Real-Time Identification of an Induction Motor using Sinusoidal PWM Voltage Signals

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Abstract
In this paper, a novel technique is presented for real-time identification of Induction Motor (IM) parameters. The method is based on the IM model in the synchronously rotating frame and it utilizes sinusoidal PWM voltage signals for parameters identification tests. The proposed algorithm provides accurate estimation of the stator and rotor parameters based on the stator current measurement and voltage command. As compare to the commonly used techniques for the tuning of the slip gain the IM torque measurement is not required. Validation simulation and experimental results are presented.

1. Introduction
The real-time identification of induction machine (IM) parameters improves the efficiency of AC drives in the changing operational environment. It is especially important for advanced control schemes without rotational transducers that rely on the accurate machine models. Another aspect of the real-time monitoring of IM parameters is the failure detection, analysis and mitigation.

The real-time IM parameters identification schemes have been reported in the open literature by many research teams [1]-[6]. The approaches in [1],[2] utilize special test signals. The schemes in [3],[4],[5] are based on the model with excluded rotor fluxes that involves high pass filter differentiation for calculation of the acceleration of the voltage and currents [3], numerical approximation of the first and second order stator current derivatives [4], or calculation of the pseudo voltage and current signals [5]. An adaptive system technique is applied in [6] using the special form of IM model with separated flux and current dynamics to estimate rotor and stator resistances.

In this paper, a novel identification scheme of IM parameters is suggested that utilizes the projection of the voltage signal on the axes related with the stator current in the synchronously rotating frame. The paper first describes in Section 1 the dynamic model of IM and control problems, the parameters estimation algorithm and identification tests are given in Section 2 with simulation and experimental results being reported in sections 3 and 4.

2. Dynamic model of an induction motor and control problems
Consider the dynamic model of an induction motor (IM) in the stator frame [7]

\[
\frac{d\Theta}{dt} = \omega \tag{1}
\]

\[
\frac{d\omega}{dt} = -\frac{3n_pM}{2mL_s}\lambda_r^TJ_i - T_L/m \tag{2}
\]

\[
\frac{d\lambda_r}{dt} = \left(-\frac{R_e}{L_r}I + n_p\omega J\right)\lambda_r + \frac{R_r}{L_r}Mi_s \tag{3}
\]

\[
\frac{di_s}{dt} = -\frac{1}{\sigma L_s L_r}\left(-\frac{R_e}{L_r}I + n_p\omega J\right)\lambda_r - \frac{1}{\sigma L_s}\left(R_s + \frac{M^2 R_s}{L_r^2}\right)i_s + \frac{1}{\sigma L_s}v_s \tag{4}
\]

where

\[
I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad J = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}
\]

\[
\lambda_r, i_s, v_s \quad \text{rotor flux, stator current and stator voltage command,}
\]

\[
\Theta \quad \text{angular position of the rotor,}
\]

\[
\omega \quad \text{angular speed of the rotor,}
\]

\[
R_r, R_s \quad \text{rotor and stator resistance,}
\]

\[
M \quad \text{mutual inductance,}
\]

\[
L_r, L_s \quad \text{rotor and stator inductance,}
\]

\[
\sigma = 1 - M^2/L_s L_r \quad \text{leakage parameter,}
\]

\[
n_p \quad \text{number of pole pairs,}
\]

\[
m \quad \text{moment of inertia of the rotor,}
\]

\[
\alpha \quad \text{damping gain,}
\]

\[
T_L \quad \text{external load torque,}
\]

\[
\mu = 3n_p M/2L_r,
\]

\[
T_e = \mu i_s^T J\lambda_r \quad \text{the electromagnetic torque.}
\]

The first control goal is for the electromagnetic torque to follow the reference value \(T_{ref}\)

\[
\lim_{t \to \infty} T_e = T_{ref}. \tag{5}
\]
To achieve the goal (5) the flux magnitude is to be kept at a certain level and the second control goal is
\[ \lim_{t \to \infty} \lambda_r \to F_{\text{ref}} \]
(6)
where \( F_{\text{ref}} \) is the flux reference value.

It is assumed that the reference values \( T_{\text{ref}}, F_{\text{ref}} \in C^1[\mathbb{R}^+] \) and they should be selected accounting for constraints on the voltage and current signals.

