Modelling of High-speed Milling for Prediction of Regenerative Chatter

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Abstract
High-speed milling is widely used to manufacture products. Examples of application areas are the aerospace industry and mould industry. Cost-related considerations place high demands on the material removal rate and surface generation rate. However, in this respect restrictions on the process parameters, determining these two rates, are imposed by the occurrence of regenerative chatter. Chatter is an undesired instability phenomenon, which causes both reduced product surface quality and tool wear. In this paper, the milling process is modelled, based on dedicated experiments on both the material behaviour of the workpiece material and the machine dynamics. Moreover, an efficient method for determining the chatter boundaries in the model is proposed and applied to the model in order to predict chatter boundaries in the process parameters, such as the spindle speed and depth of cut, which both influence the material removal rate and surface generation rate. Finally, experiments are performed to estimate these chatter boundaries in practice. Comparison of the modelled chatter boundaries with these experimental results confirm the validity of the model and the effectiveness of the stability analysis proposed.

1 Introduction

The milling process is used widely in many sectors of industry. Some examples are the fabrication of moulds and the aeroplane building industry, where large amounts of material are removed from a large structure. The milling process is most efficient if the material removal rate is as large as possible, while maintaining a high quality level. For a certain machine-tool-workpiece combination, the main factors that have influence on this removal rate are the spindle speed, the depth of cut (axial and radial) and the feed rate.

During the milling process, chatter can occur at certain combinations of axial depth of cut and spindle speed. This is an undesired phenomenon, since the surface of the workpiece becomes non-smooth as a result of heavy vibrations of the cutter. Moreover, the cutting tool and machine wear out rapidly and a lot of noise is produced when chatter occurs. Several physical mechanisms causing chatter can be distinguished [1]. Wierchigroch et al. [2; 3] and Grabec [4] showed that friction between the tool and workpiece can cause chatter. Another cause for chatter is due to the thermodynamics of the cutting process [1; 5]. In [6], the phenomenon of mode-coupling is discussed as a cause of chatter. Chatter due to these physical mechanisms is often called primary chatter. Secondary chatter is caused by the regeneration of waviness of the surface of the workpiece. This regenerative chatter is considered to be one of the most important causes of instability in the cutting process. This type of chatter will be considered in this paper.
Several studies have been made since the late 1950’s regarding regenerative chatter, by e.g. Tobias et al. [7; 8], Tlusty et al. [6], Merrit [9] and Altintas [10]. It was shown that the border between a stable cut (i.e. no chatter) and an unstable cut (i.e. with chatter) can be visualised in terms of the axial depth of cut as a function of the spindle speed. This results in a Stability Lobes Diagram (SLD). Using these diagrams it is possible to find the specific combination of machining parameters, which results in the maximum chatter-free material removal rate.

In order to predict the stability boundaries related to chatter, an accurate dynamic model for the milling process is needed. Therefore, in section 2, the modelling of the milling process is described. This model will be used to find the stability limit of the milling process. The model consists of a part which describes the machine-tool interaction and a part which describes the machine dynamics. For both model parts, dedicated experiments are performed to support such modelling. The resulting model is described by a set of linear delay differential equations. In section 3, the D-partitioning method [11; 12] is used to analyse the stability boundary (in terms of spindle speed and depth of cut) of the equilibrium point of those differential equations, which represents a stable cut. The results of such a stability analysis for the model, constructed in section 2, is compared to experimentally determined stability boundaries in section 4 for validation purposes. Finally in section 5, conclusions are presented.

2 Modelling

In the milling process, material is removed from a workpiece by a rotating tool, which has one or more cutting teeth. While the tool rotates, it translates in the feed direction at a certain speed. A schematic representation is shown in figure 1. The parameters shown are the spindle speed \( \Omega \), the feed per tooth \( f_z \), the axial depth of cut \( a_p \) and the radial depth of cut \( a_e \). As a result of the feed rate and the rotating cutter, the chip thickness is not constant, but periodic.

![Figure 1: Schematic representation of the milling process.](image-url)
The milling process is an interaction between a milling machine and a workpiece. This interaction is shown in a block diagram in figure 2. A certain displacement of the cutter, related to the feed, is dictated to the spindle of the machine. The static chip thickness ($h_{stat}$) is a result of this displacement. While achieving this displacement the cutter-workpiece interaction results in a force $F$ acting on the cutter, if the cutter is in cut. This force on the cutter causes a displacement of the cutter. This fact reflects the main difference between the milling process and e.g. the sawing process. Namely, in the sawing process a certain force is dictated to the tool, whereas in the milling process a certain displacement is dictated to the tool.

