The influence of polarization states on non-linearities in laser interferometry

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Abstract

With use of an ellipsometric measurement the polarization state of several optical components used in interferometry was measured. A model of a heterodyne laser interferometer was developed, enabling to predict the non-linearities resulting from non-ideal polarization optics.

Introduction

The principal limitation of laser interferometer systems for achieving (sub-)nm uncertainties in small displacements is in the photonic noise and in the residual non-linearities which are inherent of periodic interferometer signals and which repeat each fraction of a wavelength. These deviations are both present in the phase measurement system of the interferometer itself, but they can also result from the optics: beam splitters, retardation plates, and corner cubes can influence the polarization state in interferometers working with polarizing optics, or influence the contrast in interferometers working with non-polarizing optics. Here we deal with a heterodyne interferometer using polarizing optics.

Heterodyne laser interferometry

In figure 1 a schematical representation is given of a heterodyne laser interferometer using differential optics. A heterodyne interferometer consists of a laser source with two orthogonal polarized beams with a different frequency ($f_1$ and $f_2$). The reference signal (RS) consists of an interference signal of the initial frequency difference: $\Delta f_{R} = f_1 - f_2$. The measurement is done by splitting the two frequencies using a polarising beam splitter (PBS). One frequency is reflected by the reference mirror ($M_R$) and the second frequency is reflected from the measuring mirror ($M_m$). Since the measurement mirror is moving an extra frequency shift is gained, resulting in a phase shift in the interference signal (MS).

The principle of splitting frequencies by polarization splitting can result in non-linearities as was already published by [eg. 1, 2, 3, e.a.] when this splitting is not perfect.

Characterization of components

In the heterodyne setup polarization mixing results in frequency mixing. This mixing can occur in all polarizing and non-polarizing components. Therefore it is essential to characterize the polarization mixing of each component.
This is done by an ellipsometric setup as shown in figure 2. Here we assume no depolarization in the optical component. Further the assumption is made that the Jones representation of the component is a diagonal matrix of the form:

\[
T_{OS} = A_O \begin{bmatrix} 1 & 0 \\ 0 & T_O e^{i\delta_O} \end{bmatrix}
\]

With \( A_O \) the amplitude attenuation of the entire optical system, \( T_O \) the amplitude difference between the \( o \) and \( e \) polarization and \( \delta_O \) the phase retardation between the two polarization components.

From the laser source circular polarized light emerges through a polarizer with azimuth angle \( P \), a compensator of the Babinet Soleil type enabling a phase difference of \( \delta_C \), azimuth \( C \), both in front of the optical system and a polarizer with azimuth angle \( A \) behind the optical system. By finding a set of azimuth angles for polarizer, compensator and analyser (\( P, C \) and \( A \)) such that the light flux falling on the detector is extinguished, the Jones matrix of the optical system can be resolved. This results in the following formula [4]:

\[
T_O e^{i\delta_O} = \tan \psi e^{i\Delta} = -\tan A \left[ \frac{\tan C + T_C e^{i\delta_C} \tan(P - C)}{1 - T_C e^{i\delta_C} \tan C \tan(P - C)} \right]
\]

Where \( T_C \) is the amplitude attenuation of the two polarization directions of the compensator. A fourth variable that can be adjusted is the relative phase retardation of the compensator (\( \delta_C \)). Since the resolution to which this parameter can be measured compared to the azimuth angles is magnitudes poorer this is not done. In stead we use four zone averaging. The compensator is fixed to an azimuth of \( \pm \pi/4 \) and the relative phase retardation is fixed to \( \delta_C = \pm \pi/2 \). With \( T_C = 1 \), for each fixed azimuth of \( C \) two combinations of \( P \) and \( A \) result in an intensity minimum. Four zone averaging consists of taking the mean of these angles resulting in the two ellipsometric angles \( \psi \) and \( \Delta \). The advantage of this method is that any imperfections of the ellipsometer other than entrance and exit window birefringence are cancelled out. Therefore in our setup all ellipsometer imperfections are cancelled out.

**Measurement results**

Since the method assumes a compensator azimuth angle of \( \pm 45^\circ \) with the axis of the optical component under testing the optical system was aligned before measurement. The extinction ratio of polarizer and analyser combination was better than 1:100 000. The reproducibility of the measurement was tested using a polarising beam splitter in the setup shown in figure 2.
The results are shown in figure 3, the standard deviation for $\psi$ was 0.02°, and the standard deviation for $\Delta$ was 0.25°.

With this knowledge some measurements were done for a polarizer and a quarter wave plate under different angles around the pitch axis in figure 2. The different angles were chosen to investigate the influence of on axis rotational misalignment. For each angle 2 measurements were made, except for 0°, there 4 measurements were made. The results are shown in figure 4.

![figure 3: Reproducibility of the ellipsometric measurements](image1)

The left figure shows a minimal angle $\psi$ of 1.84°, representing an amplitude leakage of 3% in the beam splitter for the transmitted polarization component. The phase shift of the polarising beam splitter depends on the angle of the beam splitter. Therefore also the resulting polarization state after the beam splitter depends on this alignment.

In the right figure the ellipsometric angles of a quarter wave plate are shown, the leakage is less than 0.2%, however the phase difference between fast and slow axis is in a normal alignment -97.85° instead of the expected −90°. From the figure it can be seen that not only the rotational alignment around the optical axis influence the polarization state of the output, but also the alignment.

**Resulting non-linearities**

With use of Jones Matrices a modular model was built up in Matlab© in which a simulation can be made of the non-linearities resulting from non-ideal optical components, or misaligned optical components, non-ideal input parameters (like non-orthogonality of the laser beams or elliptical polarized laser beams). The model was tested using an experimental setup. Standard deviations of measurements compared to this model were 0.2 nm [5].
Simulations done with this model on a deviation equal to the standard deviation with which the optics could be measured resulted in a non-linearity of $4 \times 10^{-4}$ nm. Based on the previous measurements the calculated non-linearities of the measured optics are shown in figure 5.

![Image of figure 5: Non-linearities resulting from deviations in the polarising interferometer optics (5a) and from an additional error in the laser source polarization (5b).]

From the figure 5a can be seen that the non-linearities resulting from the deviations in the optics alone have a maximal amplitude of 0.67 nm. Addition of possible laser source ellipticity (1/4000 in intensity) and non-orthogonality (0.1°) result in a non-linearity shown in figure 5b.

**Conclusion**

With use of null-ellipsometry the deviations from ideal optical components were measured with a standard deviation of 0.02° for $\psi$ and 0.25° for $\Delta$, representing a negligible non-linearity of $4 \times 10^{-4}$ nm. The resulting non-linearities from these measured deviations were calculated using a Jones matrix formalism. The results show a minimal non-linearity in the flat mirror optics of 0.67 nm.

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**References**


