution law (the nonlocal plastic multiplier $\bar{\kappa}$ is used instead of the nonlocal equivalent plastic strain $\bar{\varepsilon}_p$):

$$\omega_p = \frac{1}{C_G} \int_{\bar{\kappa}} \left( 1 + 3.9 \frac{\sigma^h}{\sigma_{eq}} \right) \bar{\kappa}^{0.63} d\bar{\kappa}. \quad (22)$$

As opposed to the previously defined damage evolution laws, Equation (22) incorporates the triaxiality (hydrostatic stress divided by an equivalent stress), which is an important factor in ductile fracture initiation due to the presence of voids.

2.2 Numerical examples

The following examples [3] show some features of the damage model in the finite element package Crystal. The damage-plasticity model should be able to describe softening in a proper way, i.e. the result should converge to a unique solution upon mesh refinement. To check this, simulations with three different discretisations are performed. Therefore a well-known benchmark problem of a necking axisymmetric cylindrical bar is used. The geometry and boundary conditions are shown in Figure 4. Note that only half of the bar has been modelled; the symmetry plane is at the left of the figure.

![Figure 4: geometry and boundary conditions for the axisymmetric bar](image)

Because of the fixed end-grips, necking is triggered in the center of the specimen due to the large displacements. The material properties are: $E = 180000$ [MPa], $\nu = 0.3$ [-], initial yield stress $K_0 = 272$ [MPa], isotropic hardening according to a Nadai model with the following parameters: $C = 826$ [MPa] and $n = 0.246$ [-]. The damage evolution law as defined by Equation (22) is used with $C_G = 3.53$, $\ell = 1$ [mm]. Figure 5 shows that the result are mesh independent.

To illustrate the geometrical softening during loading of a tensile bar, a simulation has been performed without damage evolution [8]. The corresponding force-displacement curve is
Figure 5: resulting force-displacement curves for three different discretisations

Figure 6: resulting force-displacement curves for geometrical softening and geometrical-physical softening shown in Figure 6. This figure also contains the force-displacement curve in case of the combined geometrical-physical softening (including the presence of damage in the material). The response is more 'brittle' for the case of combined softening, which is characteristic for metals when loaded in tension.

To demonstrate the influence of the intrinsic length scale $\ell$ in the model, simulations have been performed with three different values: 0.1, 1.0 and 10.0 [mm]. This value determines the localisation width, thus a small length scale results in a more brittle behaviour, see Figure 7. In the case of $\ell = 0.1$, snapback occurred at a displacement of 4 mm, which led to convergence problems.
2.3 Experimental techniques for damage analysis

Many experimental techniques exist to measure damage or damage related events [4] [6]. It is often difficult to perform a quantitative measurement with the majority of these techniques, but several methods are worth mentioning and will be discussed briefly.

A straightforward manner to quantify the isotropic elastic damage variable is to measure the decrease of the stiffness of the material. Successive loading and unloading of the material permits to measure different stages of damage which is then computed through

\[
D = 1 - \frac{\tilde{E}}{E},
\]

(23)

where \( \tilde{E} \) is the effective elasticity modulus of the damaged material, derived from measurements, and \( E \) the Young’s modulus of the virgin material.

The variation of the elasticity modulus also has an influence on the speed of ultrasonic waves. The longitudinal wave speed \( c_L \) and the transversal wave speed \( c_T \) in an isotropic elastic medium are given by

\[
c_L = \sqrt{\frac{E}{\rho} \frac{(1 - \nu)}{(1 - \nu)(1 - 2\nu)}},
\]

(24)

\[
c_T = \sqrt{\frac{E}{\rho} \frac{1}{2(1 + \nu)}},
\]

(25)

where \( \rho \) is the density of the material. Assuming that Poisson’s ratio \( \nu \) is not affected by damage, \( D \) may be calculated from

\[
D = 1 - \frac{\tilde{E}}{E} = 1 - \frac{\tilde{\rho}c_L^2}{\rho c_L^2} = 1 - \frac{\tilde{\rho}c_T^2}{\rho c_T^2},
\]

(26)
in which $\tilde{\rho}$, $\tilde{c}_L$ and $\tilde{c}_T$ are the density, the transversal wave speed and the longitudinal wave speed in the damaged continuum respectively.

In the case of ductile damage, the defects are cavities which can be assumed to be roughly spherical. This means that the volume increases with damage, which leads to a measurable decrease of density $\tilde{\rho}$. This assumption does not hold in case of shear failure. The following relation can be derived:

$$D = \left(1 - \frac{\tilde{\rho}}{\rho}\right)^{\frac{2}{3}}.$$  \hspace{1cm} (27)

Ductile damage can also be quantified at the surface with a microhardness indentation test. A microhardness test is a process in which a pointed or rounded indenter is pressed into a surface under a given load, within a specific period of time. There are several microhardness tests and consequently also several microhardness definitions such as Brinell, Rockwell and Vickers. If $H$ is the microhardness of the material without any damage, and $\tilde{H}$ the actual microhardness value of the damaged material, then damage is determined through:

$$D = 1 - \frac{\tilde{H}}{H}.$$  \hspace{1cm} (28)

One of these techniques has to be used to determine a damage evolution law. Because no experimental set-up was available for performing a damage analysis experiment according to Equation (26), this technique was not considered as a suitable option.

To find the damage variation in the interior of the test specimen according to Equation (27), the test specimen has to be cut into smaller parts to relate the measured damage to the concerning part of the sample in order to obtain the damage variation over the sample. If the nonlocal equivalent plastic strain is known for these different parts, a damage evolution law can be obtained. A major drawback of this technique is the cutting of the material, especially in the necking zone, which is very difficult and probably influences the measured damage.

