THE EFFECT OF FLUID FLOW ON THE DETACHMENT OF DROPS IN THE WAKE OF A FLAT PLATE

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ABSTRACT

The pinch-off of liquid drops from the end of a flat plate has been studied. Gas flows parallel to the plate with mean velocity $v$, and creates a wake at the plate end. The break-up process is found to be reproducible, with the number of satellites increasing with both increasing velocity and increasing mass density. The diameter of the main drop after detachment, $d$, at Weber numbers exceeding 5 is found to be given by

$$\frac{d}{d_v} = 1 - C\, Re_{pd}$$

with $C = (5.6 \pm 0.3) \times 10^{-5}$ for ethanol and $C = (2.9 \pm 0.4) \times 10^{-5}$ for water. The diameter in quiescent gas, $d_v$, is easily predicted, and $Re_{pd}$ is the Reynolds number based on $v$ and the thickness of the plate. Diameters at smaller Weber numbers are better predicted by a similar correlation in terms of the Weber number.

Instantaneous acceleration and forces on a drop have been measured and analyzed. Hydrodynamic interaction between liquid and gas at high gas velocities at moments of strong interaction could be quantified with the usual expression for the drag force and a drag coefficient of about 1.3.

1 INTRODUCTION

The pinch-off of liquid drops from the downstream end of plates is an important re-entrainment mechanism of condensate in compact condensers in the process industry. This re-entrainment is usually to be avoided, and therefore limits the velocity of the gas from which heat is regenerated by condensation in the heat exchanger.

It is well known that the break-up of drops due to their velocity relative to the surrounding, unbounded gas (fluid) is mainly dependent on Reynolds and Weber numbers in terms of this relative velocity. The present experimental study investigates whether this is still the case for drops that are growing slowly in the downstream wake of a plate with a thickness of 3 mm. Histories of forces acting on drops will be presented and analyzed in order to facilitate interpretation of detachment results.

2 EXPERIMENTAL

The test rig consists of a circulation loop with a pump (mixed flow type) and a test section. The flow velocity is measured both optically (particle tracking of seeding particles in a laminar flow area) and using a pitot tube. Temperature is controlled via a heat exchanger, electrical heating and via a water bath; residual fluctuations were less than 0.02 °C. Temperature is measured with Pt100 thermoresistors, reproducibility 0.01 °C and accuracy 0.1 °C.

The test section comprises a 3 mm thick aluminum plate in the centre of a channel that is 40 mm wide, 200 mm long and 10 mm high (Fig. 1). The cross-sectional area upstream of the plate end varies by less that 10 % (Fig. 1). Liquid is supplied by a syringe pump to the plate end, the lower edge of the plate, through a hole 1.2 mm in diameter, and via a 3 mm tube that is located 35 mm upstream of the plate end. Gas or vapor is downward. Windows are mounted flush in the channel wall, with outer windows and gas filling to warrant safe operation at pressures up to 60 bar. The window area is 50x10 mm².

Two 200 W light sources with infrared filters and paper sheets created a diffuse lightning system (Fig. 2). High-speed video recording at frame rates between 500 and 2250 Hz was applied. A calibration grid with 0.25 mm dots at 1 mm spacing was moved in the test section prior to the actual measurements. After each measurement session, the calibration grid was replaced for a second calibration. Observations were done both from the side and from the front (Fig. 2), but comparison merely showed that most fluid topologies have a large degree of axial symmetry. Each contour has been determined with sub-pixel accuracy in a viewing window that measures 256x256 pixels², typically corresponding to 25x25 mm² actual size. Error propagation has been computed with the method of Cline and McClintock [1].

A thorough cleansing procedure has been followed, for details see Lexmond [2].

3 RESULTS

The main goal of the research reported in this paper has been to determine the influence of Reynolds and Weber numbers on the detachment radius of drops of fluids with relatively high surface tension. Drops have been suspended underneath a thin plate, with nitrogen flowing along at velocities between 0 and 20 m/s. Temperature was 21.7-24.5 °C and pressure has been varied between 1 and 51 bar.
Figure 1. Schematic of test section. Left: front view. Middle: side view. Right: cross sectional area. Horizontal and vertical axes have different scale. Lengths in mm, areas in mm$^2$.

Figure 2. Schematic of optical set-up, front view. Sizes in mm. Mirror and camera are in the horizontal plane, while vapor flows vertically along the vertical inner plate. Gravity points in flow direction, see Fig. 1.
Figure 3. Pinch-off of ethanol in otherwise quiescent nitrogen gas from the bottom edge of a vertical plate. Gravity points downward, see Fig. 1.