In indirect field oriented (IFO) torque and flux control of an IM [7] it is required to measure or estimate the rotor mechanical speed \( \omega \). Then the electrical excitation frequency \( \omega_e \) can be estimated by summing the commanded slip frequency \( \omega_s^* \) with the measured rotor speed
\[ \omega_e^* = n_p \omega + \omega_s^* \]
(7)
where
\[ \omega_s^* = \frac{i_s^*}{i_d^*} \]
(8)
and \( \eta = \frac{R_e}{L_e} \) is the slip gain, \( i_s^*, i_d^* \) are the reference values for the stator current.

Integrating the signal \( \omega_e^* \) (7) provides information about the reference frame related with the rotor flux where the separation of the flux and torque tracking is achieved. The slip gain is estimated using standard IM characterization procedure [8] or common practice is to use the dynomometer tests with the true value of \( \eta \) providing the maximum torque output given current reference commands.

In many applications it is desirable to avoid measurements of the rotor position or speed (such sensors make the system expensive and less reliable). Thus the problem of the estimation of the rotor speed from the available for the measurement stator current and stator voltage command arises
\[ \lim_{t \to \infty} \dot{\omega} \to \omega \]
(9)
where \( \dot{\omega} \) is a rotor speed estimate.

In some rotor-flux-oriented torque and flux regulation schemes the value of the rotor flux or its estimate is used. Thus the next problem is to construct an observer for the rotor flux with the convergence
\[ \lim_{t \to \infty} \dot{\lambda}_r \to \lambda_r \]
(10)
where \( \dot{\lambda}_r \) is a rotor flux estimate.

Since parameters of the IM may change during its operation and their exact values may be essential for the quality of control, the problem of online estimation of the motor parameters arises. In particular, rotor and stator resistances are to be estimated
\[ \lim_{t \to \infty} \dot{P} \to P \]
(11)
where \( P = [R_r, R_s]^T \) and \( \dot{P} \) is its estimate.

The results on first two problems of the rotor speed and flux estimation using the IM model with separated flux and stator current dynamics have been reported in [9].

The adaptive scheme that provides the rotor and stator identification if the tuning model input is persistently exciting (PE) has been designed in [6].

This paper reports a simple practical procedure of the IM identification using the projection of the voltage signal on the axes related with the stator current in the synchronously rotating frame. The identification scheme is based on the steady-state IM signals and it does not involve calculation or approximation of the current and voltage derivatives.

3. IM identification model and tests

For convenience purposes the IM model is transformed to the synchronously rotating frame (d,q) in Figure 1 and the complex vector representation of the signals is introduced [10]
\[ V_s = r_s i_s + \omega_e J \lambda_s + \frac{d\lambda_s}{dt} \]
\[ 0 = r_r i_r + (\omega_e - \omega_r) J \lambda_r + \frac{d\lambda_r}{dt} \]
(12)
Here \( V_s, i_s, \lambda_s \) are the stator voltage, current, and flux
\[ i_r, \lambda_r \] are the rotor current and flux
\[ i_r = i_{rd} + j i_{rq} \]
\[ \lambda_r = \lambda_{rd} + j \lambda_{rq} \]
(13)
\( r_s, r_r \) are the stator and rotor resistances, \( \omega_e, \omega_r \) are the stator excitation frequency and rotor electrical speed, and \( j = \sqrt{-1} \).

Using the expressions for the stator and rotor fluxes
\[ \lambda_s = L_s i_s + L_M i_r \]
\[ \lambda_r = L_r i_r + L_M i_s \]
(15)
where \( L_s, L_r, L_M \) are the stator, rotor and mutual inductions, and assuming that the IM is at the steady-state the equation (12) is transformed to
\[ V_s = r_s i_s + \omega_e J (L_s i_s + L_M i_r) \]
\[ 0 = r_r i_r + \omega_r J (L_r i_r + L_M i_s) \]
(16)
where \( \omega_s = \omega_e - \omega_r \) is the slip frequency.
In the model (16) the rotor current is the variable unavailable for the measurement, so the second equation is used to express the rotor current via the measured stator current

\[ i_r = -\frac{r_r I + \omega_2 L_r J}{r_r^2 + \omega_2^2 L_r^2} j_s \]  

(17)