Equation (28) can be investigated with indentation experiments, in which the hardness of the material with damage $\tilde{H}$ and without damage $H$ can be measured. Once the spatial distribution of the nonlocal equivalent plastic strain of the sample is determined and the positions of the indentations are known, the damage values according to Equation (28) can be coupled to the nonlocal equivalent plastic strain. Finally a damage evolution law $\omega_p(\tilde{\kappa})$ can be determined which is defined in terms of the nonlocal plastic multiplier $\tilde{\kappa}$, which is the maximum nonlocal equivalent plastic strain in each point. This method seems to be the most promising technique and therefore two different indentation experiments are performed, namely a micro Vickers hardness measurement and a micro Berkovich indentation experiment (which can also be used to determine $\tilde{E}$ in Equation (23)).
3  Tensile experiment

3.1 Experimental set-up

To perform an experimental analysis of a damage process, a specimen in which damage is produced has to be obtained. A tensile specimen, which is deformed to the point of fracture, must show a continuous variation of damaged material from $\omega_p = 0$ to $\omega_p = 1$ (in practice, the measured value for $\omega_p$ is maximally $w_p = 0.6$). Such a specimen can be obtained according to a micro tensile experiment in the microtest device that is schematically shown in Figure 8.

![Schematic representation of the microtest device with tensile module](image_url)

This microtest device consists of a firm base on which the motor (1), gearbox (2) and spindle (3) are mounted. Two traverses (4, 5) are driven by the spindle and can move from and towards each other with a constant prescribed velocity. At one traverse the loadcell (6) is positioned and at the other, the extensiometer (7) is located. To perform a tensile experiment, a tensile specimen (8) is clamped between the base plates (9, 10) and the two upper holders (11, 12). Instead of performing a tensile experiment, other modules can be mounted on the base plate for other particular test set-ups. The device has a size of about 135 x 85 x 400 mm and can therefore be placed underneath a light microscope or in a Scanning Electron Microscope (SEM). The dimensions of the tensile specimen are depicted in Figure 9.

![Dimensions of tensile specimen](image_url)
The material that has been used during all the experiments is named N004 at Philips. It is a martensite stainless chromium steel with a low carbon content. The grade is characterized by excellent formability. After a heat treatment the grade has good corrosion resistance and high toughness. The chemical composition of the material is shown in Table 1.

Table 1: chemical composition of N004 [%]

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>Si</th>
<th>Mn</th>
<th>P max</th>
<th>S max</th>
<th>Cr</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.32</td>
<td>0.2</td>
<td>0.3</td>
<td>0.025</td>
<td>0.01</td>
<td>13.5</td>
</tr>
</tbody>
</table>

Furthermore, a mechanical characterisation is performed according to a tensile experiment on a tensile specimen of material N004 with an initial rectangular cross section (length $L = 125\ mm$, width $W = 20.3\ mm$ and thickness $T = 0.5\ mm$). The force-displacement curve of the tensile test is shown in Figure 10 and is visualised as the blue curve which is determined according to a tensile experiment at the TU/e.

![Figure 10: stress-strain curves](image)

This curve covers a region of $0 < \varepsilon_p < 0.3$ and is extrapolated to an equivalent plastic strain value of $\varepsilon_p = 4$. Figure 10 also shows a green curve which is generated with data from Philips DAP. A large difference between the two curves can be observed in the region $0 < \varepsilon_p < 0.3$, see Figure 11.

It has to be remarked that the data from Philips DAP is not recently obtained and changes in the processing of the material could be an explanation for this discrepancy. Therefore the data from TU/e is used for the region $0 < \varepsilon_p < 1$ and the data from Philips DAP for the region $1 \leq \varepsilon_p < 4$ to circumvent the inaccuracy of the large extrapolation for the TU/e curve. The data from Philips DAP has not been extrapolated and, for that reason, is more reliable. This composite curve is visualised as the red curve in Figure 10.
The mechanical properties of the material are determined according to this composite curve. The strain-hardening behaviour is obtained by fitting the Nadai hardening law, which is defined as

$$\sigma_y = C (\varepsilon_p + \varepsilon_0)^n,$$

with the constants $C$, $\varepsilon_0$ and $n$ the hardening exponent. These parameters are given in Table 2.

Table 2: mechanical characterization of N004

<table>
<thead>
<tr>
<th>$\sigma_y$ [MPa]</th>
<th>$C$ [MPa]</th>
<th>$n$ [-]</th>
<th>$\varepsilon_0$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>287</td>
<td>1070</td>
<td>0.24</td>
<td>0.0016</td>
</tr>
</tbody>
</table>

3.2 ARAMIS

As was already mentioned in Section 2, the damage evolution law is a function of the nonlocal plastic multiplier $\bar{\kappa}$, which is a function of the nonlocal equivalent plastic strain $\bar{\varepsilon}_p$. The damage evolution law $\omega_p(\bar{\kappa})$, that will be experimentally determined in Section 4, has to be coupled to the nonlocal equivalent plastic strain $\bar{\varepsilon}_p$. To measure the strain fields of the tensile specimen, the software package ARAMIS is used [6] [9]. ARAMIS calculates displacement fields using digital image correlation (DIC). DIC is a method that is based on the correlation of gray values of successive digital images which are recorded during the deformation process at different deformation stages. The images are stored and the actual processing of recorded data is done after the experiment. The specimen must have a random speckle pattern on its surface to obtain a gray value distribution in the image. If the natural texture of the material does not present such a speckle pattern it can be artificially created by lightly spraying some paint onto the surface. The displacement of an arbitrary material point of the image can be determined by correlating a subset of pixels between two images. During data processing, the quality of the correlation is analysed and decorrelated measurement points are removed from
the subsequent analysis. The accuracy depends on the resolution of the camera, the quality of the speckle pattern, surface conditions during deformation and the input adjustment of the DIC software. The method is particularly well suited for localisation problems, since the precise localisation of the measurement points can be determined after the experiments.

