Today, the focus of physical scientists is shifting more to biology than ever before. Because there is no biological tissue that is not a porous medium, the porous media community should be very alert to this shift of attention. The impact that porous media mechanics had on geosciences in the past centuries, may very well be reiterated in the field of biology - probably in an amplified form. The characteristic pore size in most biological applications is close to the molecular level and hence below the Debye-Hückel scale. Not only pressure gradients and concentration gradients, but electrical gradients as well are intimately linked to fluid flow, ion flow and deformation. To give a feeling for the subject, applications are covered in this presentation: blood perfusion, the skin barrier, the mechanosensory mechanics in bone and the swelling of tissues and cells.

1 INTRODUCTION

More than half of the medical research in the Netherlands is done by technically trained scientists. The Netherlands Fund for Fundamental Research into Matter - traditionally the research fund of physicists to finance atomic physics research - has decided to redirect a good deal of its funds to the interface between physics and biology. While the 20th century was the century of the physical sciences, the 21st century is expected to become the century of biology. The uncovering of the human genetic code has now called upon the uncovering of the mechanisms through which the genetic information is translated into the macroscopic human body. As all the functional constituents are present already at the level of microsized cell, many of the challenges lie at the nano- and micromscale and require therefore advanced technical skills for both measuring and modelling. What is the role of porous media mechanics in all of this? Even on the level of a single cell, the number of molecules involved are so high that if any modelling is going to be successful, it will necessarily involve continuum mechanics. We will discuss one application on the microporescale, blood perfusion, and three applications on the nanoscalescale, the skin barrier, bone remodelling and the biomechanics of swelling.

2 BLOOD PERFUSION

Figure 1: Corrosion cast of the coronary arterial tree of a canine heart (cast is courtesy of dr. P. Santens, University of Gent)

Wilson, Aifantis and many other have been successful in using methods of multiporosity porous media mechanics on fractured rocks (Wilson and Aifantis 1982). Multiporosity theory has applications even
Coronary artery disease is believed to be number one cause of mortality in our world. The coronary vascular system is nothing but a pore structure inside a deforming solid which we call the heart muscle. The muscle is subject to large deformations. The pore structure is highly organised to deliver every heart cell from its waste materials and supply every cell of its nutrients. The way this is done is through a multiporosity structure (fig. 1). Since many centuries these porosities have names: arteries, arterioles, capillaries, venules and veins. They have their characteristic pore size, their characteristic blood velocity, their characteristic wall properties and their characteristic pathology. Every doctor in the world even knows a quantity proper to multiporosity flow: perfusion. It is the flow of fluid from one porosity to another, measured per unit volume tissue. Flow has a dimension $m^3 s^{-1}$ and as it is defined per unit volume, perfusion has a dimension of $m^{-1}$. There is evidence that blood perfusion is affected by the deformation of the heart muscle (Spaan 1985). There is evidence that blood perfusion affects the deformation, both metabolically as mechanically (Olsen et al. 1981). These findings demonstrate that the coronary system should be modelled using a multiporosity, finite deformation approach (fig. 2).

The multiporosity model was implemented in 2D, 3D and axisymmetric finite elements (Vankan et al. 1997). Animal experiments were undertaken to verify the concepts (van Donkelaar et al. 2001). Comparison of experiment and model clearly demonstrate the capabilities. The same equations were derived twice: using mixture theory (Vankan et al. 1996) and using averaging theorems (Huyghe and van Campen 1995a; Huyghe and van Campen 1995b). A micro-macro transformation was derived to compute hydraulic permeabilities from the microstructure (Huyghe et al. 1989; Huyghe et al. 1989; Vankan et al. 1997). While at the time of the research, the microstructure was hard to reconstruct, today micro-computed tomography are operational methods to reconstruct the 3D microgeometry post mortem.

3 SKIN BARRIER

As we are sitting in the room we all are porous media in contact with air. As the chemical potential of the vapour in the air is lower than the chemical potential of the water in the body, we continually loose water through evaporation. This process bears the name of transepidermal water loss. If we were to put a bath of water at 37 °C Centigrade next to us - with same contact area with air as our body, we would find the loss of fluid from the bath to be many times more than from our body. So, there is a barrier that protect us against desiccation. Minute bruises to our skin reduces the barrier very significantly, underscoring the role of the upper micrometers of the horny layer of the epidermis in maintaining the barrier. Because of interest in the skin barrier from the industry, we undertook

Figure 2: Simulated blood pressure distribution across a section $\alpha - \alpha$ of the heart wall. During diastole most of the arteriovenous pressure drop is located in the arterioles. During systole the deeper endocardial layers of the heart wall are subjecting the coronary system to high pressure because of the strong muscle contraction.