The mechanism described above, results in vibrations of the tool. This causes a wavy surface of the workpiece. The next cutting tooth encounters this wavy surface and generates its own wavy surface. The chip thickness is, therefore, the sum of the static and dynamic chip thickness. The static chip thickness is a function of the feed rate $f_z$ and the rotation angle of the cutter $\phi$: $h_{stat}(t) = f_z \sin(\phi(t))$, which expresses that the chip thickness is measured in radial direction, see figure 1. Here, the tooth path is assumed to be a circular arc. The dynamic chip thickness is a result of the vibration of the tooth in cut in $x$ and $y$-direction (see figure 1) and the vibration of the previous tooth in cut in these directions. These displacement coordinates $x$ and $y$ are gathered in a displacement column $\mathbf{x} = [x, y]^T$. The difference between the actual displacement of the cutter and the dictated displacement of the cutter causes a dynamic chip thickness $h_{dyn}(t)$. This dynamic chip thickness is defined by: $h_{dyn}(t) = \sin(\phi(t)) \cos(\phi(t)) \left( x(t) - x(t-\tau) \right)$. So, it can be seen that the force on the cutter is not only dependent on the current cutter displacement, but also on the displacement of the previous tooth. Therefore, the tooth passing period determines the delay time $\tau$ in figure 2. When the spindle speed $\Omega$ is given in rpm and $z$ is the number of teeth of the cutter, the tooth passing period is defined as

$$\tau = \frac{60}{\pi \Omega}. \tag{1}$$

Appropriate modelling of the cutting process and machine dynamics (see figure 2) is discussed in sections 2.1 and 2.2, respectively. The material model will be constructed on the basis of dedicated experiments in section 2.1. Note that in figure 1, the dynamics of the spindle system is modelled by a mass-spring-damper system. However, of course the dynamics, in both $x$ and $y$-directions, may very well be more complex and in section 2.2 a multi-modal model will be proposed, for both directions, based on experiments.

### 2.1 Material model

As was outlined in the previous section 2, the milling process is an interaction between the cutting process and the machine dynamics. In this section, we focus on the cutting process. In literature, various models have been proposed to model the radial and tangential cutting forces $F_r$ and $F_t$ as a function of the cutting parameters, such as the depth of cut and the feed rate. For some of these models, the forces acting on a single cutting tooth are shown in table 1. The parameter $K_{c1,1}$ in the model by Kienzle, is defined as the force that is needed to cut a chip of 1-by-1 mm.
that for a fixed spindle speed $\Omega$ chatter may occur for one specific feed rate. Use this exponential model, since experiments (such as described later on in this section) showed with the equations which describe the force in table 1 by a function $F_{t}(t)$ and tangential forces are zero when the tooth is not in cut. This can be modelled by multiplying this model with equation (4) gives for a single tooth:

$$h_{j}(\phi_{j}(t)) = f_{z} \sin \phi_{j}(t) + (x(t) - x(t - \tau)) \sin \phi_{j}(t) + (y(t) - y(t - \tau)) \cos \phi_{j}(t),$$

with

$$\phi_{j}(t) = \Omega t + j \vartheta, \quad j = 0, 1, \ldots, z-1,$$

where $\Omega$ is expressed in rad/s and $\vartheta = \frac{2\pi}{z}$ is the angle between two subsequent teeth. The radial and tangential forces are zero when the tooth is not in cut. This can be modelled by multiplying the equations which describe the force of table 1 by a function $g_{j}(\phi_{j}(t))$, that describes whether a tooth is in or out of cut. The tooth is in cut if $\phi_{s} \leq \phi_{j} \leq \phi_{e}$, where $\phi_{s}$ and $\phi_{e}$ are the start and exit angles, respectively. This function is given by

$$g_{j}(\phi_{j}(t)) = \frac{1}{2} (1 + \text{sign} (\sin (\phi_{j}(t) - \psi) - p)) = \begin{cases} 1, & \phi_{s} \leq \phi_{j}(t) \leq \phi_{e}, \\ 0, & \text{else} \end{cases},$$

with

$$\tan \psi = \frac{\sin \phi_{s} - \sin \phi_{e}}{\cos \phi_{s} - \cos \phi_{e}}, \quad p = \sin (\phi_{s} - \psi).$$

