Computational Mechanics

Modelling the Ballistic Impact Behaviour of Polyethylene-Fibre-Reinforced Composites

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With ref.
Subject headings: ballistic impact ; polyethylene-fibre-reinforced composites.
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April 1996
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Summary

The purpose of this research is to analyze the high velocity impact behaviour of polyethylene-fibre-reinforced composites, focusing on the influence of the matrix material and laminate structure on the damage evolution and energy absorbing mechanisms. A finite element program is used to model a composite plate, which is being hit by a non-deformable, reversed fragment simulating projectile (17 grain FSP). To describe the influence of stacking sequence, the modelled composite plate is subdivided into three layers, each representing a cross-ply of more than ten plies. Orthotropic stiffness properties of all layers are determined using micromechanical models (e.g. the rule of mixtures) and laminate plate theory. Necessary fibre and matrix properties at high strain-rates are determined using Dynamic Mechanical Thermal Analysis (DMTA). In the finite element model so called 'Rayleigh' damping is introduced by taking the damping proportional to the stiffness matrix. Measured damping properties (tan δ values), obtained from DMTA-experiments, are used to determine the Rayleigh-coefficients. To show the damage evolution and active failure mechanisms, three different failure criteria are implemented in the model. These criteria take into account the effects of fibre, matrix and delamination failure by reducing the in- and out-of-plane mechanical properties. In the model the fibre content and type of matrix material is taken into account by adjusting the moduli of elasticity and strength values of the cross-ply laminate. The significance of the material properties of the laminate on the impact performance is shown by the velocity decay of the projectile and corresponding trauma. Moreover, the influence of progressive failure on the impact behaviour is investigated by means of a parametric study.

Numerical results show that contact forces are high within the first 10 μs after impact. Consequently, upper plies of the laminate are perforated during this time period and no longer contribute to the resistance of the laminate. Therefore, the impact behaviour of the composite laminate is actually represented by modelling only the non-perforated part of the laminate. According to ballistic experiments close to the ballistic limit, generally up to 50% of the plies of an orthotropic laminate is perforated by a rigid projectile. The post-failure material behaviour of the laminate is estimated from measured stress-strain curves at low temperatures and high strain rates. For this the slope of the post-failure stress-strain curve (softening) is chosen twice the slope of the pre-failure stress-strain curve (modulus). We observed that the impact behaviour of cross-ply HPPE laminates depends on the in-plane tensile and shear strength of the cross-ply laminates. Therefore, in-plane shear failure and fibre breakage are the predominant failure mechanism. Consequently, matrix systems with high damping and ductile material behaviour at impact rates, having a low glass transition temperature \( T_g \) in the order of -50°C, improve ballistic performance of cross-ply laminates as a result of higher in-plane tensile strength values. However, it is shown that the influence of damping in itself on the impact behaviour is negligible at impact rates. For orthotropic laminates both the predicted trauma and trauma diameter compare well with measured trauma values, which were obtained by ballistic experiments.
Surprisingly, the impact behaviour is hardly influenced by the moduli of a cross-ply laminate when failure mechanisms are present, whereas if failure mechanisms are neglected higher in-plane moduli and especially out-of-plane shear moduli improve ballistic performance. Hence, the post-failure behaviour of the laminate dominates the impact performance. Consequently, HPPE fibres on the backside of the laminate should be optimized to render maximum strength at impact rates instead of maximum modulus. However, differences in ballistic performance due to failure processes occur only after a time period of 10 μs when contact forces are much lower, indicating that the in-plane tensile and shear strength of the laminate are less important during the punching process. Hence, fibres on the impact side of the laminate should be optimized to possess a maximum modulus, to store maximum elastic energy before punching. Moreover, the transverse motion of the fibres improves for higher moduli, which reduces contact forces. Consequently, the number of perforated plies should decrease, improving ballistic performance strongly.

The influence of lay-up is analyzed by comparing the impact behaviour of an orthotropic and quasi-isotropic laminate structure. For a quasi-isotropic laminate structure in-plane shear failure is less pronounced compared to an orthotropic laminate, whereas in-plane compressive failure is more pronounced. The in practise observed reduced ballistic performance for the quasi-isotropic laminate structure is possibly caused by the low in-plane compressive strength of HPPE laminates. Nevertheless, in our calculations predicted ballistic performance improved for the quasi-isotropic laminate structure. The reason for the difference in experimental and numerical results is caused by the fact that a quasi-isotropic laminate structure no longer deforms as a rigid plate, as a consequence of extensive delamination. However, numerical results make clear that as long as the stacked plies deform/behance as a rigid laminate plate, a more quasi-isotropic laminate structure gives optimum ballistic performance.
Contents

Summary

Chapter 1  Introduction

1.1 General introduction to impact 1
1.2 Some aspects concerning ballistic impact of laminates and types of projectiles 3
1.3 Modelling the ballistic impact behaviour of composites 7
1.4 Outline of this report 8

Chapter 2  General aspects of wave propagation

2.1 Introduction 9
2.2 Wave propagation in fibres 9
   2.2.1 Transverse wave propagation in fibres 9
   2.2.2 Longitudinal and transverse wave propagation in fibres 10
2.3 Wave propagation in elastic solids 12
   2.3.1 Longitudinal waves 14
   2.3.2 Transverse waves 15
2.4 Conclusions 16

Chapter 3  Material properties of HPPE laminates at impact rates

3.1 Introduction 17
3.2 Material properties of fibres and matrix 17
   3.2.1 Fibre properties 18
   3.2.2 Matrix properties 18
3.3 Material properties of unidirectional lamina 22
3.4 Material properties of cross-ply laminates 24
   3.4.1 Moduli of elasticity of cross-ply laminates 24
   3.4.2 In-plane tensile strength of cross-ply laminates 26
3.5 Conclusions 33

Chapter 4  Modelling the elastic behaviour of HPPE laminates during ballistic impact

4.1 Introduction 34
4.2 The finite element model 34
4.3 Modelling of damping 37
4.4 Numerical results 38
4.5 Conclusions 41
### Chapter 5  Modelling the failure behaviour of HPPE laminates during ballistic impact

5.1 Introduction \hspace{1cm} 45  
5.2 Contact modelling \hspace{1cm} 47  
5.3 Failure mechanisms \hspace{1cm} 49  
5.3.1 Failure criteria \hspace{1cm} 49  
5.3.2 Modelling progressive failure \hspace{1cm} 51  
5.4 A numerical parametric analysis \hspace{1cm} 54  
5.4.1 Influence of laminate strength \hspace{1cm} 54  
5.4.2 Additional numerical results \hspace{1cm} 58  
5.5 Experimental validation \hspace{1cm} 67  
5.6 Conclusions \hspace{1cm} 71

### Chapter 6  Conclusions & recommendations

6.1 Conclusions \hspace{1cm} 72  
6.2 Recommendations for material applications \hspace{1cm} 73  
6.3 Recommendations for further research \hspace{1cm} 74  

### References 75

### Appendix A  Standard for fragment simulating projectiles 78

### Appendix B  Material properties of HPPE laminates and wave propagation velocities at impact rates 79

- Material properties of epoxy and S-I-S rubber based laminates 79
- Longitudinal and transverse wave velocities \(c_1\) and \(c_2\) 80

### Appendix C  Test results ballistic experiments 81

- \([0/90]_{12s}\) laminate structure 81
- \([0/90]_{15}[-45/45]_{30}[0/90]_{30}[-45/45]_{30}[0/90]_{15}\) laminate structure 82

### Appendix D  Failure Program 83
Chapter 1

Introduction

1.1 General introduction to ballistic impact

Impact is a very broad research area [1]. Various industries are involved such as the automotive, aviation and military industry. In general, the possibility of impact on a structure, system or component requires additional specifications for a product on top of the conventional ones. Automotive vehicles are nowadays designed to guarantee good crashworthiness capabilities for passengers. Aircrafts are not supposed to crash because of the consequences of a bird impact. In military service, helicopters, vehicles and personal armour [2] (for instance, helmets and bullet-proof vests) are exposed to several types of ammunition, causing typical high velocity impact problems. Impact problems in the automotive and aviation industry as mentioned above are low velocity impact problems. All these impact problems have one aspect in common, viz. protected structures should be as light as possible to avoid loss of mobility. For this reason impact resistance of materials is always defined by comparing achieved protection versus structural weight [2-5].

Impact behaviour of materials is mainly categorised by the low and high velocity impact regime [6]. These regimes are characterised by either the projectile velocity (kinetic energy) or the duration of the impact event. In the low velocity regime (<10 ms⁻¹) the whole impacted structure experiences the impact. Kinetic energy of the projectile is mainly absorbed by elastic deformation of the target. Boundary conditions and geometry of the target have a significant influence on the, more or less, quasi-static behaviour of the target. At high velocity impact (>300 ms⁻¹), due to the much shorter duration of the event, only material in the vicinity of the projectile experiences deformation. Inertial effects and wave propagation phenomena control the transient dynamic behaviour of the target [7,8]. Geometry and boundary conditions are less important as long as the generated longitudinal strain waves are not reflected from the edges of the target. Kinetic energy of the projectile is absorbed by the local deformation and destruction of the target material, deformation of the projectile and local acceleration of the target. No clear boundary between both velocity regimes exists, since there is a transition which is target material dependent. There is another important difference between the impact regimes. In many low velocity impact problems structures should still be capable of bearing stresses after impact, i.e. bird-impact of an aircraft. Whereas, at high velocity impact, a structural element gives a requested protection level and is replaced after impact, because of irreversible damage, i.e. ballistic impact of armoured car door.
Traditionally steel armour is widely used for high velocity impact problems. However, in recent years steel armour has been successfully replaced by composites to achieve weight advantages [2,4]. Energy dissipation mechanisms are quite different for various materials. For example, high velocity impact of steel or composite armour gives completely different results. Composites are not capable of bearing the extremely high hydrostatic stresses, near the point of impact, observed for steel armour. At high impact rates, steel armour behaves more or less like a high viscous fluid due to yielding, clearly visible from the created bulges on top and bottom of the target [9]. Composites lose integrity due to various failure processes [10,11] whereas the fibres hardly show any yielding behaviour at high strain rates [12].

Composite materials are suitable for use in problems involving high velocity impact because they combine lightness with high mechanical strength [2]. However, impact of composite materials is a very complex process. The impact resistance of a composite depends on several aspects such as: fibre properties (elastic moduli, failure strain and viscoelastic properties), fibre architecture (unidirectional lamina, woven fabrics), matrix properties, composite areal density etc. Many failure phenomena occur during impact such as: fibre and matrix breakage, delamination and fibre pull-out. These failure processes interact very strong and are strain rate dependent, which makes it difficult to determine which mechanism is predominant. At high loading frequencies or impact rates, material properties are different from those observed at normal loading conditions as a result of viscoelastic properties [13,14]. As a consequence of this the amount of energy absorbed by these separate failure mechanisms depends on the velocity of the projectile [15]. Furthermore, many of these failure processes are strongly dependent on environmental effects such as temperature and moisture [15,16]. Therefore, fibre, matrix and interface properties are still optimized by trial and error using ballistic experiments [17,18].

A project between TUE and DSM, producer of the super strong polyethylene fibre Dyneema, was initiated in the area of predictive modelling of the high velocity impact behaviour of polyethylene-fibre-reinforced composites. The purpose of this two-years project was to better understand the mechanism by which a projectile is stopped by a composite laminate. Using FEM, the influence of fibre and matrix properties on the damage evolution and energy absorbing mechanisms is investigated. Once these mechanisms are known, the effects of new developed composite systems can be analyzed. Moreover, such methods can be used to analyze the influence of the laminate structure on the ballistic performance.
1.2 Some aspects concerning ballistic impact of laminates and types of projectiles

High-performance polyethylene (HPPE) fibres possess unique properties. The toughness and viscoelastic character of HPPE fibres makes them highly-suited for applications where impact resistance \( [15,16,19] \) and vibrational damping \([20]\) are required. Especially, unidirectional sheets of polyethylene fibres, embedded in different matrix materials, and fabrics of these fibres are used to design optimum protective materials against small calibre projectiles \([18,21]\). Flexible bullet-proof vests consist of a pack of such woven fabrics or unidirectional sheets. Fig. 1 shows the construction of such a cross-ply (Dyneema® UD). Rigid shaped products such as (military) helmets can also be produced by impregnating a pack of fabrics after forming. In so-called ‘soft’ ballistic applications, fabrics or lamina are separated by two low-density polyethylene sheets. The low friction coefficient between these adjacent LDPE sheets allows the final product to be flexible (soft). For ‘hard’ ballistic applications these intermediate layers are omitted. To obtain a rigid plate with a high bending stiffness, the sheets or ‘prepregs’ are hot-pressed at a temperature of 125°C. Combinations of laminates, stacked unidirectional sheets or impregnated fabrics, and metallic plates (aluminum, steel) or ceramic tiles are used to defeat projectiles travelling at very high velocities \((800 \text{ ms}^{-1})\) \([3,22]\). Examples are the armouring of vehicles and the reinforcement of vests for police forces. Vests in service are exposed to various types of small calibre projectiles. Table 1 gives an overview of frequently used projectiles, according to the German Police Standard.

At high impact velocities, laminates give, when impacted with non-deforming fragment ammunition, a better performance than a pack of soft fabric with the same areal density of fibre material, whereas at lower impact velocities \((\pm 400 \text{ ms}^{-1})\) the opposite occurs \([1,17]\). In soft fabrics, the primarily impacted yarns interact with other yarns due to the existing cross-over points \([7,23]\). The positive effect of this mechanism is that the energy will be absorbed over a relatively large area.
Table 1. German Police Standard for small calibre projectiles (bullets)

<table>
<thead>
<tr>
<th>Class</th>
<th>Calibre</th>
<th>Ammunition type</th>
<th>Mass</th>
<th>Impact velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>9 mm Para</td>
<td>VMR/WK</td>
<td>8.00 g</td>
<td>365 ± 5 m/s</td>
</tr>
<tr>
<td>I</td>
<td>9 mm Para</td>
<td>VMR/WK</td>
<td>8.00 g</td>
<td>410 ± 10 m/s</td>
</tr>
<tr>
<td>II</td>
<td>.357 Magnum</td>
<td>MsF</td>
<td>4.00 g</td>
<td>570 ± 20 m/s</td>
</tr>
</tbody>
</table>

VMR/WK Full Metal Jacketed
MsF Brass Flat Nose Bullet

The disadvantage of the yarn coupling in woven fabrics is that the cross-over points reflect part of the strain waves, which increases the amplitude of the strain wave, as shown in Fig. 2. The amplitude of the strain wave in a single fibre increases if the impact velocity of the projectile is higher [24]. From a certain projectile velocity, fibre failure is accelerated if cross-over points are present. Therefore, stacked unidirectional sheets give a better performance than fabrics at high impact velocities, although the energy absorbing area is only controlled by the primarily impacted fibres due to the absence of the intra-ply fibre coupling. In this report only the impact behaviour of stacked unidirectional sheets is analysed.

![Reflections of strain waves at cross-over points.](image)

A very important aspect in high velocity impact of HPPE composites is the hardness and mass of the projectile. Soft core ammunition, such as lead bullets, deforms heavily upon impact reducing the projectiles kinetic energy up to 50% before significant panel deflection is observed [21]. Non-deformable projectiles, such as fragments of an exploding shell, use this amount of energy to further destruct the target.
Furthermore, geometrical effects such as the shape and size of projectiles have a strong influence on the (ballistic) performance of materials [25]. Conical shaped and small projectiles penetrate an impact protection panel much easier than hemispherical and large ones. Another important aspect for the targets impact performance is the angle of incidence of the projectile [21]. For instance, fragments of an exploding shell will penetrate or perforate a military helmet dependent on the fragments velocity, mass and angle of incidence [17]. Fragments have various shapes, mass and velocities. They are categorised by means of the standard for fragment simulating projectiles (FSP), as mentioned in Appendix A.

To compare the level of protection of several materials against various types of projectiles, the impact performance has to be characterised. The literature describes three different methods of measuring anti-ballistic performance of materials. Firstly, the $V_{50}$-method, which is a measure for the initial speed of the projectile ($V_{50}$) if 50% of the samples are penetrated under identical conditions [5]. Secondly, the critical velocity method, which is a measure for the maximum speed ($V_c$) at which a material can be loaded without introducing failure, calculated from the product of the strain-at-break and the sonic velocity in the material [26]. Finally, the weight-efficiency method compares different materials by the required weight (often equivalent to thickness) necessary to stop a certain projectile at identical conditions [4]. The above mentioned methods can give different results for the ballistic performance of identical materials, since even small changes in test conditions can dramatically affect ballistic performance. In this report the $V_{50}$-method is preferred to characterize the performance [18].

Ballistic and structural laminates have different properties, as shown in Table 2. In ballistic laminates the fibre content is high, giving a poor fibre impregnation, and the adhesion moderate [27]. A higher matrix volume content and too good adhesion gives poor ballistic behaviour, whereas in structural laminates this is necessary to utilize the full strength of the material. The difference in behaviour can be explained by the fact that fibres during impact have to deform to absorb the energy of the projectile. After high velocity impact, generated strain waves propagate much faster in the fibre than the matrix material [8,24]. Therefore, the present fibre/matrix interaction limits the deformability of the fibre. At low velocity impact problems, these wave propagation phenomena are not present, so the fibre/matrix interaction contributes to the resistance of (structural) laminates. On the other hand if the matrix is omitted in ballistic laminates, fibres are easier pushed aside and the bending stiffness of the laminate, necessary to deform the projectile, will be negligible. Due to the higher bending stiffness, contact forces increase and more energy is stored in deforming the projectile. As mentioned before, this aspect is very important, since deformation of the projectile absorbs a large amount of energy and increases the effective diameter of the projectile [3,21], thus more fibres are loaded. Therefore, 'hard' laminates show a better performance than 'soft' laminates if deforming projectiles are used.
However, tests with non-deforming fragment simulating projectiles (FSP’s) show the opposite. The degree of punching is then higher for 'hard' than for 'soft' laminates, which reduces their performance [18]. The reason for this is that fibres absorb less energy by punching/cutting than by stretching up to failure. In service, ballistic laminates are exposed to deformable (bullets) and non-deformable (fragments) projectiles. Therefore, a ballistic laminate is always a compromise, which depends on the speed, shape and hardness of the expected projectiles.