Figure 3: Hydraulic permeability profile across the skin (van Kemenade et al.).
the challenge to model transepidermal water loss using Biot’s theory coupled to an air vapour diffusion model. The finite element model was compared to \textit{in vivo} data obtained from volunteers. A sharp drop in hydraulic permeability (fig. 3) and a sharp increase in capillary effects towards the skin surface explains the barrier function satisfactorily.

4 \textbf{BONE MECHANORESPONSE}

Figure 4: Longitudinal section of a femur. The spongy bone orients its porous structure according to the local stress field (Williams et al. 1989)

Bone is known to adapt its pore structure to local mechanical load (fig. 4). Load-related alignment and mass of bone are the foundations of the Law of Bone Transformation (Wolff’s Law), formulated by Julius Wolff in 1892. Computer simulations confirmed his postulate, that bone adapts its form according to rules of mathematical design (Huiskes et al. 2000). However, the cellular mechanism that underlies the typical secondary bone structure is as yet unexplained. Bone remodelling involves groups of cells of 3 different types, which collaborate in so-called basic multi-cellular units: osteoblasts, osteoclasts and osteocytes Basic multicellular units proceed by tunnelling, during which osteoclasts excavate a canal that is partly refilled by osteoblasts, thus forming an osteon (fig. 5). The osteocytes reside in lacunae inside the bone matrix and thus have a good position for mechanosensing. With their long slender protrusions that run through canaliculi they form a three-dimensional network that reaches to the bone surface, which allows them to signal (pre-) osteoclasts and (pre-) osteoblasts. In order to respond to mechanical stimuli, bone necessarily needs to sense the mechanical load it adapts to. The most accepted hypothesis on how bone senses its own stress field, involves fluid flow induced by mechanical load (Cowin et al. 1991). In other words it involves a consolidation process, similar to the consolidation of geomaterials. Burger and Klein Nulend (Burger and Nulend 1999) have demonstrated through several experimental studies in vitro that osteocytes react to fluid flow. As an early response they release nitric oxide (NO) and prostaglandins (PG) E2 and I2, followed by expression of the enzyme cyclo-oxygenase 2 (COX-2), which allows for continued release of prostaglandins. NO, PGE2 and COX-2 play a crucial role in the induction of bone formation (Chambers et al. 1999), while NO and PGE2 also inhibit osteoclasts. Large tissue strains thus conceivably lead to osteoblast recruit-
Figure 6: Local fluid content change in an osteon during walking as computed by Biot’s theory. In front of the cutting cone appears an area of volumetric expansion that effectively inhibits the exchange of fluid between the bone matrix and the vascular porosity (Smit et al.). Conversely, decreased stimulation of osteocytes may activate osteoclasts, possibly by the mechanism of cell death. As osteocytes are likely stimulated by strain-induced fluid flow, my colleagues of the Free University of Amsterdam and myself aimed to determine the local pattern of fluid flow at a remodelling site (Smit et al.).

Cortical bone has two systems of interconnected channels. The largest of these is the vascular porosity consisting of Haversian and Volkmann’s canals, with a diameter of some 50 μm, which contains a.o. blood vessels and nerves. The smaller system consists of the canaliculi and lacunae. The canaliculi are at the submicron level and house the protrusions of the osteocytes. When bone is loaded, fluids within the solid matrix sustain a pressure gradient that drives a flow. It is generally assumed that the flow of extra-cellular fluid around osteocytes plays an important role not only in the nutrition of these cells, but also in the bone’s mechanosensory system. The interaction between the deformation of the bone matrix and the flow of fluid is modeled using Biot’s theory of poroelasticity. However, because of the inhomogeneity of the bone matrix and the scale of the porosities, it is not possible to experimentally determine all the parameters that are needed for numerical implementation. Smit et al. (Smit et al.) derive these parameters using composite modelling and experimental data from literature. A full set of constants is estimated for a linear isotropic description of cortical bone as a two-level porous medium. An axisymmetric finite element model of the minerised bone surrounding a haversian canal is constructed. The lacuno-canaliculur porosity is modelled as a pore space saturated with compressible fluid. The axial load depends on time according to the typical loading pattern of a femur during walking. Along the cylindrical part of the tunnel (the closing cone), fluid is pressed out of the bone matrix. At the tip (the cutting cone), however, fluid flows into the bone matrix, because of a local, superficial area of volumetric expansion. The amplitude of this fluid flow at the cutting cone is about six times lower than the flow at the closing cone. At unloading of the bone tissue, the fluid flow pattern is reversed, resulting in a fluid outflow at the cutting cone, and an inflow along the rest of the osteonic tunnel wall. Inside the bone matrix, the outflow pattern along the closing cone damps out at a depth of some 100 μm. This 100 μm is just about the distance from the deepest osteocytes to the bone surface both in human compact and trabecular bone, which confirms a role for the transport of nutrients. At the cutting cone, however, a different phenomenon is observed: here the fluid flow direction is reversed at a depth of some 10 μm, because the volumetric expansion is only a superficial phenomenon and the deeper layers experience volumetric compression (Smit and Burger). The area of volumetric expansion in front of the cutting cone not only reduces the flow amplitude around the osteocytes located there, but inhibits the exchange of fluid between the bone matrix and the osteonic lumen as well. The bone tissue layer immediately in front of the cutting cone thus appears as an area of local disuse with lack of fluid transport. This is precisely the bone layer that osteoclasts erode. By contrast, behind the cutting cone the tissue deformation and the canalicular fluid flow are increased; there the osteocytes are inhibited from further excavation, while osteoblasts are recruited in order to refill the tunnel. So by excavating a tunnel,
osteoclasts create a local area of increased canalicular fluid flow that leads to osteoblast recruitment, and at the same time create an area of disuse in the loading direction that guides their continued resorption activity. This explains the progression of osteon formation along the principle loading direction (Smit and Burger). From the simulated flow pattern, the time course of the streaming potential across the shaft of the osteon is computed (fig. 7) assuming a vanishing streaming current. The hydraulic permeability is chosen so as to reproduce experimentally measured data from the literature (Otter et al. 1992). The above mechanism explains the orientation of the bone structure along the principle stress direction and the tuning of bone mass according to the magnitude of the stress and hence provides a cellular basis for Wolff’s law (Smit and Burger). In this mechanism, Biot’s theory is a key element, indicating the strong need for communication between the porous media community and biologists.

5 SWELLING TISSUES AND CELLS

![Figure 8: Chemical potential of the fluid as a function of time during swelling of a one dimensional ionised medium. The solution from a 3D finite element code (van Loon et al.) is compared to the analytical solution (van Meerveld et al.).]

Since antiquity, the phenomenon of swelling of tissues has been closely related to health and disease. Biological, synthetic and mineral porous media often exhibit swelling or shrinking when in contact with changing salt concentrations. This phenomenon, observed in clays, shales, cartilage and gels, is caused by a combination of electrostatic forces and hydration forces (Lai, Hou, and Mow 1991). In case of biological tissue, electrostatic forces are often dominant.

6 Donnan Osmosis

When an ionised medium is in contact with a monovalent salt solution, diffusion of salt ions and flow of fluid take place between the medium and the salt solution until equilibrium is reached:

\[
\begin{align*}
\mu^+ &= \bar{\mu}^+ \\
\mu^- &= \bar{\mu}^- \\
\mu^f &= \bar{\mu}^f
\end{align*}
\]}

