Tests of the geometrical description of blood vessels in a thermal model using counter-current geometries

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Abstract. We have developed a thermal model, for use in hyperthermia treatment planning, in which blood vessels are described as geometrical objects; 3D curves with associated diameters. For the calculation of the heat exchange with the tissue an analytic result is used. To arrive at this result some assumptions were made. One of these assumptions is a cylindrically symmetric temperature distribution. In this paper the behaviour of the model is examined for counter-current vessel geometries for which this assumption is not valid. Counter-current vessel pairs intersecting a circular tissue slice are tested. For these 2D geometries vessel spacing, tissue radius and resolution are varied, as well as the position of the vessel pair with respect to the discretized tissue grid. The simulation results are evaluated by comparison of the different heat flow rates with analytical predictions. The tests show that for a fixed vessel configuration the accuracy is not a simple decreasing function of the voxel dimensions, but is also sensitive to the position of the configuration with respect to the discretized tissue grid.

1. Introduction

For the prediction of the local thermal effects of large blood vessels in local hyperthermia, discrete modelling of these vessels is needed (Lagendijk 1982, Kolios et al 1995). Numerical models that describe the thermal effects of vessels individually have been developed and used by, among others, Lagendijk et al (1984), Mooibroek and Lagendijk (1991), Chen and Roemer (1992), Rawsnsley et al (1994) and Chan (1992). Earlier (Kotte et al 1996) we presented a discrete vessel thermal model in which vessels are described as geometrical objects. These geometrical objects exist apart from the finite-difference grid describing the tissue and employ an analytic expression for the calculation of the heat exchange with the tissue. In this model vessels are free to curve and to branch. Advantages of the geometrical description of the vasculature are the independence from the tissue grid (one vessel description suffices for all tissue resolutions) and the high level of freedom it offers to the vessels.

The analytical result used to calculate the tissue–vessel interaction is derived assuming thermally developed flow in the vessel and a cylindrically symmetric stationary temperature distribution around the vessel. It was shown that the model gives highly accurate results if conditions conform to the assumptions made in the calculation of the tissue–vessel interaction: straight vessels embedded in tissue cylinders, modelled at high resolution (Kotte et al 1996). The accuracy of the model was also investigated for resolutions low compared with the vessel diameter and for curved vessels (Van Leeuwen et al 1997). In the present paper the attention is focused on violation of another assumption. Here,
the accuracy of the model for non-cylindrically-symmetric temperature distributions is investigated. This is done using counter-current vessel geometries. For close vessels the deviation from cylindrical symmetry in the surrounding tissue temperature distribution can be pronounced. This fact, and the availability of analytical results (see e.g. Baish et al 1986b, Wissler 1988, Zhu and Weinbaum 1995) make this geometry a suitable test site. Furthermore, the occurrence of counter-current vessel pairs in tissue (Weinbaum et al 1984) and their suggested important role in the thermal behaviour of vascularized tissue (Chato 1980, Weinbaum and Jiji 1985) adds relevance to this geometry.

For the model tests in this paper a two-dimensional set-up is used. The reason is that we are particularly interested in the behaviour of the vessel–tissue and vessel–vessel heat transfers, which take place in 2D. The accuracy of modelling the axial heat transport in the vessel has already been investigated (Van Leeuwen et al 1997) and is not an issue here, nor is the axial conduction in the tissue. Apart from reducing the simulation times, using a 2D set-up has the advantage that the analytical results used in the comparisons do not involve the extra approximations that are used to derive analytical 3D results.

2. Methods

2.1. Thermal model description

A brief description of some of the key concepts of the model relevant to this study is given here. For a more comprehensive description of the model we refer to the article by Kotte et al (1996).

The conductive heat transport in the tissue is accounted for using the finite-difference method. Convective heat transport by the blood is modelled by description of the thermal effects of individual vessels. The discrete vasculature is described by a number of vascular trees. These arterial or venous trees are made up from vessel segments, which are defined as vessel sections between branching points. The segments are described using parametrized curves in 3D and a diameter. The axial temperature profile along the segment is described by a finite number of samples. These samples are stored in lists of so-called buckets, separate from the tissue grid. Each bucket describes a sub-segment of a vessel, holding the temperature and taking care of the interaction of the blood sample with the surrounding tissue. The bucket temperature represents the local vessel mixing cup (velocity weighted average) temperature. For the calculations the bucket must know its position in space, the vessel radius, the volume flow, and the blood density, blood specific heat and blood thermal conductivity.