The output that is generated by Aramis consist of the Von Mises strain, i.e. the equivalent strain measure, according to

\[
\varepsilon_{vm} = \sqrt{\frac{2}{3}(\varphi_1^2 + \varphi_2^2 + \varphi_3^2)}
\]

(30)

where \(\varphi_1\) is the major strain, \(\varphi_2\) is the minor strain and \(\varphi_3\) is the thickness reduction. The major and minor strain are found in the following way. First, the stretch tensor \(U\) is determined. The eigenvalues of the stretch tensor \(U\) are calculated, leading to two stretch ratios \(\lambda_1\) and \(\lambda_2\). These stretch ratios are the two principal strains or major and minor strain \((\varphi_i = \ln(\lambda_i))\).

The thickness reduction is calculated while making use of incompressibility, thus \(\lambda_1 \cdot \lambda_2 \cdot \lambda_3 = 1\).

Now, the equivalent plastic strain has to be determined. Aramis generates the total effective Von Mises strain. Because the elastic region of the concerning material is very small compared to the plastic region, the effective Von Mises strain is assumed as the local equivalent plastic strain:

\[
\varepsilon_{vm} \approx \varepsilon_{vm,p} = \varepsilon_p
\]

(31)

The output of Aramis is shown in Figure 12 and represents a complete tensile sample and the corresponding local effective plastic strain.

Figure 12: local effective plastic strain for a complete sample

Figure 13 shows the local effective plastic strain for the region of interest for damage evolution, namely the necking zone. It shows that the method clearly picks up the localised plastic straining in the neck, with a maximum effective plastic strain of 0.48.
3.3 Determination of the nonlocal strain distribution

Now, the local equivalent plastic strain over the domain of the tensile specimen is known. Although the damage evolution law is defined in terms of the nonlocal equivalent plastic strain \( \bar{\varepsilon}_p \), the value of the damage parameter has to be known as function of the local equivalent plastic strain \( \varepsilon_p \) when the damage evolution law is coupled to the hardening law (Equation (8)). This subsection elaborates the differences between the local and nonlocal equivalent plastic strain for a physically motivated length scale \( \ell \). To compare the local and nonlocal equivalent plastic strain, the nonlocal equivalent plastic strain has to be extracted from the results generated with ARAMIS. For this purpose, Equation (15) is solved by the finite element method.

The derivation of the nonlocal equivalent plastic strain \( \bar{\varepsilon}_p \) is started with Equation (15) and (16) which are reproduced here:

\[
\bar{\varepsilon}_p - \ell^2 \nabla^2 \bar{\varepsilon}_p = \varepsilon_p, \\
\nabla \bar{\varepsilon}_p \cdot \vec{n} = 0 \quad \text{on } \Gamma.
\]

This PDE (15) can be transformed into its weak form using the weighted residuals approach. For this purpose the following class of weight functions is defined:

\[
W_\varepsilon = \{ w_\varepsilon | w_\varepsilon \in [C^0] \}, \tag{32}
\]

Multiplying Equation (15) with the weight function \( w_\varepsilon \) and integrating the equation over \( \Omega \) yields:

\[
\int_{\Omega} w_\varepsilon (\bar{\varepsilon}_p - \ell^2 \nabla^2 \bar{\varepsilon}_p) \, d\Omega = \int_{\Omega} w_\varepsilon \varepsilon_p \, d\Omega. \tag{33}
\]
Substitution of the following expression

\[ w_\varepsilon \ell^2 \nabla^2 \bar{\varepsilon}_p = \nabla \cdot \left( w_\varepsilon \ell^2 \nabla \bar{\varepsilon}_p \right) - \nabla w_\varepsilon \cdot \ell^2 \nabla \bar{\varepsilon}_p, \]  

in Equation (33) leads to:

\[ \int_{\Omega} \left( w_\varepsilon \bar{\varepsilon}_p + \nabla w_\varepsilon \cdot \ell^2 \nabla \bar{\varepsilon}_p - \nabla \cdot \left( w_\varepsilon \ell^2 \nabla \bar{\varepsilon}_p \right) \right) \, d\Omega = \int_{\Omega} w_\varepsilon \bar{\varepsilon}_p \, d\Omega. \]  

(35)

Applying the divergence theorem of Gauss

\[ \ell^2 \int_{\Omega} \nabla \cdot \left( w_\varepsilon \nabla \bar{\varepsilon}_p \right) \, d\Omega = \ell^2 \int_{\Gamma} w_\varepsilon \nabla \bar{\varepsilon}_p \cdot \bar{n} \, d\Gamma, \]  

(36)

and incorporation of the boundary condition (16) gives the weak form of Equation (15):

\[ \int_{\Omega} \left( w_\varepsilon \bar{\varepsilon}_p + \nabla w_\varepsilon \cdot \ell^2 \nabla \bar{\varepsilon}_p \right) \, d\Omega = \int_{\Omega} w_\varepsilon \bar{\varepsilon}_p \, d\Omega. \]  

(37)

Now, the Finite Element Method (FEM) is used to solve equation (37). For this reason, the domain \( \Omega \) is divided in \( n \) elements. Furthermore, the weight function and the nonlocal effective plastic strain are defined within each element according to:

\[ w_\varepsilon = N_\varepsilon w_\varepsilon^e. \]  

(38)

\[ \bar{\varepsilon}_p = N_\varepsilon \bar{\varepsilon}_p^e. \]  

(39)

\( N_\varepsilon \) contains the shape functions and is formulated in the isoparametric coordinate system (\( \xi, \eta \)) for two-dimensional bilinear elements:

\[ N_\varepsilon = \begin{pmatrix} N_\varepsilon^1 & N_\varepsilon^2 & N_\varepsilon^3 & N_\varepsilon^4 \end{pmatrix} \]  

(40)

Furthermore, the gradient terms can be denoted as:

\[ \nabla w_\varepsilon \cdot \nabla \bar{\varepsilon}_p \Rightarrow (w_\varepsilon^e)^T B_e^T B_e \bar{\varepsilon}_p \]  