\(\mu^+\) is the electrochemical potential of the cations, \(\mu^-\) is the electrochemical potential of the anions and \(\mu^f\) the chemical potential of the fluid in the medium. The corresponding overlined symbols refer to chemical potentials in the outer solution. Standard expressions for (electro)chemical potentials are found in the literature (Richards 1980). If we assume incompressibility for each constituent, i.e. same partial molar volumes in either solution, we find:

\[
\begin{align*}
\mu^+ &= \mu_0^+ + \frac{1}{\overline{V}^+}(RT\ln a^+ + F\xi) \\
\mu^- &= \mu_0^- + \frac{1}{\overline{V}^-}(RT\ln a^- - F\xi) \\
\mu^f &= \mu_0^f + p + \frac{RT}{\overline{V}^f}\ln a^f
\end{align*}
\]
where $\xi - \bar{\xi}$ is the Donnan potential between the inner and outer solution. If we define $c^f$ as the fixed charge density per unit fluid volume of the inner solution, taken positive for positive charges and negative for negative charges, we can write the electroneutrality conditions as:

$$c^- = c^+ + c^f$$

$$c^- = \bar{c}^+ = c^f$$

$c^+$ and $c^-$ are the cationic and anionic concentrations per unit fluid volume in the inner solution, while the corresponding overlined symbols pertain to the outer solution. From the previous equations we derive the Donnan equilibrium concentration of the ions:

$$2c^+ = - c^f + \sqrt{(c^f)^2 + 4f^2\bar{c}^2}$$

$$2c^- = c^f + \sqrt{(c^f)^2 + 4f^2\bar{c}^2}$$

with

$$f^2 = \frac{\bar{f}^+ f^-}{f^+ f^-}$$

and $f^\beta = \frac{\phi^\beta}{\phi^\alpha}$, $\beta = +, -$ the activity coefficient of component $\beta$. Eqs. (11-12) show that the cationic concentration jumps to a higher and the anionic concentration to a lower value when entering the porous medium. These concentration jumps are responsible for the attraction of water into the porous medium during swelling and for the associated osmotic pressure $\pi$. Using eq. (6) one can derive Van ’t Hoff relation from (3):

$$\pi = p - \bar{p} = RT[\phi^f (c^+ + c^-) - 2\bar{\phi}^f \bar{c}]$$

provided that the molar fractions of the ions are small compared to the molar fraction of the fluid. $\phi^f$ and $\bar{\phi}^f$ are the osmotic coefficients:

$$\phi^f = \frac{\partial \bar{n}^f}{\partial n^f}$$

$$\bar{\phi}^f = \frac{\partial \bar{n}^f}{\partial \bar{n}^f}$$

$x^f = \frac{\bar{c}^f}{\bar{n}^f}$ and $x^f = \frac{\bar{\bar{c}}^f}{\bar{\bar{n}}^f}$ are the molar fractions of the fluid in the inner and outer solution. It may be clear from the above considerations that physical phenomena occurring in the porous medium are a combination of mechanical, chemical and electrical effects. The interrelationship between these effects are well known for membrane processes (Staverman 1952). The purpose of this paper is to generalise these relationships for porous media subjected to threedimensional finite deformation. The four phases that we consider in the medium are: solid (superscript s), fluid (superscript f), monovalent anions (superscript -) and monovalent cations (superscript +). Assuming all components intrinsically incompressible and excluding mass transfer between phases, the mass balance of each phase is written as:

$$\frac{\partial n^\alpha}{\partial t} + \nabla \cdot (n^\alpha \bar{\sigma}^\alpha) = 0, \quad \alpha = s, f, +, -$$

in which $n^\alpha$ is the volume fraction and $\bar{\sigma}^\alpha$ the velocity of phase $\alpha$. As we assume saturation

$$n^s + n^f + n^+ + n^- = 1$$

and as we neglect the volume fraction of the ions relative to the other volume fractions, summation of the eqs. (17) yields the mass balance of the mixture:

$$\nabla \cdot \bar{\sigma}^s + \nabla \cdot (n^f (\bar{\sigma}^f - \bar{\sigma}^s)) = 0$$

It is useful to refer current descriptors of the mixture with respect to an initial state of the porous solid. We define the deformation gradient tensor $F$ mapping an infinitesimal material line segment in the initial state of the solid onto the corresponding infinitesimal line segment in the current state of the solid. The relative volume change from the initial to the current state is the determinant of the deformation gradient tensor $J = det F$. If we introduce volume fractions

$$N^\alpha = J n^\alpha$$

per unit initial volume, we can rewrite the mass balance equation (17) as follows:

$$\frac{D^s N^\alpha}{Dt} + J \bar{\nabla} \cdot [n^\alpha (\bar{\sigma}^\alpha - \bar{\sigma}^s)] = 0$$

when using the identity:

$$\frac{D^s J}{Dt} = J \bar{\nabla} \cdot \bar{\sigma}^s$$

Neglecting body forces and inertia, the momentum balance takes the form:

$$\nabla \cdot \sigma^\alpha + \bar{\pi}^\alpha = 0, \quad \alpha = s, f, +, -$$
Figure 9: Electrochemical potential of anions and cations as a function of time during swelling of a one dimensional ionised medium. The solution from a 3D finite element code (van Loon et al.) is compared to the analytical solution (van Meerveld et al.).

