Stability of viscoelastic flows: Issues of constitutive modeling
Arjen Bogaards, Anne Grillet, Gerrit Peters and Frank Baaijens
Eindhoven University of Technology,
Faculty of Mechanical Engineering,
Section Materials Technology / Dutch Polymer Institute,
P.O. Box 513, 5600 MB Eindhoven, the Netherlands

Introduction
Unstable flows in polymer processing can result in severe limitations on production rates and final product properties. Any attempt to study the mechanism of these instabilities or to determine critical flow conditions is ‘as good as’ the applied rheological model.

Problem definition
Before moving on to complex flows [1], we first study the dynamics of planar Poiseuille flows. We consider the widely used exponential Phan-Thien Tanner (PTT) model and the recently proposed Pom-Pom model [2, 3]. Figure 1 shows viscometric functions for the strain hardening parameter settings used.

Numerical Tools
In general, a flow is called stable or unstable depending on whether small perturbations $\delta(x,t) = \delta(x) \exp(\sigma t)$ decay or grow in time according to $M \delta + K \delta = 0$. Two different strategies can be followed to obtain the maximum growth rate $\sigma_r(= \text{real}(\sigma))$ and the most dangerous eigenmode $\delta(x)$:

1. Solve 1D Generalized Eigenvalue Problem using a Chebyshev-Tau method $(\sigma M + K)\delta = 0$,
2. Direct time integration using a 2D Finite Element Technique.

Results
Typical eigenspectra are shown in figure 2 for different wavenumbers ($k$). Obviously, the PTT model is unstable for a number of wavelengths. This is also shown in figure 3 where a critical Weissenberg number is computed for the PTT model whereas the Pom-Pom model remains stable. Comparison of the unstable eigenmode of the PTT model using both the eigenvalue analysis and the finite element analysis proves to be consistent (figure 4).

Conclusions
Although the steady characteristics of both models are quite similar (at least qualitatively), there is a large difference in the temporal behavior. Obviously, the choice of constitutive model must be made with great care!

References: