THE ACOUSTIC RESPONSE OF BURNER-STABILIZED PREMIXED FLAT FLAMES

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The behavior of acoustically driven flat flames has been analyzed experimentally. In this study, pressure transducers and laser Doppler velocimetry are used to characterize the acoustical waves upstream and downstream of a flat flame stabilized on a flame holder. Two different flame holders have been used, a perforated brass plate and a ceramic foam, exhibiting very different surface temperatures. From these experiments, the acoustical transfer function can be derived. This transfer function shows a resonance-like behavior, of which the shape and peak frequency is governed mainly by the surface temperature of the burner and the velocity of the unburned mixture. The brass burner exhibits a resonance frequency around 140 Hz, where the resonance of the ceramic burner seems to have shifted to much higher frequencies and is much more damped. All results can be understood very well with an analytical model in terms of Zeldovich number, standoff distance, and heat conductivity. Apart from the analytical model for the brass flame holder, numerical simulations with detailed chemistry have also been performed. Again, the correspondence is good. The most interesting application is the acoustic behavior of central heating systems, in which these burners are frequently used. For the purpose of modeling the acoustical behavior of complete boiler systems, the analytical model can be used with minor adjustments to the Zeldovich number and heat conductivity, yielding a fairly accurate semiempirical model describing the transfer function.

Introduction

Since the introduction of low-NOx premixed burners with a large modulation range, severe noise problems hamper further developments of modulating domestic heating boilers. In recent years, a fundamental change has occurred in boiler design. In the past, the burner operated only at one or a few discrete thermal loads. Noise problems could be solved by trial and error methods. Current developments, however, use designs where the burner load is allowed to vary continuously. The major drawback of this approach is that many different acoustic instabilities are now triggered, and phenomena like high whistling noises and low vibrational excitations of the complete system occur.

To analyze these instabilities, the whole system must be considered. A way to do this is by using the transfer matrix method [1], in which every acoustic element is characterized by a (frequency-dependent) transfer matrix which couples the acoustic fields on both sides of the element. At relatively low frequencies (<1000 Hz), the acoustic field in a boiler system can be described as a planar wave of fluctuations in pressure ($p'$) and velocity ($u'$):

$$u = \bar{u} + u'$$

$$p = \bar{p} + p'$$

(1)

The average values of the velocity and pressure are denoted by $\bar{u}$ and $\bar{p}$, respectively.

For a burner-stabilized flat flame, it can be shown [2] that the transfer matrix has (in the approximation that the Mach number $Ma \rightarrow 0$) the following form:

$$ \begin{bmatrix} p' \\ u' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & T_{22} \end{bmatrix} \begin{bmatrix} p' \\ u' \end{bmatrix} $$

(2)

which means that the complete transfer matrix is determined by $T_{22}$, the coupling between the velocity fluctuations upstream ($u'_a$) and downstream ($u'_b$) of the flame.

Since the acoustical properties of flat burner-stabilized flames are important for practical applications, the availability of an (analytical) model describing the transfer function is needed. On the subject of acoustics and combustion, extensive theoretical and experimental studies have been performed; for an overview, see the paper by Raun [3]. Work on the forced oscillation of duct-confined free flames has been done by Fleifil et al. [4], while Searby [5] worked on spontaneous oscillations of duct-confined free flames. Dowling [6] also studied self-excitation, but of a duct-confined turbulent flame. Mitsui [7] worked on the forced oscillation of arrays of Bunsen flames, and Ducruix et al. [8] studied the forced oscillation of a single Bunsen flame.

The subject of this paper, forced oscillation of burner-stabilized flames, was covered by McIntosh...
and coworkers during several years’ time. This resulted in a rather complex but mathematically rigorous analytical model [9,10]. The basis of their analysis is formed by a one-step chemistry description of the combustion and an ideal flame holder, that is, a flame holder with an infinite heat transfer coefficient and an infinite heat capacity. Consequently, the flame holder has the same temperature as the upstream unburned gases. Their results were obtained via large activation energy asymptotics, coupling the different acoustic zones, and show good correspondence with experiments obtained in a Rijke tube burner. Recently, Rook and De Goey derived an alternative analytical model [11,12], which is also based on one-step chemistry and the infinite heat transfer assumption, but allows for the flame holder having a higher temperature than the gas upstream of the flame holder. Their analysis is based on the decoupling of the unsteady conservation equations into the G-equation, describing the motion of the flame, and a flamelet system, describing the inner structure and the mass burning rate. Additionally, it is assumed that the Lewis number is unity, which is of minor importance as long as methane/air flames are considered. Recent experimental work has been performed by Chen and Chen [13,14], but their focus was on the admittance of the burner and not on the transfer coefficient. Also, no direct comparison was made with a model.