which after summation over the four phases, yields:

\[ \nabla \cdot \sigma = \nabla \cdot \sigma^s + \nabla \cdot \sigma^f + \nabla \cdot \sigma^+ + \nabla \cdot \sigma^- = 0 \]  

(24)

if use is made of the balance condition:

\[ \pi^s + \pi^f + \pi^+ + \pi^- = 0 \]  

(25)

\( \sigma^\alpha \) is the partial stress tensor of constituent \( \alpha \), \( \pi^\alpha \) is the momentum interaction with constituents other than \( \alpha \). Balance of moment of momentum requires that the stress tensor \( \sigma \) be symmetric. If no moment of momentum interaction between components occurs, the partial stresses \( \sigma^\alpha \) also are symmetric. We assume all partial stresses to be symmetric. Under isothermal and incompressible conditions, the entropy inequality for a unit volume of mixture reads (Huyghe and Janssen 1997): We introduce the strain energy function as the Helmholtz free energy of a mixture volume which in the initial state of the solid equals unity. \( \psi^\alpha \) is the Helmholtz free energy of constituent \( \alpha \) per unit mixture volume. The inequality for the entropy production per initial mixture volume reads:

\[ - \frac{D}{Dt} W + J\sigma : \nabla \psi^\alpha + \] 

\[ J\nabla \cdot \sum_{\beta = f, i, +, -} [(\psi^\beta - \psi^\alpha) \cdot \sigma^\beta - (\psi^\beta - \psi^\alpha) \psi^\beta] \geq 0. \]  

(26)

Introducing a Lagrange multiplier \( p \) for the saturation condition and \( \lambda \) for the electroneutrality condition, the entropy inequality transforms into:

Figure 10: Meridional section of a cylinder subject to a stepwise decrease of the salt concentration of the external solution. From top to bottom sections of deformed meshes are shown on time t = 0 s, t = 250 s, t = 500 s, t = 1700 s, t = 40000 s. The vertical displacement of the lower base of the cylinder is held fixed. Along the upper and lower base of the cylinder a no flow boundary condition is prescribed for water, cations and anions. Along the jacket of the cylinder the chemical potential of the fluid and the electrochemical potential of the anions and cations are prescribed, equal to their values in the external salt solution. Pressure, electrical potential and ion concentrations are not prescribed along the jacket because these quantities exhibit deformation dependent jumps across the boundaries. Computation is done using a 3D 27 node finite elements (van Loon et al.). The scale of the displacements and the positions are the same. Although the overall response to a lowering of the external salt concentration is swelling, initial shrinking is observed at t = 250 s. This is explained by the friction between ions and water and the initial outflux of ions convecting water with it against the gradient in chemical potential of the water.
The Lagrange multiplier $p$ can be interpreted as the effective stress of the medium. We choose as independent variables the Green strain $E$, the Lagrangian form of the volume fractions of the fluid and the ions $N^\beta$, and of the relative velocities $v^{\beta s} = F^{-1} \cdot (v^\beta - v^s)$, $\beta = f, +, -$. We apply the principle of equipres- sure as the chain rule for time differentiation of $W$:

$$\frac{D}{Dt} W + J \sigma^{eff} : \nabla \sigma^\alpha$$

$$+ J \sum_{\beta = f, +, -} \left[ \sigma^\beta + ((p + \frac{z^\beta \lambda}{V^\beta}) n^\beta - \psi^\beta) I \right]$$

$$: \nabla (v^\beta - v^s)$$

$$+ J \sum_{\beta = f, +, -} \left( v^{\beta s} - v^s \right) \cdot \left[ -\nabla \psi^\beta + (p + \frac{z^\beta \lambda}{V^\beta}) \nabla n^\beta \right]$$

