Calibration of a 2-D grid using 1-D length measurements

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Abstract
In this paper a calibration method is described for two-dimensional grid intersections using one-dimensional distance measurements. The evaluation of the measurements requires a non-linear least-squares approach for which an iteration schema is developed. The type-A uncertainties are estimated using a Monte-Carlo method. It is shown that for the distances in the grid the same uncertainty is achieved as the uncertainty in the measurements along one line. As an example a measurement of a ball-plate is described with which an uncertainty of 0,35 μm has been achieved.

Keywords: Dimensional measurements, Least-squares method, grid, ball-plate

1. INTRODUCTION
Two-dimensional grids are in use to check or to calibrate coordinate measuring instruments, such as Coordinate Measuring Machines (CMM’s), and measuring microscopes. In the case of measuring microscopes this consists of a glass plate on which a grid is applied. In the case of a CMM this consists of a ball plate of which only the x-y coordinates of the balls are important. The calibration of such a grid can be carried out by a swing-around method using a 2-D or 3-D CMM. In this case mainly the reproducibility of the CMM measurements limits the accuracy.

In this paper a method is described in which only 1-D measurements are required. Distances in 1-D can usually be measured more accurately than coordinates in 2 or 3 dimensions.

2. MEASUREMENT PRINCIPLE
The mutual distances between the grid points are measured in the x direction, the y direction and along two diagonal directions. Figure 1 gives the lay-out for measurements on a 6 x 4 grid. In this example, 68 measurements are taken from which 48 coordinates are to be derived. The method in which this calculation is carried out is given in section 3.

Fig. 1 Example of a measured grid. The arrows indicate the measurement directions
3. EVALUATION METHOD

The appropriate method to derive point coordinates from the method is the least-squares method. With this method, the squared sum of differences between measurements and the derived quantities is minimized.

Assuming a standard deviation of unity in the measurements, the 'squared sum' $Q^2$ can be written as:

$$Q^2 = \sum_{i=1}^{k} \left[ (x_a - x_b)^2 + (y_a - y_b)^2 - m_i \right]^2$$

where $m_i$ is the measured distance $n^i$ between the points with coordinates $(x_a, y_a)$ and $(x_b, y_b)$, where $a$ and $b$ change with $i$, and $k$ is the total number of measurements. Putting the derivative of $Q^2$ to $x_a$ (or any other coordinate) equal to zero does not lead to a set of linear equations. Therefore, the rigorous method for linear least squares problems cannot be used here.

To come to an iterative procedure which minimizes equation (1), one point is considered which is surrounded by 8 neighbouring points. 8 measurements are available giving the distance between the point and its neighbours. This is illustrated in figure 2.

Assuming that all neighbouring points are fixed, equation (1) reduces to:

$$Q^2 = \sum_{i=1}^{8} \left[ (x_i - x)^2 + (y_i - y)^2 - m_i \right]^2$$

The problem can be further reduced considering the two measurements in the x-direction: $m_1$ and $m_5$. In this case, a small variation of $x$ has a much larger influence on $Q^2$ than a small variation of $y$. Neglecting the terms $(y_1 - y)^2$ and $(y_5 - y)^2$ in $Q^2$ and putting $dQ^2/dx$ equal to zero leads directly to the solution:

$$x = \frac{m_1 + m_5}{2}$$

This means that the optimum position of the solution is in-between the points which are predicted from the right and the left. So it is the average of the positions determined by $m_1$ and $m_5$. The same applies for the y-direction considering measurements $m_3$ and $m_7$. Also, this applies for the diagonal directions when the coordinate system is rotated 45°. This consideration leads to the following definition of the new position $(x, y)$:

The best estimate of the coordinates $x$ and $y$ are the averages of the 8x2 coordinates which are determined taking each measurement into account and the coordinates of the surrounding point to which the distance has been measured.
The iteration procedure consists of adjusting all the positions \((x,y)\) and repeating this for the complete grid until all
the positions satisfy the above condition. Then, also the squared sum \(Q\) from equation (1) reaches a minimum.
When this procedure is carried out in practice, the following points are to be noted:
1. All points are given nominal values as starting points.
2. The measurements in the x-direction predict an unchanged y-coordinate and v.v. This acts as an
underrelaxation effect and does not affect the final solution.
3. The points at the edges and the corners of the grid are treated in an analogous way.
4. After each iteration in which all grid points are adjusted, the orientation is recovered by transforming the
coordinates such that the lower-left point is \((0,0)\) and the y-coordinate of the lower-right point is 0.
5. The iteration is repeated until the relative change in \(Q^2\) is less than \(10^{-4}\) between successive iterations.
The minimum value of \(Q^2\) is called the Chi-square value \(\chi^2\). When the standard-deviation of the
measurements is unity, the expected value of \(\chi^2\) will be about \((2n-k-3)\), where \(n\) is the number of grid-points. Each
coordinate can be considered as an average of \((2n/k)\) measurements. This enables us to make an estimate of the
standard deviation \(s_1\) in the distance between two neighboring points:

\[
s_1 = \sqrt{\frac{\chi^2}{2n - k - 3}} \cdot \frac{2n}{k}
\]