In table 1, the material model used by Stépán et al. [15] is shown. From now on, we will use this exponential model, since experiments (such as described later on in this section) showed that for a fixed spindle speed $\Omega$ chatter may occur for one specific feed rate $f_{z}$ where it may not for another feed rate. At low feed rates (0.08 or 0.12 mm/tooth) chatter occurred at some spindle speeds, while for higher feed rated, the chatter did not occur at the same spindle speed. Such feed rate dependency is is not modelled by the linear model of Altintas [14; 10], whereas the exponential model takes a feed rate dependency into account. As will be shown the estimated stability limit indeed increases if the feed rate increases when the exponential model is used. This behaviour was also found empirically when performing the cutting tests described in the next section. Multiplying this model with equation (4) gives for a single tooth:

$$F_{t_{j}}(t) = K_{t} a_{p} h_{j}(t)^{x_{r}} g_{j}(\phi_{j}(t))$$

$$F_{r_{j}}(t) = K_{r} a_{p} h_{j}(t)^{x_{r}} g_{j}(\phi_{j}(t)),$$

where $0 < x_{r} < 1$. Using (2), (4) and (6) an expression for the cutting forces can be derived:

$$F(t) = \begin{bmatrix} F_{t}(t) \\ F_{y}(t) \end{bmatrix} = a_{p} \sum_{j=0}^{z-1} g_{j}(\phi_{j}(t)) (f_{z} \sin \phi_{j}(t) + x(t, t - \tau) \sin \phi_{j}(t) + y(t, t - \tau) \cos \phi_{j}(t))^{x_{r}}$$

$$\begin{bmatrix} -K_{t} \cos \phi_{j}(t) - K_{r} \sin \phi_{j}(t) \\ K_{t} \sin \phi_{j}(t) - K_{r} \cos \phi_{j}(t) \end{bmatrix},$$

$$\begin{array}{|c|c|}
\hline
\text{Author} & \text{Model} \\
\hline
\text{Kienzle [13]} & F_{t} = a_{p} K_{c1} h \hat{h}^{1-m} \\
\text{Altintas et al. [14; 10]} & F_{t} = a_{p} K_{tc} h + a_{p} K_{te}, \quad F_{r} = a_{p} K_{rc} h + a_{p} K_{re} \\
\text{Stépán et al. [15; 16]} & F_{t} = a_{p} K_{c} h^{x_{r}}, \quad F_{r} = a_{p} K_{r} h^{x_{r}} \\
\hline
\end{array}$$

As was stated before, the static chip thickness is approximated by $h_{\text{stat}} = f_{z} \sin \phi_{j}$, where $\phi_{j}$ is the rotation angle of tooth $j$. The dynamic chip thickness is assumed to be the difference between the vibration $x$ and $y$ of the current tooth at rotation $\phi_{j}(t)$ and the vibration $x$ and $y$ of the previous tooth at $(t - \tau)$, when $\phi_{j}(t) = \phi_{j-1}(t - \tau)$. Consequently, the chip thickness $h$ encountered by tooth $j$ can be described as a function of rotation angle $\phi_{j}$ by $(h = h_{\text{stat}} + h_{\text{dyn}})$

$$h_{j}(\phi_{j}(t)) = f_{z} \sin \phi_{j}(t) + (x(t) - x(t - \tau)) \sin \phi_{j}(t) + (y(t) - y(t - \tau)) \cos \phi_{j}(t),$$

$$\begin{array}{|l|}
\hline
\text{Author} & \text{Model} \\
\hline
\text{Kienzle [13]} & F_{t} = a_{p} K_{c1} h \hat{h}^{1-m} \\
\text{Altintas et al. [14; 10]} & F_{t} = a_{p} K_{tc} h + a_{p} K_{te}, \quad F_{r} = a_{p} K_{rc} h + a_{p} K_{re} \\
\text{Stépán et al. [15; 16]} & F_{t} = a_{p} K_{c} h^{x_{r}}, \quad F_{r} = a_{p} K_{r} h^{x_{r}} \\
\hline
\end{array}$$