(41)

where \( B_e \) can be written as

\[ B_e = \begin{pmatrix} \frac{\partial N_\varepsilon^1}{\partial x} & \frac{\partial N_\varepsilon^2}{\partial x} & \frac{\partial N_\varepsilon^3}{\partial x} & \frac{\partial N_\varepsilon^4}{\partial x} \\ \frac{\partial N_\varepsilon^1}{\partial y} & \frac{\partial N_\varepsilon^2}{\partial y} & \frac{\partial N_\varepsilon^3}{\partial y} & \frac{\partial N_\varepsilon^4}{\partial y} \end{pmatrix} \]  

(42)

Using the introduced spatial discretisation, equation (37) is transformed to element level according to:

\[ (w_\varepsilon^e)^T \int_{\Omega_e} \left( N_\varepsilon^T N_\varepsilon + \ell^2 B_e^T B_e \right) \bar{\varepsilon}_p^e \, d\Omega_e = (w_\varepsilon^e)^T \int_{\Omega_e} N_\varepsilon^T N_\varepsilon \varepsilon_p \, d\Omega_e. \]  

(43)

At last, equation (43) can be written in the following form:
\[ K \varepsilon_p^e = f \] (44)

with

\[ K = \int_{\Omega_e} \left( N^T \varepsilon N + \ell^2 B^T B \right) \, d\Omega_e, \] (45)

and

\[ f = \int_{\Omega_e} \left( N^T \varepsilon \varepsilon_p \right) \, d\Omega_e. \] (46)

First matrix \( K \) and column \( f \) are calculated at element level. Then the element matrices and columns are assembled into a global matrix and a global column. Finally, the whole system (Equation (44)) is solved producing the nonlocal effective plastic strain \( \varepsilon_p^e \) in the nodal points for all elements.

To test the implementation of Equation (44), two problems are defined by prescribing the value of \( f \) over a domain spanned by 100 respectively 40 elements which are positioned in a row. The first case consists of a prescribed local equivalent plastic strain according to a Dirac function:

\[ \delta(x, x^*) = \begin{cases} \alpha & \text{if } x = x^* \\ 0 & \text{otherwise} \end{cases} \] (47)

This function is theoretically infinite in \( x^* \), thus \( \alpha = \infty \). However, to validate Equation (44), several values for \( \alpha \) have to be prescribed and convergence to the analytical solution has to be proved. Instead of the local equivalent plastic strain, the Dirac function forms the input in Equation (15):

\[ \varepsilon_p(x) - \ell^2 \nabla^2 \varepsilon_p(x) = \delta(x, x^*). \] (48)

The following solution (the nonlocal equivalent plastic strain in Equation (15)) is found:

\[ \varepsilon_p(x^*) = \frac{1}{2} e^{-\frac{|x-x^*|}{\ell}} \] (49)

Figure 14 shows the analytical solution and the output of Equation (44) for the concerning problem. The form of the analytical solution has been described correctly, but the value of the peak at \( x^* \) has not been calculated precisely.

To test the Equation (44) quantitatively, another mathematical function is chosen as input for the local equivalent plastic strain, namely a Heaviside function:

\[ H(x, x^*) = \begin{cases} 0 & \text{if } x < x^* \\ 1 & \text{if } x > x^* \end{cases} \] (50)

Substitution of Equation (50) in equation (15) leads to:
The solution for this equation on an infinite domain reads:

\[
\bar{\varepsilon}_p(x) = \begin{cases} 
\frac{1}{2} e^{\frac{(x-x^*)}{l}} & \text{if } x < x^* \\
1 - \frac{1}{2} e^{-\frac{(x-x^*)}{l}} & \text{if } x > x^*
\end{cases}
\]  

(52)

Figure 15 shows the analytically determined value of \( \bar{\varepsilon}_p(x) \) as well as the numerically obtained solution.

This figure shows very accurate results and it can be concluded that the implementation and derivation of Equation (44) is performed correctly.
3.4 Nonlocal equivalent plastic strain of the tensile specimen

In this subsection, the spatial distribution of the nonlocal equivalent plastic strain of the tensile specimen is discussed. The procedure mentioned above is applied to the ARAMIS output. Figure 16 shows the local equivalent plastic strain and the nonlocal equivalent plastic strain for a length parameter of $\ell = 0.1 \text{ mm}$.

The relative difference, which is defined as

$$\text{relative difference} = \text{abs} \left( \frac{\bar{\varepsilon}_p(x) - \varepsilon_p(x)}{\bar{\varepsilon}_p(x)} \right),$$

has the value 0.1 [-] for the center of the necking zone and varies around the value 0.03 [-] for the rest of the domain.

Figure 17 shows the output for the same problem with a length parameter of $\ell = 1 \text{ mm}$. For this case, the maximum relative error is 0.89 [-] and the mean relative error is 0.31 [-].

It was already mentioned in Section 2 that the length parameter $\ell$ determines the size of the volume which effectively contributes to the nonlocal quantity. This parameter is related to the scale of the microstructure of the material, e.g. the width of the grains. According to this, a value of $\ell = 0.1 \text{ mm}$ is thus more representative than $\ell = 1 \text{ mm}$ [4].

The relative error between the local and nonlocal equivalent plastic strain is sufficiently small for a length scale of $\ell = 0.1 \text{ mm}$. It can be concluded that it is justified to continue the determination of the damage evolution law with the use of the local effective plastic strain instead of the actual nonlocal effective plastic strain.
Figure 17: local and nonlocal equivalent plastic strain ($\ell = 1\text{mm}$)
4 Indentation experiments

4.1 Micro Vickers hardness measurement

The Vickers hardness test consists of applying a standard pressure to the surface of the material for a standard length of time by means of a pyramid-shaped diamond, see Figure 18 for the tip geometry.

![Figure 18: geometry of Vickers indent tip](image_url)

The diagonals $d_1$ and $d_2$ of the resulting impression are measured under a light microscope and the Vickers hardness value $HV$ can then be read from a conversion table. The Vickers hardness is calculated using the following formula [11]:

$$HV = 1.854 \frac{F}{A},$$

with $F$ being the applied load and $A$ the total surface area of the indentation.