Figure 4. Pinch-off of a water drop from the downstream edge of a vertical plate; mean velocity of nitrogen gas on each side of the plate is 0.53 m/s. Gravity points downward, see Fig. 1.
3.1 Drop diameter after pinch-off in otherwise quiescent nitrogen

As liquid is continually fed into a drop that is pendant from the downstream edge of the test plate, drop size increases gradually. Ethanol and water drops cover the entire thickness $D$ of the plate (3 mm), but only a part of the 40 mm-width of the plate, as indicated in Fig. 2. With increasing drop size, the shape of the liquid ligament changes gradually and slightly. At certain shape, surface tension forces fail to counterbalance gravity in some part of the liquid ligament. This results in elongation and eventually contraction and drop detachment. In the contraction stage, liquid is mainly in the lower liquid volume, that has a shape close to that of a sphere, but liquid is also present in a slim fluid column in the center as well as in a foot remainder on the plate, see Fig. 3. The thin column yields satellite drops in usual, instability-driven manner.

3.2 Drop diameter after detachment in forced convection

For both ethanol and water, the following three measurement series have been carried out:

- Gas velocity $0.65 \pm 0.05$ m/s at pressures between 1 and 51 bar
- Pressure 1 bar and velocities 0-20 m/s
- Pressure 51 $\pm 0.3$ bar and gas velocities between 1 and 1.3 m/s.

This variety of conditions will prove sufficient to draw conclusions regarding the effect of the main governing flow parameters.

![Figure 5](image1.png)

Figure 5. The effect of gas velocity on the size of the main drop after pinch-off from vertical 3 mm thick plate at elevated pressure. Water: $T=22.2 \pm 0.1^\circ C, p=50.9 \pm 0.2$ bar; ethanol: $T=23.0 \pm 0.2^\circ C, p=50.4 \pm 0.2$ bar.

At low velocities, droplet pinch-off naturally resembles pinch-off in quiescent gas. With increasing velocity, the shape of the drops becomes more irregular and the column between drop and remainder at the plate thickens, see for example Fig. 4. Break-up time of the drop, starting at the time the contraction sets in and ending with the formation of the last satellite drop, increases significantly with increasing nitrogen velocity. For example, this detachment process takes 30 ms for ethanol in quiescent vapor, but 200 ms at 1.1 m/s. This increase in detachment time is caused by the increased complexity of the break-up process, as explained below.

At gas velocities exceeding 1 m/s, no clear distinction between foot, column and main drop can be made. The droplet is stretched into a column that elongates and oscillates in horizontal direction, i.e. in the direction normal to the plate axis. The thicker end of the fluid column, formerly designated as main drop, is irregular in shape. Small droplets have been observed to pinch off from the fluid column even before the main drop pinches off. With increasing vapor velocity, more and bigger satellites are formed, quite often with a size more than half that of the main drop. This holds for both water and ethanol. At the same conditions, main water drops are usually bigger than main ethanol drops.

![Figure 6](image2.png)

Figure 6. The effect of gas velocity on the size of the main drop after pinch-off from vertical 3 mm thick plate at 0.1 MPa. $T=21.0 \pm 0.5^\circ C$.

The effect that the mean gas velocity at either side from the test plate has on the drop size of the largest drop is shown in Fig.’s 5 and 6, for 5.1 MPa and 0.1 Mpa respectively. Note that the velocity range in Fig. 6 is 2.5 times that of Fig. 5. Because of the increasing satellite size, main drop size decreases with increasing velocity.

![Figure 7](image3.png)

Figure 7. The effect of the gas mass density on the size of the main drop after pinch-off from 3 mm thick plate. Water: $T=22.0 \pm 0.3^\circ C, v=0.65 \pm 0.6$ m/s; ethanol: $T=23.1 \pm 0.6^\circ C, v=0.65 \pm 0.6$ m/s.

With increasing system pressure also the mass density of the gas, $\rho_v$, increases. The measurements exhibit the same trends with increasing pressure (at constant velocity) as with increasing gas velocity (at constant pressure). Total break-up time, for example, increases with increasing mass density. Figure 7 shows the effect of mass density on main drop size after detachment. The errors indicated in these figures correspond to the measuring accuracies of individual drop recordings, see Kline and McClintock [1]. Reproducibility decreases with increasing gas velocity. Each data point of Fig’s 5-7 is the mean of several recordings, but the number of
recordings was too low to accurately estimate the reproducibility error (standard deviation).