$\begin{array}{|l|}
\hline
\text{Year} \\
\hline
1950's \\
1995, 2000 \\
2000, 2001 \\
\hline
\end{array}$
with \( \mathbf{q}(t,t-\tau) = \mathbf{q}(t) - \mathbf{q}(t-\tau) \). This equation can also be linearised around \( \mathbf{q} = \mathbf{0} \), which corresponds to a stable cut without chatter, resulting in \( \mathbf{F} = \mathbf{F}(\mathbf{q} = \mathbf{0}) + \Delta \mathbf{F} \). The linearised force \( \Delta \mathbf{F} \) can then be written as:

\[
\Delta \mathbf{F} = \mathbf{a}_p \mathbf{k}(t) \mathbf{q}(t, t - \tau),
\]

with the matrix \( \mathbf{k}(t) \) defined by

\[
\mathbf{k}(t) = \begin{bmatrix}
k_{xx} & k_{xy} \\
k_{yx} & k_{yy}
\end{bmatrix} = \sum_{j=0}^{z-1} \int_z^{x_F} g_j (\phi_j(t)) \sin^{x_F} \phi_j(t)
\]

\[
- (K_t \cos \phi_j(t) + K_r \sin \phi_j(t)) \quad \sin^{-1} \phi_j(t) \cos \phi_j(t) \quad (K_t \cos \phi_j(t) + K_r \sin \phi_j(t))
\]

\[
(\sin^{x_F} \phi_j(t) \cos \phi_j(t) \quad - \sin^{-1} \phi_j(t) \cos \phi_j(t) \quad K_t \sin \phi_j(t) - K_r \cos \phi_j(t))
\]

(9)

If the spindle is modelled as a two-dimensional linear mass-spring-damper system, the milling process is described as

\[
M \ddot{\mathbf{x}}(t) + \mathbf{B} \dot{\mathbf{x}}(t) + \mathbf{C} \mathbf{x}(t) = \mathbf{a}_p \mathbf{k}(t) \mathbf{q}(t, t - \tau),
\]

with \( M, \mathbf{B} \) and \( \mathbf{C} \) the mass, damping and stiffness matrices, respectively. It should be noted that, in section 2.2, a higher-order model for the machine dynamics is constructed.

In order to validate the model described above and to find the values of the material parameters \( K_t, K_r \) and \( x_F \) of the model, experiments are performed. A schematic representation of the experimental setup is shown in figure 3. An aluminium 6082 (DIN 3.2315.71) workpiece is mounted on a dynamometer type Kistler 9255. Using this dynamometer and a charge amplifier, the forces in \( x, y \) and \( z \) direction, can be measured. The dynamometer is placed on the machine bed in such a way that \( x \) is the feed direction, \( y \) is the normal direction and \( z \) the axial direction. At full immersion, a cut is made and the forces are measured. The cuts have been made at TNO Industrial Technology using a Mikron HSM 700 milling machine and a Kelch shrink-fit toolholder. The tool used is a 2-teeth, 10 mm diameter Jabro Tools JH420 cutter. The spindle speed has been varied from 5000 rpm to 40000 rpm with increments of 5000 rpm. At each spindle speed the feed rate has been varied from 0.08 mm/tooth to 0.24 mm/tooth with increments of 0.04 mm/tooth. Two cuts have been made at each combination of spindle speed and feed rate. The sampling frequency of the measurements is 30 kHz for spindle speeds below 35000 rpm and 50 kHz at a spindle speed of 35000 and 40000 rpm.

![Figure 3: Schematic representation of the setup for the cutting experiments.](image-url)
For an experiment performed at 20000 rpm, a fit of the exponential model using the mean cutting forces, \( \bar{F}_x = \int_0^\tau F_x(t)dt \), \( \bar{F}_y = \int_0^\tau F_y(t)dt \), is performed taking all measurements at that spindle speed into account. The result is shown in figure 4. The material parameters of this exponential model can be found in table 2. Corresponding measured forces and the modelled forces, at a feed rate \( f_z = 0.16 \text{ mm/tooth} \) are shown in figure 5. If the spindle speed is changed, also the values for the material parameters change to certain extent. In figure 6, the material parameters of the exponential model are shown as a function of spindle speed.

2.2 Machine model

The second part of the block diagram of figure 2 represents the modelling of the machine dynamics, i.e. the tool, toolholder and spindle. In this section, the machine dynamics will be modelled based on dedicated experiments.