As an intuitive model for the interaction between the flame and the acoustic waves, consider Fig. 1. Following the G-equation description, a variation in the standoff distance of the flame (induced by the acoustic velocity fluctuations) results in a displacement of the complete flame structure, except for the temperature. The temperature remains fixed at the flame holder. Due to a varying fuel concentration at the flame holder, the enthalpy varies with it. Thus, an oscillating flame position results in enthalpy waves traveling toward the flame front. When this wave has the right phase relationship with respect to the motion of the flame front, amplification or damping of the flame front motion can occur. Some kind of resonance behavior is then expected for certain frequencies. As an estimation for the lowest mode, consider the upstream flow velocity (on the order of 10 cm/s) and the standoff distance (on the order of 1 mm), yielding 100 Hz as an order-of-magnitude estimate for the position of the resonance. From this intuitive picture, it can be seen that one of the most important parameters is the surface temperature of the flame holder, since this determines the standoff distance to a great extent. Higher harmonics are not expected, since high-frequency enthalpy waves damp quickly due to diffusion.

In this paper, an experimental characterization of the $T_{22}$ element of the transfer matrix is presented for a burner-stabilized flat flame in a tube with fixed diameter. The diameter of the tube is chosen to be much smaller than the acoustical wavelengths of interest, and a one-dimensional description of the acoustics is valid. Two burner types are used: a perforated brass plate, closely resembling the ideally cooled burner as assumed in the model by McIntosh, and a ceramic foam plate, which has a significantly higher surface temperature. The results are compared to numerical calculations obtained with a model derived previously by Rook et al. [2,16], with the analytical model derived by McIntosh and Rylands [10], and with the simple analytical model derived previously by Rook et al. [11,12].

**Experiment**

The measurement of the transfer matrix element coupling the acoustic velocities before and after the flame-burner combination involves the time-correlated measurement of the velocity upstream and downstream of the flame. An established way of determining pressure waves inside a tube is by means of multiple pressure transducers fitted in the wall of the tube [1,15]. If the medium in the tube has constant properties (density and temperature), two microphones suffice to characterize the waves traveling in both directions. From the pressure wave and the properties of the medium and the tube, the velocity wave can be reconstructed.

The upstream region of the flame does have the desired constant density and temperature, but the downstream region does not, because the hot gas cools down rapidly. Therefore, we have chosen to use the two-microphone method in the upstream region, but a direct measurement of the velocity in the downstream region by means of laser Doppler velocimetry (LDV). A difficulty that arises when using
as entrance for the premixed CH$_4$/air mixture. Some is closed with a flange, in which a small hole serves tube with a diameter of 5 cm (Fig. 2). The bottom length of the tube below the flame holder is approximately part of the tube are (thermostatically) cooled. The total diameter, and the flame holder. The flame holder and the upper transducer, the laser beams of the laser Doppler velocim-
ter, and the flame holder. The flame holder and the upper transducer, the laser beams of the laser Doppler velocim-
tion of the tube itself have been studied, and it was found that when the tube is excited by broadband noise, while some frequencies are coupled in more effectively than others, no clear resonance shows up. Since measurements are only performed at a single frequency, this can be compensated for by an adjustment of the amplifier gain. The absence of any influence of the tube characteristics on the transfer coefficient is supported by a measurement without flame, which (within the experimental error) yields unity for all frequencies used.

Two materials are used for the flame holder. The first is a perforated plate made of brass with a thickness of 2 mm. The perforation pattern is hexagonal, with a hole diameter of 0.5 mm and a pitch of 0.7 mm. The hole size is small enough that a flat methane/air flame stabilizes on top of it. The other is made of a ceramic foam, with a porosity of 90% and an average pore size of 45 pores/in. The ceramic foam is used in commercial applications.