The calculation of the tissue–blood heat transfer is performed outside the finite-difference scheme. This heat flow rate is estimated using the analytic result derived for a vessel embedded in tissue with cylindrically symmetric boundary conditions and uniform power distribution. For such a vessel the stationary heat flow rate density at the vessel wall is given by (Kotte et al 1996)

$$\phi|_{r=r_{ves}} = \frac{1}{2k_{tis} / Nu k_b + \ln(R/r_{ves})} \left\{ k_{tis} \left( T_{tis}(R) - T_{ves} \right) + P r_{ves} \left( \frac{1}{4} \left( \frac{R}{r_{ves}} \right)^2 - 1 \right) \right\}.$$

This expression neglects axial conduction in the tissue as well as in the blood. $k_{tis}$ and $k_b$ are the thermal conductivities of tissue and blood, $r_{ves}$ is the radius of the vessel, $R$ ($> r_{ves}$) is the radial distance to the vessel axis, $P$ is the local absorbed power density as
delivered by the hyperthermia system and $T_{\text{ves}}$ is the vessel mixing cup temperature. The Nusselt number, $Nu$ describes the heat transfer in the vessel, from vessel wall to mixing cup temperature.

$$\phi = -\frac{Nu k_b}{2r_{\text{ves}}} (T(r_{\text{ves}}) - T_b) \left( \text{J s}^{-1} \text{ m}^{-2} \right).$$  \hspace{1cm} (2)

For the implementation of the heat flow rate estimation and the heat exchange itself, two sets of voxels are employed by the vessel (see figure 1). For each bucket, the set of heat exchange voxels comprises all voxels that have their centres inside the bucket or (for small vessels) contain part of the heart line of the bucket, and neighbour at least one voxel of surrounding tissue. The estimation set consists of the voxels in the surrounding tissue that neighbour at least one exchange set voxel, and do not belong to the exchange set of any other vessel.

For each voxel in the estimation set an estimate for the heat flow rate density at the vessel wall can be calculated using (1). These estimates are averaged and multiplied by the bucket surface to yield the heat flow rate into the bucket. In case of a cylindrically symmetric temperature distribution around the vessel all estimates in a radial plane will be approximately equal and accurate. For an asymmetric distribution, however, both the estimation of the heat flow rate using (1) as well as the process of averaging the heat flow rates may be inaccurate. From the heat flow rate and the length of the time step the change in bucket temperature can be calculated. Of course the program must also account for the heat exchange in the tissue. This is done using the voxels of the exchange set, each of which loses the same amount of heat. This rather bold distribution of the heat flow rate over the tissue may cause further inaccuracies, for both asymmetric and symmetric temperature distributions. It is these inaccuracies due to the assumptions and simplifications in the calculation of the tissue–vessel heat exchange and its effectuation in the tissue that are investigated in this paper. The flow of the blood is accounted for by interpolation of the tissue–blood heat flow rates between consecutive buckets and translation of the vessel temperature profile. The accuracy of this scheme for modelling the axial heat transport in the vessel, however, is not an issue here.

It is again stressed that the vessel temperatures are maintained in bucket structures that are additional to the 3D tissue grid. Voxels on vessel locations in the grid (in many cases exchange set voxels) do not represent the blood temperatures. These voxel temperatures just suit the heat flow rates between vessel and neighbouring surrounding tissue voxels.

2.2. Simulation set-up and analysis

The temperature distribution around a blood vessel is not cylindrically symmetric if there is a counter-current vessel in the vicinity (figure 2). For equal (with respect to vessel radius and blood velocity) counter-current vessels crossing a plane with an isothermal circular boundary an analytic solution for the stationary temperature distribution in the plane can be obtained. The temperature distribution is a superposition of a distribution with equal vessel (mixing cup) temperatures and a distribution with opposing vessel temperatures and tissue wall temperature zero (see figure 12). The solutions for both distributions are derived (Baish et al 1986b) using four equivalent point heat sources/sinks. In these solutions the temperature is taken to be constant over the circumference of each vessel wall. After this approximation the heat transfer in the vessels themselves is described by a fixed Nusselt number. This approach is quite close to the approach used in our computer model, but may result in inaccurate results for vessel pairs that are closely spaced. For closely spaced
vessels we will therefore employ a second analytic description for the comparison with our numerical model (see section 3.4).

For given relative dimensions of vessel radius, vessel spacing and tissue radius the total heat flow rate into the two vessels is proportional to the difference between average tissue temperature and the average of the two vessel mixing cup temperatures. Consequently a so-called conduction coupling constant (also known as shape factor) $\sigma_\Sigma$ can be defined (Baish et al 1986b) (see (3) below) where $\Phi'$ denotes the heat flow rate per unit length into a vessel (W m$^{-1}$) and $\langle T_{\text{tis}} \rangle$ is the average temperature in the tissue disc. Analogously a coupling constant $\sigma_{\Delta}$ can be defined that describes the heat exchange between the two vessels (4). Since the analytical values of $\sigma_\Sigma$ are derived using an approximation for the calculation of the average tissue temperature, we prefer to use for our model tests the alternative coupling constant $\kappa_{\Sigma}$, as defined by (5), the calculation of which does not involve this approximation.