In literature [17; 18; 19], the machine system is assumed to be a 1DOF or 2DOF linear second-order (mass-spring-damper) system. However, experiments will show that a higher-order model is necessary to describe the machine dynamics. The dynamics can be described by the transfer function matrix between cutting forces \( \mathbf{F} = [F_x \ F_y]^T \) and displacements of the cutter \( \mathbf{x} = [x \ y]^T \):

\[
\mathbf{X}(s) = H(s)\mathbf{F}(s) = \begin{bmatrix} H_{11}(s) & H_{12}(s) \\ H_{21}(s) & H_{22}(s) \end{bmatrix} \mathbf{F}(s),
\]

where \( \mathbf{X}(s) = \mathcal{L}(\mathbf{z}(t)) \) and \( \mathbf{F}(s) = \mathcal{L}(\mathbf{F}(t)) \) with \( \mathcal{L}(\cdot) \) the Laplace operator. Each entry of this transfer function matrix can be modelled using the following model form:

\[
H_{ij}(s) = \frac{b_m s^m + b_{m-1}s^{m-1} + \ldots + b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0},
\]

Table 2: Material parameters for cutting at 20000 rpm.

<p>| | | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>( K_t )</td>
<td>473.7 N/mm(^{1+x_F} )</td>
<td></td>
</tr>
<tr>
<td>( K_r )</td>
<td>127.5 N/mm(^{1+x_F} )</td>
<td></td>
</tr>
<tr>
<td>( x_F )</td>
<td>0.78</td>
<td></td>
</tr>
</tbody>
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\( 6 \)
where the coefficients $a$ and $b$ differ for different $i$ and $j$. It will be assumed that $H_{ij} = 0$ for $i \neq j$. In other words, the dynamics in $x$ and $y$-direction is decoupled. These transfer functions describe the dynamics of the cutter (mill), due to flexibility of the mill, and the dynamics of the spindle and toolholder, from now on called the spindle dynamics. Firstly, experiments are performed to measure the spindle dynamics at different spindle speeds. Secondly, experiments are performed, at $\Omega = 0$, to measure the dynamics of the mill. Let us now discuss these experiments performed to retrieve $H_{11}$ and $H_{22}$.

In order to measure the dynamic behaviour of the spindle system, consisting of the tool, toolholder and spindle, and the influence of the spindle speed on this dynamic behaviour, impulse tests are performed. Using these tests it is possible to find a frequency response function between the force applied to the tool and the displacement, which provides information on the dynamical behaviour of the spindle system. These impulse tests are performed at various spindle speeds, in order to study the spindle speed dependency of the machine dynamics. A schematic representation of the setup is shown in figure 7. A 50 mm long 10 mm diameter carbide cylinder is mounted in a Kelch shrink-fit toolholder and used on a Mikron HSM 700 milling machine. Since the spindle is rotating while being hit by the impulse hammer, a mill can not be used. The first natural frequency of the cylinder is approximately 4 kHz and the dynamics of toolholder and spindle is related to a lower frequency range. Therefore, the dynamics of the toolholder and spindle can be distinguished from the dynamics of the cylinder and identified separately.

The displacement of the cylinder in $x$ or $y$ direction is measured with an LMI Laser Twin Sensor (LTS 15/3). An impulse force hammer is used to hit the cylinder and to measure the force applied to it. A Siglab dynamic signal analyser is used for data acquisition purposes. The spindle speed has been varied from 0 to 25000 rpm, with increments of 5000 rpm. At each spindle
speed, 20 impacts have been performed in both $x$ and $y$ direction, while the laser has been placed opposite to the place of impact.

In figure 8, the absolute value of the measured frequency response function $H_{11}(i\omega)$ is depicted at different spindle speeds (mean of 20 measurements). Statistical significance tests showed that the differences between these frequency response functions at different spindle speeds are significant, especially in the frequency range $[750, 1750]$ Hz, in which the most important resonances related to the spindle dynamics are situated. Since the natural frequency of the cylinder lies in the order $4$ kHz and the measured natural frequencies are much lower, the latter frequencies are not the natural frequencies of the cylinder but are related to the toolholder and spindle. Identical experiments are performed to measure $H_{22}(i\omega)$.