A meaningful measurement is only obtained when the indenter tip penetrates the material perpendicular to the surface. Therefore, the samples have to be processed before the indentations are performed to prevent that the indenter tip penetrates the surface at the slope of a scratch. Grinding (step 1 and 2) and polishing (step 3 and 4) are used to remove scratches and to smooth the rough surface by means of abrasive grains attached to a polishing wheel. Those abrasive grains and polishing wheels are different for each step. Table 3 shows the successive steps, according to [12].

The grinding and polishing procedures are performed on an automatic polishing set-up. Step 1 is primarily used to ensure that the surface is perpendicular over the whole sample. The last three steps are used to smoothen the surface of the sample. Therefore the abrasive, which is a spray that consists of very small particles (see grain size), is applied to the surface of a typically polishing wheel. The MD-Allegro surface consists of a metallic disc, while the MD-Dac and MD-Chem discs are covered with cloth. During the grinding and polishing steps, the sample has to be lubricated with water and an alcohol based solution respectively. The adjustment for the set-up is also given in Table 3.
Table 3: grinding and polishing steps

<table>
<thead>
<tr>
<th>step</th>
<th>surface</th>
<th>abrasive</th>
<th>grit/grain size</th>
<th>lubricant</th>
<th>[rpm]</th>
<th>force [N]</th>
<th>time [min]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>abrasive paper</td>
<td>sand</td>
<td>400</td>
<td>water</td>
<td>300</td>
<td>manual</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>MD-Allegro</td>
<td>DP-suspension</td>
<td>9 µm</td>
<td>alcohol based</td>
<td>150</td>
<td>180</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>MD-Dac</td>
<td>DP-suspension</td>
<td>3 µm</td>
<td>alcohol based</td>
<td>150</td>
<td>180</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>MD-Chem</td>
<td>OP-S</td>
<td>-</td>
<td>-</td>
<td>150</td>
<td>90</td>
<td>2</td>
</tr>
</tbody>
</table>

The results of the micro Vickers indentations are shown in Figure 19. This figure consist of three parts. The lower part shows an image of the tensile specimen just before total fracture occurred. The middle part of the figure consists of the output of Aramis for the concerning image and represents the local equivalent plastic strain $\varepsilon_p$. The corresponding scale of the local equivalent plastic strain can be seen at the right. The positions of the indentations are marked, so the corresponding local equivalent plastic strain values of the indentations are known (from Aramis). The upper part of the figure shows the micro Vickers hardness values $HV$ at the different indentation positions. Now the local effective plastic strain values and the corresponding micro Vickers hardness values are known, a figure can be generated in which the hardness is plotted as a function of the local equivalent plastic strain, see Figure 20.

![Figure 19: results of micro Vickers indentations](image)

Figure 20 shows no significant effect of the plastic strain on the hardness. It is assumed that there must be a variation of the hardness in case of damage evolution. For that reason another indentation method for determination of the hardness is used, namely an automated Berkovich indentation system.
4.2 Micro Berkovich hardness measurement

The method described above is the classical technique of measuring the hardness. Nowadays, another relative simple method exists that is much more automated than the microscope based method named above. Furthermore, the set-up provides the ability to position the indenter tip with a resolution of 1 micron. The system that has been used is the MTS Nano Indenter XP system, equipped with Test Work 4 Professional level software [13].

In short, the method works as follows. After an indentation experiment, the contact area $A_c$ is determined as a function of the indentation depth. As can be seen in Figure 21, two different depths can be defined. Under full load the indenter penetrates the surface to a total depth $h_t$, which can be explicitly taken from the load-displacement curve. After complete unloading, the remaining depth $h_r$ can also be taken directly from the load-displacement curve.

The most widely used method for calculating the contact area is developed by Oliver and Pharr [14]. This data analysis procedure begins by fitting the load-displacement data acquired during the unloading to the power-law relation:
in which \( P \) is the indentation load measured during the experiment and \( B \) and \( m \) are empirically determined fitting parameters. However, for the calculation of the contact area the so-called contact depth \( h_c \) is required. According to the theory of Oliver and Pharr \( h_c \) is given as:

\[
h_c = h_t - \epsilon \frac{P}{S},
\]

with \( \epsilon \) being a geometry dependent constant. The parameter \( S \) is the unloading stiffness which is determined by fitting the upper 25 % to 50 % of the unloading curve, see also Figure 21. A Berkovich tip has been used which has a value of \( \epsilon = 0.75 \). This tip has a three-sided pyramid shape and the center-line to face angle is 65.3°, see also Figure 22.

\[
P = B(h_t - h_r)^m,
\]

Finally \( A_c \) is easily calculated by evaluating an empirically determined area function in terms of the contact depth \( h_c \):

\[
A_c = a_0h_c^2 + a_1h_c + a_2h_c^{\frac{1}{3}} + a_3h_c^{\frac{1}{3}} + \ldots ,
\]

where the variables \( a_0 \ldots a_n \) are determined according to a calibration of the indenter tip. Once the unloading stiffness and the projected contact area have been determined, the hardness \( H \) and the Young’s modulus \( E \) can be calculated. The hardness of the material can be calculated, according to the simple relation:

\[
H = \frac{P}{A_c}.
\]

The Young’s modulus is determined using the following expression:

\[
\frac{1}{E_r} = \left(1 - \nu^2\right)\frac{1}{E} + \left(1 - \nu_i^2\right)\frac{1}{E_i},
\]

where \( E_r \) is the reduced Young’s modulus, given by:

\[
E_r = \frac{(\sqrt{\pi} S)}{2\beta \sqrt{A_c}},
\]
with $\beta$ a constant which depends only on the geometry of the tip. For triangular cross sections of the indenter, like the Berkovich tip, $\beta = 1.034$. Furthermore, $E_i$ and $\nu_i$ in Equation (59) are the Young’s modulus and Poisson’s ratio, respectively, of the indenter tip. For diamond, the Young’s modulus $E_i = 1141$ GPa and Poisson’s ratio $\nu_i = 0.07$ are used.

