Laser Polarization State Measurement in Heterodyne Interferometry

Precision Engineering section, Department of Mechanical Engineering,
Eindhoven University of Technology, Eindhoven, The Netherlands

Abstract
In heterodyne interferometry the accuracy is partly limited by non-linearity errors. These errors are caused by laser polarization state errors and errors of optical components, including alignment. In this paper we present two independent methods to measure the actual polarization state properties of the laser beam. Uncertainties and differences between the methods are discussed. Measurement of a commercially available laser source show ellipticity up to 1:170 in the E-fields and 0.3° deviation of orthogonality. This may cause a non-linearity calculated in a heterodyne interferometer of 0.6 nm.

Keywords:
interferometry, measurement, non-linearity errors

1 INTRODUCTION
As the industrial demands for higher accuracy in length measurements are continuously increasing, high accuracy laser interferometry is widely used now for accurate length measurement [1]. In Figure 1, the setup of a heterodyne interferometer is presented schematically. The laser source produces two polarized beams, with different frequencies, which are ideally linear polarized and orthogonal. In the interferometer, these two frequencies are separated according to their polarization state by the polarizing beam splitter. One beam serves as the reference beam and the other as the measurement beam.

A displacement of the corner cube in the measurement arm will result in a phase difference. From this phase difference the displacement is calculated. The displacement of the corner cube in the measurement arm is linearly proportional to the phase difference.

Ever since the development of the interferometer, error sources that limit the accuracy have been investigated [2]. Currently non-linearities are one of the limiting factors in high precision laser interferometry. Non-linearity errors are influenced by the following sources [3]:
- Elliptically polarized laser beams
- Non-orthogonality in the polarized laser beams
- Rotational errors in the alignment of laser and polarizing beam splitter
- Rotational errors in the alignment of the mixing polarizer
- Different transmission coefficients in the beam splitter

This non-linearity has a cyclic behavior with 1st and 2nd harmonics in distance measurements [4]. From experiments, it is known that the input beam is not perfect [5]. Therefore it is necessary to measure the ellipticity and non-orthogonality of the beam, before it enters the optics in the interferometer. These results are the input for the non-linearity calculation in a heterodyne interferometer [3]. This is an alternative for a direct measurement of non-linearities in an optical system, as e.g. shown in [6].

In this paper two independent methods are described to measure the ellipticity and the non-orthogonality of the laser beam. For the two methods a calibration and an error analysis is carried out, in order to give an uncertainty for the measured quantities.

Figure 1: Heterodyne interferometer, with frequency $f_1$ and $f_2$, PB the polarizing beam splitter, FC the fixed corner cube and MC the moveable corner cube.

Figure 2: Presentation of the laser polarization state
1.1 Definitions
In Figure 2, an imperfect heterodyne laser beam is presented, where the E-fields $E_1$ and $E_{01}$ have frequency $f_1$ and E-fields $E_2$ and $E_{02}$ have frequency $f_2$. The non-orthogonality is represented by angle $\varepsilon$. The phase difference between the main E-field vector $E_1$ and $E_2$ with their complementary vector is defined as $\pi/2$. This makes the beam elliptically polarized. In the described methods, the relative ratios $E_1/E_{01}$ and $E_2/E_{02}$ and the non-orthogonality $\varepsilon$ are measured.

2 METHOD 1: CARRIER FREQUENCY METHOD

The setup for this experiment is shown in Figure 3. The heterodyne laser emits generally two elliptically polarized frequencies, these are mixed with a circular polarized reference source. The use of a circular reference source with carrier frequency $f_3$ allows measurements of beatsignals through polarizing optics. The mixed beam propagates through a polarizer on the AC-detector, which is read out by a spectrum analyzer.

Because of the high ratios to be measured, a Glan-Thompson polarizer with an extinction ratio of 1:200 000 in intensity is used [7]. The rotation of the polarizer, see Figure 4, will give an intensity variation at the beat frequencies. The measured frequencies are difference frequencies between the circular polarized reference source and the heterodyne laser beam with magnitude $|f_1-f_3|$, $|f_2-f_3|$. The intensities of these beat frequencies are proportional to $E_1E_3$, $E_2E_3$, $E_{01}E_3$ and $E_{02}E_3$.