<table>
<thead>
<tr>
<th>Laminate type</th>
<th>fibre volume content</th>
<th>fibre impregnation</th>
<th>adhesion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ballistic Laminate</td>
<td>70 - 85 %</td>
<td>poor</td>
<td>moderate</td>
</tr>
<tr>
<td>Structural Laminate</td>
<td>50 - 60 %</td>
<td>good</td>
<td>good</td>
</tr>
</tbody>
</table>

At high impact velocities and strain rates, the energy storage correlates very strong with the number of broken fibres [15]. Ballistic tests with, by irradiation, weakened fibres showed that the ballistic efficiency of HPPE cross-ply laminates increases linearly with the energy-to-break of the fibre [21]. Whether delamination is the predominant mechanism under impact conditions is still controversial [10,11]. Tests on composites with deliberately introduced delaminations show that the total energy absorption is hardly changed compared to the untreated composites [6]. However, panels with good penetration resistance always are heavily delaminated, indicating that a considerable amount of energy was absorbed by delamination.

Much research has been devoted to optimization of fibre properties. Impact measurements on HPPE yarn show that fibres have to possess a high modulus of elasticity, strain-at-break and tensile strength [28]. In general, a smaller yarn and filament diameter increases the ballistic performance of laminates. However, these optimized fibres do not automatically yield composite materials with optimum impact resistance, due to the influence of the matrix material and fibre/matrix adhesion on the failure process of composites. For example, previous studies showed that: (i) improved fibre/matrix adhesion significantly reduces impact performance of HPPE/epoxy composites [19]; (ii) the ballistic limit $V_{50}$ of the laminate depends on the used matrix material, as shown by ballistic test on aramid and polyethylene fibre based laminates; (iii) aramid laminates have a (10%) higher ballistic performance if as matrix material polycarbonate is used instead of polypropylene; (iv) HPPE based laminates have an (10%) increase in ballistic performance if a polystyrene matrix is replaced by a S-I-S (Styrene-Isoprene-Styrene) block polymer matrix [29]. This means that, under impact conditions, the toughness of the matrix material has an influence on the ballistic performance of laminates.
A proper choice of the matrix in combination with an optimized laminate structure will therefore give maximum ballistic performance of the HPPE composites. However, in the literature hardly any explanation is given for effects of matrix, interface and laminate structure on ballistic performance of composite materials.

1.3 Modelling the ballistic impact behaviour of composites

Fibre, matrix and interface properties are mainly optimized by trial and error, using ballistic experiments. Although such experiments are essential, research based on other techniques, such as numerical methods, may provide more fundamental insight. The literature describes three types of models to analyze the impact behaviour of fibres, fabrics and laminates. First, empirical models obtained from results of numerous real ballistic test, which are in general used to determine the laminate thickness necessary for the desired level of protection [17]. These models represent the observed impact behaviour very well, but are only valid for the tested structures and materials at test-conditions. Secondly, analytical models describing the dynamic mechanical material behaviour present during impact, are more promising in explaining the impact behaviour because they allow a variety of parametric analyses [22,30]. However, analytical methods are often based on rough assumptions and simplifications in order to be able to solve the formulated differential equations. In general, these models do not take material failure into account. Finally, numerical analyses (finite element methods) are attractive tools to optimize ballistic design of laminate composites because they can show the main features of impact problems [31-33]. These techniques can show the influence of different material parameters on the impact behaviour independently in a parametrical study.

All models need reliable input data such as: the impact and post-impact velocity of the projectile, and the material properties (moduli, strain-at-break and tensile strength values) of the target at impact rates. Velocities are measured by means of several techniques such as: high-speed photography, optical transducers, Doppler radar techniques and pressure transducers in the barrel of the projectile accelerator apparatus [8]. Good results for strength and strain-at-break values at high strain rates (up to 1000 s\(^{-1}\)) can be obtained using Hopkinson bar techniques, as shown by Ruiz and Harding [34].

Only a few numerical models are described in literature for the treatment of high velocity impact problems on composite materials. These macroscopic models assume that fibre and matrix behave as one homogeneous orthotropic material. Most models take damage criteria into account such as the Tsai-Wu or Tsai-Hill criteria [31,32]. Damage is generally introduced by reducing the orthotropic material properties. The problem of numerical optimization methods is to define an appropriate model which includes representative failure mechanisms. For example, different failure criteria are used to successfully describe the development of delaminations in transverse impacted laminates. Some authors showed that mainly transverse shear stresses are responsible for the development of delaminations, whereas others suggest that delamination effects primarily occur due to the tensile stress in the out-of-plane direction of the laminate [10,11].
1.4 Outline of this report

In this report a macroscopic finite element model is used to describe the impact behaviour of HPPE based laminates. To analyze the laminate target response only, non-deforming projectiles are used for simulations and experiments. To avoid complex contact procedures, laminates are impacted normal to the target surface. First, the theoretical wave propagation velocities of longitudinal and transverse waves in fibres and solids are deduced in Chapter 2. To check numerical solutions, obtained transverse and longitudinal wave velocities are compared with theoretical values. The influence of high strain rates on material properties of HPPE composites is investigated in Chapter 3. Since the time-temperature equivalency theory holds for the used materials, this theory is used to determine moduli, damping properties and in-plane strength data present at impact strain rates up to $10^6 \text{s}^{-1}$ [13]. In Chapter 4 the impact behaviour of HPPE laminates is investigated using a numerical model without taking failure mechanisms into account. The influence of the fibre content, matrix type and laminate structure on the impact behaviour of HPPE laminates is analyzed. Next, in Chapter 5 the model is adapted to show effects of damage evolution due to fibre, matrix and delamination failure. By means of a parametric analysis the influence of the different failure processes is investigated. Predicted maximum deflection values of the laminate are compared with experimental results from ballistic tests to verify the model. Finally some conclusions and recommendations are given in Chapter 6.
Chapter 2

General aspects of wave propagation

2.1 Introduction

After impact of a composite, transverse and longitudinal waves propagate away from the point of impact. When no failure processes are present, the kinetic energy of the projectile is transformed into elastic and kinetic energy of the composite. The size of the in tension loaded area of the composite is controlled by the longitudinal wave propagation. The lateral movement of the composite is controlled by the transverse wave propagation. Since we analyze the composite as a homogenous medium, the theoretical longitudinal and transverse wave velocities are deduced for elastic solids. However, the transient dynamic behaviour of a single fibre is comparable to the behaviour of a string when the influence of fibre/matrix adhesion and intra-ply interaction is neglected. Therefore, first the impact behaviour of a linear elastic string is treated.

2.2 Wave propagation in fibres

2.2.1 Transverse wave propagation in fibres

When a stretched string fixed at both its ends, held at a constant tension $T$, and of a density $\rho$ per unit of its initial length is given a transverse displacement as shown in Fig. 3a, a transverse wave will propagate along the string.

Figure 3. (a) Lateral displacement $v$ of a stretched string, (b) loads on a infinitesimal small string.
After applying Newton’s second law for the lateral motion to the infinitesimal small string element of Fig. 3b we obtain [35]:

$$T \sin(\theta + \frac{\delta \theta}{\delta x} dx) - T \sin(\theta) = \left( \rho \cos(\theta) ds \right) \frac{\delta^2 \nu}{\delta t^2}$$  \hspace{1cm} (1)

where we can write the length of the element $ds$ in terms of $dx$:

$$ds = (dx^2 + dv^2)^{\frac{1}{2}} = \left( 1 + \left( \frac{\delta v}{\delta x} \right)^2 \right)^{\frac{1}{2}} dx$$  \hspace{1cm} (2)

If we assume that the slope of the string relative to the x-axis is small, then $ds = dx$, $\theta = \delta v/\delta x$. Moreover, according to a Taylor approximation $\sin(\theta) = \theta + o(\theta)$ and $\cos(\theta) = 1 + o(\theta^2)$, where the errors $o(\theta)$ and $o(\theta^2)$ are small with respect to the value of $\theta$. Therefore, we can write the Equation of motion in the form:

$$\frac{\delta^2 \nu}{\delta x^2} = \frac{1}{c_2^2} \frac{\delta^2 \nu}{\delta t^2}, \quad c_2 = \sqrt{\frac{T}{\rho}}$$  \hspace{1cm} (3)

Which is the one-dimensional wave equation. The solution of this wave equation is given by the one-dimensional wave function [36]:

$$\nu(x,t) = f(x - c_2 t) + g(x + c_2 t)$$  \hspace{1cm} (4)

where the functions $f(x - c_2 t)$ and $g(x + c_2 t)$ determine the specific nature of the wave, depending on the initial conditions. The function $f(x - c_2 t)$ represents a wave propagating in positive x-direction with velocity $c_2$, while the function $g(x + c_2 t)$ represents a wave, which propagates with the same velocity in the opposite direction. We conclude that small lateral motions of a stretched string propagate with a velocity $c_2$, which depends on the tension and mass of the string.

2.2.2 Longitudinal and transverse wave propagation in fibres

In the previous paragraph only the transverse wave velocity is deduced. However, after impact both longitudinal and transverse wave types occur simultaneously. Cole et al [24] developed a nonlinear theory to describe the dynamic behaviour of transverse impacted linear elastic strings. Next some observations of their work are mentioned.
Wave propagation

After transverse impact, a longitudinal wave propagates with constant velocity $c_1$ away from the point of impact, as shown in Fig. 4. Next the material between the longitudinal wave front and the point of impact starts to move into the direction of the point of impact with a constant velocity $W$:

$$W = c_1 \epsilon$$  \hfill (5)

where $\epsilon$ is the constant strain behind the longitudinal wave front. Simultaneously, a transverse wave propagates through the strained string with velocity $c_2$, shown by the formed triangle in Fig. 4. In general, the transverse wave speed is smaller than the longitudinal wave speed. At the front of the transverse wave, the material stops to move into the direction of the impact point. At this point, the full horizontal string movement is translated to and used for the lateral string movement, since the strain level in the total string is constant behind the longitudinal wave front. All material behind the transverse wave front moves into the same direction and at the same speed as the projectile, which results in the typical triangular shaped transverse wave.

![Figure 4. Longitudinal and transverse waves in a fibre.](image)

Cole at al [24] showed that the speed of longitudinal and transverse waves is related to the strain and tension in the string. The differential equation of motion are formulated again by applying Newton's second law of motion to an infinitesimal element of string. Solutions to the equations of motion were given for the propagation of waves of constant strain and tension. The longitudinal and transverse wave speeds are respectively:

$$c_1 = \sqrt{\frac{1}{\rho} \frac{\sigma}{\delta \epsilon}} = \sqrt{\frac{E}{\rho}}$$  \hfill (6)

$$c_2 = \sqrt{\frac{1}{\rho} \frac{\sigma}{1+\epsilon}} = c_1 \sqrt{\frac{\epsilon}{1+\epsilon}}$$  \hfill (7)

So, the transverse wave velocity depends on the strain level in the string.
Furthermore they showed that for a string whose initial tension is zero, the constant strain level in the string depends on the impact velocity $V$ and longitudinal wave velocity $c_1$:

$$e = 2^{\frac{2}{3}} \left( \frac{V}{c_1} \right)^4$$ (8)

This means that the ballistic limit $V_{50}$ of a fibre with a certain strain at break increases for a higher longitudinal wave velocity. Smith et al [37,38] also described the effects of plastic deformation behind the longitudinal wave front. Due to the plasticity of the material the strain level behind the longitudinal wave front increases continuously until a maximum strain level $\varepsilon_p$ is reached. As a consequence of this, the tangential modulus ($\frac{d\sigma}{d\varepsilon}$) and thus the longitudinal wave speed changes continuously while the strain level increases, see Eqn. (6). The movement of a infinitesimal string, into the direction of the point of impact at velocity $W$, depends on the longitudinal wave velocity and strain level as shown in Eqn. (5). As a result of the changing velocity $W$, a large number of subsequent longitudinal wave fronts will be present in the string. Since HPPE fibres hardly show any yielding at impact rates [12], in the remainder of this report the influence of plasticity is neglected.

2.3 Wave propagation in elastic solids

Composites are heterogeneous materials due to the different properties of the fibres and matrix. As a result of this longitudinal and transverse waves propagate at different velocities in the fibre and matrix (Eqn. 6 and 7). Due to the high modulus of the PE-fibres longitudinal waves propagate much faster in the fibre than in the matrix. This means that the fibres are loaded in tension behind the longitudinal wave front, whereas the matrix still behaves as a rigid material. However, a composite resembles to a certain extend an elastic solid due to the interaction between the stacked plies and the presence of the matrix. Therefore, also the wave propagation of longitudinal and transverse waves in elastic solids is deduced. The dynamic behaviour of homogeneous isotropic elastic solids can be described in tensor notation by the following set of equations [35]:

$$\nabla \cdot \sigma + \rho \ddot{u} = \rho \dddot{u}$$ (9)

$$\sigma = \lambda tr(e)I + 2\mu e$$ (10)

$$e = \frac{1}{2} \{ (\nabla u)^T + (\nabla u)^T \}$$ (11)

where $\sigma$ is the stress tensor, $u$ is the displacement vector, $f$ is the body force per unit mass of material, $\rho$ is the material density and $\varepsilon$ is the strain tensor.
Wave propagation

The stress tensor is symmetric, so that \( \sigma = \sigma^T \). The Lamé constants \( \lambda \) and \( \mu \) can be derived from the Young’s modulus \( E \) and Poisson’s ratio \( v \) of the isotropic material:

\[
\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}, \quad \mu = \frac{E}{2(1+\nu)} = G
\]

where \( G \) is the shear modulus.

Substitution of the expression for strain (11) into the stress-strain relation (10) and the result thereof into the equation of motion (9) gives the Navier’s equation in terms of the displacements for isotropic media:

\[
(\lambda + \mu) \nabla^2 \vec{u} + \mu \nabla^2 \vec{u} + \rho \ddot{\vec{u}} = \rho \ddot{\vec{u}}
\]

where

\[
\nabla^2 = \nabla \cdot \nabla \nabla
\]

In this report the presence of body forces such as gravity is neglected, so the governing displacement equation is given by:

\[
(\lambda + \mu) \nabla(\nabla \cdot \vec{u}) + \mu \nabla^2 \vec{u} = \rho \ddot{\vec{u}}
\]

Using the vector identity:

\[
\nabla^2 \vec{u} = \nabla(\nabla \cdot \vec{u}) - \nabla \times (\nabla \times \vec{u})
\]

and defining the dilatation \( \Delta \) of a material:

\[
\Delta = \nabla \cdot \vec{u} = tr(e)
\]

and the rotation vector \( \vec{\omega} \):

\[
\vec{\omega} = \frac{1}{2} \nabla \times \vec{u}
\]

Eqn. (15) is rewritten as:

\[
(\lambda + 2\mu) \nabla \Delta - 2\mu \nabla \times \vec{\omega} = \rho \ddot{\vec{u}}
\]

The dilatation \( \Delta \) describes the change in volume. The rotation vector \( \vec{\omega} \) describes the shear deformation. Eqn. (19) clearly shows the contributions of the dilatation and rotation to the equation of motion.
2.3.1 Longitudinal waves

If the vector operation of divergence is performed on the equation above, shear deformations are omitted in the equation, since:

$$\nabla \cdot (\nabla \times \mathbf{\omega}) = 0$$  \hspace{1cm} (20)

Using Eqn. (14) and (17), Eqn. (19) reduces to:

$$(\lambda + 2\mu)\nabla^2 \Delta = \rho \frac{\delta^2 \Delta}{\delta t^2}$$  \hspace{1cm} (21)

This equation can be rewritten into the wave equation:

$$\nabla^2 \Delta = \frac{1}{c_1^2} \frac{\delta^2 \Delta}{\delta t^2}, \quad c_1 = \left(\frac{\lambda + 2\mu}{\rho}\right)^{\frac{1}{2}}$$  \hspace{1cm} (22)

with:

$$\nabla^2 \Delta = \frac{\delta^2 \Delta}{\delta x^2} + \frac{\delta^2 \Delta}{\delta y^2} + \frac{\delta^2 \Delta}{\delta z^2}$$  \hspace{1cm} (23)

Let us for instance focus on the propagation of a longitudinal plane wave in x-direction. Further we assume that the displacements and velocities of the material in y- and z-direction are zero at $t=0$. Consequently, at the front of the longitudinal wave Eqn. (23) reduces to:

$$\nabla^2 \Delta = \frac{\delta^2 \Delta}{\delta x^2}$$  \hspace{1cm} (24)

Giving the one-dimensional wave equation for the change in volume $\Delta$:

$$\frac{\delta^2 \Delta}{\delta x^2} = \frac{1}{c_1^2} \frac{\delta^2 \Delta}{\delta t^2}$$  \hspace{1cm} (25)