It is important to note that the contact stiffness $S$ is required for the determination of the Young’s modulus and the hardness, because it is used in the calculation of the projected contact area $A_c$. The indenter system incorporates a continuous stiffness measurement option, which allows the continuous measurement of the contact stiffness during loading, and not just at the point of initial unload. This is accomplished by superimposing a small oscillation on the primary loading signal and analyzing the resulting response of the system by means of a frequency-specific amplifier. With a continuous measure of $S$, the hardness and Young’s modulus are obtained as a continuous function of the indentation depth from a single indentation experiment.

According to this method indentation experiments are performed to obtain the Young’s modulus and the hardness. Figure 23 presents the output data, namely the force-displacement curves of the indentation experiments at different positions along the axial direction of the tensile specimen. The highest curve corresponds to the undamaged region, whereas the lowest curve has been measured closest to the crack. The intermediate results follow this trend: the closer to the final crack, the lower the peak force.

![Figure 23: force-displacement curves of the indentation experiments](image)

26
As can be seen the indentation experiments are performed as follows: the indenter tip penetrates the material to a prescribed depth of 6500 nm. When this point is reached, the load on the sample is kept constant for a prescribed time. During this period creep can be observed, namely the penetration of the material under a constant load. Then the indenter withdraws during the unloading stage. At the end of this stage, the force is kept constant again and the material springs back without further unloading. This behaviour results in different remaining indentation depths (or widths) at the different regions of damage, see Figure 24.

![Figure 24: indentation series](image)

The hardness is determined at a depth of 5000 nm and Figure 23 clearly shows the variation in reaction force $F$. The surface area $A$ is almost constant for all measurements, which can be seen in the figure at the point at the end of the unloading curves where all the curves come together (if the extra penetration due to creep is subtracted). The fact that the surface area is constant can be explained according to the variation in Young’s modulus. The unloading curves show that Young’s modulus of the material in the damaged region has a smaller value compared with Young’s modulus of the material in the undamaged region. For this reason, a curve with a large reaction force and a large Young’s modulus ends at the same depth as a curve with a small Young’s modulus and a small reaction force. The variation of the hardness and the constant area function (for depths that are less than 6500 nm) finally results in a variation of the hardness $H = \frac{F}{A}$.

The hardness results are shown in Figure 25. As can be seen, the hardness varies significantly over the domain of the sample: it is approx 50% lower in the central region than away from the neck. Because the hardness varies mainly in the necking zone, another set of indentations is performed on this zone of interest. Figure 26 shows a contour plot of the local equivalent plastic strain in the necking zone of the tensile specimen. The fracture surface is located in the left part of the figure and indentation positions are visualised as blue triangles in the right part.
The indentations that are shown in Figure 24 correspond to the indentation series in Figure 26. The width $w$ of these impressions can be calculated according to the indentation depth and the geometry of the indenter tip. The width $w$ is defined as the length of one side of the three-sided pyramid and has a value of 10.87 $\mu$m, which is about 5 times more than the width of the voids that can be seen in Figure 1.
The results of the indentation series are used to obtain Figure 27, which shows the relation between the hardness $H$ and the local effective plastic strain $\varepsilon_p$.

Figure 27: Hardness as function of the local equivalent plastic strain

A clear trend can be distinguished and as a consequence, this figure will be used in Chapter 5 to determine a damage evolution law instead of the results of the micro Vickers indentations. With the knowledge of the micro Berkovich measurements, a possible explanation for constant value of the measured micro Vickers hardness can be given. Therefore two opposite cases are considered, namely case 1 which corresponds to the undamaged material and case 2 which describes the damaged material. In case 1, the hardness value is still large as well as the value of Young’s modulus. During the loading stage of an indentation experiment with a prescribed load, the relatively large value of the hardness prevents the indenter to penetrate the material to a large depth. During unloading, the large Young’s modulus leads to little spring-back and as a consequence, the remaining width of the indentation is small. On the other hand, the small value of the hardness in case 2 leads to a large indentation depth (or width). However the small value of the Young’s modulus leads to more spring-back during unloading and as a consequence the width of the indentation will finally be small too. It seems that these two opposite effects led to a constant value of the indentation widths for all micro Vickers hardness measurements.

In the following, the decrease of the Young’s modulus is elaborated. This effect was already observed in Figure 23. To verify this, the indentation system is used to calculate the Young’s modulus. The value of the Young’s modulus as function of the position on the tensile specimen is shown in Figure 28 and Figure 29 shows the relation between the Young’s modulus and the local equivalent plastic strain.
It is obvious that the value of the Young’s modulus varies as a function of the local equivalent plastic strain. However, the value of the measured Young’s modulus does not correspond to the initial value of the Young’s modulus of material N004, which is $E = 205$ GPa. The value for the Young’s modulus is obtained at the same point as the value for the hardness, which is 5000 nm. During the indentation experiment, the value of the Young’s modulus decreases rapidly, see Figure 30. A possible explanation for this decrease might be the growth of damage due to the tensile loading, which further increases during the indentation experiment. However, it is unlikely that this process can lead to a damage evolution from 0 to 0.85 during an indentation experi-
Figure 30: decreasing Young’s modulus as a function of the indentation depth

iment, particularly in the undamaged part of the specimen. A numerical simulation of the indentation experiment should be performed to study the mechanical behaviour of damaged material during an indentation.

The rapid decrease can probably be explained by an inaccurate calibration of the indenter tip. This calibration determines the variables in Equation (57). As a consequence, the Young’s modulus is calculated wrongly. Several calibrations have been performed to circumvent this problem, but accurate results were not obtained.

Determination of the Young’s modulus at small indentation depths, where the value corresponds to the initial value, is not possible because the values would be very inaccurate due to measurement errors at the point that the indenter touched the surface. Furthermore, the width of the indentations would be too small in comparison with the width of the micro voids.

Nevertheless, to determine a damage evolution law the relative values of the measured data ($H$ or $E$) are important. In other words, a qualitative damage evolution law can be obtained without quantitatively correct data.