The frequency difference of the heterodyne laser will show two peaks on the spectrum analyzer. The frequency difference between these peaks is the frequency difference of the heterodyne laser. The relative ratios $P_1$ and $P_2$ are calculated by dividing the maximum intensities by the minimum intensities [8]:

$$R_1 = \frac{I_{ac,190}}{I_{ac,10}} = \frac{E_1 \cdot E_3}{E_{01} \cdot E_3} = \frac{E_1}{E_{01}} \quad (1)$$

$$R_2 = \frac{I_{ac,2x}}{I_{ac,290+\varepsilon}} = \frac{E_2 \cdot E_3}{E_{02} \cdot E_3} = \frac{E_2}{E_{02}} \quad (2)$$

Where $I_{ac,190}$ is the maximum intensity for a polarizer angle of 90° and $I_{ac,10}$ is the minimum intensity for a polarizer angle of 0°, both for difference frequency $|f_1-f_3|$. The maximum intensity at polarizer angle $\varepsilon$ is defined as $I_{ac,2x}$ and $I_{ac,290+\varepsilon}$ is the minimum intensity both for difference frequency $|f_2-f_3|$.

From the measurements the relative ratios and the ellipticities of the heterodyne laser beam are calculated. The use of a second laser gives a frequency shift in the beat frequency and an amplitude distortion, which results in a somewhat higher uncertainty. Alignment of the heterodyne beam on the circular beam is also critical because a maximum intensity is obtained for complete mixing of the two beams.

The circular polarized reference source is calibrated because with this method at two different polarizer angles the intensity is measured and an imperfect circular polarized beam will not give the same intensity at these two angles. This is realized by placing a polarizer in the circular polarized beam and the intensity is measured for the polarizer angle range used in the experiment described in this section.

Measurements are corrected for the errors in the circular reference source. The circular reference source, together with the non-polarising beamsplitter, used in our experiments has an intensity variation of 1.3%, without correction this results in a ratio error of about 9%.

Figure 3: Setup of measurement method 1.

Figure 4: Vector presentation of method 1.
2.1 Calibration

The calibration of this method is done by placing a precisely rotatable 1/4 waveplate [7] between the heterodyne laser and the non-polarizing beam splitter. With this optical device it is possible to make a defined elliptical polarized beam. It is hereby possible to create a ratio between infinity and 1 for circular polarized light because the phase difference between the main vector and its complementary is $\pi/4$. The ellipticity is measured at several angles of the waveplate, this is represented in Figure 6 on the horizontal axis. On the vertical axis the calibration results for the measurements method are shown. The difference between the two ratios occurs to the non-orthogonality of the polarization of the two frequencies. This calculated offset is for this laser $0.3^\circ$.

The measured ellipticity exactly matches the theoretically predicted, taking into account this angle and the mixing already present at 0$. The ellipticity is measured exactly at the points described in the following equations [8]:

$$I_{ac,0} = \sqrt{(\sin(x))^2 E_{01}^2 + (\cos(x))^2 E_{02}^2} \quad (4)$$

$$I_{ac,45^\circ + \epsilon} = \frac{1}{\sqrt{2}} \left[ E_{01}^2 E_{02}^2 + E_{01}^2 E_{02}^2 \sin(2\epsilon) + 4 E_{01} E_{02} \cos(2\epsilon) \right] \quad (5)$$

$$I_{ac,90^\circ + \epsilon} = \sqrt{(\sin(x))^2 E_1^2 + (\cos(x))^2 E_2^2} \quad (6)$$

In the experiments, the components $E_{01}$ and $E_{02}$ are again relatively small compared to the $E_1$ and $E_2$ components. The square of the small components is even smaller, therefore these are neglected. This results in the following simplified equations for the ratios:

$$2 \cdot P_1 = \frac{I_{ac,45^\circ + \epsilon}}{I_{ac,0}} = \frac{E_1}{E_{01}} \quad (7)$$

$$2 \cdot P_2 = \frac{I_{ac,45^\circ + \epsilon}}{I_{ac,90^\circ + \epsilon}} = \frac{E_2}{E_{02}} \quad (8)$$

These ratios are the same as in method 1 apart from a factor of 2, but in this method a simplification was made. This simplification results in an error of approximately 0.04% of the ratios. The frequency shift in this experiment is relatively small compared to the E1 and E2 components.

2.2 Results

With this method, the ellipticity and non-orthogonality of a commercially available laser is measured. The frequency of both lasers shift. This results in a shifting beat frequency, due to the modulation of both lasers. During a measurement the amplitude of the different beat frequencies at the specific polarizer angle is measured. From this data the maximum ratio is calculated. This laser has its minimum for frequency $f_1$ at a polarizer angle of $0^\circ$ and frequency $f_2$ has its minimum at $90.3^\circ$. Therefore the non-orthogonality of the beams for this laser is $0.3^\circ$.

