Task Specific Uncertainty Estimation in Dimensional Metrology

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Abstract

With the acceptance of ISO standard 14253-1 [1], the requirement for the traceability and uncertainty in dimensional measurements is extended from the measuring instruments to artefacts or industrial products which are measured with those instruments. For the calibration and uncertainty estimation of length standards and instruments, various methods are available for which usually an uncertainty budget according to the GUM [2] can be set up. However, especially where many measured data are involved, such as in CMM measurements, but also in typical dimensional geometry measurements such as roughness, roundness and flatness measurements, setting up an uncertainty budget according to the GUM for each measurement can be tedious and is a rather impossible demand.

In this paper it is shown that the ‘mainstream’ GUM-uncertainty budget can be modified in several ways to allow for more complicated measurements. This can lead to a measuring instrument where, next to the measured value, also the uncertainty is displayed. As an example of how measured deviations can be treated and what difficulties this may give the treatment of measured scale deviations is briefly given.

Mainstream GUM approach

The term ‘mainstream GUM’ was used in a paper on Monte-Carlo methods [2], to characterise the method which is described in the main text of the GUM in contrast to its alternatives. This approach is repeated here briefly. In general, a measurement result \( Y \) is a function of \( n \) input quantities \( x_i \) (\( i = 1, 2, \ldots, n \)). This leads to the general functional relationship, known as the ‘model function’ [1]:

\[
Y = f(x_1, x_2, \ldots, x_n) \tag{1}
\]

The model function incorporates the measurement and the calculation procedure. It can be an analytical function, but also a complicated, iterative, computer algorithm. The measurement data \( x_i \) can be estimated either by a type A (statistic) approach or by a type B approach which accounts for all kind of known and unknown systematic deviations and uncertainties.

However this grouping does not influence the uncertainty evaluation; it is just essential that a standard uncertainty \( u_{x_i} \) is attributed to any influencing quantity \( x_i \).

The quantity \( Y \) is best approximated by using the best approximations for \( x_i \), which are usually the measured data, in equation (1). Now, the uncertainty \( u_Y \) can be written as:

\[
u_Y^2 = \sum_i \left( \frac{\partial f}{\partial x_i} \right)_y^2 \cdot \langle \Delta_i^2 \rangle + 2 \cdot \sum_{i,j} \frac{\partial^2 f}{\partial x_i \cdot \partial x_j} \cdot \langle \Delta_i \cdot \Delta_j \rangle \tag{2}\]

where \( \Delta_i \) and \( \Delta_j \) are the deviations from the true value of \( x_i \) and \( x_j \) respectively; and \(<=>\)
denotes the average over a large ensemble. The squared expected deviation of \( x_i \), \( <\Delta_i^2> \), is known as the variance, which is the square of the standard uncertainty \( u_i \). So \( u_i^2 = <\Delta_i^2> \). The product \( <\Delta_i \cdot \Delta_j> \) is known as the covariance of the deviations in \( x_i \) and \( x_j \). In the case of uncorrelated measurement data \( x_i \) and \( x_j \), equation (2) reduces to:

\[
\sum_i \left( \frac{\partial f}{\partial x_i} \right)_Y \cdot u_{x_i}^2 \]

\( \text{(3)} \)

### Direct variational calculation

The calculation of the derivative can be replaced by two evaluations of the model function \( Y \), which is equivalent in first order. The correspondence with Monte-Carlo methods which are discussed later is illustrated by rewriting equation (3) as:

\[
\sum_i \left( f(x_1, \ldots, x_i + r \cdot u_{x_i}, \ldots, x_n) - Y \right)^2
\]

where \( r \) is a random number with average 0, a standard deviation of 1 and an absolute value of 1, so \( r = \pm 1 \). This probability distribution is known as the bi-modal distribution which we introduce here to illustrate the consistency between the mainstream GUM approach and the Monte-Carlo technique. An example of its use is given in [4] where it is explained how the effect of the uncertainty in the probe diameter \( R \) in a roughness measurement on the uncertainty in the roughness parameter \( Ra \) is calculated.