However, it is more interesting to have an uncertainty estimate for a distance between two arbitrary points in the
grid or for each coordinate. The method in which this is achieved is outlined in the next section.

4. UNCERTAINTY ESTIMATION

4.1 Estimation method

The influence of a known uncertainty of the measurements on the final results is estimated by a Monte
Carlo method. This is carried out as follows. First, nominal values of all coordinates are assumed. From these
assumed coordinates simulated measurements are derived which, in first instance, can be regarded as 'ideal'
measurements:

\[
m_i = \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2}
\]

where \(m_i\) is the distance between point \(a\) and \(b\). These measurements are ideal because they will give a \(\chi^2\) value
(see eqn. 1) equal to zero. The basic simulation method is that these measurements are randomized by adding a
Gaussian distributed random number \(r\) with zero mean and a unit variance multiplied by the input uncertainty \(s_i\):

\[
m_i = \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2 + r \cdot s_i}
\]

This method is modified to give a closer correspondence with the measuring process. The measuring
process is different in the sense that measurements along one line are taken and that each measurement consist
of measuring the distance to a hypothetical zero-value which is the same for each line. E.g. when the
measurements of 4 points along one line in the x-direction are simulated, the derived measurements are:

\[
\begin{align*}
m_1 &= (x_2+r_2/\sqrt{2})-(x_1+r_1/\sqrt{2}) \\
m_2 &= (x_3+r_3/\sqrt{2})-(x_2+r_2/\sqrt{2}) \\
m_3 &= (x_4+r_4/\sqrt{2})-(x_3+r_3/\sqrt{2})
\end{align*}
\]

Where \(x_1..x_4\) are successive x-coordinates and \(r_1..r_4\) are four random numbers. This leads to simulated
measurements which have a standard deviation of unity but which are correlated along the lines along which the
measurements take place.

Using these simulated measurements, the calculations as described in section 3 are carried out again
which gives a new set of \((x,y)\) coordinates. The square of the difference between each calculated coordinate and
the nominal value gives an estimate for the variance in each x- and y-coordinate. By repeating this calculation
many times with simulated measurements, and taking the average variance in each coordinate, a reliable
estimation for the uncertainty in each coordinate is obtained. The same procedure is used for estimating the
uncertainty in the distance between two arbitrary points. Further, the average value of \(\chi^2\) is determined.
The latter can be used to estimate the measurement uncertainty $s_m$ from the calculated uncertainty $s_c$:

$$s_m = s_c \sqrt{\chi^2_m / \chi^2_c}$$  \hspace{1cm} (7)

where $\chi^2_c$ is the average value of $\chi^2$ resulting from the simulations and $\chi^2_m$ is the value of $\chi^2$ resulting from the measurements.

### 4.2 Results of the simulations

The standard deviations in the coordinates for a 5x5 equidistant grid are shown graphically in figure 3. In this figure, the scale for the coordinates can be considered as mm. The standard deviations (the error bars in the figure) are in $\mu$m assuming that $s_i$ is 1 $\mu$m.

The RMS (Root-Mean-Square) standard deviation in each x- or y-coordinate is 1.16 times the input uncertainty $s_i$. The figure shows that the largest standard deviation is in the x-coordinate at position (0,16). This is due to the fact that the y-coordinate of point (16,0) is put equal to zero. The choice of a more stable reference point and reference orientation would diminish the uncertainty somewhat. A choice could be: define the center of all coordinates as (0,0) and take the average slope of the least-square lines in the x-direction as the reference direction. In this case, however, all coordinates differ from zero which gives less insight in their positions.

A measure of the uncertainty which is independent of the orientation of the grid is the standard deviation in the distance between two points. In the above-mentioned example this standard deviation proves to be nearly equal to the standard deviation of the input measurements $s_i$. The results for different grid sizes are summarized in table 1.