Next, the same type of experiments are performed on a non-rotating mill in order to capture the dynamics of the real cutter tool. Since the flexibility modes of the tool do not depend on the spindle speed, only measurements at 0 rpm are performed. The total dynamics of the toolholder and spindle and the mill can now be constructed by superposition of the individual dynamics of the toolholder and spindle on the one hand and the dynamics of the mill on the other hand, see figure 9. In doing so, up to $3000$ Hz the spindle dynamics is taken into account and above $3000$ Hz the mill dynamics is taken into account.

Figure 7: Schematic representation of the setup for the impulse tests.

Figure 8: $|H_{11}|$ at different spindle speeds. Dotted line: 95% confidence interval at 0 rpm.
Hz the mill dynamics is dominant.

The total machine dynamics, see figure 9, can now be modelled using (11) and (12). In order to obtain the parameters of this model, the distance between the measured and modelled frequency response functions in the complex plane is minimised using an optimisation routine and a least squares type object function. In figure 9, the resulting modelled frequency response function is compared to the measured frequency response function, for \( \Omega = 0 \). When performing this identification step, the highest level of importance was assigned to the highest resonance peaks in the absolute value of the experimental frequency response function, because these resonances are dominant in the stability analysis, as will be shown in section 4. Therefore, the mismatch in the frequency range of [500 1000] Hz is of no concern for the result of the stability analysis. Of course, the same parametric identification procedure was performed for other spindle speeds, resulting in different dynamic models for different spindle speeds.

### 2.3 The total milling model

The models for the material behaviour and the dynamics of tool, toolholder and spindle can be combined to form a model for the milling process using the block diagram of the milling process as depicted in figure 2. This leads to the following description of the milling process in the Laplace domain:

\[
\begin{align*}
H_\text{F}^{-1}(s)X(s) &= a_p \overline{K(s) \ast \left(1 - e^{-sT}\right)} X(s),
\end{align*}
\]

where \( H_\text{F}^{-1}(s) \) is defined by (11) and (12), the matrix \( \overline{K(s)} = \mathcal{L}(\bar{k}(t)) \) is the Laplace transform of the matrix \( \bar{k}(t) \) related to the material properties, and \( \ast \) denotes the convolution operator.

It should be noted that (13) represents the Laplace transform of a set of linear non-autonomous delay differential equations, where the non-autonomous nature is due to the explicit time-dependency of \( \bar{k}(t) \). For the remainder of this paper, \( \bar{k}(t) \) will be approximated by its zero-th order Fourier
approximation:
\[ k = \frac{1}{\tau} \int_0^\tau k(t) \, dt, \]  
(14)

which, since \( k(t) \) is only non-zero if a tooth is actually in cut, equals
\[ k = \frac{1}{\vartheta} \int_{\phi_0}^{\phi_0 + \vartheta} k(\phi) \, d\phi, \]  
(15)

where \( \vartheta \) is the angle between two subsequent teeth, \( \phi = \Omega t \) and \( k(t) \) is defined by (9) for the linearised exponential material model. This approximation is quite common in literature and dramatically simplifies the stability analysis since it transforms the model into an autonomous model [10]. Consequently, the convolution operation in (13) changes to a normal multiplication:
\[ H^{-1}F(s)X(s) = a_p K \left( 1 - e^{-s\tau} \right) \]  
(16)

where \( K \) is a constant matrix.

3 Stability analysis of the milling system

In this section, the model of the milling system proposed in the previous section (see (13)) will be used for the purpose of stability analysis. The linear autonomous delay differential equation describing the dynamics of (16) in the time-domain exhibits one unique equilibrium point: \( x = 0 \), which corresponds to the desired no-chatter situation. Therefore, the stability of this equilibrium point corresponds to the stability of the milling process and instability of the equilibrium point corresponds to a response with chatter.

Here, the method of D-partition [12] will be used to assess the stability of this equilibrium point. This method was used by e.g. Stépán [11; 20] to investigate the stability of the milling process using a single-degree-of-freedom, single-mode milling model. Note that, for a given spindle speed \( \Omega \), the stability of the equilibrium point depends on the axial depth of cut \( a_p \). So, the stability analysis will aim at finding the critical value for \( a_p \), at a given spindle speed \( \Omega \), which forms the stability boundary, allowing for the construction of so-called stability lobes diagrams.