$$\Phi'_{\text{vein}} + \Phi'_{\text{art}} = k_{\text{tis}} \sigma_\Sigma (\langle T_{\text{tis}} \rangle - (T_{\text{art}} + T_{\text{vein}})/2)$$

$$\Phi'_{\text{vein}} - \Phi'_{\text{art}} = 2k_{\text{tis}} \sigma_{\Delta} (T_{\text{art}} - T_{\text{vein}})$$

$$\Phi'_{\text{vein}} + \Phi'_{\text{art}} = k_{\text{tis}} \kappa_{\Sigma} (T_{\text{tis, wall}} - (T_{\text{art}} + T_{\text{vein}})/2).$$

The values of the coupling constants are easily found, both analytically (Baish et al 1986b, see the appendix) and from the simulations. From the errors in the simulated coupling constants it is possible, with the substitution of the Nusselt relation in (4) and (5), to estimate the errors in the simulated implicit vessel wall temperatures. For all simulated geometries the relative (with respect to the driving temperature difference) errors in the vessel wall temperatures are smaller than the relative errors in the coupling constants. For example, the worst case $\sigma_{\Delta}$ is about 3.0. For a pure dipole temperature distribution $\Phi'_{\text{vein}} = -\Phi'_{\text{art}}$, so

$$\sigma_{\Delta} = \frac{\text{Nu} k_b \pi (T_{\text{art, wall}} - T_{\text{art}})}{k_{\text{tis}} (T_{\text{vein}} - T_{\text{art}})}.$$

Using a Nusselt number of 4.0, it can be seen that for $\sigma_{\Delta} = 3$ the difference between the arterial wall temperature and the mixing cup temperature is approximately 0.25 of the temperature difference between vein and artery. To first order the relative error in the vessel wall temperatures is therefore approximately one-quarter of the relative error in the coupling constant $\sigma_{\Delta}$. For some tissue voxels close to the vessels the error may be larger.

**Figure 1.** Two vessels with associated sets. The estimation voxels are used for the calculation of the tissue–vessel interaction. In the tissue the heat exchange is effected in the exchange voxels.
than this; close to the tissue boundary the errors in the tissue temperatures will again be small, because of the proximity of the fixed temperature boundary and the small temperature gradient. The point is that generally for most of the tissue the accuracy of the simulated temperatures is (much) better than the accuracy of the coupling constants. This observation and the physical significance of the coupling constants for the thermal behaviour of the vascularized tissue make the accuracy of the coupling constants a good means of testing the thermal model.

As stated before, the analytical solution was derived under the assumption that the temperature is constant over the circumference of the vessel walls. In our model, in calculating the vessel–tissue interaction a cylindrically symmetric temperature distribution is assumed in deriving (1), but there is no explicit wall temperature. The approximation of constant wall temperature is replaced by the less severe approximation of identical heat loss in all the exchange set voxels.

Since the analytical results are 2D the simulations were performed using just one slice in a 3D volume. Both coupling constants for a given counter-current geometry can be found from just one simulation run. For instance, defining the tissue circular boundary temperature as zero and choosing the vessel inflow temperatures 0 and 1 K will yield a stationary temperature distribution that is a superposition of two distributions with equal and opposing vessel temperatures respectively that are both non-trivial, i.e. not constant everywhere. The heat flow rates into the vessels were determined using the outflow temperatures of the vessels.

The geometry of the modelled system is characterized by three parameters: the tissue radius, the vessel radius and the vessel spacing. A further parameter influencing the accuracy of the simulations is the resolution. With respect to the accuracy only three of these four parameters are independent in the 2D counter-current tests: doubling all geometrical parameters as well as the voxel dimensions will result in exactly the same simulated coupling constant. We performed tests in which vessel spacing, tissue radius and resolution were varied. For very close vessels the interaction between vessels may depend significantly on the positions of the vessels with respect to the tissue grid. Therefore the behaviour of the model was examined in more detail for close vessels, also varying the position of the model geometry with respect to the discrete grid.