Substituting (17) into (16) gives the model that relates only the measured or known variables and IM parameters

\[ V_s = (r_s + r_r \frac{\omega_2 \omega_s L_M^2}{r_r^2 + \omega_2^2 L_r^2}) i_s + \omega_2 (L_s - L_r \frac{\omega_2^2 L_M^2}{r_r^2 + \omega_2^2 L_r^2}) j_s \]  

(18)

Note that the first term in (18) is parallel to the stator current vector and the second term is orthogonal to it. By introducing notation

\[ V_{sd}^i = \frac{V_s i_s}{|i_s|} \]

\[ V_{sq}^i = \frac{V_s j_i_s}{|i_s|} \]  

(19)

and by projecting (18) on vectors \( i_s \) and \( j_i_s \) the two equations for d-q components of the stator voltage follow

\[ V_{sd}^i = (r_s + r_r \frac{\omega_2 \omega_s L_M^2}{r_r^2 + \omega_2^2 L_r^2}) |i_s| \]

\[ V_{sq}^i = \omega_2 (L_s - L_r \frac{\omega_2^2 L_M^2}{r_r^2 + \omega_2^2 L_r^2}) |i_s| \]  

(20)

3.1. Parameters estimation algorithm

Transform first equations (20) to the regression form

\[ Y_1(t) = P_1 \varphi_1(t) \]  

(21)

where \( Y_1(t), \varphi_1(t) \) are the d-component of the stator voltage and the current amplitude

\[ Y_1(t) = V_{sd}^i(t) \]

\[ \varphi_1(t) = |i_s(t)| \]  

(22)

and \( P_1 \) is the parameter to be estimated

\[ P_1 = r_s + r_r \frac{\omega_2 \omega_s L_M^2}{r_r^2 + \omega_2^2 L_r^2} \]  

(23)

To improve the estimator robustness to the noise in the current measurement the parameter estimation can be obtained as a solution of the continuous set of equations (21) over a given time interval. The usual strategy is to put the maximum weight on the recent measurements and to gradually discount the previous measurements. Minimizing the integral mean error

\[ \int_0^t e^{-\gamma s} |Y_1(t-s) - P_1 \varphi_1(t-s)|^2 ds \to \min \]  

(24)

with the discount factor \( e^{-\gamma s} \) leads to the following estimation scheme for the parameter \( P_1 \)

\[ \frac{dY_1}{dt} + gY_1 = gY_1 \]

\[ \frac{d\varphi_1}{dt} + g\varphi_1 = g\varphi_1 \]

\[ P_1 = \frac{Y_1}{\varphi_1} \]  

(25)

Similarly the second equation in (20) is represented as

\[ Y_2(t) = P_2 \varphi_2(t) \]  

(26)

where

\[ Y_2(t) = V_{sq}^i(t) \]

\[ \varphi_2(t) = \omega_2 (t)|i_s(t)| \]

\[ P_2 = (L_s - L_r \frac{\omega_2^2 L_M^2}{r_r^2 + \omega_2^2 L_r^2}) \]  

(27)

and the observer for \( P_2 \) follows

\[ \frac{dY_2}{dt} + gY_2 = gY_2 \]

\[ \frac{d\varphi_2}{dt} + g\varphi_2 = g\varphi_2 \]

\[ P_2 = \frac{Y_2}{\varphi_2} \]  

(28)

Note that the observers (25), (28) are equivalent to the low pass filtering of the input and output signals of the regression models (21), (26).

3.2. Parameters estimation tests

Test 1: Estimation of parameters \( P_1, P_2 \) at no load condition (the slip frequency equals to zero) defines the value of the stator resistance and inductance

\[ r_s = \hat{P}_1 |_{\omega_s = 0} \]

\[ L_s = \hat{P}_2 |_{\omega_s = 0} \]  

(29)

Note that (25),(28),(29) are further transformed to

\[ r_s = \frac{\int_0^t e^{-\gamma (t-s)} V_{ds}^i(s)ds}{\int_0^t e^{-\gamma (t-s)} |i_s(s)|ds} \]  

(30)

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that includes exponential forgetting factor in the real-time update of the stator resistance and inductance.