3.1 Method of D-partition

The method of D-partition uses the criterion that an equilibrium point of a system, described by a linear, autonomous delay differential equation, is asymptotically stable if and only if all the roots of its characteristic equation lie in the open left-half complex plane. It should be noted that a delay-differential equation has an infinite number of poles. The characteristic equation corresponding to (16) is given by
\[ \det \left( H^{-1}F(s) - a_p K \left( 1 - e^{-s\tau} \right) \right) = 0. \]  
(17)

A certain choice for the systems parameters (for example \( a_p \)) determines the number of poles in the open left-half complex plane. The parameter space can be divided into domains \( D(k, n - k) \), \( 0 \leq k \leq n \) which contains all the points with poles with \( k \) negative real parts and \( n - k \) positive real parts. This is called D-partitioning. The domain of asymptotic stability is the domain \( D(n, 0) \).

An increase of the number of roots with positive real parts can only occur if a certain pole crosses the imaginary axis from the left to the right. This corresponds with the situation that a certain point in parameter space moves from the domain \( D(k, n - k) \) to \( D(k - 1, n - (k - 1)) \). Therefore, the borders of the \( D \)-partitions are the map of the imaginary axis \( s = i\omega, \) with \( -\infty < \omega < +\infty \) on the parameter space.
Let us introduce a new complex variable \( S = s\tau \) and use this to rewrite the characteristic equation (17) to
\[
\det \left( H^{-1} \frac{S}{\tau} - a_p K (1 - e^{-S}) \right) = 0. \tag{18}
\]
Now, we will not use this equation to determine the poles of the system for given parameters, but we will determine the values for the parameter \( a_p \) for which at least one pole lies on the imaginary axis \((s = i\omega)\). Using the fact that \( H_{x,F}(s) \) is given by (11), with \( H_{12} = H_{21} = 0 \), and choosing \( S = i\omega^* \), with \( \omega^* = \omega\tau \), which corresponds to \( s = i\omega \), (18) transforms to
\[
a_0 a_p^2 + a_1 a_p + 1 = 0, \tag{19}
\]
with
\[
a_0 = (1 - \cos \omega^* + i \sin \omega^*)^2 H_{11}(\omega^*) H_{22}(\omega^*)(k_{xx}k_{yy} - k_{xy}k_{yx}), \tag{20}
\]
\[
a_1 = -(1 - \cos \omega^* + i \sin \omega^*)(k_{xx}H_{11}(\omega^*) + k_{yy}H_{22}(\omega^*)). \tag{21}
\]
The axial depth of cut as a function of \( \omega^* \), \( a_p(\omega^*) \), can then be found by
\[
a_p(\omega^*) = \frac{a_1 \pm \sqrt{a_1^2 - 4a_0}}{2a_0}. \tag{22}
\]
The critical axial depth of cut (with respect to stability) is defined as the depth of cut \( a_p \) in the parameter set defined by \( \{ a_p(\omega^*) : \Im a_p(\omega^*) = 0 \land \Re a_p(\omega^*) > 0 \} \), for which \( |\Re a_p(\omega^*)| \) has its minimum value, since for \( a_p = 0 \) all poles are in the open left-half complex plane when all the modes of the machine dynamics are damped, which is always the case in practice. The value for \( \omega^* \) for which this occurs, is the dimensionless chatter frequency \( \omega^* = \omega_c^* \). The real chatter frequency that corresponds to dimensionless chatter frequency is \( \omega_c = \frac{\omega_c^*}{\tau} \). In summary, the following steps that need to be taken in order to use the method of D-partition to find the chatter boundary in terms of \( a_p \) (for a specific value of the spindle speed):

1. Choose a certain spindle speed \( \Omega \), and calculate the delay factor \( \tau \) by equation (1);
2. Choose a domain for \( \omega^* \);
3. In the characteristic equation (18), substitute \( S = i\omega^* \);
4. Solve equation (19) for \( a_p \). Now, \( a_p(\omega^*) \) is known, but \( a_{p,\text{crit}} \) still has to be found;
5. \( \omega^* = \omega_c^* \) if \( \Im a_p(\omega_c^*) = 0 \), \( \Re a_p(\omega_c^*) > 0 \) and \( |\Re a_p(\omega_c^*)| \) has its minimum value. By scanning the positive real axis, it is the point where a D-curve crosses the real axis for the first time. This is shown in figure 10. In this figure, the dotted lines represent the boundaries between the domains \( D(k, n - k) \);
6. Calculate the chatter frequency \( \omega_c = \frac{\omega_c^*}{\tau} \);
7. Repeat all steps for different spindle speeds.