$$+ \nabla \cdot \sigma^\beta] \geq 0. \quad (27)$$

in which $z^\beta$ is the valence of constituent $\beta$, $\sigma^{eff}$ is the effective stress of the medium. We choose as independent variables the Green strain $E$, the Lagrangian form of the volume fractions of the fluid and the ions $N^\beta$, and of the relative velocities $v^{\beta s} = F^{-1} \cdot (v^\beta - v^s)$, $\beta = f, +, -$. We apply the principle of equipres- sure as the chain rule for time differentiation of $W$:

$$\left( J \sigma^{eff} - F \cdot \frac{\partial W}{\partial E} \cdot F^c \right) : \nabla \sigma^\alpha + \sum_{\beta = f, +, -} \left( \frac{\partial W}{\partial v^{\beta s}} \cdot \frac{D}{Dt} v^{\beta s} \right)$$

$$+ J [ \sigma^\beta + (\mu^\beta n^\beta - \psi^\beta) I ] : \nabla (v^\beta - v^s)$$

$$+ J (v^{\beta s} - v^s) \cdot \left[ -\nabla \psi^\beta + \mu^\beta \nabla n^\beta + \nabla \cdot \sigma^\beta \right] \geq 0. \quad (28)$$

in which $\mu^\beta$ are the electrochemical potentials of fluid and ions:

$$\mu^f = \frac{\partial W}{\partial N^f} + p$$

$$\mu^+ = \frac{\partial W}{\partial N^+} + \frac{\lambda}{V^+} + p$$

$$\mu^- = \frac{\partial W}{\partial N^-} - \frac{\lambda}{V^-} + p \quad (29)$$

The Lagrange multiplier $p$ can be interpreted as the fluid pressure and $\lambda$ as the electrical potential of the medium multiplied by the constant of Faraday. By a standard argument (Coleman and Noll 1963), we find:

$$\sigma^{eff} = \frac{1}{J} F \cdot \frac{\partial W}{\partial E} \cdot F^c$$

$$\frac{\partial W}{\partial v^{\beta s}} = 0 \quad (30)$$

$$\sigma^\beta = (\psi^\beta - \mu^\beta n^\beta) I \quad (31)$$

$$\sigma^\beta = (\psi^\beta - \mu^\beta n^\beta) I \quad (32)$$

Eq. (30) indicates that the effective stress of the mixture can be derived from a strain energy function $W$ which represents the free energy of the mixture. Eq. (31) shows that the strain energy function cannot depend on the relative velocities. Thus, the effective stress of a quadriphasic medium can be derived from a regular strain energy function, which physically has the same meaning as in single phase or biphasic media, but which can depend on both strain and ion concentrations in the medium. According to eq. (32) the partial stress of the fluid and the ions are scalars. Transforming the relative velocities to their Lagrangian equivalents, we find in stead of (33):

$$\sum_{\beta = f, +, -} \left( v^{\beta s} - v^s \right) \cdot \left[ -\nabla_0 \psi^\beta + \mu^\beta \nabla_0 n^\beta + \nabla_0 \cdot \sigma^\beta \right] \geq 0. \quad (34)$$

in which $\nabla_0 = F^c \cdot \nabla$ is the gradient operator with respect to the initial configuration. If we assume that the system is not too far from equilibrium, we can express the dissipation (34) associated with relative flow of fluid and ions as a quadratic function of the relative velocities:

$$-\nabla \psi^\beta + \mu^\beta \nabla n^\beta + \nabla_0 \cdot \sigma^\beta = \sum_{\gamma = f, +, -} B^{\beta \gamma} \cdot \sigma^{\gamma s} \quad (35)$$

$B^{\beta \gamma}$ is a positive definite matrix of frictional tensors. Substituting eq. (32) into eq. (35) yields Lagrangian forms of the classical equations of irreversible thermodynamics:

$$-n^\beta \nabla_0 \psi^\beta + \mu^\beta \nabla_0 n^\beta + \nabla_0 \cdot \sigma^\beta = \sum_{\gamma = f, +, -} B^{\beta \gamma} \cdot \sigma^{\gamma s} \quad (35)$$