The solution of this wave equation is also given by the one-dimensional wave function:

$$\Delta(x, t) = f(x - c_1 t) + g(x + c_1 t)$$  \hspace{1cm} (26)

where, similar to Eqn. (4) the functions $f(x - c_1 t)$ and $g(x + c_1 t)$ determine the specific nature of the wave. We can conclude that a change in volume $\Delta$, due to tensile or compressive stresses, propagates at the velocity:

$$c_1 = \left(\frac{\lambda + 2\mu}{\rho}\right)^{\frac{1}{2}}, \quad c_1 = \sqrt{\frac{E(1-v)}{\rho(1+v)(1-2v)}}$$  \hspace{1cm} (27)
2.3.2 Transverse waves

If the operation of curl is performed on the equation of motions (19) we obtain:

$$(\lambda+2\mu)\nabla \times (\nabla \Delta) - 2\mu \nabla \times (\nabla \times \omega) = \rho \nabla \times \ddot{u}$$

(28)

Since:

$$\nabla \times (\nabla \Delta) = 0$$

(29)

the dilatation can be omitted in Eqn. (28). Using the vector identities:

$$\nabla^2 \omega = \nabla (\nabla \cdot \omega) - \nabla \times (\nabla \times \omega)$$

(30)

$$\nabla \cdot \omega = \frac{1}{2} \nabla \times (\nabla \times \omega) = 0$$

(31)

and recalling Eqn. (18), we obtain for Eqn. (28):

$$2\mu \nabla^2 \omega = 2\rho \frac{\delta^2 \omega}{\delta t^2}$$

(32)

or:

$$\nabla^2 \omega = \frac{1}{c_2^2} \frac{\delta^2 \omega}{\delta t^2}, \quad c_2 = \sqrt{\frac{\mu}{\rho}}$$

(33)

Thus, transverse waves propagate with a velocity:

$$c_2 = \sqrt{\frac{E}{2\rho(1+v)}}, \quad c_2 = \sqrt{\frac{G}{\rho}}$$

(34)

This theoretical analysis shows that in elastic 3D media only two types of waves can propagate, each with a specified wave velocity. In a longitudinal or dilatational wave the material of the medium moves in the direction of propagation, perpendicular to the wave front. For a transverse or distortional wave the material moves in the direction perpendicular to the direction of propagation, parallel to the wave front.
2.4 Conclusions

The propagation of longitudinal and transverse waves in elastic strings and continua has been deduced. With respect to the wave propagation phenomena in strings or fibres we can conclude that a high modulus and low density gives high longitudinal and transverse wave velocities. A higher longitudinal wave velocity gives an increased energy absorption, since a larger part of the string is loaded in tension. Furthermore, we observed that a high longitudinal wave velocity reduces the strain level in the string at a certain impact velocity. So, a high modulus and low density in combination with a high strain at break are desirable properties for a ballistic fibre. HPPE fibres possess the highest specific modulus of all man-made fibres due to the combination of a high modulus and low density. Since the presence of the matrix reduces the in-plane modulus of the laminate and thus the longitudinal wave velocity, the fibre content is chosen as high as possible in ballistic laminates. Furthermore we can conclude that the specific Young’s and shear moduli of an elastic solid (composite) have a strong influence on the transverse and longitudinal wave velocity. Higher wave velocities enlarge the energy absorbing area during impact, which improves the elastic and kinetic energy storage of an elastic solid. However, Young’s and shear moduli of a composite laminate are strain rate dependent due to the viscoelastic character of fibres and matrix. Therefore, next the influence of high strain rates on material properties of HPPE laminates is discussed in Chapter 3.
Chapter 3

Material properties of HPPE laminates at impact rates

3.1 Introduction

To analyze the impact behaviour of HPPE laminates, it is very important to know the actual moduli and strength values of these laminates at impact conditions. Especially the influence of the matrix material should be investigated, since improvements in impact performance of 10% are reported for HPPE laminates if ductile matrix systems are used instead of brittle matrices [15,29]. During impact of HPPE laminates, strain rates up to $10^5$ s$^{-1}$ do occur in fibres and matrix. Material properties can be quite different at such high strain rates. In this Chapter, first the influence of such high strain rates on the moduli of fibres, matrix and laminate is investigated. Tensile tests, such as the Hopkinson bar method, are suitable to measure the moduli and strength of a laminate at strain rates up to $10^3$ s$^{-1}$. This technique has been successfully applied to determine the properties of carbon-fibre-reinforced laminates [34]. However, no reliable results for the material properties of HPPE laminates have been reported using this method. In this report another technique is used to determine the material properties at impact rates. By means of dynamic mechanical thermal analysis (DMTA) fibre and matrix properties can be obtained at low loading frequencies and various temperatures. Since the used fibres and matrix materials obey the rules of time-temperature superposition, this method is used to determine the dynamic properties, i.e. modulus and damping, of the HPPE fibre, matrix and laminates at impact rates. Based on the micromechanical analyses (e.g. rule of mixtures) and laminate plate theory, the material properties of a laminate are calculated from the fibre and matrix material properties for different fibre contents and matrix materials. To verify the calculations, predicted laminate properties are compared with experimental values, which have been obtained using high frequency sonic as well as quasi-static measurements. Finally, based on the time-temperature equivalence theory, tensile tests at low temperatures and high strain rates are used to determine the in-plane moduli and strength of different laminates at impact strain rates.

3.2 Material properties of fibres and matrix

Dynamic properties of HPPE fibres and matrix materials can be studied using dynamic mechanical thermal analysis. By means of this technique, the storage modulus $E'$ and loss modulus $E''$ can be determined at frequencies of 1 to 50 Hz and temperatures between -100 and 23°C. The storage modulus represents the elastic component of cyclic loaded viscoelastic materials in uniaxial extension, whereas the loss modulus stands for the viscous contribution to the dynamic response of the material at a certain loading frequency and temperature.
The damping $\tan \delta$ of a viscoelastic material is defined by dividing the loss modulus by the storage modulus. As a result of isothermal frequency scans, curves for the storage and loss moduli are obtained as function of the (angular) frequency for various temperatures. According to the time-temperature superposition principle for viscoelastic materials, curves measured at different temperatures can be shifted to a master curve at a reference temperature by adapting the time or frequency scales. It was shown that time-temperature superposition is allowed for HPPE fibres and the used matrix systems [13,39]. Hence, master curves of $E'$, $E''$ and $\tan \delta$ can be constructed for a chosen reference temperature, by shifting and matching of adjacent curves along the frequency-axis. These master curves are suitable to predict the actual fibre or matrix behaviour at a reference temperature (i.e. $23^\circ$C) up to frequencies of $10^6$ Hz. Therefore, results from DMTA measurements can be used to predict the moduli and damping properties of fibres, matrix and composite at impact strain rates up to $10^6$ s$^{-1}$.

3.2.1 Fibre properties

At high strain rates, that is short loading times, the dynamic modulus of the HPPE fibre is hardly affected by the load frequency [15,39]. Fig. 5 shows the master curve of the actual dynamic modulus \(E = \sqrt{(E''^2 + E'^2)}\) and damping $\tan \delta$ versus the logarithm of the angular frequency. Since the master curve of the HPPE fibre at a reference temperature of $30^\circ$C shows a plateau value for the dynamic modulus at frequencies higher than approximately 1 Hz, the viscoelastic fibre behaves during the impact process as an elastic material. The material properties of the Dyneema® SK60 and SK66 HPPE fibre are listed in Table 3. From tensile tests and Fig. 5 it is shown that the Dyneema® (SK60) SK66 HPPE fibres possess a high longitudinal modulus in the order of (85) 100 GPa at normal strain rates and (100) 120 GPa at high strain rates [2]. The transverse modulus ($E_{yy}$) and shear moduli ($G_{xy}, G_{xz}$) of the HPPE fibre, obtained from off-axis tensile tests at normal strain rates on ultra-high molecular weight HPPE tapes, are in the order of 1 GPa [40]. The unknown value for the out-of-plane shear modulus $G_{yz}$ of the HPPE fibre is assumed to be one third of the hypothetical value for a polyethylene crystal, since the measured value of the in-plane shear modulus $G_{xy}$ of the HPPE tape (or fibre) is also one third of the perfect crystal value [40]. The fibre properties are used to calculate the laminate properties at high strain rates or high frequencies.

3.2.2 Matrix properties

To show the difference in impact behaviour of composites with a glassy or rubbery matrix material, properties of an epoxy and S-I-S (Styrene-Isoprene-Styrene) rubber based laminate are determined. From DMTA measurements it was shown that the viscoelastic behaviour of the epoxy resin is negligible [20]. Therefore, material properties of the epoxy matrix at high frequencies can be obtained from tensile tests at room temperature and normal strain rates. The material properties of the epoxy matrix are also listed in Table 3 [41]. However, the modulus and damping of the S-I-S rubber are clearly strain rate dependent, as shown by DMTA measurements.
Table 3. Used material parameters for fibre and matrices at a loading frequency of 1 MHz and a reference temperature of 23 °C.

<table>
<thead>
<tr>
<th>Material parameters</th>
<th>HPPE fibre SK60, SK66</th>
<th>epoxy (isotropic)</th>
<th>S-1-S rubber (isotropic)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{xx}$ [GPa]</td>
<td>100, 120</td>
<td>3.4</td>
<td>0.3</td>
</tr>
<tr>
<td>$E_{yy}$ [GPa]</td>
<td>3.4</td>
<td>3.4</td>
<td>0.3</td>
</tr>
<tr>
<td>$G_{xy}$ [GPa]</td>
<td>1.0</td>
<td>1.24</td>
<td>0.11</td>
</tr>
<tr>
<td>$G_{xz}$ [GPa]</td>
<td>0.6</td>
<td>1.24</td>
<td>0.11</td>
</tr>
<tr>
<td>$v_{xy}$ [-]</td>
<td>0.29</td>
<td>0.37</td>
<td>0.33</td>
</tr>
<tr>
<td>$\rho$ [kg.m$^{-3}$]</td>
<td>970</td>
<td>1200</td>
<td>950</td>
</tr>
<tr>
<td>tan $\delta$ [-]</td>
<td>0.01</td>
<td>0.04</td>
<td>1.26</td>
</tr>
</tbody>
</table>

Figure 5. Master curves of the dynamic modulus and tan $\delta$ vs. the angular frequency for a reference temperature of 30°C of: • Dyneema® SK60; ○ Dyneema® SK66; and Δ Spectra® 1000.
Materials and mechanical testing

DMTA measurements are performed on a S-I-S (Styrene-Isoprene-Styrene) block polymer matrix material. From a flat S-I-S rubber membrane with a thickness of 2.5 mm, rectangular samples have been cut which were 28 mm long and 5 mm wide. This membrane was produced by compressing rubber beads in a mould at a temperature of 125°C and a pressure of 30 bar during 20 minutes. The dynamic properties of the S-I-S rubber are determined using a DMTA MKIII from Polymer Laboratories. Isothermal frequency scans were performed at frequencies ranging from 1 to 50 Hz and temperatures between -100 and 23°C using a dynamic strain amplitude of 1%. To avoid buckling of the samples as a result of the dynamic strain amplitude, a small static load was applied to obtain a pre-strain of 1%. A master curve for the storage modulus \( E' \) and damping \( \tan \delta \) was constructed from the isothermal frequency scans at a reference temperature of 23°C using the WLF-equation [42], as shown in Fig. 6. To determine the glass-rubber transition temperature \( T_g \) the dynamic properties \( E' \), \( E'' \) and \( \tan \delta \) are measured for different temperatures at a constant frequency of 1 Hz, as shown in Fig. 7.

Experimental results

From the master curve of Fig. 6 we observe that the glass-rubber transition has a strong influence on the dynamic behaviour of the S-I-S rubber at high impact rates or frequencies. As a result of this the values of the storage modulus \( E' \) and damping \( \tan \delta \) at high frequencies are quite different compared to the values at low loading frequencies, e.g. 1 Hz. For instance, the value of the modulus increases from 1 MPa to 0.3 GPa at a frequency of 1 MHz. Surprisingly, Fig. 6 makes clear that at a reference temperature of 23°C and a loading frequency of 1 MHz (impact rates) the S-I-S rubber still behaves as an ductile (elastic) material due to the position of the \( \tan \delta \) curve. The glass transition temperature \( T_g \) of the rubber matrix can be determined from thermal DMTA scans at a constant frequency by the position of the maximum of the \( \tan \delta \) curve. From such measurements, as shown in Fig. 7 for a frequency of 1 Hz, we observe that the glass transition temperature \( T_g \) of the S-I-S rubber is approximately -50°C. Hence, matrix materials with a \( T_g \) well below room temperature (~ -50°C) show still a ductile material behaviour at impact rates.

Since the duration of an impact event is about 20 to 100 μs, only frequencies higher than 0.01 MHz are relevant. Moreover, strain rates are generally lower than 10⁶ s⁻¹, which implies that frequencies of 0.01 to 1 MHz dominate the impact behaviour. This means that the characteristic frequency for the determination of the material parameters of S-I-S rubber at impact rates should be within this region. The values for the modulus and damping of the rubbery matrix, as listed in Table 3, are read from Fig. 6 at a frequency of 1 MHz. This characteristic frequency is chosen to compare the finally calculated material properties of a cross-ply laminate with experimental results obtained from sonic measurements performed at this frequency.
Figure 6. Master curve of the storage modulus ($E'$) and $\tan \delta$ ($E''/E'$) vs. the logarithm of the angular frequency for an S-I-S rubber at a reference temperature of 23°C.

Figure 7. (*) Storage modulus ($E'$) and (△) $\tan \delta$ ($E''/E'$) for an S-I-S rubber versus temperature.
3.3 Material properties of unidirectional lamina

The material properties of the HPPE fibre and matrix materials at strain rates up to $10^6$ s$^{-1}$, as mentioned in Table 3, are used to calculate the properties of a unidirectional lamina by means of micromechanical models. At high fibre volume fractions ($V_f > 70\%$) the influence of voids in the lamina should be taken into account. Therefore, the equations of the used micromechanical models are slightly adapted by defining the sum of the fibre and matrix volume fraction smaller than one ($V_f + V_m < 1$). The moduli of elasticity are formulated according to a cartesian coordinate system, as shown in Fig. 8 for a unidirectional lamina.

![Figure 8. Cartesian coordinate system of a unidirectional lamina (UD)](image)

For the longitudinal and transverse modulus of a unidirectional lamina we obtain using the rule of mixtures (ROM):

$$E_{xx \text{ud}} = V_f E_{xx f} + V_m E_{xx m} (1 - V_m^2)$$  \hspace{1cm} (35)

$$\frac{V_f + V_m}{E_{yy \text{ud}}} = \frac{1}{V_f + V_m} \left( \frac{V_f}{E_{yy f}} + \frac{V_m}{E_{yy m} (1 - V_m^2)} \right)$$  \hspace{1cm} (36)

Indexes 'f', 'm' and 'ud', denote fibre, matrix and unidirectional respectively. The shear moduli of a unidirectional lamina are calculated using the parallel micromechanical model:

$$\frac{V_f + V_m}{G_{xy \text{ud}}} = \frac{1}{V_f + V_m} \left( \frac{V_f}{G_{xy f}} + \frac{V_m}{G_{xy m}} \right)$$  \hspace{1cm} (37)

$$\frac{V_f + V_m}{G_{yz \text{ud}}} = \frac{1}{V_f + V_m} \left( \frac{V_f}{G_{yz f}} + \frac{V_m}{G_{yz m}} \right)$$  \hspace{1cm} (38)

$$G_{xz \text{ud}} = G_{xy \text{ud}}$$  \hspace{1cm} (39)
Material properties

The Poisson’s ratios of a HPPE lamina are given by the micromechanical analyses using:

\[
\nu_{xy \, ud} = \frac{1}{V_f + V_m} (V_f \nu_f + V_m \nu_m) \quad (40)
\]

\[
\nu_{yz \, ud} = \frac{E_{yy \, ud}}{2G_{yz \, ud}} - 1 \quad (41)
\]

\[
\nu_{xz \, ud} = \nu_{xy \, ud} \quad (42)
\]

The density of the unidirectional lamina is given by:

\[
\rho_{ud} = V_f \rho_f + V_m \rho_m \quad (43)
\]

In uniaxially loaded lamina, the storage and loss moduli can be described in a similar manner as the ROM for the Young’s modulus of unidirectional composites [20]:

\[
E'_{xx \, ud} = V_f E'_{xx \, f} + V_m E'_{xx \, m} \quad (44)
\]

\[
E''_{xx \, ud} = V_f E''_{xx \, f} + V_m E''_{xx \, m} \quad (45)
\]

where \(E'\) is the storage modulus and \(E''\) is the loss modulus. The damping of the unidirectional lamina is related to the storage and loss moduli by:

\[
\tan \delta_{ud} = \frac{E''_{xx \, ud}}{E'_{xx \, ud}} \quad (46)
\]