Figure 2. Simulated geometry: equal counter-current vessels in a tissue cylinder.
3. Results

3.1. Vessel spacing

First the dependence of the accuracy on vessel spacing was investigated. The tissue cylinder was chosen to have a diameter of 20 mm while in this set of simulations the vessel inner diameter was 0.5 mm. We varied the spacing between one-fifth of the vessel diameter (close counter-current vessels) and 6.5 mm. The voxel x and y dimensions were 0.5 mm, equal to the vessel diameter. The centre of the tissue cylinder coincided with the centre of the central
Figure 5. Temperature profile on line through both vessels. The wall temperatures according to the program are indicated by open symbols.

 voxel, and the discretization of the tissue was such that the two main axes of the planar grid were respectively parallel and perpendicular to a line segment connecting the vessel axes. As the thermal model gives the freedom to position vessels anywhere with respect to the grid, the spacing was, in particular for close vessels, slowly varied. For each spacing the coupling constants $\sigma_1$ and $\kappa_5$ were determined. The results are shown in figure 3. The shape of the graphs that depict the results from the simulations is quite remarkable, but can be explained. Both graphs show regularly spaced discontinuities. These can be attributed to transitions in the exchange and estimation sets of the vessels (see figure 4). For given exchange and estimation sets both simulated coupling constants have a minimum value generally close to the spacing for which the vessels are positioned central in their respective voxels. The behaviour around this minimum is essentially quadratic. An explanation is found in the manner in which the heat flow into the vessel is calculated and the withdrawal of heat from the tissue is effected. If a vessel is placed away from its voxel centred position the distance to one of the neighbouring estimation set voxels will decrease, leading to a larger estimate for the local heat flow rate density (1). To first order in the displacement from the voxel centre this increase in heat flow is countered by a decrease in the estimates for the heat flow rate density for the other estimation set voxels. However, the terms that are of second order in the displacement yield a net increase in average heat flow rate density. The calculated heat flow that must be withdrawn from the tissue is removed in the single exchange set voxel. Because there is no specifically larger heat flow to the vessel from the most nearby estimation set voxel, the temperature of this voxel will remain relatively high, sustaining an increased heat flow into the vessel. This is expressed by higher coupling constants for pairs of vessels that are eccentrically located in their respective voxels. For vessels with a diameter smaller than the voxel size this effect will be less pronounced, due to the relatively smaller differences in heat flow rate estimates for the different estimation set voxels.
If the results from the simulations are compared with the analytical curves it is seen that generally they agree quite well: errors are typically just below 2%. For small vessel spacings, however, there are considerable differences between simulated and analytical coupling constants. This is especially true for \( \sigma_1 \), which describes the heat transfer between the vessels. In section 3.4 the modelling of closely spaced vessels as well as the analytical calculation of the mutual interaction will be further investigated. The on average slightly too low simulated coupling constant \( \kappa_2 \) for large spacings is caused by the discretization of the tissue, which also accounts for longer equilibration lengths for single vessels (see Kotte et al. 1996).

For two different spacings stationary temperature distributions were determined in order to relate the accuracy of the tissue temperatures to the accuracy of the coupling constants. Mixing cup temperatures were set to \(-1\) and \(1\) K with respect to the reference temperature on the tissue outer boundary, a situation described by coupling constant \( \sigma_\Delta \). In figure 5, depicting the temperature profile on a line going through both vessels, a distinctive property of our model can be noticed: the temperatures of the (exchange) voxels at the vessel locations are different from the vessel temperatures. The simulated vessel wall temperatures are in very good accordance with the analytically determined values. As anticipated the error in the wall temperatures relative to the driving temperature difference \((T_{\text{art}} - T_{\text{vein}})\) is small with respect to the relative error in the coupling constant \( \sigma_\Delta \). For the closely spaced pair the simulated \( \sigma_\Delta \) was 4.5% smaller than the theoretical value of 1.69 (A14). The difference between the simulated and theoretical vessel wall temperatures was just 0.013 K, a mere 0.7% of the temperature difference between artery and vein. Just outside the vessel a maximum temperature difference between simulation and theory of 0.08 K was recorded.
3.2. Resolution

The influence of the resolution on the accuracy was investigated in similar fashion. For different vessel spacings simulations were performed using various resolutions. The diameter of the vessel was again 0.5 mm. The voxel y and z dimensions were varied between 0.25 and 1.5 mm. As was shown in the previous subsection the position of the vessel within a voxel does matter. Therefore each geometry was simulated twice at each resolution, the difference between the two simulations being a shift of the discretization origin over half the voxel length in the direction of the line connecting the two vessels. For the chosen spacings this meant that often the vessel axes were located either in a voxel centre or on a voxel boundary. The results are shown in figures 6 and 7.

For the voxel sizes smaller than the vessel diameter the accuracy is good to very good. For voxels approximately the size of the vessel diameter the results do depend considerably on chance, in the sense that a different origin for the discretization, or a slightly different vessel geometry (see also figure 3) can result in a significant change in the simulated interactions. This change can be as much as 30% in the heat exchange between close vessels (figure 7). This is not unexpected. For two vessels that are so close that the exchange voxels are adjacent, our methods of estimation and implementation of the heat exchange are rather crude. As the temperatures of the exchange voxels do not represent actual temperatures, our program excludes exchange voxels of neighbouring vessels from the estimation sets. As a consequence the heat flow estimates into each one of the two close vessels are based on tissue temperatures in all directions except the one in which the neighbour vessel is to be found. Nonetheless, for two close vessels with a large temperature difference, the conductive heat flow in the tissue grid will largely take place directly between
the exchange voxels. So due to our unconventional approach the direction of largest heat flow is not considered here in the analytical estimate of the heat flow, which makes this mechanism of heat exchange quite unrefined.