The momentum balance equation (24), the mass balance eq. (21), the frictional eqs. (35), the constitutive relationships for the electrochemical potentials (29) and of the effective stress (30) form a set of partial differential equations. The boundary conditions are given by a no-jump condition of the electrochemical potential of the ions and the fluid across the boundary and the momentum balance of the boundary. In our application we choose the mixing part of the energy
plied by the weighing functions multiplied by arbitrary, time independent weighing divergence theorem, we find, equation and equation for electroneutrality are multiplied. After partial integration and applying the weighing function as

\[ W(N^f, N^+, N^-) = \mu_0^f N^f + \mu_0^+ N^+ + \mu_0^- N^- \]

\[ -RT T \left( \frac{N^+}{\bar{V}^+} + \frac{N^-}{\bar{V}^-} \right) \ln \left( \frac{N^f}{\bar{V}} \right) \]

\[ + RT N^+ \left( \ln \left( \frac{N^+}{\bar{V}^+} \right) - 1 \right) \]

\[ + RT N^- \left( \ln \left( \frac{N^-}{\bar{V}^-} \right) - 1 \right) \] (36)

Rearranging equation (29) yields,

\[ \mu^\alpha - p - \frac{z^\alpha}{\bar{V}^\alpha} \lambda = \sum_{\beta = f, +, -} C_{\alpha \beta}^{-1} N^\beta \] (37)

in which,

\[ C_{\alpha \beta} = \left[ \frac{\partial W(N^f, N^+, N^-)}{\partial N^\alpha \partial N^\beta} \right]^{-1} \]

is the inverse of the Hessian of the mixing energy. To obtain the weak formulation the equations are multiplied by arbitrary, time independent weighing functions and integrated over the volume of the mixture (Ω). The momentum equation is multiplied by a weighing function \( w_x \). The saturation condition, mass equation and equation for electroneutrality are multiplied by the weighing functions \( w_p, w^\alpha \) and \( w_\xi \), respectively. After partial integration and applying the divergence theorem, we find,

\[ \int_{\Omega} \left( \nabla \bar{w}_x \right)^c : \sigma \, d\Omega = \int_{\Gamma} \bar{w}_x \cdot (\sigma \cdot n) \, d\Gamma , \]

\[ \int_{\Omega} w_p \left( \nabla \cdot v^\alpha \right) \, d\Omega - \int_{\Omega} w_p \sum_{\beta} \frac{D^\beta N^\beta}{Dt} \, d\Omega = 0 , \]

\[ \int_{\Omega} w^\mu \frac{1}{J} \frac{D^\beta N^\beta}{Dt} \, d\Omega + \int_{\Omega} \left( \sum_{\alpha} K_{\alpha \beta} \nabla \mu^\alpha \right) \cdot \nabla w^\mu \, d\Omega \]

\[ = \int_{\Gamma} w^\mu \left( \sum_{\alpha} K_{\alpha \beta} \nabla \mu^\alpha \right) \cdot \bar{n} \, d\Gamma , \]

\[ \int_{\Omega} \xi \left( \frac{1}{J} \sum_{\beta} \frac{z^\beta F D^\beta N^\beta}{Dt} \right) \, d\Omega = 0 . \] (39)

\[ \Gamma \]

The time discretization scheme can be varied easily from implicit Euler (\( \theta = 1 \)) to explicit Euler (\( \theta = 0 \)). The Newton-Raphson iteration procedure is used to determine a sequence of approximate solutions of the non-linear equations. Quadratic interpolation functions (\( \bar{\Phi} \)) are used for the position field and weighing function \( \bar{w}_x \). Linear interpolation functions (\( \bar{\Phi} \)) are taken for the discretization of the pressure, electrochemical potentials, electric potential, volume fractions and their corresponding weighing functions. The predictor is a set of linearized equations,

\[ \vec{\nu} = F_\Delta \cdot \vec{\nu}_n \] (41)

As the total deformation is divided, the volume ratio is divided in a similar way. From the definition of \( J \) it follows that,

\[ J = J_\Delta J_n \] (42)

Time discretization of the material time derivatives for \( J_\Delta \) and \( N^\beta \) yields,

\[ J_\Delta \left( \nabla \cdot v^\alpha \right) \quad \dot{J}_\Delta = \frac{J_\Delta - 1}{\Delta t} \] (43)

\[ \frac{D^\beta N^\beta}{Dt} = \frac{N^\beta - N^\beta_n}{\Delta t} \] (44)