4 ANALYSIS

Body forces, only gravitational in our experiments, capillary forces and fluid stresses all affect drop size at detachment. Without gas flow, the mean curvature \( \kappa \) must satisfy the Young-Laplace equation

\[ \kappa \sigma = \Delta p + g \Delta \rho X. \]

(1)

at each height \( X \), of the liquid-gas interface. Here \( \sigma \) denotes the surface tension coefficient, \( \Delta p \) the pressure drop over the interface and \( \Delta \rho \) the mass density difference between liquid and gas. Detachment occurs when no shape can be found that satisfies the Young-Laplace equation everywhere. Assuming that the main detached drop with equivalent radius \( r = d_{\text{det}}/2 \) results from a mass of fluid that was too heavy to be kept to the remainder by surface tension forces, the following criterion is easily derived:

\[ d_{\text{det}} = 2 \sqrt{\frac{3 \sigma D}{4 \Delta \rho}} \]

(2)

Here \( d_{\text{det}} \) is the drop diameter at zero gas velocity. Substituting \( \Delta \rho = 800 \text{ kg/m}^3 \) and \( \sigma = 22 \text{ mN/m} \) for ethanol-nitrogen and \( \Delta \rho = 1000 \text{ kg/m}^3 \) and \( \sigma = 72 \text{ mN/m} \) for water-nitrogen one predicts with Eq. (5) a drop volume of 27 \( \mu \text{l} \) for ethanol and one of 69 \( \mu \text{l} \) for water. Measured drop volumes are 29\( \pm 3 \) \( \mu \text{l} \) and 65\( \pm 5 \) \( \mu \text{l} \) respectively. Measured and predicted drop volumes correspond well.

With gas flowing, fluid stresses may be important. It is well known that in laminar flow free droplets break up when the Weber number exceeds a threshold value, see e.g. Frohn & Roth [3] or Lefebvre [4]. The Weber number is defined by \( \text{We} = \rho v^2 d/\sigma \), where \( v \) in unbounded fluids is to be interpreted as the difference in velocity between the drop and the surrounding gas. The flow situation downstream of a plate edge is obviously different from that around a drop in an unbounded fluid, but our expectation is still that in some circumstances the Weber number would be a governing parameter. The measurement results of section 3 will now be used to assess the importance of the Weber number for break-up downstream of a flat plate with a thickness of 3 mm. Velocity \( v \) is now the mean velocity component of the gas parallel to the plate.

It is readily seen that the measured break-up diameters are not represented by a criterion of the form \( \text{We} = \rho v^2 d/\sigma \) constant. This is easily understood from the importance of gravity, especially at low Weber numbers, and from the fact that velocity \( v \) is not the actual relative velocity in the rather complex flow area in the wake of the plate. The effect of flow at low velocities on detachment diameter, \( d \), is conveniently measured as a deviation from \( d_{\text{det}} \), with \( d_{\text{det}} \) well predicted by Eq. (2). It therefore makes sense to use \( d_0 \) as reference length scale, and correlate \( d/d_{\text{det}} \), with respect to \( \text{We}_0 = \rho v^2 d_{\text{det}}/\sigma \). For low velocities, i.e. for \( \text{We}_0 < 10 \), \( d/d_{\text{det}} \) is close to 1 and a simple relation between the ratio \( d/d_{\text{det}} \) and the Weber number \( \text{We}_0 \) is expected to be appropriate. A fit of the coefficient \( C \) in the equation

\[ d/d_{\text{det}} = 1 - C \text{We}_0 \]

(3)

to the data presented in section 3 yields \( C = 0.11 \pm 0.01 \) for water. The error indicated is the 95 % estimate, and the fit accuracy is given by the value of the correlation coefficient, \( r^2 \), which amounts to 0.83, and by \( F=45 \). The ethanol data with \( \text{We}_0 < 4 \) have been correlated with \( C = 0.072 \pm 0.005 \), with \( r^2=0.93 \) and \( F=106 \). However, the ethanol data at 0.1 MPa, for velocities up to 15 m/s, yield Weber numbers up to 80 for detachment drop diameters in the same range as those corresponding to low Weber numbers. More specific, for given \( d/d_{\text{det}} \) -ratio, Weber numbers at 0.1 MPa differ from those at higher pressures (and lower velocities) by two orders of magnitude.