For the lowest resolutions it is even possible that closely spaced vessels pass through one or more of the same voxels and consequently share some exchange set voxels. This is allowed in the model but highly accurate results may not be expected. Nonetheless, the results were quite good for the tested geometries with identical exchange sets. In fact the simulated heat exchange between the two vessels was significantly better than obtained using the same resolution but with the different origin of discretization. Yet, the estimates of the heat flow rates into each vessel are based on four temperatures at locations on a circle that surrounds both vessels. Consequently the accuracy of the heat exchange observed in these simulations is largely coincidental. Also it must be noted that the analysis of the relation between the accuracy of the local temperatures and the accuracy of the coupling constant given in section 2.2 loses some of its validity. For large vessel spacings the accuracy of the model at low resolution is still moderate to good.

3.3. Tissue radius

For a number of vessel spacings the tissue radius was varied. In these sets of simulations the vessel diameter was 0.5 mm, as was the size of the voxels. It was found (see figures 8 and 9), that reducing the tissue radius does not really have a profound effect on the accuracy of the simulations as long as the radius is sufficiently large compared to the vessel spacing and the resolution.

3.4. A detailed study of close vessels

In the vicinity of close counter-current vessels there are large differences in conductive heat flow that occur on a length scale that is small with respect to typical voxel dimensions. Therefore it can be expected that for close vessels the results of simulations will depend considerably on the exact geometry. To see how much the accuracy of the results can be affected two sets of simulations were carried out: one set featuring vessels of fixed diameter and fixed spacing in which the resolution was varied, another set at fixed resolution and fixed axis to axis vessel spacing in which the vessel diameter was varied.

Before the results of the simulations are discussed we will make a second stab at calculating $\sigma_1$. In the analytical calculation of the heat exchange between two vessels using the method indicated in section 2.2 it is assumed that the temperature is constant over the circumference of each vessel wall. This assumption will have a significant impact on the calculated interaction if the vessels are very closely spaced. The modelling of the tissue boundary on the other hand can expected to be not really critical. Therefore for close vessels the coupling constant $\sigma_1$, which describes the mutual heat exchange between two vessels, was also calculated using the solution given by Zhu and Weinbaum (1995). In this solution heat sources are placed at the vessel centres, and the temperatures of the vessel walls are not constant over the circumference. The considered tissue geometry however is somewhat different from that simulated, the vessel pair being part of a 1D array of counter-current pairs in a thin tissue slab. If the distance between vessel pairs is chosen to be $15r_{ves}$ and the distances to the (isothermal) slab top and bottom surfaces are chosen as $20r_{ves}$, the coupling constant $\sigma_1$ calculated from the temperature distribution is, over a range of not extremely small spacings ($> 0.5$ mm), equal to the $\sigma_1$ calculated according to (A14). For smaller spacings however significantly smaller coupling constants are found. As the assumptions
underlying this solution correspond better to the reality this solution is considered to be the most accurate in this regime.

For two vessels with a diameter ($\phi$) of 0.5 mm and an in between spacing of 0.1 mm the coupling constants were determined from temperature distributions calculated using different resolutions. As in section 3.2, for each coupling constant the value was determined for two different origins of the discretization. These were chosen so that in the second simulation all grid points were shifted one half voxel length parallel to the line connecting the vessels.

It is seen (figure 10) that for the high resolutions the simulated heat exchange between the vessels increases considerably with decreasing spacing, although not as much as predicted by the analytical solution ($\sigma_x \approx 15\%$ less). For voxel dimensions comparable to the vessel diameter the result is sensitive to the exact geometry and the simulation of the coupling constant can yield values as much as 35% below the analytical value. For low resolution the coupling constant hardly depends on the spacing and is quite consistently much too low.

In a second set of simulations the positions of the vessel axes were fixed, while the vessel diameter was varied. In all the simulations the axes of both the vessels ran straight through a voxel centre. The extra interest in this type of simulation originates from the larger likelihood for a vessel segment produced by our vessel network generation program (Van Leeuwen et al 1995) to pass through the centre of a voxel. In the simulations, with voxel $x$ and $y$ dimensions 0.5 mm, the vessel axis to vessel axis distances were either 0.5 or 1.0 mm. The vessel diameters ranged from 0.2 mm up to 0.45 and 0.9 mm respectively (almost touching). As can be seen in figure 11 the heat exchange between vessels that are positioned in neighbouring voxels is underestimated by 25% up to 45% for the largest vessel diameter. The underestimation can be explained qualitatively by the fact that the heat flow for each vessel is estimated on the basis of three surrounding tissue temperatures with

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**Figure 8.** The tissue radius dependence of the coupling constant $\kappa_x$. 
disregard of the large temperature gradient in the direction of the other vessel (see 3.2). For vessels positioned 1.0 mm apart there is one ‘free’ tissue voxel between the vessels which is part of the estimation sets of both vessels. For this larger spacing the heat exchange is modelled much more accurately; for $\sigma_1$ a maximum deviation from the analytical value of about 7% is found for the largest modelled vessel diameter.