For the mass balance a time discretization scheme is applied,

\[ \bar{X} = \theta \bar{X}(t_n + \Delta t) + (1 - \theta) \bar{X}_n \] (45)
permeability is

\[ S = \begin{bmatrix} \tilde{K}_{\alpha} & -\tilde{K}_{\beta} & 0 & 0 \\ -\tilde{K}_{\beta} & \tilde{K}_{\alpha} & 0 & 0 \\ 0 & 0 & \tilde{K}_{\alpha} & \tilde{K}_{\beta} \\ 0 & 0 & \tilde{K}_{\beta} & \tilde{K}_{\alpha} \end{bmatrix} \]

\[ L = \int_\Omega B_\psi^T \Psi d\Omega \]

\[ C_{\alpha\beta} = \sum_a \int_\Omega \frac{1}{J_{n+1}} \tilde{C}_{\alpha\beta} \Psi d\Omega \]

\[ \tilde{K}_{\alpha\beta} = \theta \sum_a \int_\Omega \frac{B_\mu^T K_{\alpha\beta} B_\mu}{\tilde{J}_{\Delta}} d\Omega \]

\[ R = \int_\Omega B_\psi^T \Psi d\Omega \]

\[ T_1 = \frac{1}{J_{n+1} T_{\Delta}} \int_\Omega \frac{B_\mu^T F}{\tilde{J}_{\Delta}} K_{\alpha\beta} B_\mu d\Omega \]

\[ Q_{\beta} = \int_\Omega \Psi \frac{1}{J_{n+1} \Delta} \Psi d\Omega \]

The matrices \( B_\psi \), \( B_\mu \), and \( B_\nu \) result from the linearisation of \( \Psi \), \( \Psi^T \), and \( J_{\Delta} \) respectively. The matrices \( B_\psi \) and \( B_\mu \) contain the derivatives of the quadratic and linear interpolation functions respectively. The column \( B_\nu \) also contains derivatives of the quadratic interpolation functions. For calculation a 27-node brick element is chosen with 3 displacements in every node. In each corner of the brick one pressure, 3 chemical potentials and an electric potential is calculated, resulting in a total of 121 degrees of freedom per element. The code is verified using analytical solutions of the linearised equations for a 1D medium subject to stepwise change in external salt concentration (van Meerveld et al.). The comparison is shown in fig. 8 for the chemical potential of the fluid and in fig. 9 for the electrochemical potentials of the cations and anions. The analytical solution is obtained by reducing the linearised equations to 3 diffusion equations. Both the numerical solution as the analytical solution clearly show two time constants, one for the diffusion of the ions and the other for the pressure diffusion. As an example of application we computed the response of a cylindrical mesh of ... elements subject to a stepwise change in external salt concentration (fig. 10). Diffusion coefficients of the ions are \( 5 \times 10^{-10} m^2 s^{-1} \). Hydraulic permeability is \( 1 \times 10^{-16} m^2 s^{-1} \). The formation factor is 0.4. Neo-Hookean material behaviour is assumed with a shear modulus of 0.06 MPa and a bulk modulus of 0.08 MPa. Initial fixed charge density is -0.2 Meq and an initial fluid volume fraction of 0.7.

A two way response is calculated; an initial shrink is followed by swelling to an new equilibrium.

The initial shrinking is associated with negative osmosis, because the outflux of ions towards the lowered external salt concentration convects water away from the sample, against the gradient of chemical potential of the water. Non-uniform electrical potential changes within the medium are computed. Present research has focussed on the experimental verification of the predicted electrical potential changes during swelling and consolidation. In order to have reproducible and homogenous samples, we do a good deal of the research on synthetic hydrogels which exhibit properties similar to biological tissues.

7 CONCLUSIONS

The application of porous media mechanics in biology is growing field of interest, that presents challenges which in many aspects are similar to those encountered in geomechanics. At small pore scales, electrical effects become very significant. However, the biological materials have a structure that is far more sophisticated. Hence, cooperation with many more disciplines is mandatory in order to cover the whole range expertise needed to address all issues. It is vital that the community of porous media experts contribute their share to understanding of our own bodies and those of other species.

8 ACKNOWLEDGEMENT

The authors acknowledge financial support from the Technology Foundation STW, the technological branch of the Netherlands Organisation of Scientific
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