The Weber number is apparently not the proper parameter to unify all velocity results. The flow field downstream of the plate depends on the width of the plate, 3 mm, and is characterized by the Reynolds number \( \text{Re}_p = v 0.003 / v \), with \( v \) the kinematic viscosity of the gas in m\(^2\)/s and \( \nu \) the mean gas velocity in m/s. A fit of the coefficient \( C \) in the following equation

\[ d/d_{\text{det}} = 1 - C \text{Re}_p \]

(4)

to all the ethanol data presented in section 3 yields \( C = (5.6 \pm 0.3) \times 10^{-5} \) with \( r^2 \)-value 0.92 and \( F=145 \). In view of the variation in velocity and in \( \text{We}_0 \) this is a surprisingly good fit. The water data have been fitted to the same equation (4), with the results \( C = (2.9 \pm 0.4) \times 10^{-5} \) with \( r^2 \)-value 0.65 and \( F=32 \). This accuracy is less than that given by Eq.(3), but for water no high-velocity data are available.

It is noted that the amount of liquid in satellites depends on \( \text{Re}_p \), as well. If we were to compute the equivalent diameter corresponding to the total mass of main drop and of satellites, still the same dependency of detachment diameter on \( \text{Re}_p \) would result. So the Reynolds dependency of Eq. (4) at high Weber numbers does not depend on the selection of drops. This dependency on \( \text{Re}_p \) is worth further investigating, which is done below. The definition of the Weber number implies that at high \( \text{We}_0 \), the relative importance of surface tension is small. The success of Eq. (4) in fitting detachment data indicates that at high \( \text{We}_0 \) fluid stresses exerted by the gas are dominant. This is further investigated below.

Summarizing, the following prediction procedure for main detachment diameter is recommended:

- First estimate \( d_{\text{det}} \) from Eq. (2).
- Then compute \( \text{We}_0 \).
- If \( \text{We}_0 < 5 \), use Eq. (3), with appropriate \( C \)-value,
- If \( \text{We}_0 \geq 5 \), use Eq. (4).
To improve our understanding of break-up of liquid filaments in the wake of a flat plate, forces acting on the near-sphere from which a main drop is to emerge have been analyzed. Figure 8 gives a typical example of force histories during detachment. In the test of Fig. 8, gravity points in a direction normal to the test plate, from which water was pinching off in air. The contributions of gravity and surface tension have been subtracted from the measured acceleration in order to quantify the drag and lift forces, together denoted by ‘hydrodynamic’ in Fig. 8, exerted by the fluid. The procedure is discussed below. Results like those in Fig. 8 show that the flow pattern downstream of the plate (and Re_pl) is indeed important for detachment. Suppose that gas would exert form drag on a liquid filament protruding into the gas stream, see Fig. 4. As a result, this filament would obviously move downward and back into the wake of the plate again. If the flow pattern of the gas promotes this kind of interaction, it affects break-up in obvious manner. In the example of Fig. 8, hydrodynamic forces on the liquid filament are highest around times 0.104 s and 0.121 s, see the RHS of Fig. 8. At these times, the liquid protrudes into the gas stream, at the points 0.0017 and 0.0045 m at the LHS of Fig. 8, and the ‘drop’ quickly alters direction. In the example of Fig. 8, gravity in itself is relatively small (0.22 mN), but assists by bringing the liquid into contact with the gas. As a result, the effect of gas stresses is clearly observed and these stresses are related to drop detachment. In the experiments with water and ethanol of section 3, gravity acts parallel to the flow, and instabilities in gas flow and/or liquid position, and an appropriate flow pattern, are required to establish interaction between gas and liquid. Fluid stresses still have bearing to detachment, however, which explains the importance of the Reynolds number and the success of Eq. (4) in correlating high-velocity data.

The forces have been analyzed in the following way. Let \( \mathbf{m} \) be the unit vector normal to the downstream edge of the plate, pointing downward in the case of the flat edged plate of Fig. 1. The force balance on a drop on the plate can be expressed as

\[
\rho_l V \frac{dv_f}{dt} = \left( m A_{foot} \frac{2\sigma}{R} + F_{e} \right) + \left( \rho_l - \rho_g \right) V \left[ g - m(m.g) \right] + F_H
\]  

(5)

Here \( F_H \) denotes the sum of the hydrodynamic forces, \( V \) is the volume of the drop, \( g \) the acceleration due to gravity, \( F_e \) the total surface tension force at the foot of the drop, \( v_f \) the velocity of the drop, \( A_{foot} \) the area of the foot of the drop. Curvature \( 1/R \) is the average of the (to the drop foot) projected mean Gaussian curvatures of the drop surface. For hemispheres, \( F_e = -2\pi \sigma R m \) and \( A_{foot} \) equals \( \pi R^2 \), making the first term on the RHS of Eq. (5) equal to zero. Would the plate edge have been vertical, \( m \) would have been horizontal and \( m.g \) would have been zero. In that case a hemispherical drop on a vertical plate end in a quiescent gas would experience only gravity and buoyancy, \( i.e. \left( \rho_l - \rho_g \right) V g \), and would need to accelerate downward, or to deform. The plate end in the experiments of section 3 is horizontal, making \( m(m.g) = g \) and the second term on the RHS of Eq. (5) equal to zero. The first term on the RHS of Eq. (5), the one with \( R \), comprises the well-known pressure correction force, which corrects for the pressure difference between liquid in the drop and gas in the vicinity of the drop foot, as well as gravity acting on the drop. The form (5) in which the force balance of the drop has been cast facilitates experimental assessment of force contributions from measured drop shapes.

Using this approach, the hydrodynamic force has been quantified; see the example of Fig. 8. With the aid of measured mean gas velocities parallel to the plate, \( v \), maximum values of the magnitude \( F_H \) (for example those around times 0.104 s and 0.121 s in Fig. 8) have been correlated by the fitting de parameter \( C_{pl} \) in the equation

\[
F_{H, max} = \frac{1}{2} A_d C_{pl} \rho, v^2
\]  

(6)
Here $A_f$ denotes the frontal area of the drop that is in contact with the gas, estimated from the width of the drop and the distance that the drop protrudes into the vapor flow (about 0.6 mm in the position at 0.121 s of Fig. 8). Parameter $C_D$ will be named drag force coefficient but is actually a lumping parameter of all hydrodynamic action on an adhering liquid drop. The fitting result is $C_D = 1.3 \pm 0.4$. The error estimate accounts for the spread in the maximum values of $F_H$ measured.

The value of drag force coefficient $C_D$ is large as compared to more common drag force coefficients. This shows the significance of hydrodynamic interaction for the detachment of drops from the end of a plate.

5 CONCLUSIONS

Experiments with water/nitrogen and with ethanol/nitrogen have been performed at pressures between 0.1 and 5.1 Mpa. Gas flows symmetrically downward along both sides of a 3 mm thick flat plate at velocity $v$; total channel width is 10 mm. A single drop at the time has been made to grow steadily and slowly underneath the plate that has a width of 40 mm. With the aid of high-speed video recordings the detachment process has been studied, and satellite formation and mean drop sizes have been quantified. The pinch-off of drops has been found to be a reproducible process, with predictable interface topologies, satellite formation and main drop diameter at detachment. $d$. Satellite formation is found to increase, and $d$ is found to decrease, with increasing gas mass density and increasing gas velocity.

In order to predict the diameter of the largest drop after detachment, it is recommended to first predict the drop size in the absence of gas flow, $d_{\omega 0}$, using Eq. (2), and subsequently determine the Weber number $We_0 = \rho_l v^2 d_{\omega 0}/\sigma$. If $We_0$ is less than 5, a correlation for $d/d_{\omega 0}$ of the form $d/d_{\omega 0} = 1 – C We_0$ is to be used. If $We_0$ exceeds 5, surface tension forces are relatively small and a correlation of the form $d/d_{\omega 0} = 1 – C Re_{pl}$ is to be employed, where the Reynolds number is based on velocity $v$ and the plate thickness (3 mm in the present experiments).

The importance of gas velocity and the flow pattern downstream of a plate has been further investigated by analyzing the forces acting on a liquid filament that is connected to the plate. Its is shown that moments of strong gas-liquid interaction occur, that eventually lead up to detachment. If all the hydrodynamic action at these moments is assumed to be represented by an expression like $F_H = \frac{1}{2} A_f C_D \rho_l v^2$, rather high values (around 1.3) of the ‘drag force coefficient’ $C_D$ are found. This all goes to show that hydrodynamic interaction is significant, and that $Re_{pl}$ is indeed an appropriate parameter to correlate detachment data at relatively high gas velocities.

No comparison with literature results could be made since no forced convection experiments with break-up in a wake were found.

REFERENCES


NOMENCLATURE

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<th>Symbol</th>
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<td>$A_f$</td>
<td>Frontal area of drop</td>
<td>m²</td>
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<tr>
<td>$A_{foot}$</td>
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<td>$C$</td>
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Here $\rho_l$ denotes the mass density of the liquid.