As before, the orientation of the discretization grid was such that the two main axes of the planar grid were respectively parallel and perpendicular to the line connecting the two vessel axes. If the grid is rotated by 45° with respect to the centre of the tissue cylinder while the vessel positions remain fixed, the modelled situation stays the same but the two vessels are now placed in voxels that are positioned on one diagonal of the tissue grid. To see how the heat exchange between the vessels is affected by the resulting changes in exchange and estimation sets a number of simulations were carried out for closely spaced vessels with a vessel diameter of 0.5 mm and same voxel size. The coupling constant $\sigma_1$ was found to vary very little for different vessel spacings that have identical exchange and estimation sets (not pictured). As a result the simulated $\sigma_1$ does not reproduce the analytical spacing dependence of $\sigma_1$ on a length scale smaller than the voxel size. Other than that the results of the simulations do agree quite well with the analytical results.

4. Discussion and conclusions

The tests discussed in this paper deal with the method of modelling the thermal effects of individual blood vessels using geometrical objects presented by Kotte et al (1996). The geometrical vessel representation offers a very high level of freedom in the spatial definition of vascular structures. This freedom results partly from the fact that the vessels are
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Figure 10. The coupling constant $\sigma_A$ for close counter-current vessels with $\phi = 0.5$ mm at different resolutions.

represented independently from the finite-difference grid which describes the temperatures in the tissue and in which the conductive heat transport is realised. Calculation of the vessel–tissue interaction is a responsibility of the vessel structures, which must of course sample surrounding tissue temperatures in the finite-difference grid. Ergo, the resolution at which the tissue is modelled is involved in the accuracy of the simulations in two ways: in the description of the conductive heat flow and in the calculation of the vessel–tissue heat exchange. Naturally, spatial dependences of the temperature distribution on a scale smaller than the grid voxel size will not always be modelled accurately. This however, is not necessarily a problem; spatial deviations in the temperature on a sub-resolution length scale are often of not much interest. If, on the other hand, the simulated heat flow into the vessels is systematically incorrect this might result in significant errors. An incorrect vessel–tissue heat exchange is particularly likely if the temperature distribution around a vessel is not cylindrically symmetric, as it is for counter-current vessels. The test simulations described in this paper show the sometimes limited accuracy for close counter-current vessels, and give some insight into the factors influencing this accuracy.

For voxel dimensions smaller than the diameter of the modelled vessels the error in the simulated heat exchanges for the vessels was found to be a few per cent or less, except for very small spacings. For larger voxel sizes the accuracy is not only less, but also more dependent on the exact model geometry. Because of the way in which the vessel–tissue interaction is calculated not only the parameters describing the vessel configuration, but also the position and orientation with respect to the discretization grid, are important. For instance, the error in the simulated heat exchange between two close vessels can be tens of per cent for one set-up, while after a translation of the tissue grid by half a voxel a simulation of the same vessel configuration yields a highly accurate answer. For even larger voxel sizes the accuracy is not dramatically poorer, albeit that the errors tend to be more persistent.
Generalizing, the calculation of the heat exchange between tissue and vessel in situations with non-cylindrically-symmetric boundary conditions can result in serious inaccuracies. These inaccuracies arise under conditions in which the length scale of the heat flow variations is small in relation to the voxel dimensions. Specifically this can occur for close counter-current vessels. If the deviation from the cylindrically-symmetric distribution is caused by another, less local source, the accuracy of the simulation will be higher. This is indicated by the good accuracy of the results obtained for two not very closely spaced vessels.

One might consider to improve the model accuracy for close counter-current vessels by using adjusted values for the Nusselt number to compensate partially for systematic errors. However, the fact that the simulation error in the heat exchange between two very close vessels is both difficult to predict and generally different in sign from the error in the heat exchange between the vessels and the tissue makes this approach seem not very promising. Another way to better the accuracy might be the design of a new geometrical object specifically for the description of counter-current vessel segments. In analogy with the single-vessel case, analytical results (Baish et al. 1986b, Zhu and Weinbaum 1995) must be employed to calculate the heat flow rates. For the heat exchange between two close vessels this does not look to be very complicated. On the other hand, the calculation of the heat exchanges with the tissue, and the realization of these heat exchanges in the tissue, is not straightforward. Furthermore, the description of complicated arterio-venous networks (partly) using these counter-current vessel objects might prove difficult.