To allow for the use of the two-microphone method, two pressure transducers are mounted on the wall of the tube. Optical access to the downstream region of the flame is somewhat difficult, since the flame holder is placed 7 cm before the open end. Three small holes have been made in the downstream part of the burner. Their size is small enough not to influence the acoustical wave in the tube. Two holes serve as an entrance for the two LDV laser beams, and through the third hole the scattered laser light is detected. In this way, the velocity is measured in the middle of the tube at 4 mm above the flame holder.

In principle, one does not measure the transfer matrix element of the flame in this way, but the transfer matrix element of the flame combined with the flame holder. Test measurements showed, however, that the transfer matrix element of the flame holder itself is very close to unity for all materials at the frequencies of interest and can be neglected.

The influence of the surface temperature of the burner, however, cannot be neglected. Since the flame is burner stabilized, some heat loss will occur via the flame holder.

For the perforated brass burner, the temperature is mainly determined by the temperature at the edge. Heat loss from the flame to the burner is conducted away via the flame holder. This also means that the temperature in the middle of the flame holder will be somewhat larger than at the edge. To calculate this radial dependence of the temperature ($T(r)$), consider the equation describing the heat

\[ \frac{\partial T}{\partial r} + \frac{2}{r} \frac{\partial T}{\partial r} = \frac{\partial^2 T}{\partial r^2} \]

For the burner essentially consists of a 50 cm long tube with a diameter of 5 cm (Fig. 2). The bottom is closed with a flange, in which a small hole serves as entrance for the premixed CH$_4$/air mixture. Some grids are fitted right after the entrance to settle the flow. In the lower part of the tube, a hole is made in the side and coupled with a flexible hose to a loudspeaker. The top is an open end, to allow the exhaust gas to escape. The open end acts nearly as a perfect reflector with, in first the order, a velocity antinode at the opening. A small portion of the acoustic energy, however, is radiated out, and a small transfer region exists in which the velocity fluctuations decrease strongly from the values associated with the wave inside the tube to the values outside the tube. The length of this region is approximately equal to the radius of the tube. The burner plate is placed approximately 7 cm below the exit, at sufficient distance from the transition zone. The part of the tube downstream of the flame holder and the flame holder itself are water cooled at nominally 50 °C to avoid condensation of water. The acoustical properties of the tube itself have been studied, and it was found that when the tube is excited by broadband noise, while some frequencies are coupled in more effectively than others, no clear resonance shows up. Since measurements are only performed at a single frequency, this can be compensated for by an adjustment of the amplifier gain. The absence of any influence of the tube characteristics on the transfer coefficient is supported by a measurement without flame, which (within the experimental error) yields unity for all frequencies used.
flow through the flame holder in cylindrical coordinates,

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} T \right) = - \frac{\rho u_c}{d} (T_{ad} - T_b)$$

in which $d$ is the thickness of the flame holder, $\lambda$ the effective heat conductivity of the flame holder in the radial direction, $\rho$ the density, $u$ the flow velocity of the flame of interest, $c_p$ the (mixture-averaged) heat capacity, and $T_{ad}$ and $T_b$ the temperatures of an adiabatic flame and the flame of interest, respectively. The temperature $T_b$ can be derived from the well-known empirical relation between the burning velocity and the flame temperature using data of an adiabatic flame \cite{17},

$$u = K \exp \left( - \frac{E_a}{2RT_b} \right)$$

with $E_a$ the effective activation energy.

When temperature variations perpendicular to the flame holder are neglected and a perfect cylindrical symmetry is assumed, a solution to equation 3 can be written as

$$T(r) = T(R) + (R^2 - r^2) \frac{Q}{4\pi d}$$

This solution is based on the boundary condition that the temperature at the edge ($r = R$) of the flame holder ($T(R)$) is fixed and known. For our perforated brass burner, it turns out that $T(0) - T(R)$ is approximately 40 K higher than at the edge. The non-uniform temperature distribution across the flame holder requires, in principle, a two-dimensional treatment, but since the downstream velocity is measured very close to the flame holder, we will assume a surface temperature ($T_{surf}$) equal to the temperature directly below the measurement point, that is, $T_{surf} = T(0)$.