**Test2:** The second test is performed when the slip frequency is high. Selecting locked rotor condition to maximize the slip frequency and assuming that the slip frequency is much higher than the rotor time constant

\[
\omega_s = \omega_e \\
\omega_s \gg \frac{r_r}{L_r}
\]

(33)

give the estimation of the rotor resistance

\[
r_r \approx \frac{L_r^2}{L_m^2} (\hat{P}_1|_{\omega_s=\omega_e} - \hat{P}_1|_{\omega_s=0})
\]

(34)

The second parameter provides the estimation of the sum of the rotor and stator self inductances

\[
L_{1s} + L_{1r} \approx \hat{P}_2|_{\omega_s=\omega_e}
\]

(35)

Equalities (29),(34),(35) define the set of the IM parameters.

Note that when the rotor speed is measured then the estimations (34),(35) can be performed at any value of the slip frequency by resolving nonlinear expressions (23),(27). From (23) follows that the second addendum in (23) has maximum value for fixed excitation frequency when the slip frequency equals to

\[
\omega_s^{\text{max}} = \frac{r_r}{L_r}
\]

(36)

Selecting the slip frequency close to the expected value of the rotor time constant increase the accuracy of the rotor resistance estimation.

### 4. Simulation results

The observer model was developed in Matlab/Simulink and it includes the IM model, a dynamometer, and the IM parameters observer. The parameters identification technique is tested on the low scale dynamometer using 1kW specialty induction machine. The IM parameters to be identified are given in the Table 1

<table>
<thead>
<tr>
<th>$R_s$</th>
<th>$R_r$</th>
<th>$M$</th>
<th>$L_{1r}$</th>
<th>$L_{1s}$</th>
<th>$n_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>.023</td>
<td>.012</td>
<td>.00064</td>
<td>.00063</td>
<td>.00058</td>
</tr>
<tr>
<td>$P$</td>
<td>.0228</td>
<td>.0118</td>
<td>.00063</td>
<td>.00063</td>
<td>.000575</td>
</tr>
</tbody>
</table>

Table 1

The excitation voltage signal is selected to have the amplitude and frequency

\[
V_{so} = 5V, \quad \omega_e = 100 \text{rad/ sec}
\]

(37)

The parameters estimation results are given in Figure 2 and the estimations of the $r_s$ and $L_s$ coincide with the expected values from Table 1.

For the locked rotor test the rotor flux is not aligned with the stator current and the flux amplitude is lower $\lambda_r = .015Wb$. The $P_1, P_2$ estimates in Figure 3 give the values for the nonlinear terms

\[
r_r \frac{\omega_e^2 L_M^2}{r_r^2 + \omega_e^2 L_r^2} \approx .0098
\]

(38)

\[
L_s - L_r \frac{\omega_e^2 L_M^2}{r_r^2 + \omega_e^2 L_r^2} \approx 1.15e^{-4}
\]

(39)

Finding roots of the nonlinear functions (38),(39) defines the values for the rotor resistance and the leakage parameter that are close to the expected values from the Table 1.
5. Experimental results

The real-time parameters identification algorithms were implemented on the low-scale dynamometer shown in Figure 4 that includes the specialty induction motor and a permanent magnet synchronous motor (PMSM) coupled through the shaft with a torque sensor. The dSPACE ACE controller performs signal conditioning and run the IM inverter. The Xmath/SystemBuild graphical environment in the laptop PC is used for the generation of the controller C code and system development. To make the controller code more efficient, all computation processes are divided into two classes: slow and fast. The slow processes are implemented in the outer loop with a sampling frequency of 1kHz, and the fast ones run at 10kHz in the inner loop. The fast processes include implementation of the field-oriented (FO) control, generation of the sinusoidal voltage signals for identification algorithms and updates of the current and speed signals. The IM parameters identification is performed in the outer loop with the frequency 1kHz.

The results of the stator resistance identification are summarized in the Figures 5 that shows the voltage $V_s$ as a function of the current $I_d$ measured at different rotor speeds $\omega_r = 0, 300, 500$rpm. For the least-mean-squares (LMS) approximation of the curves in Figure 5 the Matlab function POLYFIT($X, Y, N$) finds the coefficients of a polynomial $P(X)$ of degree $N$ that fits the data, $P(X(I)) = Y(I)$, in a least-squares sense. The LMS approximation of the curves in Figure 5 gives the following values for the stator resistance and the voltage offset

$$R_s = 0.023\Omega, \ V_0 = 0.146V$$

with the result of $R_s$ identification being not dependent on the rotor speed.

