DESIGN TOOL FOR A 6-DOF PLANAR MOTOR WITH MOVING PERMANENT MAGNETS AND STANDSTILL COILS

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Abstract—This paper describes a fast design tool, which can be used for the rapid modelling, evaluation and optimisation of a moving magnet planar motor. It can be used as an alternative to slow three-dimensional finite element simulations and can calculate forces and torques on the moving platform, which can be positioned in 6-DoF with respect to the stationary part. The force calculations are validated using finite element simulations.

Key words—analytical analysis, 6-DoF, planar motor.

1. INTRODUCTION

In many industrial apparatus, like pick-and-place machines, wafer scanners and inspection systems, objects are positioned and moved in a horizontal plane. Most multi-dimensional positioning stages in these systems have long strokes in the \( x \)- and \( y \)-directions and smaller (limited) strokes in one or more other directions. These stages are often constructed by stacking several 1-DoF linear drives. The disadvantage of such a concept is that the specification of the individual drives is determined rather by the weight of the propelled drives, than the weight of the object, propelled by the positioning stage. Besides, these drives cannot operate in vacuum because of the roller or air bearings.

The 6-DoF planar motor with long strokes in the \( x \)- and \( y \)-directions is an alternative to these drives. The concept of the earliest solutions is based on a motor with moving coils and a stationary permanent magnet array [1]. However, the cable slab assembled to the moving platform causes dynamic disturbances, and the necessity of water-cooling increases the weight of the moving platform. Therefore, a planar motor with the inverted structure, i.e. moving magnets and standstill coils, which can operate in deep vacuum, is investigated.

The use of three-dimensional finite element analysis for the evaluation of topologies, the design and optimisations of the planar motor structure is unfeasible because of the simulation time. For that reason, a design tool is developed, based on the analytical solutions of the magnetic field of a permanent magnet and numerical integration of the Lorentz force over the coil volume.

This tool calculates all forces and torques on the moving platform of the motor as functions of the position of the moving platform with respect to the stationary coils, as well as the self- and mutual inductances of the coils.

2. PLANAR MOTOR TOPOLOGY

Different planar motors with moving magnets can be found in literature [2], [3]. The disadvantage of these topologies is that the stroke of the planar motor is limited to a few centimetres. The topology in [4] has a stroke, which is only limited by the size of the coil array. A simplified topology, derived from this, has been used to test the tools. Fig.1 shows this topology, consisting of a Halbach magnet array and four coils. These coils form a two-dimensional two-phase winding. This set of coils cannot provide stable levitation of the magnet array but can produce a position independent force on the magnet array.

Assumed that the flux linkages of the different coils, \( \psi_{i,j} \), from the permanent magnets are equal to:

\[
\psi_{i,j} = \psi_0 e^{-\alpha z} \cos\left(\frac{\pi x}{\tau} - \frac{\pi}{2}\right) \cos\left(\frac{\pi y}{\tau} - \frac{\pi}{2}\right) \tag{1}
\]

where \( \psi_0 \) is the flux linkage at the surface of the magnet array, \( \alpha \) is a geometric constant related to the magnet size, \( \tau \) is the pole pitch of the magnet array, \( x \) and \( y \) are the coordinates of the position of the centre of the magnet array and \( z \) is the length of the gap between the magnets and the coils.

Figure 1: Planar motor concept
A DQ0-decomposition can be applied to the coils. In Fig. 1 the coil 0,0 is located at the D-axis. In this three-dimensional case there are two Q-axes, one is in the x-direction (coil 1,0) and another is in the y-direction (coil 0,1). Coil (1,1) is located at the 0-axis. The total current in the coils, \( I_{ij} \), can be written as a combination of the currents in the different axes: \( I_x \), \( I_y \), and \( I_z \). If the currents in the coils are equal to:

\[
I_{ij} = I_x \sin \left( \frac{\pi x}{\tau} - \frac{\pi i}{2} \right) \cos \left( \frac{\pi y}{\tau} - \frac{\pi j}{2} \right) \\
+ I_y \cos \left( \frac{\pi x}{\tau} - \frac{\pi i}{2} \right) \sin \left( \frac{\pi y}{\tau} - \frac{\pi j}{2} \right) \\
+ I_z \cos \left( \frac{\pi x}{\tau} - \frac{\pi i}{2} \right) \cos \left( \frac{\pi y}{\tau} - \frac{\pi j}{2} \right),
\]