In the second step of this procedure, a choice for the domain of \( \omega^* \) should be made, such that \( \omega_c^* \) lies in this domain. For a milling model, in which the lowest angular eigenfrequency of the machine dynamics is denoted by \( \omega_l \) and the highest eigenfrequency by \( \omega_h \), a suitable choice for this domain is: \( 0.5\omega_l \tau < \omega^* < 1.5\omega_h \tau \).
4 Results

The model of the milling process, constructed in the previous section, will be used to pursue a stability analysis. Stability lobe diagrams can be computed using the analysis method illuminated in section 3. In order to study the accuracy of the model, the stability boundaries obtained in this way need to be validated with the stability border determined in practice. The model used in this analysis uses the material parameters given in table 2. Moreover, the spindle speed dependency of the machine dynamics is taken into account in the following way. At discrete values for the spindle speed $\Omega = 0, 5000, 10000, 15000, 20000, 25000$ rpm, models for the machine dynamics were constructed. When the stability analysis is performed for an arbitrary spindle speed (not at one of the discrete values for which the models were constructed) in order to find the critical value for the depth of cut $a_p$ at that spindle speed, the dynamic model corresponding to the closest spindle speed is used. In this way, the spindle speed dependency of the machine dynamics is accounted for in the stability analysis.

For validation purposes, the stability boundary, in terms of $a_p$ as a function of $\Omega$, has to be determined experimentally. Hereto, a series of cuts have been made at the Mikron HSM 700 milling machine using an aluminium 6082 (DIN 3.2315.71) workpiece, a Kelch shrink-fit toolholder and a 2-teeth, 10 mm diameter Jabro Tools JH420 cutter. For every spindle speed investigated, an initial depth of cut is chosen such that no chatter occurs. Whether chatter is occurring or not is detected using the experimental setup depicted in figure 11 and the software program Harmonizer. This program scans the sound of the cutting process measured using the microphone. If the energy of
the measured sound signal at a certain frequency exceeds a certain threshold, the cut is marked as exhibiting chatter. This threshold can be set automatically by Harmonizer, but it can also be set manually. Using Harmonizer, also the chatter frequency is measured. The frequency at which the energy level exceeds the threshold level is marked as the chatter frequency.

In figure 12, the experimental results are compared with the modelled stability lobes. In figure 13, the measured chatter frequencies are compared with the modelled chatter frequencies and the natural frequencies of the spindle and tool. It can be concluded that the dynamics of the non-rotating mill highly influences the stability lobes, since the chatter frequency is always close to a resonance frequency of the mill. Clearly, the prediction of the stability lobes diagram is good. Moreover, the modelled chatter frequencies also resemble the measured chatter frequencies very well. The natural frequencies of the spindle are much lower. This indicates that the dynamics of the spindle, which is spindle speed dependent, is less important than the dynamics of the mill, which is spindle speed independent. It can be concluded that for this specific, rather slender, tool this speed dependency of spindle dynamics does not influence the stability lobes diagram to a great extent. However, when a shorter, thicker mill would have been used (resulting in extremely high resonance frequencies for the mill dynamics), the spindle dynamics may become dominant. In such a case, the spindle speed dependency of spindle dynamics is important to be included in the stability analysis.

5 Conclusions

In this paper, a dynamic model for the milling process has been constructed based on dedicated experiments. The model comprises a material model and a model for the machine dynamics. For the material model, an exponential model (of the cutter forces in terms of the chip thickness) is used to account for feed rate dependencies in the stability lobes diagram. A linear, higher-order model for the machine dynamics has been constructed, including dynamic modes of the tool and the spindle and toolholder. The spindle dynamics appeared to be spindle speed dependent. The total model is described by a linear, higher-order delay differential equation.

The method of D-partition is used to analyse the stability of the unique equilibrium point of
this differential equation, corresponding to a process situation without chatter. In this way, the stability lobes diagram is constructed. An advantage of this method over the method used by [10] is that while using the method of D-partition the critical depth of cut can be found for a specific spindle speed, whereas while using the method of Altintas the critical depth of cut for a specific chatter frequency, which is an unknown response variable, is found. This allows us, in case of the method of D-partition, to account for spindle speed dependencies of the milling model, whereas this is not possible in the method of [10].

It was shown that the constructed model provides an accurate prediction for the stability lobes diagram when it is compared to an experimentally determined stability lobes diagram.

References