It has to be remarked that the value of the Young’s modulus is constant in the presented damage model in Section 2. The assumption that the Young’s modulus is not influenced by damage does not hold for the considered material. However, it can easily be implemented in the finite element package by coupling the damage evolution law $\omega_p$ to the Young’s modulus according to

$$\tilde{E} = E(1 - \omega_p).$$

(61)

This effect is incorporated in the elastic damage model that is described in Appendix A.


5 Damage evolution law

5.1 Determination of a damage evolution law

Chapter 2 described the damage incorporation in the plasticity model by including the softening behaviour in the post-yield function. Since the hardness is related to this post-yield function (according to an empirical expression), the hardness measurement results will be used to determine a damage evolution laws instead of results of the Young’s modulus. It has to be remarked that the measured damage during the hardness measurements does not exactly describe the softening behaviour as was visualised in Figure 2, but rather shows a combination of elastic and plastic damage models, see Figure 31.

Figure 31: incorporation of softening behaviour into post-yield function and Young’s modulus

Nevertheless, the experimentally obtained relation between the micro Berkovich hardness and the local equivalent plastic strain will be used to determine a damage evolution law according to Equation (28)

\[ \omega_p = 1 - \frac{\tilde{H}}{H}, \]

where \( \tilde{H} \) is experimentally determined and \( H \) has to be obtained by an extrapolation procedure, because the hardness value is not constant, but a function of strain-hardening. Figure 32 shows the same data set as in Figure 27 in Section 4.

The data set that is indicated with blue diamonds corresponds to the hardness of the damaged material \( \tilde{H} \). The red diamonds represent the hardness of the undamaged material \( H \) and will be used to extrapolate the micro Berkovich hardness curve in the region where damage occurred. It is assumed that damage evolution started at \( (\varepsilon_p > 0.06) \). The green line represents the extrapolation curve which is determined according to a interpolation fit in MATLAB and represents \( H \) in the region where damage occurred. According to Equation 28 the damage values are calculated and shown in Figure 33 as the blue marks.
The damage evolution laws $\omega_p$ that are mentioned in Section 2 are fitted to the damage curve in Figure 33 to obtain the most representative damage evolution law for the concerning material. The laws are visualised by the red lines in Figure 34.

It is obvious that both the modified power law as well as the exponential damage evolution law describe the damage curve accurately. Both equations are reproduced in Equations (62) and (63):

$$\omega_p = 1 - \left( \frac{\kappa_i}{\kappa} \right)^\beta \left( \frac{\kappa_c - \kappa}{\kappa_c - \kappa_i} \right)^\alpha. \tag{62}$$

$$\omega_p = 1 - \left( \frac{\kappa_i}{\kappa} \right) \left( 1 - \delta \right) + \delta \exp^{-\gamma(\kappa - \kappa_i)}. \tag{63}$$

The material parameters for these two damage evolution laws are: $\kappa_i = 0.06$, $\kappa_c = 3.53$, $\alpha = 1.73$, $\beta = 0.351$, $\gamma = 3.52$ and $\delta = -2.68$. 

33
Figure 34: several fitted damage evolution laws.

It has to be remarked that the determined damage evolution laws can possibly under- or overestimate the exact damage evolution. Regarding Figure 29, the relation between the Young’s modulus and the local equivalent plastic strain can be calculated and according to Equation (23) a damage evolution law can be defined, see Figure 35.

Figure 35: Young’s modulus as a function of the local equivalent plastic strain and the corresponding damage evolution law.
The left part shows the initial Young’s modulus $E$ (blue line) and the Young’s modulus of the damaged material $\tilde{E}$ (red marks). The right part of the figure shows the damage evolution law according to Equation (23). It is already mentioned that the values of the Young’s modulus are very low. A possible explanation is a wrong calibration of the area function. Furthermore, an extra damage evolution caused by the penetration of the material during the indentation can also lead to a small contribution in the decrease of the Young’s modulus. The damage evolution laws are determined with the assumption that this extra damage evolution has not been influenced by the damage that was already present in the material due to the tensile experiment. This situation is visualised in Figure 36 as the green line that describes the evolved damage during the tensile experiment.

![Figure 36: under- and overestimation of damage evolution law.](image)

This is the case when no indentation experiment has been performed and the black arrow describes the damage due to the indentation experiment. It is thus assumed that the damage evolution during an indentation is independent of the damage variation in the material. It might be possible that the actual damage evolution due to the tensile experiment describes the purple curve in the left part of Figure 36. In this case, the damage that was already present in the material does influence the damage evolution during an indentation. The corresponding damage evolution law is shown in the right part as the purple curve. In this case, the determined damage evolution law underestimates the actual damage at the beginning and overestimates the damage at the end. At the moment, it is unknown if such an effect has occurred.
Another damage evolution law has been mentioned in chapter 2 and is reproduced here, namely Equation (22):

\[ \omega_p = \frac{1}{C_G} \int_\kappa \left( 1 + 3.9 \frac{\sigma_h}{\sigma_{eq}} \right) \bar{\kappa}^{0.63} \, d\bar{\kappa}. \] (64)

This damage evolution law incorporates a ductile fracture threshold value \( C_G \), which is a critical value for crack initiation. To implement Equation 22 in a finite element package, the material parameter \( C_G \) has to be determined according to a relative simple tensile experiment: first a tensile test has to be performed and when fracture has occurred, the thickness of the specimen is measured (i). The next step is to simulate the concerning tensile experiment (ii). The simulation is stopped when the thickness of the material reached the measured value obtained with the tensile experiment. At this point Equation 21 can be calculated and \( C_G \) has been determined (iii).

To perform this procedure, a tensile experiment has to be simulated accurately. Therefore 3D or plane stress elements have to be available. At the moment, ARTEMIS does not incorporate these element types. 3D elements are incorporated in CRYSTAL, but 3D boundary conditions have not been implemented yet. Therefore, the determination of the parameter \( C_G \) has not been performed.
6 Discussion

6.1 Conclusions

- The digital image correlation package Aramis proved to be a very useful tool to describe the local equivalent plastic strain distribution in the necking zone of a tensile specimen, which is the region of damage evolution.