In the evaluation of predictions made by the model it must be realized that the inaccuracies introduced by the model in the description of the discrete vessels are often not the only inaccuracies. In the clinic many relevant parameters are not precisely known and/or change during hyperthermia treatment. This will often seriously limit the achievable precision of the model predictions. Increasing the resolution of a simulation, at the cost of expanding calculation time, will be of little use as soon as highly inaccurate or incomplete
input data sets are the dominant source of errors. In particular, 3D imaging techniques are not sufficiently powerful to enable the construction of vessel networks describing the patient’s complete thermally significant vasculature. Consequently the thermal effects of the missing vessels ($\phi \lesssim 1$ mm) have to be described in an alternative way. The aptness of this alternative description will also influence the accuracy of the model predictions. The two principal thermal effects of the missing vessels that must be accounted for are the completion of the thermal equilibration of the arterial blood (Pennes 1948), and the apparent increased thermal conductivity due to the net heat transport by blood flowing in opposing directions with different temperatures (Chen and Holmes 1980, Weinbaum and Jiji 1985).

In the theoretical study of these alternative (continuum) descriptions a discrete vessel model can be used (cf Huang et al 1996).

Vessel networks that are constructed to develop heat transfer theory may or may not be intended to give a realistic description of an actual anatomy. Especially in the case of rudimentary artificial vessel configurations it might be possible to minimize simulation errors by positioning the modelled geometry optimally with respect to the finite-difference grid. As the model predictions need only be related to the completely known input data sets, the accuracy of correlations found between these basal vessel configurations and the thermal behaviour is here solely determined by the accuracy of the thermal model. On the other hand, in the numerical study of the thermal behaviour of realistic vascularized tissue the realism of the results will depend also on how well the modelled parameters reflect actual tissue. One of the important parameters for counter-current vessels is the vessel spacing. The strong influence of this spacing is demonstrated in the spacing dependence of the coupling constants (figure 3). These describe the respective heat exchanges in simple counter-current geometries and can be instrumental in calculating an effective thermal conductivity coefficient (Baish et al 1986a). The vessel spacing also influences the accuracy of the simulations, small spacings being the most difficult to model accurately. The usefulness of the model is therefore dependent on actual vessel spacings. Geometrical vessel data can be found in the work of Weinbaum and co-workers (Weinbaum et al 1984, Zhu et al 1995). For example, typical values for a first-generation artery–vein pair in rat cremaster muscle are: artery diameter 120 $\mu$m, vein diameter 200 $\mu$m and inter-vessel distance 25 $\mu$m (Zhu and Weinbaum 1995). For these small spacings the model can easily underestimate the heat exchange between the vessels. In the simulations this will lead to slower thermal equilibration with the tissue of the arterial blood as well as to a higher thermal effective conductivity. The realism of the simulations is furthermore influenced by the branching parameters of the modelled vessel networks. In particular, if the thermal effects of the blood are modelled exclusively through the discrete vessels, the number of vessel generations is important. Insufficient detail in the model vasculature will result in incomplete thermal equilibration, and consequently a violated heat balance, as well as possibly a lower thermal effective conductivity. This can be a serious problem, because although generation by computer facilitates the construction of highly detailed artificial vessel networks (Van Leeuwen et al 1995), in practice the amount of detail in the artificial vasculature is inconveniently restricted by the available computer memory.

In conclusion, although precise quantitative results are not viable for all realistic vasculatures, the geometric description of the vessels provides an accuracy for the thermal simulations that is good enough for the thermal model to be a very valuable tool.

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Appendix

This section presents the results of Baish et al (1986b) for the counter-current coupling constants, and adds the result for the new coupling constant $\kappa_\Sigma$. No more explanation is given than is necessary to deduce the analytical result for $\kappa_\Sigma$ given the results for the conventional coupling constants $\sigma_\lambda$ and $\sigma_\Sigma$.

The stationary temperature distribution for two vessels crossing a plane with an isothermal circular boundary can be decomposed into two distributions, one with both the vessels at the mean of the actual vessel temperatures, and one with the two vessels having the same temperature difference between them (see figure A1). The sum and the difference of the two vessel mixing cup temperatures are denoted by $\Sigma$ and $\Delta$ respectively. The temperature is assumed to be constant over the circumference of each vessel wall. The stationary temperature distributions for the two separate problems are both approximated by superpositions of four line sources: two inside and two outside the tissue disc. The positions and strengths of the sources can be chosen so as to nearly comply to the boundary conditions. The isothermal condition on the tissue outer boundary is satisfied but the vessel wall temperatures aimed at are only produced at the points where the vessel wall intersects the line through the vessel axes. The isotherms through these points are not exactly circular, so the boundary conditions on the vessel walls are only approximately satisfied. The strengths of the sources inside the tissue cylinder yield the heat flow rates $\Phi'$.