Predicted moduli \((E_{xx}, E_{yy}, G_{xy})\), Poisson’s ratio \((\nu_{xy})\) of a unidirectional epoxy-based lamina \((V_f = 55\%)\) compare well with experimental values, as shown in Table 4 of the next paragraph. Since all material properties of a lamina can be approximated using Eqn.’s (35) to (42), the constitutive equation for elastic orthotropic media can be used to determine the mechanical behaviour of a lamina. The constitutive equation of an orthotropic unidirectional lamina is given by [34,43]:

\[
\begin{bmatrix}
\sigma_{xx \, ud} \\
\sigma_{yy \, ud} \\
\sigma_{xy \, ud}
\end{bmatrix} = \frac{1}{(1-\nu_{xy} \nu_{yx})} \begin{bmatrix}
E_{xx \, ud} & \nu_{yx} E_{yy \, ud} & 0 \\
\nu_{xy} E_{xx \, ud} & E_{yy \, ud} & 0 \\
0 & 0 & (1-\nu_{xy} \nu_{yx}) G_{xy \, ud}
\end{bmatrix} \begin{bmatrix}
\epsilon_{xx \, ud} \\
\epsilon_{yy \, ud} \\
\epsilon_{xy \, ud}
\end{bmatrix} \quad (47)
\]

\[
\begin{bmatrix}
\sigma_0 \\
\epsilon_0
\end{bmatrix} = S_0 \begin{bmatrix}
\sigma_0 \\
\epsilon_0
\end{bmatrix} \quad (48)
\]

where \(\sigma\) is the stresses, \(\epsilon\) the strains and \(S_0\) is the orthotropic stiffness matrix of a 0° ply, with the fibres oriented in the 0° direction (x-coordinate).
3.4 Material properties of cross-ply laminates

3.4.1 Moduli of elasticity of cross-ply laminates

Macroscopic material properties of a symmetric [0/90]s cross-ply laminate can be calculated from the material properties of the unidirectional lamina, with the fibres oriented in the 0° and 90° directions, using laminate plate theory [34,43]. Applying this theory to such a laminate finally results in the constitutive equation (49):

\[
\begin{bmatrix}
N \\
M
\end{bmatrix} = \begin{bmatrix}
A & B \\
B & D
\end{bmatrix} \begin{bmatrix}
\varepsilon_0 \\
\kappa
\end{bmatrix}
\]

where \(N\) and \(M\) are respectively the resultant tractions and moments, \(\varepsilon_0\) the midplane strains and \(\kappa\) is the plate curvatures. The matrices \(A, B\) and \(D\) are the extensional stiffness matrix, the coupling stiffness matrix and the bending stiffness matrix, respectively. For a symmetrically stacked laminate the coupling stiffness matrix \(B\) is zero. Since the 0 and 90 plies have the same thickness, the extensional stiffness matrix is calculated by the average of the individual stiffness matrices \(S_0\) and \(S_{90}\) of the two basic plies:

\[
A = \frac{d}{2}S_0 + \frac{d}{2}S_{90}
\]

where \(d\) is the thickness of the laminate. According to laminate plate theory the bending stiffness (matrix \(D\)) of cross-ply laminates can be calculated from the individual stiffness matrices \(S_0\) and \(S_{90}\) using Eqn. (51).

\[
D = \frac{2}{3} \left( \frac{d}{2} \right)^3 \left[ c_0 S_0 + c_{90} S_{90} \right]
\]

where the coefficients \(c_0\) and \(c_{90}\) are a function of the number of layers \(n\) as shown in Fig. 9.

\[\text{Figure 9. The coefficients } c_0 \text{ and } c_{90} \text{ as a function of } n \text{ for a [0/90]s laminate.}\]
Material properties

The values of these coefficients depend on the position of the 0° and 90° plies. The solid line represents \( c_0 \) for a laminate with the 0° plies positioned on the outside of a symmetric [0/90]_\text{ss} laminate, whereas the dashed line represents a stacking sequence with the 90° plies on the outside of the laminate (Fig. 9). The coefficients \( c_0 \) and \( c_{90} \) approach a value of 0.5 if \( n \) is higher than five. Material properties of the cross-ply laminate follow then from the stiffness matrix \( \mathbf{S} \) of Eqn. (52).

\[
\mathbf{S} = \frac{1}{2} \left( \mathbf{S}_{0} + \mathbf{S}_{90} \right) \tag{52}
\]

From Eqn. (51) we conclude that the difference in bending stiffness between the two possible stacking sequences for a symmetric [0/90]_\text{ss} laminate can be neglected if the number of layers \( n \) exceeds five, which makes a total of ten plies. Hence, matrices \( \mathbf{A} \) and \( \mathbf{D} \) can be determined using stiffness matrix \( \mathbf{S} \) of Eqn. (52). This means that the material properties of a laminate, with more than ten plies, may be approximated by applying the rule of mixtures to the 0° and 90° plies (see Eqn. (52)), giving for the in-plane moduli of the laminate:

\[
E_{xx \text{ lam}} = \frac{1}{2} \left( E_{xx \text{ ud}} + E_{yy \text{ ud}} \right) \tag{53}
\]

\[
E_{yy \text{ lam}} = E_{xx \text{ lam}} \tag{54}
\]

\[
G_{xy \text{ lam}} = G_{xy \text{ ud}} \tag{55}
\]

Indexes 'ud' and 'lam' denote unidirectional and laminate respectively. The out-of-plane shear moduli are calculated from the basic plies using again the parallel micromechanical model:

\[
\frac{1}{G_{yz \text{ lam}}} = \frac{1}{2} \left( \frac{1}{G_{yz \text{ ud}}} + \frac{1}{G_{yz \text{ ud}}} \right) \tag{56}
\]

\[
G_{xz \text{ lam}} = G_{yz \text{ lam}} \tag{57}
\]

Since:

\[
\nu_{xy \text{ ud}} = \nu_{yz \text{ ud}} \frac{E_{xx \text{ ud}}}{E_{yy \text{ ud}}} \tag{58}
\]

we obtain for the Poisson’s ratios of the laminate:

\[
\nu_{xy \text{ lam}} = \nu_{yz \text{ ud}} \frac{E_{xx \text{ ud}}}{E_{yy \text{ lam}}} = \nu_{xy \text{ ud}} \frac{E_{yy \text{ ud}}}{E_{yy \text{ lam}}} \tag{59}
\]

\[
\nu_{yz \text{ lam}} = \frac{1}{2} \left( \nu_{yz \text{ ud}} + \nu_{xz \text{ ud}} \right) \tag{60}
\]

\[
\nu_{xz \text{ lam}} = \nu_{yz \text{ lam}} \tag{61}
\]

The density of the laminate is given by:

\[
\rho_{\text{lam}} = V_p \rho_p + V_m \rho_m \tag{62}
\]
Chapter 3

We assume that the damping in the separate plies of a cross-ply laminate is not influenced by the laminate structure. Hence, Eqns.'s (44) to (46) are also used to determine the damping properties of a cross-ply laminate, whereas the value of $\tan \delta$ for the HPPE laminate is equal to the damping of the fibres. The reason for this is that the damping of the unidirectional lamina is dominated by the HPPE fibres due to the large differences in axial stiffness of the fibres and matrices, see Eqn. (46).

The calculated moduli ($E_{xx}$, $E_{yy}$, $G_{xy}$, $G_{yy}$, $G_{xz}$), Poisson's ratios ($v_{xy}$, $v_{yz}$, $v_{xz}$) and damping ($\tan \delta$) of an unidirectional epoxy-based lamina ($V_f = 55\%$) and an S-I-S rubber based cross-ply laminate ($V_f = 70\%$) are shown in the third and fifth columns respectively of Table 4. These values compare well with experimental data, supplied by DSM High Performance Fibres BV, shown in the second and fourth columns. The material properties of the epoxy-based unidirectional laminate are obtained from tensile and shear tests at room temperature and normal strain rates. The material properties of the S-I-S rubber based cross-ply laminate are obtained from measured wave propagation velocities, using ultrasonic experiments at a frequency of 1 MHz. As shown in Chapter 2, elastic moduli of a cross-ply laminate at high frequencies can be deduced from measured longitudinal and transverse wave propagation velocities. The material properties of an arbitrarily unidirectional or cross-ply composite can now be calculated as a function of fibre and matrix properties and the volume fractions ($V_f$, $V_m$). In Appendix B the calculated material properties of HPPE laminates are shown for the used matrix systems with different fibre contents. Moreover, the theoretical wave propagation velocities are also shown for the different laminates.

### 3.4.2 In-plane tensile strength of cross-ply laminates

To perform a numerical analysis of the impact behaviour of HPPE laminates, reliable strength values have to be determined by experiments at impact rates. Experiments in this high velocity regime are difficult to perform, due to the required high strain rates and present waves propagation phenomena. Therefore, the in-plane strength of HPPE laminates at impact rates is often predicted using the failure strain of the HPPE fibre at normal strain rates and the modulus of the fibre at impact rates, which are respectively 3.7% and 120 GPa [2]. In this experimental research the tensile strength and modulus of HPPE laminates is analyzed using tensile tests at low temperatures (-50°C) and high strain rates (0.72 s$^{-1}$) [44]. Since the time-temperature equivalency theory holds for the HPPE fibre and used polymer matrices, the influence of strain rate and temperature on the material properties of HPPE laminates can be measured using these quasi-static experiments. Although wave propagation phenomena between fibre and matrix and subsequent layers are not present, these experiments show the consequences of impact strain rates on the tensile strength and modulus for laminates with a brittle glassy (polystyrene) or ductile rubbery (S-I-S) matrix.
Material properties

Table 4. Experimental data and model predictions using laminate plate theory of HPPE composites based on an epoxy and rubber matrix.

<table>
<thead>
<tr>
<th>Laminate parameters</th>
<th>Tensile tests epoxy UD60</th>
<th>Model prediction epoxy UD60</th>
<th>Sonic tests rubber UD66 cross-ply (1 MHz)</th>
<th>Model prediction rubber UD66 cross-ply</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{xx}$ [GPa]</td>
<td>46.6</td>
<td>46.6</td>
<td>32.8</td>
<td>36.0</td>
</tr>
<tr>
<td>$E_{yy}$ [GPa]</td>
<td>3.6</td>
<td>3.62</td>
<td>32.8</td>
<td>36.0</td>
</tr>
<tr>
<td>$G_{xy}$ [GPa]</td>
<td>1.1</td>
<td>1.10</td>
<td>0.1</td>
<td>0.30</td>
</tr>
<tr>
<td>$G_{xz}$ [GPa]</td>
<td>-</td>
<td>-</td>
<td>0.3</td>
<td>0.28</td>
</tr>
<tr>
<td>$G_{yz}$ [GPa]</td>
<td>-</td>
<td>-</td>
<td>0.3</td>
<td>0.28</td>
</tr>
<tr>
<td>$v_{xy}$ [-]</td>
<td>0.32</td>
<td>0.32</td>
<td>-0.0065</td>
<td>0.0089</td>
</tr>
<tr>
<td>$v_{yx}$ [-]</td>
<td>-</td>
<td>-</td>
<td>0.61</td>
<td>0.53</td>
</tr>
<tr>
<td>$v_{xz}$ [-]</td>
<td>-</td>
<td>-</td>
<td>0.61</td>
<td>0.53</td>
</tr>
<tr>
<td>$\rho$ [kg.m$^{-3}$]</td>
<td>-</td>
<td>1074</td>
<td>952</td>
<td>954.5</td>
</tr>
<tr>
<td>$V_f$ [%]</td>
<td>55</td>
<td>55</td>
<td>70</td>
<td>70</td>
</tr>
<tr>
<td>$V_m$ [%]</td>
<td>45</td>
<td>45</td>
<td>29</td>
<td>29</td>
</tr>
<tr>
<td>$\tan \delta$ [-]</td>
<td>-</td>
<td>0.01</td>
<td>-</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 5. Material properties of fibre and matrices at normal strain rates and temperature, $T=23$ °C.

<table>
<thead>
<tr>
<th>Material parameters</th>
<th>HPPE fibre (orthotropic) SK60, SK66</th>
<th>polystyrene (isotropic)</th>
<th>S-I-S rubber (isotropic)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{xx}$ [GPa]</td>
<td>85, 100</td>
<td>3.4</td>
<td>0.002</td>
</tr>
<tr>
<td>$E_{yy}$ [GPa]</td>
<td>3.4</td>
<td>3.4</td>
<td>0.002</td>
</tr>
<tr>
<td>$\sigma_{\text{max}}$ [MPa]</td>
<td>3000</td>
<td>~50</td>
<td>~15</td>
</tr>
<tr>
<td>$\varepsilon_{\text{max}}$ [%]</td>
<td>3.7</td>
<td>3.5</td>
<td>~800</td>
</tr>
<tr>
<td>$T_\delta$ [°C]</td>
<td>-120</td>
<td>95</td>
<td>-50</td>
</tr>
</tbody>
</table>
Chapter 3

Materials and mechanical testing

The cross-ply laminates [0/90] used in this study are supplied by DSM High Performance Fibres BV. The volume fraction of the (SK66) fibres in the laminates is approximately 70%. To analyze the material properties of the HPPE laminates, cross-ply laminates were available with a polystyrene or S-I-S rubber matrix. The material properties of the fibre and matrices at normal strain rates and temperature are shown in Table 5. Notice that the material properties of the glassy polystyrene matrix are comparable to the properties of the glassy epoxy matrix. The [0/90] cross-ply, which is delivered on a 1 m wide roll, is the basic material of thick laminates. Rectangular specimens have been cut from this basic material, which were 600 mm long and 15 mm wide. Due to the cutting process a fluctuation in the wideness of the specimens is introduced of approximately 5%. The length-direction of the specimen was always taken in the 0° (role-) direction. 'Hard' ballistic laminates are obtained after hot pressing for 10 minutes at 30 bar and 10 minutes at 80 bar at a temperature of 125 °C. The thickness of the specimens was after consolidation 0.135 mm.

Tensile tests on the laminates were performed on a servo-hydraulic Zwick REL tensile tester equipped with a thermostatically controlled oven and pneumatic fibre-clamps. A tensile force reduction in the clamping unit has been accomplished as a result of the circular entrance geometry (radius of 8 mm), which is necessary to avoid slippage. The length of the clamps was chosen 30 mm. The specimen was folded in the clamping units to eliminate slippage. The gauge length (Lo) of the specimen between the clamps was 275 mm. Tensile experiments were performed on the two types of composites at a strain rate of 0.72 s⁻¹ and temperatures ranging from -50 to 23°C (-50, -30, -10, and 23°C). Each experiment is performed at least five times to be able to determine the mean values and standard deviations. Measurements were performed after the specimen had a stable temperature for 5 minutes. From all stress-strain curves secant moduli were derived between 0.5 and 1.0% strain.

Experimental results

From the stress-strain curves at low temperatures, as shown in Fig. 10, we can conclude that the failure behaviour of HPPE fibres in the glassy and rubber matrix is quite different. HPPE fibres in the rubber matrix break almost simultaneously when the maximum stress is reached, whereas the failure process of HPPE fibres in the glassy matrix starts earlier making the fibres break in a sequential manner. So, in a ductile rubber matrix the fibre failure is delayed and less scattered, which is in agreement with the effect of adhesion for composites as determined by acoustic emission [45]. These differences in failure behaviour are probably caused by the rather poor adhesion between the fibre and the polystyrene matrix, and the brittleness of the matrix. This is confirmed by the fact that polystyrene based laminates loose rigidity in a dramatic manner after failure at low temperatures, as shown in Fig. 11.
Material properties

Figure 10. Typical stress-strain curves of HPPE fibres in both a S-I-S rubber and a polystyrene matrix at a temperature of -50 °C and a strain rate of 0.72 s⁻¹.

Figure 11. Polystyrene based specimens before and after tensile test at a strain rate of 0.72 s⁻¹ and a temperature of -50°C.
The average secant modulus for both types of laminates is given in Fig. 12 as a function of the temperature at a high strain rate (0.72 s\(^{-1}\)). Errorbars have been plotted for each temperature, where the total length of the errorbar is two times the standard deviation at that temperature. The solid lines are regression lines according to the method of least squares. Slightly higher values for the mean secant moduli are found for HPPE fibres in a rubbery matrix than for HPPE fibres in a glassy matrix. Since the errorbars of both materials overlap, there is no significant difference in the secant moduli for the two composites. Hence, it can be concluded that as expected the modulus of the laminate is hardly influenced by the type of matrix. Thus, the modulus of the laminate depends on the viscoelastic behaviour of the HPPE fibres [12]. Notice that the experimental values for the modulus at a temperature of \(-50^\circ\)C are in good agreement with the measured values using sonic tests, as listed in Table 4. Hence, quasi-static tensile experiments at high strain rates and low temperatures can be used to determine the material properties at impact conditions and normal temperatures.

In Fig. 13 the temperature dependence of the average ultimate strength is presented for both laminates at a strain rate of 0.72 s\(^{-1}\). Again errorbars and least squares regression line are shown for both laminates. From Fig. 13 we can conclude that the ultimate strength is higher for the laminates based on the rubber matrix. The strength of the S-I-S rubber based laminates increases at lower temperatures due to the viscoelastic behaviour of the HPPE fibres, as shown in Fig. 14 [12]. Smith [14] observed that the ultimate strain of a rubber shows a maximum at temperatures close to the glass transition temperature \(T_g\). At temperatures well below \(T_g\) or high strain rates the ultimate strain of the rubber decreases dramatically (Fig. 15). Consequently, a polystyrene rubber with a \(T_g\) of 95°C is even brittle at normal strain rates and room temperature, which initiates fibre failure. However, the S-I-S rubber matrix possesses a \(T_g\) of \(-50^\circ\)C, giving a maximum ultimate strain at this temperature and normal strain rates (see Fig. 7). Hence, the increase of the ultimate strength at low temperatures or high strain rates depends on the ultimate strain of the matrix material. Consequently, rubber matrices with a glass transition temperature well below room temperature improve the laminate strength at low temperatures or impact rates. The in-plane strength at a temperature of \(-50^\circ\)C or impact rates (Fig. 6) of respectively an S-I-S rubber and polystyrene based HPPE laminate is approximately 1100 MPa and 900 MPa.
Figure 12. Modulus as a function of temperature at a strain rate of 0.72 s⁻¹ for an S-I-S rubber (♦) and polystyrene (△) matrix.