then the force, generated in each direction (x-, y-, z-), is only proportional to a force constant in that direction, \( K \), and the current component in that direction (Lorentz principle):

\[
F_x = \sum_{i=0}^{n} \sum_{j=0}^{m} I_{ij} \frac{\partial \psi_{ij}}{\partial x} = K_i I_x, \\
F_y = \sum_{i=0}^{n} \sum_{j=0}^{m} I_{ij} \frac{\partial \psi_{ij}}{\partial y} = K_i I_y, \\
F_z = \sum_{i=0}^{n} \sum_{j=0}^{m} I_{ij} \frac{\partial \psi_{ij}}{\partial z} = K_i I_z.
\]

If the magnet array is above at least three of these coil groups, stable levitation is possible.

3. DESIGN TOOL

A design tool has been developed for fast analysis, optimisation and simulation of the planar motor structure. In a number of studies dedicated to planar motors, [2] and [5], the method of space harmonics’ field analysis has been used to analyse a planar motor. This method does not take the end-effects into account because it assumes the magnet array to be infinitely long. However, in this moving magnet planar motor topology a number of coils will be partly covered by the magnet array. Therefore, the design tool is based on the three-dimensional analytical solution of the magnetic field of a permanent magnet and numerical integration of the Lorentz force over the coil volume [6].

3.1. Magnetic field of a permanent magnet

The permanent magnet is modelled by two surface charges with opposite polarity [7]. These surface charges represent the sides of the magnet perpendicular to the magnetization direction. Fig. 2 shows the charged surfaces of a permanent magnet with the lengths 2a, 2b and 2c in x, y and z-directions, respectively. The magnetic surface charge, \( \sigma \), is equal to the remanence of the magnets, \( B_r \). The magnetization direction is in the positive z-direction. The relative permeability is assumed to be equal to 1 in and outside the permanent magnet.

If the coordinates of the centre of the permanent magnet, defined in global space, are \( (u, v, w) \), the magnetic flux density, \( B \), in a point in- or outside the permanent magnets with coordinates \( (x, y, z) \) in global space, can be calculated:

\[
B_x = \sum_{i=0}^{n} \sum_{j=0}^{m} (-1)^{i+j} \ln(R - T), \quad (6)
\]

\[
B_y = \sum_{i=0}^{n} \sum_{j=0}^{m} (-1)^{i+j} \ln(R - S), \quad (7)
\]

\[
B_z = \sum_{i=0}^{n} \sum_{j=0}^{m} (-1)^{i+j} \tan(\frac{\pi ST}{RU}), \quad (8)
\]

where ‘atan2’ is a four-quadrant arctangent-function and:

\[
R = \sqrt{S^2 + T^2 + U^2}, \quad (9)
\]

\[
S = (x - u) - (-1)^i a, \quad (10)
\]

\[
T = (y - v) - (-1)^j b, \quad (11)
\]

\[
U = (z - w) - (-1)^k c. \quad (12)
\]

The magnetic field caused by an array of permanent magnets can be calculated by adding the field contributions of all magnets in the array.

3.2. Force and torque calculation

The fastest way to calculate forces and torques in the planar motor topology is to use the Lorentz force calculations, as the estimation of the magnetic field of the coils is not necessary. To be able to simulate the behaviour of the planar motor in 6-DoF, both the coils and magnets are defined in their own coordinate systems. The coils are defined in the global coordinate system, the magnets are defined in a local coordinate system, which is attached to the centre point of mass of the magnets.

The force, \( F \), and torque, \( T \), in the local coordinate system of the moving platform is calculated, according:

\[
F = -\iiint_{c} \sigma^T J \times B dV, \quad (13)
\]

\[
T = -\iiint_{c} r \times \sigma^T J \times B dV, \quad (14)
\]

where, \( J \) is the current density in the coils, \( r \) is the vector from the mass centre point of the magnets to the coil volume element and \( \sigma^T \) is the transformation matrix from the global space of the coils to the local space of the magnets.

The transformation matrix is calculated from the position and orientation of the local coordinate system of the moving platform. If the position of the local coordinate system in global space is \( (p_x, p_y, p_z) \) and the rotation angles about the \( x-, y-, \) and \( z- \)axes are \( \phi, \theta, \varphi \), consequently, with respect to the global reference, a roll-pitch-yaw transformation can be applied [9]:

![Diagram of a magnetized magnet in z-direction](Image)
\[
\tau = \{T((x,y,z), (p_1, p_2, p_3)) R(y, \theta) R(z, \varphi)\}^{-1},
\]

where:

\[
T((x,y,z), (p_1, p_2, p_3)) = \begin{bmatrix}
1 & 0 & 0 & p_1 \\
0 & 1 & 0 & p_2 \\
0 & 0 & 1 & p_3 \\
0 & 0 & 0 & 1
\end{bmatrix},
\]

\[
R(x, \phi) = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos(\phi) & -\sin(\phi) & 0 \\
0 & \sin(\phi) & \cos(\phi) & 0 \\
0 & 0 & 0 & 1
\end{bmatrix},
\]

\[
R(y, \theta) = \begin{bmatrix}
\cos(\theta) & 0 & \sin(\theta) & 0 \\
0 & 1 & 0 & 0 \\
-\sin(\theta) & 0 & \cos(\theta) & 0 \\
0 & 0 & 0 & 1
\end{bmatrix},
\]

\[
R(z, \varphi) = \begin{bmatrix}
\cos(\varphi) & -\sin(\varphi) & 0 & 0 \\
\sin(\varphi) & \cos(\varphi) & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

The volume integrals for force and torque are solved numerically using an algorithm based on the Legendre-Gauss quadrature. The coil volume is divided into smaller volume elements, as shown in Fig. 3. Fig. 4 shows the cross-section of one of those elements with the current direction along the y-axis.

The surface integral over that cross-section is approximated in nine points with coordinates: \((0,0), (0, \pm \sqrt{3/5}h), (\pm \sqrt{3/5}h, 0), (\pm \sqrt{3/5}h, \pm \sqrt{3/5}h)\). The volume integral over the element is equal to:

\[
\iiint f(x,y,z)dx dy dz = l4h^2 \sum_{i=0}^{9} w_i f(x_i, y_i, z_i),
\]

where \(f\) is the integrated function. The weight factors for each point, \(w_i\), are given in Fig. 4. For non-square and large conductor sections, extra points are added in the cross-section, in order that the points are distributed evenly and the integration is accurate.

4. F INITE ELEMENT CALCULATION

The non-optimised planar motor structure in Fig. 1 has been used to compare the force calculation results of the CAD design tool, which has been implemented in Matlab R12, with finite element calculations in FLUX 3D. The height of the magnet array is 10 mm, the remanence of the magnets is 1.3 T and the height of the coils is 5 mm. The currents in the coils are changed with the position according to (2) with \(I_x = 500\) A and \(I_y = 0\) A. The distance between magnets and coils is 1 mm.

The magnets are moved over 90 electrical degrees in both the x- and y-directions within 16 steps. Fig. 1 shows the initial position, Fig. 5 shows the final position of the magnet array. Both pictures are drawn in the local coordinate system of the magnets.

Fig. 6 shows the three force components calculated by the designed Matlab CAD tool at 16 points. The calculation time is about 40 seconds per step. The figure shows that a rather position independent force can be generated on the magnets. Fig. 7 shows the error of the calculation with respect to the finite element calculation for \(F_x\) and \(F_z\). This calculation takes 45 minutes per step with 650,000 elements. The design tool calculates a force, which is 0.5% lower than the finite element simulation. In Fig. 8 the absolute error for \(F_y\) is shown because the finite element package cannot calculate forces smaller than \(10^{-3}\) N accurately. However, the error is less than 0.01 N.
The design tool proves to be a fast and accurate alternative for the finite element tools. The small error between the tool and the finite element calculation is probably related to the numerical integration algorithm, which is subject to further research. If realistic NdFeB magnets are used for the finite element calculations (B_r = 1.3 T, \mu_r = 1.033) the calculations of the design tool are correct for F_x and F_y, and 3.3% lower for F_z.

5. CONCLUSION

A design tool has been developed, which calculates forces and torques on the moving platform of a planar motor, which can be positioned in 6-DoF with respect to the stationary part. The tool is based on the numerical calculation of the Lorentz force and torque, using the analytical solution of the magnetic flux density of permanent magnets. This design tool is a fast and accurate alternative to finite element calculations and therefore, it is very useful for the design, evaluation and optimisation of planar motor topologies. In the future it will be adapted for dynamic simulation of planar motors.

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REFERENCES