- For the performed tensile experiment, the difference between the local and nonlocal equivalent plastic strain is very small in case of a physically motivated length scale $\ell$.

- The results of the micro Vickers hardness measurement did not show the expected variation of the hardness as function of the local equivalent plastic strain. The small hardness value of the damaged material led to a large indentation depth. However this effect is annulled by the large spring back due to a small Young’s modulus. On the other hand, the large hardness value for undamaged material led to a small indentation depth. But now, the large value of the Young’s modulus led to little spring-back. Therefore, the remaining indentation width is the same for both cases. The hardness, which it is calculated according to the remaining indentation width only, shows no variation. Thus this method has not been used for determination of a damage evolution law $\omega_p$.

- The micro Berkovich hardness measurement led to useful results for determination of the damage evolution laws. A relation between the hardness and the local equivalent plastic strain was found.

- In contradiction to the assumption of the continuum damage model of Chapter 2, the value of the Young’s modulus $E$ decreases as a function of the local equivalent plastic strain for the concerning material. Experiments showed that the damage apparently influences both the plastic as well as the elastic mechanical properties.

- The modified power law

$$\omega_p = 1 - \left( \frac{K_i}{K} \right)^{\beta} \left( \frac{K_c - K}{K_c - K_i} \right)^{\alpha},$$

and the exponential damage law

$$\omega_p = 1 - \left( \frac{K_i}{K} \right) \left( (1 - \delta) + \delta \exp^{-\gamma(K-K_i)} \right),$$

probably describe the damage evolution in the concerning material accurately. However, it has to be remarked that these laws do not incorporate the triaxiality ($\frac{\sigma_h}{\sigma_{eq}}$). The material parameters for these two damage evolution laws are: $k_i = 0.06$, $k_c = 3.53$, $\alpha = 1.73$, $\beta = 0.351$, $\gamma = 3.52$ and $\delta = -2.68$.

- A phenomenologically better damage law is defined by [7]:

$$\omega_p = \frac{1}{C_G} \int \left\langle 1 + 3.9 \frac{\sigma_h}{\sigma_{eq}} \right\rangle \bar{K}^{0.63} d\bar{K}.$$
As opposed to the previous defined damage evolution laws, this law incorporates the triaxiality, which is an important value for ductile damage evolution. This damage evolution law can be determined according to a coupled experimental and numerical simulation.

- Damage evolution is probably also created during the indentation experiments. The relation between the damage evolution during an indentation experiment and the amount of damage that was already present in the material is unknown. For this reason, the determined damage evolution laws can under- or overestimate the damage evolution.

6.2 Recommendations

- The damaged material was obtained according to a tensile experiment. The two determined damage evolution laws are therefore probably only applicable for processes in which the loading conditions compare to the loading conditions of a tensile test. In case of a simulation of an industrial process that incorporates other loading conditions, the damage evolution laws have to be determined under process conditions that agree with the industrial process. In general, such an identification process has to be avoided. The damage evolution law according to [7] overcomes this problem.

- More research is needed on the influence of the second process, i.e. the indentation experiment. A relation between the damage evolution during an indentation and the damage present in the material has to be found.

- Furthermore, numerical simulations of the indentation experiment have to be performed to describe the plastic material behaviour of the material during an indentation experiment. To validate the damage evolution law Equation (22), a numerical simulation of a tensile experiment has to be performed too. Therefore, 3D or plane stress elements have to be available. Experimental results, i.e. a force-displacement curve, are already present.

- The rapid decrease of the Young’s modulus has to be investigated. This effect is probably based on a wrong calibration of the geometry of the indenter tip.

- Future research is also needed for the formulation of a damage model that incorporates both plastic and elastic damage evolution instead of the models that are present nowadays, which only describe plastic or elastic damage.

- In general, more tensile and indentation experiments have to be performed to cancel out measurements errors.

- Before the decrease of material parameters due to the tensile experiment can be measured, two processes took place, namely the polishing and grinding steps and the indentation experiment. The effect of the polishing and grinding steps for the concerning material have to be found out.
A Elastic damage model

An isotropic elastic damage constitutive relation can be formulated according to:

\[ \sigma = (1 - \omega_p)^4 C : \epsilon_e, \]  \hspace{1cm} (65)

where \( \epsilon \) denotes the linear strain tensor. The scalar damage variable \( \omega_p \) degrades the elastic stiffness of the material. A typical uniaxial stress-strain curve for elastic damage is shown in Figure 37. Upon unloading the material follows the elastic path with a reduced stiffness.

\[ [1 - \omega_p (\bar{\epsilon})] E \]

Figure 37: Unloading in damage for an elastic damage model
References


Acknowledgements

First of all, I would like to thank Olaf van der Sluis, who gave me the opportunity to fulfil my internship at a very interesting group at Philips CFT. His coaching was very instructive and I learned a lot during my period at CFT. Although he is very experienced in computational mechanics, his comments on the experiments were always well thought-out and critical. His interpretation of 'schade' was sometimes ambiguous...

My coach at TU/e, Ron Peerlings, for his critical and pleasant supervision. His comments were always stimulating and I appreciated his helpful coaching.

Furthermore, I would like to thank:

Peter Janssen for his skillful instruction of the digital image correlation package ARAMIS and the micro tensile test device as well as the instruction on the preparation of the samples. He was always ready to help and spend a lot of time on teaching me some experimental skills.

Willem-pier Vellinga and Christophe Pelletier, who thought along with me about the interpretation of the output of the Berkovich indentation experiments. Their instruction on the indentation set-up was very useful.

Mark van Maris, who helped me with the microscopic devices in the multi-scale lab at the TU/e as well as with the micro-Vickers set-up.

Cees Meesters for his instruction of the tensile testing machine and the micro-Vickers hardness measurement set-up.