Figure 13. Ultimate strength as a function of temperature at a strain rate of 0.72 s⁻¹ for an S-I-S rubber (♦) and polystyrene matrix (△).
Figure 14. Failure stress of the SK60 fibre versus temperature for various strain rates.

Figure 15. The variation of the ultimate strain versus the logarithm of the strain rate for a polystyrene rubber.
3.5 Conclusions

The material properties of the HPPE laminate are determined at impact conditions using DMTA and tensile test. DMTA is a very suitable method to determine the moduli and damping properties of fibre, matrix and unidirectional laminate. Micromechanical models (ROM) and laminate plate theory can be used to calculate material properties of symmetric cross-ply’s at impact conditions, i.e. high strain rates. DMTA measurements on the strain rate dependent S-I-S rubber make clear that during impact conditions this matrix material still behaves as a rubber due to the low glass transition temperature of -50°C. Furthermore, it is shown that at high frequencies or impact rates the modulus increases from 1 MPa to 0.3 GPa. By means of tensile test at low temperature (-50°C) and high strain rates (0.72 s⁻¹) the ultimate properties of the laminate are measured at impact rates. The temperature and strain rate dependence of the modulus and ultimate strength of cross-ply laminates with a rubber (S-I-S) and a glassy (polystyrene) matrix were evaluated. From the experiments it can be concluded that as expected the modulus is fully determined by the viscoelastic behaviour of the HPPE fibres. Similar to the behaviour of the single fibres, the ultimate strength of laminates increases with decreasing temperature for ductile matrix materials. For brittle matrices, however, the effect is diminished by the low ultimate strain of the matrix, which initiates fibre failure. Hence, the increase of the ultimate strength at low temperatures or high strain rates depends on the ultimate strain of the matrix. To avoid brittle material behaviour of the matrix at impact rates, the glass transition temperature of a rubbery matrix should be well below room temperature (~-50°C). The difference in strength values of the laminate for brittle and ductile matrices at low temperatures, or high strain rates, is approximately 17%. This value indicates, that the difference in impact performance as a result of the type of matrix could be explained from differences in the in-plane strength of the laminate. Therefore cross-ply laminates with a brittle matrix seem to be less suitable for applications which demand a high energy absorption. The calculated material properties for the moduli and the measured in-plane strength values are used in Chapter 4 and 5 to simulate the impact behaviour by means of finite element methods.
Chapter 4

Modelling the elastic behaviour of HPPE laminates during ballistic impact

4.1 Introduction

A ballistic composite plate consists of a large number of unidirectional sheets with different fibre orientations. From a computational point of view, it is impracticable to model all individual layers. Therefore, a so-called 'macroscopic model' is used to simulate the impact behaviour of a laminate. Within this macroscopic model no microstructural details, such as the geometry and number of fibres, are taken into account because the laminate is assumed to behave as a homogeneous orthotropic media. In Chapter 2 we deduced that the density and moduli of elasticity of isotropic elastic media, i.e. Young’s and shear moduli, determine the wave propagation velocities and thus the size of the energy absorbing area. The influence of all orthotropic material properties of a laminate on the energy absorption can be investigated using finite element methods. After impact, due to high contact forces, fibres break immediately at the contact surface between the projectile and the target. However, the elastic response of the laminate still depends on the impact behaviour of the non-damaged part of the composite plate. Therefore, a model without the incorporation of failure mechanisms will give some insight in the behaviour of a composite laminate just after impact, i.e. until ~20 μs after impact. The transient dynamic behaviour of the HPPE laminates is investigated using FEM for different fibre contents, matrix materials and laminate structures by changing the orthotropic material properties of the laminate. Moreover, the presence of damping is taken into account using so-called 'Rayleigh' damping [36]. As a result of these finite element calculations the material properties of the HPPE laminate can be classified with respect to their influence on the kinetic and elastic energy absorption. To verify the numerical solutions, obtained values for the longitudinal and transverse wave velocities are compared with the theoretical values as mentioned in Appendix B.

4.2 The finite element model

Non-penetrating ballistic experiments using deformable 9 mm Parabellum hand-gun ammunition (see Table 1 of Chapter 1) on a 10 mm thick cross-ply laminate showed that the duration of such an impact event is approximately 100 μs [30]. The maximum deflection at the point of impact (trauma) is approximately 30 mm.
Within the first 20 µs of such an impact event more than 50% of the kinetic energy of the bullet has been absorbed. It was shown that the maximum obtainable energy from straining the fibres passing through the 9 mm wide impact zone (Fig. 20), is equivalent to the kinetic energy lost by the projectile at this time [21]. Moreover, the pyramid shaped out-of-plane deflection of the cross-ply is small, so the kinetic energy stored within the pyramid is small. Hence, the energy absorption within the first 20 µs depends on the in-plane straining and out-of-plane shear deformation of the laminate, which is controlled by the longitudinal and transverse wave propagation velocities, as shown in Chapter 2. It appears that during the first 20 µs a critical response occurs, which decides whether the projectile is stopped or the target is perforated [21]. Since no failure processes are present in the model, only the first 20 µs of the impact event are analyzed.

When a cross-ply HPPE laminate is impacted by a projectile, a longitudinal strain wave will travel through the material with a velocity of approximately 5.10^3 ms⁻¹. After 20 µs, the front of the strain wave has travelled 100 mm away from the point of impact, similar to the transversal wave pattern which occurs when a stone is thrown in a pool. This implies that the material outside a circle of 100 mm around the point of impact does not experience an impact. This simple analysis shows that the finite element model, necessary to describe the behaviour of a HPPE based composite plate, can be limited to a circular mesh with a diameter of 200 mm. Since the material is 'motionless' at a radius of more than 100 mm, all degrees of freedom at the circular edge of the finite element model can be suppressed.

Since the laminates are orthotropic and impacted normal to the surface, a three-dimensional stress-state occurs due to the longitudinal strain and compression waves in respectively the in-plane and out-of-plane directions. Three-dimensional (3D) models have the advantage of being able to describe out-of-plane stresses and strains in the vicinity of the impact. However, the ballistic behaviour of laminates in a non-destructive analysis is controlled by the longitudinal and transverse wave propagation, which can be described using two-dimensional (2D) models. Moreover, two-dimensional models are more cost-effective and easier to interpret. However, for thin plates, a 2D and 3D model will show similar results [46]. In this research, the composite plate is modelled using an 2D isoparametric thick-shell element with four nodes, 24 degrees of freedom and a parabolic shear distribution [47]. The material properties of the plate in the out-of-plane direction are accounted for by the out-of-plane shear moduli and Poisson’s ratios.

In order to take the influence of stacking sequence into account, the modelled composite plate consists of three layers, each representing a cross-ply laminate with more than 10 plies. The material properties of the [0/90]ₙₙ layers were deduced in Chapter 3 for different fibre volume fractions and matrix materials, as mentioned in Appendix B. Each [0/90]ₙₙ layer is given its own orientation (i.e 0° or 45°), using the composite option of the finite element program MARC [47]. The final properties of the composite plate are calculated with this laminate plate theory based option.
The orientations of the three cross-ply laminates are selected as [0/90]_as or [45/-45]_as. The composite plate is kept symmetric and the thickness of the middle cross-ply layer is taken as 50% of the plate thickness. Table 6 gives an overview of the composites structure.

Table 6. Lay-up of the composite laminate.

<table>
<thead>
<tr>
<th>Lay-up</th>
<th>outer cross-ply</th>
<th>middle cross-ply</th>
<th>outer cross-ply</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of lamina</td>
<td>2n</td>
<td>4n</td>
<td>2n</td>
</tr>
<tr>
<td>orientation</td>
<td>β</td>
<td>β</td>
<td>β</td>
</tr>
<tr>
<td>thickness</td>
<td>25%</td>
<td>50%</td>
<td>25%</td>
</tr>
</tbody>
</table>

The composite plate is impacted by a rigid bar at a velocity of approximately 500 ms⁻¹ (Fig. 16a). After impact, only the circular edge of the projectile will make contact with the plate since the plate bends and the projectile is assumed to be rigid. The projectile is modelled by attaching point-masses, represented by the black points in Fig. 16b, to the nodes of the finite element mesh along this circular edge. Consequently, all nodes of the mesh which are struck by the projectile acquire an initial velocity of 500 ms⁻¹. All point-masses possess the same out-of-plane displacement as a result of applied nodal tyings. The real initial velocity of the projectile before impact can be calculated using the law of linear momentum.

Figure 16a. Schematic representation of a plate struck by rigid bar.

Figure 16b. Cross-section of the plate after impact. Black points denote point-masses.
Modelling elastic behaviour

The geometry of the composite plate and projectile are: a plate diameter of 200 mm, a plate thickness of 10 mm, a projectile diameter of 10 mm and a projectile mass of 10 g. Due to the presence of symmetry-planes, only a quarter of the entire laminate has to be analyzed. To avoid artificial reflection of part of the strain waves as a consequence of changing finite element geometry [48], an equidistant finite element mesh is used.

4.3 Modelling of damping

Since HPPE fibres are viscoelastic materials [39], the FEM model should take the frequency dependence of the material properties into account. From a computation point of view it is impracticable to take the full relaxation spectrum by means of complex viscoelastic models into account [41]. Therefore, a single relaxation time approximation is employed, that is able to describe the storage modulus $E'$ and damping $\tan \delta$ of the laminate at impact conditions (1 MHz). Since the damping of the laminate is dominated by the HPPE fibres, see Eqn. (46) of Chapter 3, the damping can be chosen proportional to the stiffness of the laminate. In the finite element model, this can be accomplished using proportional or 'Rayleigh' damping [36,47]. The necessary 'Rayleigh' damping coefficient $\gamma$ can be related to the experimental damping values (tan $\delta$) using the single Kelvin-Voigt viscoelastic model [41,43]. The constitutive equation for viscoelastic materials according to a single Kelvin-Voigt element is given by:

$$\sigma(t) = k \epsilon(t) + \eta \dot{\epsilon}(t)$$  \hspace{1cm} (63)

where $\sigma$ is the stress, $\epsilon$ is the strain, $\dot{\epsilon}$ is the strain rate, $k$ is the spring constant and $\eta$ is the damping constant. If we apply the Laplace transformation to Eqn. (63), the constitutive equation is translated to the frequency domain, giving:

$$\sigma(\omega) = k \epsilon(\omega) + i\omega \eta \epsilon(\omega)$$  \hspace{1cm} (64)

where $i$ is the imaginary number and $\omega$ the angular loading frequency. In general, Eqn. (64) is rewritten using the storage modulus ($E'$) and loss modulus ($E''$):

$$\sigma(\omega) = E' \epsilon(\omega) + iE'' \epsilon(\omega)$$  \hspace{1cm} (65)

Hence,

$$k = E' ; \eta = \frac{E''}{\omega}$$  \hspace{1cm} (66)

In case of proportional damping we obtain:

$$\eta = \tau_R k$$  \hspace{1cm} (67)

where $\tau_R$ or 'Rayleigh' coefficient $\gamma$ is the characteristic relaxation time of the single viscoelastic model.
As shown in Chapter 3, the value of \( E^0 \) and \( E'' \) are known at impact conditions (~1 MHz). Hence, the relaxation time \( \tau_r \) of the material can be calculated at a characteristic angular frequency \( \omega_r \) of ~10^6 s\(^{-1}\). Using Eqn. (66) and (67) and \( \tan \delta = E''/E' \), we obtain:

\[
\gamma = \tau_r = \frac{\tan \delta}{\omega_r}
\]  

(68)

According to Eqn. (68) and the value of \( \tan \delta \) for the HPPE laminate (~0.01), the value of \( \tau_r \) is approximately equal to 10^8 s at impact conditions (\( \omega_r \sim 10^6 \) s\(^{-1}\)). In terms of finite element terminology, the equations of equilibrium for a finite element system in motion can be written as:

\[
M \ddot{U} + C \dot{U} + K U = F, \quad C = \gamma K
\]  

(69)

where \( U, \dot{U} \) and \( \ddot{U} \) are the displacement, velocity and acceleration vectors; \( M, C \) and \( K \) are respectively the mass, damping and stiffness matrices; \( F \) is the external force vector of the finite element assemblage. Similar to the damping constant \( \eta \) of Eqn. (67), the damping of the laminate can be approximated by choosing matrix \( C \) proportional to the orthotropic stiffness matrix \( K \), using a 'Rayleigh' coefficient \( \gamma \) or single relaxation time \( \tau_r \) of 10^8 s.

### 4.4 Numerical results

Ballistic performance of the laminates is characterised by the velocity decay of the projectile and the deflection at the centre of the laminate (Trauma). Impact behaviour of two laminate structures ([0/90]\(_{12n}\) and ([0/90]\(_{30}\)[-45/45]\(_{60}\)[0/90]\(_{30}\)) with an epoxy or S-I-S rubber matrix is analyzed for fibre volumes fractions of 55%, 70% and 80%. At higher fibre contents the in-plane moduli are increased. Notice that the out-of-plane shear modulus of the fibre exceeds the shear modulus of the rubber matrix (Table 3). Therefore, at higher fibre volume fractions higher out-of-plane shear moduli are obtained for the laminate with a rubber matrix. However, for epoxy matrix composites the opposite occurs, so the out-of-plane shear moduli decrease with increasing fibre contents and thus lower matrix contents (see Appendix B). Although, the out-of-plane shear moduli of the laminate are always higher for epoxy matrix composites.

Numerical results show for all fibre contents that a glassy (epoxy) matrix gives a better performance than an S-I-S rubber matrix, due to the higher out-of-plane shear moduli (Fig. 17). At higher fibre contents both laminate structures show in the case of a rubber matrix improved ballistic performance (Fig. 18). Whereas, for epoxy matrix composites ballistic performance decreases at higher fibre contents. Consequently, the energy storage at a fibre volume fraction of 55% is even higher than for epoxy matrix composites with a fibre volume fraction of 80%. Although the latter type of composite possesses a higher modulus of elasticity. Hence, during the first 20 \( \mu s \) of an impact event the impact behaviour of a laminate plate is dominated by the value of the out-of-plane shear moduli.
Figure 17. (a) The velocity decay of the projectile and (b) Trauma values for a \([0/90]_{120}^9\) laminate \((V_f=70\%)\) with an epoxy (dashed line) and S-I-S rubber (solid line) matrix.

Figure 18. The velocity decay of the projectile for a quasi-isotropic laminate structure, \([0/90]_{30}^9[-45/45]_{90}^9[0/90]_{30}^9\), with an S-I-S rubber matrix and a fibre volume percentage \(V_f\) of 55\% (solid line) and 80\% (dashed line).
When proportional damping is present, ballistic performance is hardly affected for 'Rayleigh' damping coefficients $\gamma$ of $10^5$, $10^7$ and $10^8$ s. Notice that a $\gamma$ value of $10^6$ s corresponds to a very high $\tan \delta$ value of 1 for the fibres and laminate (Eqn. (68)). In Chapter 3 it was shown that the values of $\tan \delta$ for the fibres (Fig. 5) and laminate actually are much smaller than 1. Hence, the influence of damping at impact conditions or high loading frequencies is negligible. Consequently, the presence of the dissipative term can be neglected in the constitutive equation (Eqn. (69)). As a result of this, the total energy within the model is constant and equal to the initial kinetic energy of the projectile. The kinetic energy lost by the projectile is for approximately 50% stored in elastic energy and 50% in kinetic energy of the composite plate (Fig. 19).

![Figure 19. The kinetic energy of the projectile (solid line) and the elastic and kinetic energy (dashed lines) of the composite plate for a [0/90]$_{120^\circ}$ laminate ($V_f=70\%$) with an S-I-S rubber matrix.](image)

Due to the high longitudinal wave velocity, the longitudinal strain wave propagates rapidly through the medium. Fig. 20 shows the deformation pattern in a [0/90]$_{120^\circ}$ laminate with a S-I-S rubber matrix at 10 $\mu$s after an impact with a round bar, using the first component of the principal strain in the neutral plane. Fig. 21 shows the corresponding energy density. It is clear that the longitudinal strain wave propagates faster in the high modulus directions, i.e. the 0° and 90° directions. Surprisingly, the energy density pattern propagates mainly with the speed of the transversal wave. Hence, the energy absorption in the laminate plates is dominated by the transverse wave propagation and thus the out-of-plane shear modulus (Eqn. (34) of Chapter 2).
To analyze the influence of the stacking sequence, calculations are performed using an orthotropic ([0/90]_{20}) and quasi-isotropic ([0/90][{-45/45}][0/90]_{30}) laminate structure. Numerical results show that the quasi isotropic composite laminate gives a better ballistic performance as a result of the larger energy absorbing area (Fig. 22). Due to the higher Poisson's ratio $v_{xy}$ of the total quasi-isotropic laminate structure, a circular shaped deformation zone (Fig. 22) is observed instead of a cross-shaped deformation zone (Fig. 20). However, local stresses and strains near the projectile are higher for quasi-isotropic laminate structures and may initiate failure.

Numerical and theoretical wave propagation velocities are compared to check the numerical solutions. The numerical longitudinal and transverse wave velocities $c_1$ and $c_2$ in the $0^\circ$ direction of the S-I-S rubber based laminate ($V_r=70\%$) are respectively 5980 m$^s^{-1}$ and 630 m$^s^{-1}$. These values have been obtained by measuring the wave propagation during a time interval of 10 $\mu$s, i.e 5 and 15 $\mu$s after impact. These wave propagation velocities compare well with the theoretical $c_1$ and $c_2$ values, which are respectively 6140 m$^s^{-1}$ and 540 m$^s^{-1}$ (see Chapter 2 and Appendix B). Hence, the predicted numerical solutions are in agreement with theoretical observations, as mentioned in Chapter 2.

### 4.5 Conclusions

Within the assumptions of the macroscopic model, the impact behaviour of composite laminates is investigated. The analyses overestimate the ballistic performance of the laminates since no failure mechanisms are present in the model. However, the model clearly shows the influence of the different material properties.

From the numerical results it can be concluded that the out-of-plane shear modulus should be as high as possible. The positive effect of a higher modulus of elasticity on the performance can be diminished by a reduction of the out-of-plane shear moduli $G_{xz}$ and $G_{yz}$. However, in order to avoid premature failure and consequently a reduction in out-of-plane shear stiffness, a ductile matrix with a high failure strain is preferred.

The effect of a high modulus of elasticity on the propagation of the energy density is surprisingly low. The energy density remains high in the vicinity of the impact due to the large out-of-plane shear strains and propagates with the transverse wave velocity. Hence, it is believed that experimentally observed improvements in ballistic performance for high in-plane moduli are not solely caused by the longitudinal propagation of energy. The reduction of the strain wave amplitude, as a result of higher in-plane moduli and longitudinal wave speed (Eqn. (8)), is definitely more important since it postpones the initiation of failure. With respect to the influence of laminate structure we can conclude that a more isotropic lay-up gives a better performance due to the larger energy absorbing area. Furthermore, numerical results make clear that at impact conditions (proportional) damping has hardly any influence on the ballistic performance of HPPE laminates. To analyze the consequences of failure, in Chapter 5 the existing model is adapted by taking failure mechanisms into account.
Figure 20. First component of principal strain for a [0/90]_{120} laminate (V_f=70%) and a rubber matrix, 10 µs after impact.
Figure 21. Energy density in neutral layer for a $[0/90]_{120}$ laminate ($V_f=70\%$) and a rubber matrix, 10 $\mu$s after impact.
Figure 22. First component of principal strain for a $[(0/90)_{20}, (45/45)_{60}, (0/90)_{30}]_1$ laminate ($V_f=70\%$) and a rubber matrix, 10 $\mu$s after impact.
Chapter 5

Modelling the failure behaviour of HPPE laminates during ballistic impact

5.1 Introduction

Examination of the post-impact damage and deformations of HPPE laminates reveals that two characteristic mechanisms are present to stop the projectile, i.e. punching and deflection of layers (Fig. 23). Immediately after impact, the projectile losess speed due to punching of fibres, friction against the punched boundary and the out-of-plane compression of the laminate. This punching process depends on the speed, shape and mass of the projectile and the interply-interaction (rigidity) within the composite laminate. For example, rigid 'hard' ballistic laminates show a higher degree of punching than 'soft' laminates after impact with non-deformable FSP's (fragment simulating projectiles) [18]. When more plies are perforated frictional forces increase and deflection of the laminate becomes easier. Furthermore, contact-stresses in the impact zone are distributed over a larger area of compressed fibres. Consequently, for a sufficient thick laminate there will be a moment, depending on the shape of the projectile and the laminate structure, at which the punching process stops. If the punching process stops, energy will be absorbed by deflection of the non-perforated plies in front of the projectile [21]. In this second stage of the impact process most of the kinetic energy of the projectile is absorbed, because the loaded HPPE fibres can utilize their full strength.

Since the laminate is impacted perpendicular to the surface, the fibres in the first plies are loaded in the out-of-plane transverse direction. The transverse compressive and shear strength of the HPPE fibres is rather low (~30 MPa). As a result of this, fibres in the first plies are easily cut/punched by the projectile, whereas only a small amount of energy is absorbed. Therefore, if the projectile is arrested at impact velocities close to the ballistic limit of the laminate, a large amount of the kinetic energy of the projectile (up to 80%) should be absorbed by deflection of the non-perforated plies. In fact, the ballistic performance depends strongly on the number of perforated plies and the behaviour of the non-perforated part of the laminate. Ballistic tests showed that in general up to 50% of the (upper) plies of a laminate are perforated at impact velocities just below the ballistic limit \( V_{50} \) of a laminate [29]. We believe that if the punching process stops, the transverse oriented cracks propagate as a delamination, as schematically represented in Fig. 23. As up to 50% of the plies is perforated, this delamination should be initiated close to the middle of the laminate. Note that due to the local maximum in the out-of-plane shear stresses \( \sigma_{xy} \), delaminations occur most likely in the middle of the laminate [10,11].
In this research the 2D model of Chapter 4 is used to analyze the impact behaviour of the composite laminates by modelling only the non-perforated plies. For this we assumed that 50% of a laminate will be perforated by a rigid cylindrical projectile, at impact velocities close to the ballistic limit ($V_{50}$) of a laminate. This means that the energy absorption as a consequence of pure punching of the upper layers is neglected in the analysis. This assumption is supported by the huge difference in the shear (~30 MPa) and tensile (~3000 MPa) strength of the HPPE fibre.

During the impact event the laminate will be damaged as a consequence of high tensile and shear stresses within the laminate. For this reason the existing model of Chapter 4 is expanded by taking failure processes into account. The influence of delamination and fibre and matrix failure on the impact behaviour of laminates is investigated using damage mechanics. When failure is detected, the orthotropic material properties of the laminate are reduced as a function of damage parameters. Moreover, a contact procedure is introduced to improve the interaction between the projectile and the target.
5.2 Contact modelling

At the moment of impact, the contact surface between the flat rigid cylindrical projectile and the target is equal to the bottom surface of the projectile. After impact, this contact surface might change because the laminate bends. Moreover, contact forces are no longer equally distributed within the contact area. To analyze the full impact event a contact procedure is necessary, which is able to describe the out-of-plane compression of the laminate and the possible changes of the contact surface. Notice that the effect of out-of-plane compression is not taken into account by the modelled composite plate itself, as shown by the constitutive equation of the used thick shell elements:

\[
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{xy} \\
\sigma_{yz} \\
\sigma_{xz}
\end{bmatrix} = \begin{bmatrix}
E_x & v_x E_y & 0 & 0 & 0 \\
v_y E_x & E_y & 0 & 0 & 0 \\
0 & 0 & (1-v_y v_z)G_{xy} & 0 & 0 \\
0 & 0 & 0 & (1-v_x v_z)G_{yz} & 0 \\
0 & 0 & 0 & 0 & (1-v_x v_y)G_{xz}
\end{bmatrix} \begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\varepsilon_{xy} \\
\varepsilon_{yz} \\
\varepsilon_{xz}
\end{bmatrix}
\]

(70)

where \( \sigma \) is the stress, \( \varepsilon \) the mid-plane strain, \( E \) the tensile modulus, \( G \) the shear modulus and \( v \) the Poisson’s ratio.

The principle of the used contact procedure is based on the distance (in the fixed out-of-plane direction) between the rigid projectile and a point (node) of the composite plate (finite element mesh). When this distance is less than a certain constant gap-distance, contact occurs between the projectile and the laminate. During contact the applied so-called 'gap-elements' behave as linear springs to describe the compression of the laminate. The 'gap-distance' (10^-4 m) was chosen as such, that the moment of impact occurs within the first timestep (5*10^-7 s) at the applied impact velocities (600-700 m/s). The projectile is modelled using a single point-mass which is connected with 13 'gap-elements' to the finite element mesh (Fig. 24). The outer 'gap-elements' are positioned at the circular edge of the projectile’s bottom. The diameter of the contact surface is chosen 5.4 mm conform the size of the standard 17 grain FSP, as used later in the ballistic experiments. Due to the small projectile and the high longitudinal wave propagation velocity within the laminate, a fine and wide mesh had to be used consisting of 2500 elements.
The stiffness of the 'gap-elements' is approximated using an out-of-plane modulus of the laminate in the order of 3 GPa, since the compressive modulus in the out-of-plane direction is not exactly known at impact rates. The stiffness of the applied 'gap-elements' (13 elements), which are connected to a quarter of the composite plate, is calculated according to:

\[
k_{\text{gap}} = \frac{1}{13} \frac{E A / 4}{d / 4}
\]  

(71)

where \( A \) is the circular contact surface and \( d \) is the laminate thickness. The compressed area of the laminate is assumed to be equal to 25% of the laminate thickness \( d \). From Eqn. (71) we obtain, that the stiffness of the 'gap-elements' for a 10 mm thick laminate can be approximated by choosing \( k_{\text{gap}} \) in the order of \( 6.0 \times 10^5 \) N/m.

Figure 24. Finite element mesh for a quarter of the composite plate including 'gap-elements'.
5.3 Failure mechanisms

Due to the heterogeneity of composites, various failure mechanisms can occur such as fibre and matrix failure and delamination. Moreover, these different failure mechanisms can interact very strongly. Hence, failure in fibre composites is a very complex process that starts well below the ultimate strength by debonding and matrix failure and ends when all fibres are broken. During this process the modulus and strength of the material is reduced continuously because of progressive damage. This softening behaviour is of special interest for thick laminates.

To model the failure process in laminates using finite element methods (FEM) criteria have to be formulated that can predict failure from the acting stresses and/or strains. In the FEM model, softening can be incorporated by reducing the mechanical properties and strength values of the laminate as a function of damage parameters. The reduced moduli and Poisson’s ratios define finally the constitutive behaviour of the laminate (Eqn. (70)). Due to the lack of experimental information about the failure process at impact conditions, both the criteria and the corresponding softening behaviour have to be estimated.

5.3.1 Failure criteria

All used criteria of the failure model, as mentioned in Appendix D (subroutine ufail), are modified versions of the Tsai-Wu failure criterion [34,49]. The stress based quadratic failure criteria are formulated using the stresses $\sigma_{xx}$, $\sigma_{yy}$, $\sigma_{xy}$ and $\sigma_{zz}$ of the constitutive equation, Eqn. (70). In the failure model four types of failure criteria are included, viz. fibre breakage, in-plane compressive failure, out-of-plane shear failure and delamination.

Fibre breakage

Similar to the Tsai-Wu criterion [34], we assume that fibre breakage occurs if the in-plane tensile stresses ($\sigma_{xx}>0$, $\sigma_{yy}>0$, $\sigma_{xy}$) exceed the in-plane tensile and shear strength values. The fibre breakage criterion is formulated by:

$$\left(\frac{\sigma_{xx}}{S_{tt}}\right)^2 + \left(\frac{\sigma_{yy}}{S_{tt}}\right)^2 + \left(\frac{\sigma_{xy}}{S_{tt}}\right)^2 \geq 1$$ (72)

where $S_{tt}$ is the in-plane tensile strength and $S_{tt}$ is the in-plane shear strength. As a result of the symmetric stacking sequence of the $0^\circ$ and $90^\circ$ plies the tensile strength $S_{tt}$ is identical in the $0^\circ$ and $90^\circ$ directions.
When fibre failure is detected, both the in-plane and out-of-plane mechanical properties are reduced, meaning that, the failure model checks whether $\sigma_{xx}$ or $\sigma_{yy}$ is dominant. If $\sigma_{xx}$ is predominant, the value of $E_{xx}$, $G_{xy}$, $G_{xz}$ and the coupling terms $\nu_{yx}E_{yy}$ and $\nu_{xy}E_{xx}$ of Eqn. (70) are degraded. Moreover, the corresponding in-plane tensile and shear strength ($S_{it}$, $S_{is}$) and out-of-plane shear strength ($S_{os}$) of the laminate are reduced. If $\sigma_{yy}$ is predominant, in the above index 'x' and 'y' should be exchanged.

### In-plane compressive failure

In-plane compressive failure may be caused in the outer plies by bending of the laminate. The in-plane compressive failure criterion is given by:

$$\left(\frac{\sigma_{xx}}{S_{ic}}\right)^2 \geq 1 \quad ; \quad \left(\frac{\sigma_{yy}}{S_{ic}}\right)^2 \geq 1$$

(73)

To distinguish compressive failure as a result of $\sigma_{xx}$ and $\sigma_{yy}$, two criteria are introduced. If the compressive stresses ($\sigma_{xx}<0$ or $\sigma_{yy}<0$) exceed the in-plane compressive strength ($S_{ic}$), the in-plane moduli ($E_{xx}$) and compressive strength ($S_{ic}$) of the laminate are adapted. During the analysis the failure program checks whether the stresses are positive (tension) or negative (compression), using the appropriate components of the stiffness matrix and strength values ($S_{it}$ or $S_{is}$).

### Out-of-plane shear failure

As a result of matrix cracks in the laminate, the out-of-plane shear moduli will be reduced. These cracks occur as a consequence of out-of-plane shear stresses ($\sigma_{yz}$, $\sigma_{xz}$) and in-plane tensile stresses ($\sigma_{xx}>0$, $\sigma_{yy}>0$). Since the macroscopic model can not distinguish the fibre and matrix failure separately, the in-plane tensile strength ($S_{it}$) of the laminate is used to detect matrix failure. The out-of-plane shear failure criterion is formulated by:

$$\left(\frac{\sigma_{xx}}{S_{it}}\right)^2 + \left(\frac{\sigma_{yy}}{S_{it}}\right)^2 + \left(\frac{\sigma_{yz}}{S_{oz}}\right)^2 + \left(\frac{\sigma_{xz}}{S_{os}}\right)^2 \geq 1$$

(74)

where $S_{os}$ is the out-of-plane shear strength of the laminate. Due to the symmetric stacking sequence of the $0^\circ$ and $90^\circ$ plies the out-of-plane shear strength $S_{os}$ is identical for the $0^\circ$ and $90^\circ$ directions. In case of out-of-plane shear failure, the failure program checks whether $\sigma_{yz}$ or $\sigma_{xz}$ is dominant. If for example $\sigma_{yz}$ is predominant, only the out-of-plane shear modulus $G_{yz}$ (Eqn. (70)) and the corresponding shear strength ($S_{os}$) are reduced.
Delamination

The separation of lamina depends on the out-of-plane shear stresses \((\sigma_{yx}, \sigma_{xy})\) [10,11]. Therefore, the delamination failure criterion is described by selecting the contributions of the out-of-plane shear stresses:

\[
\left(\frac{\sigma_{yx}}{S_{yx}}\right)^\nu + \left(\frac{\sigma_{xy}}{S_{xy}}\right)^\nu \geq 1
\]  

(75)

When in practice plies are separated because of delamination, all out-of-plane material properties of the laminate are affected. Hence, both out-of-plane shear moduli \((G_{yx}, G_{xy})\) and shear strength values \((S_{yx}, S_{xy})\) are adapted when delamination failure is detected.

5.3.2 Modelling progressive failure

In the failure model, we assume that the failure process is controlled by the momentary stress state which is time independent. We observed that different components of the stiffness matrix of Eqn. (70) and strength values have to be reduced. Which strength values and components of the stiffness matrix are changed, depends on the violated criterion. The degradation of the components of the stiffness matrix is performed using damage variables \((D)\) for each criterion. When a failure criterion is violated, the damage is raised per time step \((5 \times 10^{-7} \text{ s})\) by the damage parameter \((\delta D)\). In the next increment the mechanical properties and strength values are reduced as a result of the increased damage level \((D + \delta D)\). Due to the usage of several failure criteria, the criterion with the highest damage value \((D)\) is preferred to reduce a certain component of the stiffness matrix or strength value.

Stiffness reduction

The degradation of the components of the stiffness matrix follows a Weibull distribution, giving an exponential progressive failure behaviour [49]:

\[
E_r = E e^{-\alpha D} \quad ; \quad D(t) = D(t-1) + \delta D
\]  

(76)

where \(E_r\) is the residual component of the stiffness matrix, \(E\) is the undegraded component of the stiffness matrix, \(\alpha\) is a material dependent parameter and \(t\) is the increment number or time. A minimum value for \(E_r\) (Fig. 25) is used, because the finite element program no longer converges to numerical solutions for zero or small components of the stiffness matrix. Note that the value of damage parameter \(\delta D\) determines the maximum degradation in the next time step.
Hence, the present stresses within the laminate can exceed the momentary strength values. As the time step is very small, the failure model responds very rapidly to fluctuations in stress. The degradation of the components of the stiffness matrix \( (E_{i}/E) \) is shown in Fig. 25 for two values of \( \alpha \) as a function of the accumulative damage parameter \( (D) \). The Weibull approach for the reduction of the material properties is based on experimental observations that state that failure will progress exponentially in fibre composites [49].

![Figure 25. Reduction of the components of the stiffness matrix \( (E/E) \) as a function of the accumulative damage \( (D) \) for two different \( \alpha \) values according to Eqn. (76).](image)

**Strength reduction**

The residual strength values \( (\sigma_{res}) \) of the damaged material are calculated from the corresponding degradation of the components of the stiffness matrix \( (E_{i}/E) \) using the following expression:

\[
\sigma_{res} = \frac{\sigma_u (1+\beta)}{1 + \frac{\beta}{E_{i}/E}}
\]  

(77)

where \( \sigma_u \) is the ultimate strength of the undamaged material and \( \beta \) is the ratio of the post-failure curve to the modulus of the unfailed curve, as schematically represented in Fig. 26.
Selection of the damage parameters $\alpha$, $\beta$, $\delta D$, $E_r$

We assume that the difference in impact behaviour due to the type of matrix material depends solely on the value for the ultimate strength at impact rates. Therefore, the stress-strain curves of the S-I-S rubber and polystyrene matrix composites (Fig. 10 of Chapter 3) are constructed using the same strength degradation parameter $\beta$ (Fig. 26). The stress-strain curves of Fig. 10 are approximated by choosing $\beta$ equal to 2 as shown in Fig. 26. Please note that for $\beta$ is equal to 2, the incline of the post-failure curve is twice the incline of the unfailed curve. This $\beta$ value is in agreement with stress-strain curves obtained at very high strain rates in the order of $10^3 \text{ s}^{-1}$ [29].

Since the parameters $\alpha$ and $\delta D$ are exchangeable (Fig. 25), the degradation parameter $\alpha$ is for the sake of simplicity chosen equal to parameter $\beta$. We assume that a time period of 10 $\mu$s (20 increments) is reasonable to fully degrade an element. Consequently, the value of the damage parameter $\delta D$ is chosen equal to 0.1, which implies that an element can be fully degraded ($D=2$) within this time period for $\alpha=2$ (Fig. 25).

The resistance of the laminate depends on the degree of damage and the area with degraded elements. Consequently, the minimum value of the residual modulus $E_r$ and the size of the used shell elements are important with respect to the obtained stiffness reduction in the laminate. Therefore, the finite element mesh is actually also a failure parameter, which should remain unchanged for all analyses. Inevitably, the size of the shell elements has to be small to describe small projectiles ($\sigma 5.4 \text{ mm}$).
With respect to the mesh dependency we can conclude that, due to the small element size used in the present study, mesh refinement did not significantly change the resistance of the laminate. If the mesh is refined, a slight increase in resistance can be compensated by further reducing the minimum value of \( E_r \) (Fig. 25). From Fig. 10 we observe that the minimum modulus \( E_r \) of the fully degraded shell elements should be small. Numerical calculations are performed using a maximum degradation \((E_r/E)\) of 1% for the in-plane moduli and 10% for the out-of-plane shear moduli.

### 5.4 A numerical parametric analysis

#### 5.4.1. Influence of laminate strength

In Chapter 3 we deduced that the in-plane tensile strength \((S_t)\) of an S-I-S rubber based cross-ply \((V_{70\%})\) is 1100 MPa at impact rates. The in-plane shear strength \((S_{ts})\) of these \([0/90]_{tt}\) laminates is in the order of 20 MPa at normal strain rates and temperatures [29]. At impact rates the in-plane shear strength will be higher. Therefore, numerical calculations are performed using an estimated in-plane shear strength of 35 MPa. No reliable results are reported in literature for the out-of-plane shear strength \((S_{os})\) and the in-plane compressive strength \((S_{ic})\) of the laminate at impact rates. By means of a parametric analysis, as printed in italics in Table 7, the influence of the ultimate properties on the impact behaviour is investigated. As reference for the S-I-S rubber based HPPE laminate, the values for the out-of-plane shear strength and compressive strength are assumed to be equal to 100 MPa and 20 MPa, respectively.

**Table 7. Parametric analysis of the ultimate properties of the S-I-S rubber based cross-ply laminate.**

<table>
<thead>
<tr>
<th>Parametric analysis</th>
<th>(S_t) tensile</th>
<th>(S_{ts}) shear</th>
<th>(S_{ic}) compressive</th>
<th>(S_{os}) shear</th>
</tr>
</thead>
<tbody>
<tr>
<td>reference</td>
<td>1100 MPa</td>
<td>35 MPa</td>
<td>20 MPa</td>
<td>100 MPa</td>
</tr>
<tr>
<td>(S_t)</td>
<td>900 MPa</td>
<td>35 MPa</td>
<td>20 MPa</td>
<td>100 MPa</td>
</tr>
<tr>
<td>(S_{ts})</td>
<td>1500 MPa</td>
<td>35 MPa</td>
<td>20 MPa</td>
<td>100 MPa</td>
</tr>
<tr>
<td>(S_{ic})</td>
<td>1100 MPa</td>
<td><strong>350 MPa</strong></td>
<td>20 MPa</td>
<td>100 MPa</td>
</tr>
<tr>
<td>(S_{os})</td>
<td>1100 MPa</td>
<td>35 MPa</td>
<td><strong>200 MPa</strong></td>
<td>100 MPa</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10 MPa</td>
<td></td>
</tr>
</tbody>
</table>
Similar to Chapter 4, the ballistic performance of the laminates is characterised by the velocity decay of the projectile and the deflection at the point of impact (trauma). Surprisingly, numerical calculations showed that the impact behaviour is hardly influenced by the value of the out-of-plane shear ($S_{os}$) and in-plane compressive strength ($S_{ic}$). As a matter of fact, the impact behaviour depends only on the in-plane tensile strength and shear strength. Hence, the impact behaviour of the S-I-S rubber based laminate can be analyzed using the estimated out-of-plane shear and in-plane compressive strength values, as listed in the second row of Table 7 (reference data).

The velocity decay of the projectile and trauma is shown in Fig. 27a and 27b for different tensile strength ($S_{tt}$) values. We observe that the difference in tensile strength between a polystyrene ($S_{tt} = 900$ MPa) and S-I-S rubber ($S_{tt} = 1100$ MPa) based laminate only slightly changes the velocity decay of the projectile, whereas the trauma increases with about 15% for the polystyrene matrix system. During the impact process the in-plane tensile stresses in the vicinity of the projectile increase very rapidly, as shown in Fig. 28 for the different $S_{tt}$ values. First failure occurs almost simultaneously for the different strength values, due to the contribution of the in-plane shear stresses to the fibre breakage criterion (Eqn. 72). Consequently, if in-plane shear failure is suppressed by choosing $S_{tt}$ equal to 350 MPa, ballistic performance is clearly improved (Fig. 29a). This becomes even more clear if we analyze the failure process in the elements next to the circular edge of the contact surface (Fig. 24). A penetration criterion is defined by calculating the degree of damage due to fibre breakage in these elements. If the penetration criterion equals one, all elements near the circular contact zone are fully damaged. Fig. 29b shows that the failure process is clearly postponed due to the higher in-plane shear strength ($S_{tt}$). Hence, the impact behaviour of the non-perforated plies is dominated by fibre breakage (Eqn. (72)), i.e., fibre failure and in-plane shear failure.

**Degree of damage**

The degree of damage, caused by each failure criterion, can be shown in all elements and layers of the laminate. The degree of damage equals zero when no failure is detected and equals one when the element is fully degraded. In Fig. 30, 31 and 32 the degree of damage in the S-I-S rubber matrix laminate is shown 25 μs after impact ($V=700$ m/s) due to fibre breakage, delamination and through thickness shear failure (matrix failure), respectively. The strength values were chosen conform the reference data of Table 7.

**Fibre breakage**

In all layers, fibre failure occurs in the 0° and 90° directions due to high tensile stresses, and in the 45° direction due to in-plane shear deformation (Fig. 30). Damage in 45° direction is quite extensive because of the low in-plane shear strength (~35 MPa), which initiates fibre breakage and reduces the ballistic performance of the laminate (Fig. 29).
Figure 27. (a) The velocity decay of the FSP and (b) Trauma vs. time, for the [0/90]_{90x} laminate ($V_f=70\%$, $\alpha=\beta=2$) for $S_n$ equals 900, 1100 and 1500 MPa.

Figure 28. In-plane tensile stress $\sigma_{zz}$ at the backside of the [0/90]_{90x} laminate in the first element (in 0° direction) next to the circular contact surface as a function of time for $S_n$ equals 900, 1100 and 1500 MPa ($\alpha=\beta=2$).
Figure 29. (a) The velocity decay of the FSP and (b) penetration criterion (b) as a function of time for the [0/90]_{60s} laminate (α=β=2), when $S_{is}$ equals to 35 MPa (solid line) and 350 MPa (dashed line).

Compressive failure

No in-plane compressive failure is observed for the [0/90]_{60s} laminate at a compressive strength of 200 MPa. However, at lower compressive strength values, compressive failure occurs in the 0° and 90° fibre directions at the front of the transverse wave. Since the in-plane compressive strength hardly influences the impact behaviour, we can conclude that the laminates are mainly loaded in tension during impact.

Delamination

Due to the presence of a parabolic shear stress distribution over the laminate thickness [10], delamination occurs mainly in the mid-plane of the laminate (Fig. 31), whereas no delamination is observed in the outer layers of the laminate. Instead of a circular shaped failure pattern, which is expected from the (isotropic) transverse wave propagation, a more or less square shaped failure pattern is obtained in the laminate as a consequence of fibre breakage (Fig. 30). Numerical results showed that the out-of-plane shear strength hardly influences the impact behaviour. However, the delaminated area clearly increases with decreasing out-of-plane shear strength.
Out-of-plane shear failure

Through thickness shear failure patterns are also present in the outer layers of the model as a consequence of high in-plane tensile stresses (Eqn. (74)), indicating that in the outer layers the shear moduli are reduced due to matrix failure (Fig. 32).

5.4.2 Additional numerical results

In the previous section it has been shown that the velocity decay of the projectile is clearly influenced by the failure process (Fig. 27). We observed that the projectile looses speed very rapidly after impact until a velocity of approximately 300 m/s is reached. At this moment, 10 µs after impact, the impact behaviour changes as a consequence of the damage mechanisms. We believe that the punching process will stop after ~10 µs, because the contact forces (out-of-plane compression) are reduced dramatically from this time (Fig. 33b). The out-of-plane compression of the laminate is defined by subtracting the trauma of the laminate from the displacement of the rigid projectile.

Influence of out-of-plane shear modulus

In Chapter 4 we concluded, that high out-of-plane shear moduli improve the ballistic performance of laminates when no failure mechanisms are present. However, as a consequence of the higher out-of-plane shear moduli for brittle matrices (Appendix B), delamination and out-of-plane shear failure is more pronounced. Consequently, the ballistic performance is slightly lower for glassy (brittle) matrices than for rubbery (ductile) matrices at constant degradation parameters (Fig. 33a). However, the (out-of-plane) compression of the laminate increases (Fig. 33b) as a result of higher out-of-plane shear moduli (glassy matrices), giving higher contact forces for 'hard' laminates and glassy matrix system. Note that in case of deformable projectile's higher contact forces improve ballistic performance, since a larger amount of energy is absorbed by projectile deformation.

Influence of the in-plane tensile modulus

Numerical results show that the impact behaviour of the S-I-S based laminate is hardly influenced by the in-plane moduli, as shown in Fig. 34.

Influence of the laminate structure

Similar to Chapter 4, we can conclude that a quasi-isotropic {[0/90]_{13},[-45/45]_{30},[0/90]_{13}}, laminate structure gives a better performance than an orthotropic structure [0/90]_{13} (Fig. 35). The in-plane shear deformation is much smaller for the quasi-isotropic than for the orthotropic laminate structure. Moreover, the quasi-isotropic laminate is damaged due to compressive failure (Fig. 36). As a result of the reduced in-plane moduli the tensile stresses are exceeded by the compressive stresses due to bending of the laminate.
Modelling ballistic impact

Figure 30. Degree of damage due to fibre breakage at the backside of the [0/90]_{50} laminate (first layer) with an S-I-S rubber matrix, 25 μs after impact (α=β=2).
Figure 31. Degree of damage due to delamination at the mid-plane of the $[0/90]_{60s}$ laminate (second layer) with an S-I-S rubber matrix, 25 $\mu$s after impact ($\alpha=\beta=2$).
Figure 32. Degree of damage due to through thickness shear or matrix failure at the mid-plane of the \([0/90]_{60}\) laminate (second layer) with an S-I-S rubber matrix, 25 μs after impact (α=β=2).
Figure 33. (a) The velocity decay of the FSP and (b) compression in the out-of-plane direction vs. time, for a \([0/90]_{60}\) laminate \((V_r=70\%, \alpha=\beta=2)\) with a rubbery (solid line) and glassy (dashed line) matrix.

Figure 34. The velocity decay of the FSP vs. time for the \([0/90]_{60}\) laminate \((V_r=70\%, \alpha=\beta=2)\) for \(E_{xx} = 35\) GPa (solid line) and \(E_{xx} = 45\) GPa (dashed line).
Modelling ballistic impact

Figure 35. The velocity decay of the FSP vs. time, for a \([0/90]_{60}\) (solid line) and a \([0/90]_{15}[-45/45]_{30}[0/90]_{15}\) (dashed line) laminate \((V_f=70\%, \frac{E/E}{min}=0.1, \alpha=\beta=2)\).

Effect of post-failure behaviour

If failure is accelerated by increasing the material parameters \(\alpha\) and \(\beta\) from 2 to 6, the impact performance is clearly reduced (Fig. 37a and 37b). Hence, the ballistic performance depends strongly on the post failure behaviour of the non-perforated plies. When failure is accelerated, the effect of in-plane shear failure is less pronounced, whereas fibre failure in the 0° and 90° directions increases, clearly visible by comparing Fig. 30 and Fig. 39.

Effect of impact velocity

When the initial velocity of the projectile is reduced to 600 m/s, the curve of the penetration criterion is identical to Fig. 29b. This means that, in this velocity range, the failure process in the non-perforated plies is independent of the impact velocity. Although, as a result of the lower impact velocity the contact forces decrease, indicating that the degree of punching will be reduced.

Influence of the non-perforated ply thickness

Ballistic performance depends strongly on the thickness of the non-perforated laminate, as shown by the velocity decay of the projectile and trauma in Fig. 38a and 38b for a laminate thickness of 5 and 10 mm.
Figure 36. Degree of damage due to in-plane compressive failure at the backside of a \([0/90]_{15}[-45/45]_{10}[0/90]_{15}\) laminate (first layer) 25 \(\mu s\) after impact \((E/E)_m=0.1, \alpha=\beta=2\).
Modelling ballistic impact

Figure 37. (a) The velocity decay of the FSP and (b) Trauma vs. time for the \([0/90]_{90\%}\) laminate \((V_f=70\%)\) when \(\alpha\) and \(\beta\) equals 2 (solid line) and 6 (dashed line).

Figure 38. (a) The velocity decay of the FSP and (b) Trauma vs. time, for the \([0/90]_{90\%}\) laminate \((V_f=70\%, \alpha=\beta=2)\) with a thickness of 5 mm (solid line) and 10 mm (dashed line).
Figure 39. Degree of damage due to fibre breakage at the backside of the [0/90]_{60} laminate (first layer) 25 μs after impact ($\alpha=\beta=6$)
5.5 Experimental validation

Ballistic experiments are performed to measure the trauma (maximum deflection normal to the panel surface) and the trauma diameter of the laminate at impact velocities close to the ballistic limit $V_{50}$. These values are obtained by placing a 5 cm thick clay-layer behind the tested panels. Test were performed with and without an air gap of 6 mm between the panel and the clay-layer, to determine the influence of the clay on the trauma values. Measurements are performed on 9.5 mm thick 'hard' laminates with an orthotropic $[0/90]_{120}$ and quasi-isotropic $\{[0/90]_{15},[-45/45],30_1,[-45/45],30_2,0/90]\}$ structure. Both types of Dyneema® UD66 laminates possess a fibre content of 70% and an S-I-S rubber matrix system. The laminates are impacted normal to the surface using non-deformable standard 1.1 g fragment simulating projectiles ($\phi$ 5.4 mm), see Appendix A. The projectiles were reversed to obtain cylindrical rigid projectiles.

From the ballistic experiments we can conclude that the presence of an air gap between the backside of the laminate and the clay layer does not change the trauma value. Hence, the influence of the clay on the trauma values is negligible. Furthermore, the obtained trauma values are approximately the same for the reversed and normal fired FSP’s. By means of optical transducers the impact velocity of the projectile was registered. The $V_{50}$ values of the orthotropic and quasi-isotropic laminate were approximately 680 and 600 m/s respectively. At these ballistic limits trauma values and trauma diameters were measured varying from 9.5 - 10.5 mm and from 20 - 25 mm respectively for both laminate types (Appendix C). The results of the ballistic experiments are shown in Table 8.

**Table 8. Experimental trauma values**

<table>
<thead>
<tr>
<th>Structure</th>
<th>$V_{50}$</th>
<th>Trauma</th>
<th>Trauma diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>orthotropic</td>
<td>680 m/s</td>
<td>9.5 - 10.5 mm</td>
<td>20 - 25 mm</td>
</tr>
<tr>
<td>quasi-isotropic</td>
<td>600 m/s</td>
<td>9.5 - 10.5 mm</td>
<td>20 - 25 mm</td>
</tr>
</tbody>
</table>

The measured trauma values for the orthotropic laminate agree very well with the numerical calculations as shown in Fig. 27b and Fig. 30. According to the numerical calculations the trauma and trauma diameter of the failed orthotropic laminate are 9 and 20 mm respectively. Hence, the estimated post-failure behaviour ($\alpha=\beta=2$) seems suitable to simulate the impact behaviour of the S-I-S rubber based laminate at the ballistic limit $V_{50}$. However, for the quasi-isotropic laminate structure numerical results are incorrect, since the predicted ballistic performance improved for the quasi-isotropic laminate structure. We believe that this discrepancy is caused by the fact that the quasi-isotropic laminate structure no longer deforms as a rigid plate, as a consequence of extensive delamination in this type of laminate.
In Fig. 40 the perforation of the upper layers of the orthotropic \([0/90]_{120}\) laminate is shown for a normal (nr. 1) and reversed (nr. 15) fired FSP. We observe that only the fibres passing through the 5.4 mm wide impact zone are loaded, as expected from the high longitudinal wave propagation in the fibre direction (Fig. 20). The corresponding backside of the impacted laminate is shown in Fig. 41. The deflection of the rear plies is more or less isotropic as a result of the (isotropic) transverse wave propagation. The side view of the same laminate reveals that two large delaminations are present in the middle of the laminate as shown in Fig. 42. All layers below the delamination show an identical displacement into the direction of the point of impact, whereas the layers above the delamination were hardly affected. Hence, this delamination is the boundary between the perforated and deflected zone of the laminate, because deflection requires an in-plane fibre movement (see Eqn. (5) of Chapter 2). This behaviour is reproducible, since it occurred always when an impact was performed at approximately 50 mm from the edges of the plate. Thus, the thickness of the non-perforated plies is, as assumed in the model, approximately 50% of the laminate plate thickness.