$$\Phi'_\text{vein} + \Phi'_\text{art} = k_{\text{tis}} \Gamma_\Sigma (T_{\text{tis,wall}} - \Sigma_{\text{wall}}/2)$$
$$= k_{\text{tis}} \kappa_\Sigma (T_{\text{tis,wall}} - \Sigma/2)$$
$$= k_{\text{tis}} \sigma_\Sigma ((T_{\text{tis}}) - \Sigma/2)$$
$$\Phi'_\text{vein} - \Phi'_\text{art} = 2k_{\text{tis}} \Gamma_\Delta \Delta_{\text{wall}}$$
$$= 2k_{\text{tis}} \sigma_\Delta \Delta.$$

The intermediate coupling constants $\Gamma_\Sigma$ and $\Gamma_\Delta$ are given by

$$\Gamma_\Sigma = 4\pi \left[ \ln \left( \frac{C_1^2 C_3^2 / C_2^2 - C_1^2}{C_2^2 - C_3^2} \right) \right]^{-1}$$

and

$$\Gamma_\Delta = \pi \left[ \ln \left( \frac{(C_2 - C_1^2/C_3)(C_2 + C_3)}{(C_2 + C_1^2/C_3)(C_2 - C_3)} \right) \right]^{-1}$$

with the constants $C$ given by

$$C_1 = r_{\text{tis}}/r_{\text{ves}}$$
$$C_2 = d/(2r_{\text{ves}})$$
$$C_3^2 = \frac{C_1^4 + C_2^2(C_2 + 2)^2}{2[(C_2 + 1)^2 + 1]} - \frac{1}{2} \left( \frac{C_1^4 + C_2^2(C_2 + 2)^2}{[(C_2 + 1)^2 + 1]} \right)^2 - 4C_1^4$$
$$C_4^2 = \frac{C_1^4 - 4C_1^2 + C_2^2(C_2 + 2)^2}{2[(C_2 + 1)^2 - 1]} - \frac{1}{2} \left( \frac{C_1^4 - 4C_1^2 + C_2^2(C_2 + 2)^2}{[(C_2 + 1)^2 - 1]} \right)^2 - 4C_1^4.$$

If the origin of the coordinate system is chosen between the vessels and the vessel centres are located on the $x$ axis, the locations of the line sources/sinks pertaining to the symmetric boundary value problem are given by $x = \pm C_3 r_{\text{ves}}$ (sources) and $x = \pm r_{\text{tis}}/(C_3 r_{\text{ves}})$ (sinks). For the anti-symmetric boundary value problem the locations are $x = \pm C_4 r_{\text{ves}}$ and $x = \pm r_{\text{tis}}/(C_4 r_{\text{ves}})$ (alternating sources–sinks).
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Figure A1. Stationary temperature distribution as a superposition of distributions with two vessels at equal and vessels at opposing temperatures.

Using the Nusselt relation
\[ \Phi_{ves} = \text{Nu} k_b \pi (T_{ves,wall} - T_{ves}) \] (A11)
a relation between \( \kappa_\Sigma \) and \( \Gamma_\Sigma \) can be obtained.

\[ \kappa_\Sigma = \Gamma_\Sigma \left[ 1 + \frac{\Gamma_\Sigma}{2\pi} \left( \frac{k_{tis}}{k_b\text{Nu}} \right) \right]^{-1} \] (A12)

Employing this relation and (A6) yields the desired \( \kappa_\Sigma \). To find an expression for \( \sigma_\Sigma \), the average tissue temperature must be determined. An estimation is made assuming that the tissue area outside the vessel pair is much larger than the area ‘enveloped’ by the pair: \( r_{tis}^2 \gg (d/2 + 2r_{ves})^2 \). Employment of the cylindric solution with \( T(r_{tis}) = T_{tis,wall} \) and \( T(d/2 + 2r_{ves}) = \Sigma_{wall}/2 \) in the region outside this enveloped area yields the needed estimation of the average tissue temperature.

\[ \sigma_\Sigma = \Gamma_\Sigma \left[ 1 + \frac{\Gamma_\Sigma}{2\pi} \left( \frac{k_{tis}}{k_b\text{Nu}} - \frac{1}{2} \right) \right]^{-1} \] (A13)

The remaining coupling constant \( \sigma_\Lambda \) can be obtained from (A7) by use of the Nusselt relation.

\[ \sigma_\Lambda = \Gamma_\Lambda [1 + 2\Gamma_\Lambda k_{tis}/(\pi k_b\text{Nu})]^{-1} \] (A14)

References

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