For the ceramic burner, another mechanism comes into play for accommodating the heat loss from the flame to the burner. Since the conduction coefficient of the ceramic material is very low, the heat can only be radiated away. A reasonable estimate of the surface temperature can be obtained by equating the heat loss of the flame to the burner (right-hand side of equation 3) to the heat loss of the burner via radiation ($\varepsilon \sigma (T_{surf}^4 - T_{surr}^4)$). When using equation 4 to calculate $T_b$, this yields for the surface temperature \cite{18}

$$T_{surf}^4 = T_{surf}^4 + \frac{\rho u_c T_{ad}}{\varepsilon \sigma} \times \frac{2RT_{ad} \ln \left( \frac{u}{s_1} \right)}{2RT_{ad} \ln \left( \frac{u}{s_1} \right) - E_a}$$

with $\varepsilon$ the emissivity of the flame holder, $\sigma$ the Stefan-Boltzmann constant, and $T_{surf}$ the temperature of the surroundings.

### Analytical Models

The experimental results are compared to two analytical models. McIntosh et al. \cite{10} derived a model based on an exact asymptotic solution of the governing equations with one-step chemistry. When inserting the condition that $Le = 1$, the following expression can be derived for the transfer coefficient \cite{19}:

$$\frac{u_b}{u_a} \approx 1 + \frac{1}{3} \frac{T_b}{T_a} \left[ 1 + r \left( 1 - \frac{T_a}{T_b} \right) \exp \left( - \frac{1}{2} + r x_{1,f} \right) \right] \times r \exp \left( - 2x_{1,f} \right) + \frac{w}{\theta (1 - T_a/T_b)}$$

where

$$r = \sqrt{w + \frac{1}{4}}$$

In this equation, $x_{1,f}$ is the mass-weighted standoff distance or adiabaticity,
\[ x_{1f} = \ln \left( \frac{T_{ad} - T_u}{T_{ad} - T_b} \right) \] (8)

with \( T_u \) the temperature of the unburned gases. The dimensionless complex frequency \( \omega \) and the dimensionless activation energy \( \theta \) are given by

\[ \omega = \frac{i \omega}{u_b^2 \rho c_p} \quad \theta = \frac{E_a}{R T_b} \] (9)

When deriving this model, it was assumed that \( T_{surf} \) is equal to \( T_u \), but this does not reflect the experimental conditions. At first glance, the model could easily be adjusted by stating \( T_u = T_{surf} \), but a closer examination leads to the conclusion that in fact a complete workover of the model would be required. We choose not to adjust this model, and this needs to be kept in mind when comparing the experimental results to the models.

The analytical model derived by Rook et al. [11] is based on a \( G \)-equation description, yielding for the transfer coefficient

\[
\frac{\tilde{u}_b}{\tilde{u}_u} = \frac{T_{surf}}{T_u} + \frac{T_b - T_{surf}}{T_u} A(\hat{\phi}) \\
+ \frac{T_{ad} - T_u}{T_u} \exp(-\Psi) \\
\times \frac{1}{2} \left( 1 + \sqrt{1 + 4 \hat{\phi}} \right) \frac{1 - A(\hat{\phi})}{\hat{\phi}} \] (10)

with

\[ A(\hat{\phi}) = \frac{MN}{MN + \hat{\phi}} \]

\[ M = \frac{Ze}{2} \frac{1}{T_b - T_u} \]

\[ N = \frac{(T_{ad} - T_u)}{T_u} \exp(-\Psi) \]

\[ \Psi = \ln \left( \frac{T_{ad} - T_u}{T_{ad} - T_b + T_{surf} - T_u} \right) \] (11)

The frequency \( \hat{\phi} \) is defined as \( \hat{\phi} = \omega T_{surf} \). The dimensionless standoff distance \( \Psi \) is given by

The frequency dependence follows the argument in the introduction.

**Results**

In Fig. 3, the frequency dependence of the transmission coefficient is plotted for the perforated brass flame holder with \( \phi = 0.8 \) and \( \tilde{u}_u = 14 \text{ cm/s} \). One can clearly see the resonance at about 150 Hz. For low frequencies, the absolute value of the transmission coefficient tends to a value around 7, which corresponds to the stationary limit, where \( \tilde{u}_b \sim (T_b/T_u) \tilde{u}_u \). For higher frequencies, the transmission coefficient drops to values around 1. The correspondence to the simulation is good. Both the phase and the absolute value of the transmission coefficient are quantitatively close to the measured values. This result means that for low frequencies, a flat flame is inherently unstable with regard to acoustical problems. When mounted in a system, the damping of low frequencies needs special attention.