Similarly the stator inductance is identified

$$L_s = 6.4 \cdot 10^{-4}H, \ V_0 = 11.97 \cdot 10^{-4}V$$

For the locked rotor tests the values of the electrical excitation frequency and voltage amplitude are selected at

$$\omega_e = 30, 60, 100$rad/sec, \ V_0 = .6, .8, 1.0V \ (42)$$

The results of the tests are given in Figures 6, 7 where the $V_q$ voltage and the $V_o$ voltage normalized by the speed are shown as a function of the $I_d$ current. The tests results are summarized in Table 2 with parameters $P_1$ and $P_2$ being nonlinear functions of the both rotor resistance and rotor and stator leakages. At low excitation frequencies for accurate identification of $R_r$ and $L_{1s}, L_{1r}$, the Matlab constr function is used. The constr finds the $R_r$ and $L_{1s}, L_{1r}$ by minimizing the quadratic function

$$(P_1(R_r, L_{1s}, L_{1r}) - P_1^*)^2 + (P_2(R_r, L_{1s}, L_{1r}) - P_2^*)^2 \rightarrow \min$$

under constraints

$$0 < R_r \leq R_{r,\text{max}}, \ 0 < L_{1s}, L_{1r} \leq L_{\text{max}} \ (44)$$

where $P_1^*, P_2^*$ are measured values of $P_1, P_2$. The results of $R_r$ and $L_{1s}, L_{1r}$ identification are given in Table 2. Note that the results at different speeds are consistent although the low frequency tests gives the higher value for the rotor resistance and the lower value for the leakage parameter.

The approximate formulas (34),(35) at the excitation frequency $\omega_e = 100$rad/sec give the values

$$R_r \approx 0.0115\Omega, \ L_{1s}, L_{1r} \approx 5.5 \cdot 10^{-5} \ (45)$$

that are close to the values in Table 2 obtained by the constrained optimization.

<table>
<thead>
<tr>
<th>$\omega_{rad/sec}$</th>
<th>$P_1(\Omega)$</th>
<th>$P_2(\Omega)$</th>
<th>$R_e(\Omega)$</th>
<th>$L_{1s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.0307</td>
<td>0.0002675</td>
<td>0.0132</td>
<td>4.78\cdot 10^{-6}</td>
</tr>
<tr>
<td>60</td>
<td>0.0327</td>
<td>0.0001525</td>
<td>0.0127</td>
<td>5.20\cdot 10^{-6}</td>
</tr>
<tr>
<td>100</td>
<td>0.0327</td>
<td>0.0001102</td>
<td>0.0119</td>
<td>5.21\cdot 10^{-6}</td>
</tr>
</tbody>
</table>

Table 2

![Fig. 4. Experimental setup](image)

The results of the stator resistance identification are summarized in the Figures 5 that shows the voltage $V_s$ as a function of the current $I_d$ measured at different rotor speeds $\omega_r = 0, 300, 500$rpm. For the least-mean-squares (LMS) approximation of the curves in Figure 5 the Matlab function POLYFIT($X, Y, N$) finds the coefficients of a polynomial $P(X)$ of degree $N$ that fits the data, $P(X(I)) = Y(I)$, in a least-squares sense. The LMS approximation of the curves in Figure 5 gives the following values for the stator resistance and the voltage offset

$$R_s = 0.023\Omega, \ V_0 = 0.146V \ (40)$$

with the result of $R_s$ identification being not dependent on the rotor speed.

Similarly the stator inductance is identified

$$L_s = 6.4 \cdot 10^{-4}H, \ V_0 = 11.97 \cdot 10^{-4}V \ (41)$$

![Fig. 5. Stator resistance estimation no load tests](image)

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6. Conclusions

- This paper presents a novel technique for the real-time identification of the IM parameters using the sinusoidal PWM voltage test signals. The parameters estimator is based on the IM model in the synchronously rotating frame and it utilizes the projection of the voltage command on the axes related with the stator current.
- To improve the estimator robustness to the measurement noise the least-mean-squares (LMS) algorithm with the discount factor is implemented to process the measured signals. The simulation results demonstrate good performance and accuracy of the method under the current measurement noise with $std(i_s) \approx 1A$.
- The constrained optimization procedure is developed for the accurate simultaneous estimation of the leakage parameters and the rotor resistance for the operational conditions when the slip frequency is of the same order of magnitude as the machine slip gain.
- The real-time characterization of the specialty IM shows good correlation with the IM parameters previously identified using a set of the dynamometer-based tests.

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References