3 METHOD 2: DIRECT BEAT MEASUREMENT

A second method was used to verify the results obtained from the first method, as well as to offer an easier setup. This method has a simpler set-up, but a disadvantage is that both frequencies must be present in different polarization states in the beam, so a beam which passed polarizing optics can not be measured. The heterodyne laser beam is directly analyzed with a polarizer, see Figure 8, the measured beat signals is $|f_1-f_2|$. Rotation of the polarizer gives a change in beat amplitude, which is read out with the spectrum analyzer. In this experiment there are three points of interest, namely the maximum beat amplitude and the two minimum beat amplitudes. From these values the ellipticity is calculated. The intensities at these points are described in the following equations [8]:

$$\alpha = 0^\circ$$

$$\alpha = 45^\circ + \epsilon$$

$$\alpha = 90^\circ + \epsilon$$

In the experiments, the components $E_{01}$ and $E_{02}$ are again relatively small compared to the $E_1$ and $E_2$ components. The square of the small components is even smaller, therefore these are neglected. This results in the following simplified equations for the ratios:

$$2 \cdot P_1 = \frac{I_{ac,45^\circ + \epsilon}}{I_{ac,0}} = \frac{E_1}{E_{01}} \quad (7)$$

$$2 \cdot P_2 = \frac{I_{ac,45^\circ + \epsilon}}{I_{ac,90^\circ + \epsilon}} = \frac{E_2}{E_{02}} \quad (8)$$

These ratios are the same as in method 1 apart from a factor of 2, but in this method a simplification was made. This simplification results in an error of approximately 0.04% of the ratios. The frequency shift in this experiment is relatively small compared to the E1 and E2 components.
3.1 Calibration

This method is calibrated in the same way as method 1. The phase difference introduced by the waveplate is \( \pi/4 \). For an angle of 0°, the slightly elliptical polarized beam is not affected, which is the same as for method 1. In this calibration experiment, it is possible to determine the non-orthogonality. This is the difference between the two ratio curves. In Figure 9, the offset between the curves is calculated (0.3°) and the curve of ratio \( E_2/E_02 \) is corrected accordingly.

3.2 Results

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Frequency in MHz</th>
<th>Mean ratio</th>
<th>Minimum ratio</th>
<th>Maximum ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1/E01</td>
<td>2,650</td>
<td>167</td>
<td>155</td>
<td>173</td>
</tr>
<tr>
<td>E2/E01</td>
<td>2,651</td>
<td>167</td>
<td>162</td>
<td>173</td>
</tr>
<tr>
<td>E1/E02</td>
<td>2,648</td>
<td>177</td>
<td>176</td>
<td>177</td>
</tr>
<tr>
<td>E2/E02</td>
<td>2,650</td>
<td>180</td>
<td>170</td>
<td>186</td>
</tr>
</tbody>
</table>

Table 1: Calculated ratios \( E_1/E_01 \) and \( E_2/E_02 \) for a commercially available laser, with method 2.

The measured laser is the same as in method 1, the two minimum amplitudes measured with this experiments have a polarizer angle difference of 90.3°, which gives a non-orthogonality of 0.3°. The mean ellipticity (or ratios) for the two polarization states is 167 for frequency 1 and 178 for frequency 2, see Table 1. The beat frequency is stable, therefore only at two beat frequencies the maximum ratios are calculated. This is contrary to the measurements in method 1 where the calculation is made at many frequencies. Using equation 14 from [3] the measured ratio results in a maximum non-linearity error of 0.6 nm, depending on the correlation between the non-orthogonalities which can not be measured in this way.

4 ERROR ANALYSIS

The error analysis has the same approach for both methods; the uncertainties are calculated according to the rules described in [9]. Major sources of uncertainty are the reproducibility and detector non-linearity. For the ratio, the calculation gives an uncertainty for method 2, which is two times smaller than for method 1. The uncertainty in the non-orthogonality depends mainly on the reproducibility and the resolution of the rotary encoder. The non-orthogonality is in method 1: 0.30° ± 0.05° and for method 2: 0.30° ± 0.05°.

5 CONCLUSIONS

The measurement of the ellipticity and non-orthogonality of a heterodyne laser beam was described and verified with measurement results. The measured commercially available laser has a non-orthogonality of 0.30° ± 0.05°. The measured ellipticity varies between 1:21000 and 1:31000 in intensity. This results in a maximum non-linearity error for a heterodyne interferometer of 0.6 nm [3].

With both methods it is possible to measure both quantities, but the difference is the uncertainty of the ellipticity. Method 1 has an uncertainty, which is two times that of method 2 due to the introduction of the circular polarized reference source and non-linearity of the AC-detector. Also, there is a significant difference in mean ratio between the two methods, which is not covered by their uncertainty. This is probably caused by back coupling of the mixed laser beam into the circular reference source and the non-linearity of the AC-detector. The measurement method 1 has to be used for measuring a heterodyne laser beam through polarizing optics. In other setups, method 2 is preferred.

6 ACKNOWLEDGEMENTS

We acknowledge Agilent Technologies B.V. and ASML The Netherlands B.V. for their support.

7 REFERENCES