The position of the resonance is mainly governed by the standoff distance of the flame. A straightforward way to vary the standoff distance is by varying the temperature of the flame holder. Two temperature settings were achieved by varying the temperature of the cooling water which flows through the ring around the brass flame holder. Another (high) temperature is obtained by using the ceramic foam. In Fig. 4, the results are plotted.

Even for the relatively small temperature difference of 40 K, a significant shift of the resonance frequency is noticed. For the temperature of \( T_{surf} = 578 \text{ K} \), the resonance seems to be absent, but this can be regarded as being moved so far into the high-frequency region that its magnitude is severely decreased. This is supported by an inspection of the phase of the transfer coefficient (Fig. 7). This behavior follows the argument in the introduction where a higher resonance frequency is expected for shorter standoff distances.
A more complex way of changing the acoustical behavior is to change the flow velocity. In the case of the brass flame holder, experiments have been carried out for four different values of the flow velocity at $\phi = 0.8$ and with the lowest possible cooling water temperature of 50 °C (Fig. 5). In the figure, the results of both analytical models are also plotted. One can see that for both of them, the correspondence is reasonable, but the model by McIntosh underestimates the resonance frequency and the model of Rook lacks magnitude. The underestimation of the resonance frequency by the model of McIntosh is probably due to the assumption of an ideally cooled surface, which is 100 K lower than the real surface temperature. As explained above, we did not adjust $T_u$ and $\bar{u}_u$ to account for the presence of a warm flame holder.

In Fig. 6, two results for the ceramic foam flame holder are presented for values of $\bar{u}_u = 10$. The experimental results of all velocity settings below the adiabatic velocity of 23.9 cm/s are nearly indistinguishable. The model of Rook shows good correspondence for low frequencies, but overestimates the high-frequency behavior.

As noted before, the values of some quantities used in calculating the analytical results are subject to uncertainty. If, for example, the heat conductivity is decreased by a factor of 2, the model of McIntosh shows nearly perfect agreement with the measured results. Also for the model by Rook, small changes in the value of $Z_e$ result in nearly perfect agreement, as shown in Fig. 7. Although adjusting $Z_e$ seems questionable, one has to take into account that the model by Rook assumes perfect heat transfer between the flame holder and the unburned gases. Since $Z_e$ expresses the sensitivity of the flame to the enthalpy wave emerging from the flame holder, $Z_e$ can be adjusted to accommodate non-perfect heat transfer. This opens the way to accurately describing the acoustical behavior of flat flames with a semiempirical model. For this practical purpose, the model by Rook seems to be versatile, since it can accommodate elevated burner surface temperatures.

**Conclusions**

Experiments have been carried out to determine the transfer coefficient between the acoustic velocity...
upstream and downstream of a burner-stabilized flat flame. Downstream of the flame, LDV has successfully been used to measure the acoustic velocities, while the upstream acoustic amplitudes have been measured with pressure transducers. The results show a clear resonance type of behavior for a flat flame, with amplification factors as high as 18. The resonance depends strongly on the surface temperature of the burner and on the flow properties. For shorter standoff distances (i.e., higher flame holder temperature), higher resonance frequencies are observed. The resonance frequency takes on lower values for low mass flow; increases for medium mass flows, but rapidly decreases again near blowoff. The region where amplification occurs extends to about 300 Hz, indicating that flat flames are inherently unstable in this region, with more dramatic effects for cooled surface burners.

The correspondence to numerical simulations performed with a model developed by Rook et al. [2,16] is good, both qualitatively and quantitatively. The correspondence to analytical models is reasonable, but can be made nearly perfect by adjusting one or two constants within physical limits. This opens the way to describe the acoustical properties of an anchored flat flame with a semiempirical model.

REFERENCES

COMMENTS

Sébastien Ducruix, Laboratoire EM2C–CNRS, France. Can the curve giving $|u|/|k|$ as a function of the frequency be considered as the amplitude of the flame transfer function? In this case, could the peak observed at 100 Hz be seen as the result of an amplification mechanism? Resonance would then be obtained in the whole system for a proper match of the different time delays.

Author’s Reply. This curve is indeed the amplitude. The peak is the result of an amplification mechanism formed by the feedback of heat generated by the flame via the flame holder to the unburned gas.