### Monte Carlo method

With a Monte-Carlo method, parameters are varied simultaneously in some random manner. As for a specific measurements problem there are many ways of doing this, one can only speak of ‘a’ Monte Carlo method. If we define a random number \( r \) as having an average of 0 and a standard deviation of 1, so \( <r> = 0, <r \cdot r> = \delta_{ij} \) (\( \delta \) being the Kronecker \( \delta \)-symbol with \( \delta_{ij} = 1 \) for \( i=j \), otherwise 0) and \( <r^2> = 1 \), where \(<>\) denotes the average over a large ensemble, then we can simulate a measurement result by varying all input quantities at a time, as following:

\[
Y_k = f(x_1, r_1 \cdot u(x_1), x_2 + r_2 \cdot u(x_2), \ldots, m_N + r_N \cdot u(x_N))
\]

the estimate for the standard uncertainty becomes:

\[
u(y) = \sqrt{\frac{1}{K} \sum_{k=1}^{K} (Y_k - Y)^2}
\]

(6)

If one takes \( K=n \) and \( r_k = \delta_{ik} \) then equations (5) and (6) (without dividing by \( K \)) become identical to (4) which is equivalent to (3). This illustrates the consistency between the methods. The main difference is that, where all parameters are varying simultaneously in equation 5, the factor \( 1/\sqrt{K} \) in the calculation of \( u_M \) corrects for that. Next to estimating the standard uncertainty by (6), the Monte-Carlo method also enables the calculation of distributions and percentile points (confidence intervals) for \( Y \), depending on the distributions of the input parameters \( x_i \) [3]

### Geometrical deviations along an axis

A major source of errors and uncertainties in dimensional metrology are linear, rotary and straightness deviations along one axis. A dimensional measurement usually
consists of a combination of measurements along axes: from a simple length measurement along one axis, fulfilling the Abbe-principle, up to a full 3-D measurement in which 3 orthogonal machine-axes and all associated deviations are involved. We consider a measurement along one axis where at each point the axis deviation is considered. For the general case we can rewrite (2) as:

\[ u^2_y = \sum_i \left( \frac{\partial f}{\partial x_i} \right)^2 \cdot AC(0) + 2 \cdot \sum_{i \neq j} \left( \frac{\partial^2 f}{\partial x_i \partial x_j} \right) \cdot AC(x_j - x_i) \]  

(7)

Here the deviations \( \Delta \) denote the - known or unknown - scale deviations. We have used \(<\Delta,\Delta> = <\Delta_i,\Delta_i(\Delta_j-\Delta_i)> = AC(x_j-x_i)\), where \( AC(x) \) is the autocorrelation function of the deviations of the scale reading \( x \). This means that, when making Monte-Carlo simulations of measurements along a scale, the autocorrelation of the scale deviations must be maintained. This is equivalent to maintaining the power spectral content, and with this automatically the variance is maintained. This can be achieved by randomising the phases in the Fourier spectrum and transforming back [4]. For uncorrelated noise-like deviations the second term in (7) vanishes and (5) can be used straightforward.

What makes this problem less straightforward is that for a dimensional measurement, e.g. of a length, the second measurement \( x_2 \) cannot have any position along the axis once \( x_1 \) is fixed: with an axis length of \( L \) and a measurement length \( l \) the domain for \( x_2 \) is restricted to \( L - l \). This restricts the possible summations in both terms of (7). We will illustrate this for a simple length measurement along an axis with coordinates \( x_i \). For the length of an object \( l = x_2 - x_1 \), the uncertainty is, according to (7):

\[ u^2(l) = \frac{1}{N - n} \left( \sum_{i=1}^{N-n} \Delta x_i^2 + \sum_{i=n}^{N} \Delta x_i^2 - 2 \cdot \sum_{i=1}^{N-n} \Delta x_i \cdot \Delta (x_1 + l) \right) \]  

(8)

Where \( N \) is the number of measurement points \( x_i \) in the domain \( L \) and \( n \) is the number of measurement points in the domain \( L - l \). An application is illustrated in figure 1, where the length deviations of a CMM-axis are given as they are measured using a step gauge. Figure 2 gives the uncertainty in a length measurement as it is calculated from equation (8). This figure gives much more useful information than when just the standard deviations in both measurements \( x_1 \) and \( x_2 \) are added quadratically which gives here a standard uncertainty of 1.6 \( \mu \)m. Figure 2 illustrates that for measuring small sizes the uncertainty can be much smaller. When simulating these scale deviations in a Monte-Carlo manner, precautions must be taken as to maintain the autocorrelation as it is written in (8), taking into account the limited scale length and the limited range in which it can be used for a specific length measurement. The simplest way of doing this is inverting and/or flipping the deviations as depicted in figure 1; however this gives only four possible simulations. For the Fourier-analysis and phase-randomisation method [4-7], especially edge-effects must be avoided as illustrated in [4]. Note that these considerations apply to any measurement instrument which is calibrated at a number of points in a certain range. Equations (7) and (8) also hold for straightness and rotational errors of axes. For a full rotation, giving a roundness deviation, the restricted domain is not a problem.
In this case the phase-randomisation method can be used or the roundness deviation can be rotated over a random angle to obtain a randomised deviation which can be added to the measured data to obtain an uncertainty in some parameter.

References