<table>
<thead>
<tr>
<th>Error! Bookmark not defined. Grid size</th>
<th>RMS-deviation in $x$- or $y$-coordinate /$s_i$</th>
<th>RMS-deviation in distance between two points /$s_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 x 2</td>
<td>1.58</td>
<td>1.00</td>
</tr>
<tr>
<td>3 x 3</td>
<td>1.32</td>
<td>1.04</td>
</tr>
<tr>
<td>5 x 5</td>
<td>1.16</td>
<td>1.00</td>
</tr>
<tr>
<td>7 x 7</td>
<td>1.07</td>
<td>0.96</td>
</tr>
<tr>
<td>10 x 10</td>
<td>1.02</td>
<td>0.93</td>
</tr>
<tr>
<td>15 x 15</td>
<td>0.87</td>
<td>0.89</td>
</tr>
<tr>
<td>8 x 4</td>
<td>0.98</td>
<td>1.00</td>
</tr>
<tr>
<td>5 x 5 (x:y=4:1)</td>
<td>1.21</td>
<td>0.98</td>
</tr>
</tbody>
</table>

From the table it is clear that the standard deviation in the distance between two points nearly equals the input standard deviation for different grid sizes. This implies that in the case of a real measurement, equation (4) without the term $(2n/k)$ gives a reliable estimate of the type-A (statistical) uncertainty.
5. EXAMPLES OF MEASUREMENTS

5.1 Calibration grid
A calibration grid of Graticules Ltd, type 200/PGR, has been calibrated with the methods described above. The grid consists of a glass plate with lines of 10 μm width which form a grid of 6 x 4 intersections with a nominal distance between the lines of 40 mm. The measurements were taken on an Aus Jena ZKM 01-250C measuring microscope, where the line intersections were observed visually. The distance between the intersections was measured using a HP laser interferometer. The optical set-up is sketched in figure 4.

![Figure 4](image)

By taking the distance between the two corner cubes at the microscope equal to the distance of the lowest of these corner cubes to the object, the Abbe-error is compensated. Evaluation of the measurements yields a standard deviation of 0.6 μm in the distance between two intersections. Taking into account the uncertainty due to the measurements of material- and air temperature, air pressure and humidity, the uncertainty, based on 2s, is 1.2 μm + 1.0 \times 10^{-6} \cdot l, where l is the distance between two intersections. This value can be improved by using photo-electronic line detection instead of visual observation of the line position.

5.2 Ball plate
A ball plate of G.Retter, type RP05/400, has been calibrated with the methods described above. It consists of a steel plate in which 5 x 5 ceramic balls are positioned at nominal distances of 83 mm. The measurements were taken on a Zeiss UC 550 coordinate measuring machine (CMM). The balls were probed with a measuring head consisting of three balls. With the CMM it was determined that the balls were situated in one plane within 20 μm. Also, it was determined that in the x- and y directions and along the diagonals the balls were in one line within 20 μm. This means that alignment of a row of balls with the laser-beams can be achieved with a negligible ‘cosine’ error. The form deviation of the balls is usually in the order of 0.1 μm.

![Figure 5](image)
When the deviations are appreciably larger, they will affect the type-A uncertainty which is derived from the measurements. The distance between the balls was measured using a HP laser interferometer. The optical set-up is sketched in figure 5. The flat mirror was carefully aligned to be normal to the interferometer beams. The Abbe error was measured separately using an autocollimator and proved to affect the measurements less than 0.2 μm over the maximum measured distances. Evaluation of the measurements yields a standard deviation of 0.16 μm in the distance between the centres of two balls. Taking into account the uncertainty due to the measurements of material- and air temperature, air pressure and humidity, the uncertainty, based on 2σ, is 0.35 μm + 8 \times 10^{-7}l.

An independent check of the method and the uncertainties can be made as the ball plate was earlier calibrated at the PTB (Physikalisch-Technische Bundesanstalt, Germany) where a swing-around method on a CMM was applied combined with laser-interferometric distance measurements along two lines. In this certificate an uncertainty of 0.6 μm + 1.0 \times 10^{-6}l is stated. The largest difference in a distance between two balls between the NMI and PTB measurements is 0.9 μm for two balls at 332 mm distance. This difference is less than the combined uncertainty of 1.2 μm of the two measurements. All other differences are also smaller than the combined uncertainty. The RMS-difference is 0.27 μm which is less than the expected value of 0.35 μm which is obtained when neglecting the length factors. This further confirms the validity of both methods and the derived uncertainties.

5.3 Further applications

The method might offer a way to calibrate grids which are in use in AFM/STM’s. Also, the method can be applied to patterns which are less strictly square. With some adaptations it could be applied to 3-D ball structures.

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