In Fig. 43 the perforation of the upper layers of the quasi-isotropic \([0/90]_{15},[-45/45]_{30},[0/90]_{30},[-45/45]_{30},[0/90]_{15}\) laminate is shown after impact with a normal fired FSP’s. Again at the edges of the quasi-isotropic laminate plate a large delaminated area is visible, when impact test are performed at a distance of 50 mm from the edges of the plate. These delaminations, which are larger than in the orthotropic laminates, occur between the sub-laminates, i.e the \([0/90]_{15}\) and \([-45/45]_{30}\) laminates. Probably, due to the quasi-isotropic structure, the in-plane movement of the plies and the deflection at the back of the laminate is restricted. Consequently, more plies are perforated, causing the reduced ballistic performance. Therefore, we can conclude that the degree of punching and thus the ballistic performance depends on the laminate structure.
Figure 40. Perforation of the upper layers of the orthotropic \([0/90]_{120}\) laminate after impact with a normal (nr. 1) and reversed (nr. 15) fired FSP's at a velocity of 680 m/s.

Figure 41. Backside of the orthotropic \([0/90]_{120}\) laminate after impact with a normal (nr. 1) and reversed (nr. 15) fired FSP's at a velocity of 680 m/s. The projectile is placed next to test nr.1. The laminate was perforated for the reversed fired projectile (test nr. 15).
Figure 42. Side-view of the orthotropic $[0/90]_{120}$ laminate after impact with a normal (nr. 1) and reversed (nr. 15) fired FSP's at a velocity of 680 m/s. The thickness of the laminate is 9.5 mm.

Figure 43. Perforation of the upper layers of the quasi-isotropic $\{[0/90]_{19},[-45/45]_{30},[0/90]_{30},[-45/45]_{30},[0/90]_{15}\}$ laminate after impact with a normal (nr. 9) fired FSP's at a velocity of 580 m/s.
5.6 Conclusions

From ballistic experiments we observed, that at impact rates close to the ballistic limit $V_{so}$, up to 50% of an orthotropic laminate is punched by a non-deformable projectile. In the case of a quasi-isotropic laminate structure the degree of punching is much higher. It is shown that the impact behaviour of the laminate depends on the thickness and post-failure behaviour of the non-perforated part of the laminate. Consequently, differences in the degree of punching due to the laminate structure have a large influence on the ballistic performance. The impact behaviour of HPPE laminates can be subdivided into two stages. Within the first 10 µs after impact, the projectile loses speed very rapidly, i.e. speed drops from 700 to 300 m/s. During this period the out-of-plane compression and thus the contact forces are high, probably causing punching of plies. Contact forces (out-of-plane compression) increase with impact velocities, causing a higher degree of punching and even penetration of the laminate. After 10 µs, in a sufficient thick laminate the punching process will be stopped because the contact forces are much lower. In the next time interval of 30 µs the projectile is stopped by deflection of the non-perforated part of the laminate. Only within this second stage of the impact process, the post-failure behaviour of the laminate influences the velocity decay of the projectile.

Numerical results show that the failure process is dominated by fibre failure and in-plane shear failure. Consequently, high values of the tensile and shear strength at impact rates, improve the ballistic performance. The impact behaviour of the non-perforated part of the laminate is hardly influenced by the in-plane moduli. However, a high fibre content is necessary to obtain a high in-plane tensile strength. Due to the strong influence of in-plane shear failure on the impact behaviour, the presence of voids at very high fibre contents (>80%) and the usage of glassy matrices reduce the impact performance. For glassy matrix systems delamination and out-of-plane shear failure is more pronounced, causing a slightly reduced ballistic performance compared to rubbery matrices. Moreover, the maximum out-of-plane compression is higher as a result of the higher out-of-plane shear moduli, giving higher contact forces for 'hard' laminates based on glassy matrix systems. For orthotropic laminates computed values for the trauma and trauma diameter compare very well with results from ballistic experiments.

Numerical results show that in a quasi-isotropic laminate structure in-plane compressive failure occurs. Since HPPE laminates posses a rather low in-plane compressive strength, this failure mechanism could cause the in practise observed reduction in ballistic performance. However, the predicted ballistic performance improves for quasi-isotropic laminates. This means that a more isotropic laminate structure gives an optimum performance, if the non-perforated plies still behave/deform as a rigid plate (no extensive delamination), as observed for the orthotropic laminate. Therefore, the difference in orientation of the fibres between the subsequent sub-laminates, i.e. $[0/90]_n$ and $[-45/45]_n$, and the number of plies $n$ should be small (e.g. 5° instead of 45° and $n=5$), so the plies tend to deform as a rigid plate. Consequently, the advantage of a larger energy absorbing area could be used, whereas no obstruction of the in-plane movements and deflection occurs as a result of the laminate structure.
Chapter 6

Conclusions & recommendations

6.1 Conclusions

Tensile test at low temperatures and high strain rates have been successfully applied to measure the material properties of HPPE fibre-reinforced composite laminates based on S-I-S rubber and polystyrene matrices at impact rates. From the experiments we can conclude that:

- Laminates possess a higher ultimate tensile strength for a rubber matrix than for glassy matrix system, as long as the ultimate strain of the matrix exceeds the ultimate strain of the fibre at impact rates.

- A glass transition temperature $T_g$ well below room temperature ($\sim -50 ^\circ C$) is desirable to avoid brittle behaviour of polymeric matrices at impact rates.

A macroscopic finite element model is developed to analyze the impact behaviour of HPPE composites. From the numerical analyses we can conclude that:

- Predicted trauma values compare well with results from ballistic experiments for the orthotropic laminate structure.

- Within the first period ($\sim 10 \, \mu s$ after impact) contact forces are high.

- The maximum contact force is higher for glassy matrix systems and 'hard' laminates (high out-of-plane shear moduli) and higher impact speeds.

- Only within the second stage of the impact event (10 to 40 $\mu s$ after impact), differences in impact behaviour occur due to differences in the post-failure behaviour. Contact forces are much smaller within this period so the punching of plies is stopped.

- The failure process is dominated by fibre breakage and in-plane shear failure.

- As a consequence of higher out-of-plane shear moduli in the case of glassy matrices and 'hard' laminates, delamination and out-of-plane shear failure is more pronounced, reducing ballistic performance compared to rubber matrix composites.
Conclusions & recommendations

- For the impact behaviour of the non-perforated part of the laminate the in-plane moduli are less important than the ultimate tensile and shear strength.

- The impact behaviour of a laminate depends strongly on the thickness (degree of punching) and post-failure behaviour of the non-perforated part of the laminate.

6.2 Recommendations for material applications

- Fibres applied at the backside (non-perforated part) of a laminate should be optimized to their ultimate tensile strength instead of modulus. Consequently, failure is delayed and more energy is stored by deflection of the laminate.

- To improve the in-plane tensile and shear strength of the laminate at impact rates, rubber matrices should be applied with a glass transition temperature well below room temperature (~ -50°C).

- The optimization of the shear and tensile strength of the fibre, matrix and laminate can be performed using tensile tests at high strain rates (~1 s⁻¹) and low temperatures (~ -50°C).

For non-deforming projectiles the transverse deflection of the laminate is important to reduce the contact forces and thus the punching of plies. As the fibres in the upper layers are cut due to high contact forces, fibres should be optimized having a high transverse wave velocity. From Chapter 2 we concluded that the fibres possess a high transverse wave velocity for a high value of the longitudinal modulus.

- Fibres at the impact side of the laminate should be optimized to possess a maximum modulus to absorb maximum energy during the cutting/punching process.

- A 'soft' laminate structure should be used because the single fibre and thus 'soft' laminates possess a higher transverse wave velocity than rigid elastic solids (Appendix B).

For deformable projectiles high contact forces are desired to absorb a large amount of energy by deformation of the projectile. Moreover, the effective diameter of the projectile increases, improving ballistic performance strongly.

- As well fibres at the impact side of the laminate should be optimized to possess a maximum modulus.

- A 'hard' laminate structure and possibly the usage of a glassy matrix system (at the impact-side of the laminate) gives an improved ballistic performance due to the higher out-of-plane shear moduli.
Numerical results show that quasi-isotropic laminates store more kinetic energy of the projectile due to a larger energy absorbing area. Possibly a higher ballistic performance can be obtained, when the difference in orientation between the subsequent sub-laminates, i.e \([0/90]_{\text{st}}\) or \([-45/45]_{\text{st}}\), and the number of plies \(n\) are chosen small, e.g. 5° instead of 45° and \(n=5\). Consequently, a more isotropic laminate is obtained, in which in-plane shear failure (fibre breakage) is suppressed, giving an improved ballistic performance.

### 6.3 Recommendations for further research

- The existing model can easily be transformed to a 3D model to describe the punching process. However, this can also be done by adapting the thickness of the composite plate as a function of contact forces during the analysis.

- A micromechanical model can be used to investigate the post-failure behaviour of plies as a consequence of dynamic tensile and shear tests, to validate the fibre breakage criterion.
References


29. Personal communication with DSM-HPF, Heerlen, The Netherlands.


44. R.C.N. Vermeulen, Comparison of strain rate and temperature effects on modulus and ultimate strength of HPPE-laminates, WFW reportnr. 95.097, Technische Universiteit Eindhoven, (1995)
Appendix A  Standard for Fragment Simulating Projectiles

<table>
<thead>
<tr>
<th>MIL SPEC/ CODE</th>
<th>A</th>
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<th>C</th>
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<td>2.55</td>
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<td>7.50</td>
<td>8.75</td>
<td>3.20</td>
<td>2.786 ± 0.02</td>
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</tbody>
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Appendix B  Material properties of HPPE laminates and wave propagation velocities at impact rates
(for different fibre volume fractions \( V_f \))

Material properties of a \([0/90]_{ns}\) HPPE laminate for an epoxy matrix system*:

**Dyneema™ UD66 based laminate**

<table>
<thead>
<tr>
<th>( V_f , V_m ) [%]</th>
<th>( E_{xx/yy} ) [GPa]</th>
<th>( v_{xy} ) [-]</th>
<th>( v_{yz/xz} ) [-]</th>
<th>( G_{xy} ) [GPa]</th>
<th>( G_{yz/xz} ) [GPa]</th>
<th>( \rho ) [Kg/m³]</th>
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<tr>
<td>70 , 29</td>
<td>37</td>
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<td>0.91</td>
<td>1.06</td>
<td>0.85</td>
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<td>55 , 45</td>
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<td>0.82</td>
<td>1.10</td>
<td>0.91</td>
<td>1074</td>
</tr>
<tr>
<td>80 , 5</td>
<td>42</td>
<td>0.03</td>
<td>1.03</td>
<td>0.86</td>
<td>0.65</td>
<td>836</td>
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</tbody>
</table>

Material properties of a \([0/90]_{ns}\) HPPE laminate for an S-I-S rubber matrix system*:

**Dyneema™ UD66 based laminate**

<table>
<thead>
<tr>
<th>( V_f , V_m ) [%]</th>
<th>( E_{xx/yy} ) [GPa]</th>
<th>( v_{xy} ) [-]</th>
<th>( v_{yz/xz} ) [-]</th>
<th>( G_{xy} ) [GPa]</th>
<th>( G_{yz/xz} ) [GPa]</th>
<th>( \rho ) [Kg/m³]</th>
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<td>70 , 29</td>
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<td>55 , 45</td>
<td>28</td>
<td>0.008</td>
<td>0.47</td>
<td>0.22</td>
<td>0.21</td>
<td>961</td>
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</tbody>
</table>

* Fibre volume fractions \( V_f \) are denoted in order of achieved performance for each matrix system. For an epoxy and S-I-S rubber matrix a fibre volume fraction of respectively 70% and 80% gives best performance.
Theoretic values of the longitudinal and transverse wave velocities \( c_1 \) and \( c_2 \) in elastic solids, according to Eqn. (27) and (34), for different fibre volume fractions and matrix materials:

**Dyneema™ UD66 based laminate**

<table>
<thead>
<tr>
<th>( V_f ) [% ]</th>
<th>( c_1 ) [( ms^{-1} )] S-I-S matrix</th>
<th>( c_1 ) [( ms^{-1} )] Epoxy matrix</th>
<th>( c_2 ) [( ms^{-1} )] S-I-S matrix</th>
<th>( c_2 ) [( ms^{-1} )] Epoxy matrix</th>
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<td>80</td>
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</tr>
<tr>
<td>70</td>
<td>6140</td>
<td>5968</td>
<td>540</td>
<td>900</td>
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<td>55</td>
<td>5398</td>
<td>5285</td>
<td>470</td>
<td>920</td>
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</table>

Theoretic values of the longitudinal and transverse wave velocities \( c_1 \) and \( c_2 \) in the HPPE fibre at an impact velocity of 700 ms\(^{-1}\), according to Eqn. (6), (7) and (8):

**Dyneema™ SK66 Fibre**

| HPPE fibre       | \( c_1 = 11122 \) ms\(^{-1}\) | \( c_2 = 1383 \) ms\(^{-1}\) | \( \varepsilon_f = 1.6 \) [\% ] |
Appendix C  Test results ballistic experiments

Results ballistic experiments laminate structure [0/90]_{120°}

<table>
<thead>
<tr>
<th>Test no.</th>
<th>$V_{in}$ [m/s]</th>
<th>Trauma $\phi$ [mm]</th>
<th>Trauma [mm]</th>
<th>FSP 17</th>
<th>Air gap [6 mm]</th>
<th>stop / penetrate</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>696</td>
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<td>s</td>
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<td></td>
<td>normal</td>
<td>no</td>
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<td>normal</td>
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*A velocity reduction of 2.7% occurs between measurement and impact. Presented values are measurements. $V_{in}$ = impact velocity, FSP 17 = 1.1 g (17 grain) fragment simulating projectile

Code: UD66, PR 1888-1  
Thickness: 9.5 mm  
Weight: 1.40 Kg  
Areal Density: 8.75 Kg/m²
Results ballistic experiments laminate structure
\{[0/90]_{15}, [-45/45]_{10}, [0/90]_{30}, [-45/45]_{30}, [0/90]_{15}\}_s

<table>
<thead>
<tr>
<th>Test no.</th>
<th>(V_{\text{im}}) [m/s]</th>
<th>Trauma (\varnothing) [mm]</th>
<th>Trauma [mm]</th>
<th>FSP 17</th>
<th>Air gap [6 mm]</th>
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<td>6</td>
<td>587</td>
<td></td>
<td></td>
<td>reversed</td>
<td>no</td>
<td>p</td>
</tr>
<tr>
<td>7</td>
<td>576</td>
<td></td>
<td></td>
<td>reversed</td>
<td>no</td>
<td>p</td>
</tr>
<tr>
<td>8</td>
<td>598</td>
<td>27</td>
<td>8.8</td>
<td>reversed</td>
<td>no</td>
<td>s</td>
</tr>
<tr>
<td>9</td>
<td>595</td>
<td>29</td>
<td>10.0</td>
<td>normal</td>
<td>no</td>
<td>s</td>
</tr>
<tr>
<td>10</td>
<td>597</td>
<td>26</td>
<td>9.0</td>
<td>normal</td>
<td>no</td>
<td>s</td>
</tr>
<tr>
<td>11</td>
<td>550</td>
<td></td>
<td>6.0</td>
<td>reversed</td>
<td>yes</td>
<td>s</td>
</tr>
<tr>
<td>13</td>
<td>607</td>
<td></td>
<td></td>
<td>reversed</td>
<td>yes</td>
<td>p</td>
</tr>
<tr>
<td>14</td>
<td>584</td>
<td></td>
<td></td>
<td>reversed</td>
<td>yes</td>
<td>p</td>
</tr>
<tr>
<td>15</td>
<td>574</td>
<td></td>
<td></td>
<td>reversed</td>
<td>yes</td>
<td>p</td>
</tr>
<tr>
<td>16</td>
<td>597</td>
<td></td>
<td>8.2</td>
<td>reversed</td>
<td>yes</td>
<td>s</td>
</tr>
<tr>
<td>17</td>
<td>618</td>
<td></td>
<td>10.8</td>
<td>reversed</td>
<td>yes</td>
<td>s</td>
</tr>
</tbody>
</table>

* A velocity reduction of 2.7% occurs between measurement and impact. Presented values are measurements
* \(V_{\text{im}}\) = impact velocity, FSP 17 = 1.1 g (17 grain) fragment simulating projectile

Code: UD66, PR 1888-2
Thickness: 9.5 mm
Weight: 1.41 Kg
Areal Density: 8.81 Kg/m²
Appendix D  Failure program

The failure criteria are checked within each integration point of the thick shell elements at each increment and iteration step. The failure program is compatible with MARC version K61.

MARC .DAT FILE:

title  job1
sizing  20000000 2038 211721702
elements 12
elements 75
dynamic 2 0 0 0
large disp
all points
print 5
dist loads 3 2038
shell sect 3
shear
setname 5
end
$---------------------------------------------------------------
solver
 0 0 0
optimize 9
connectivity

 1 75 1 2 48 47
 2 75 2 3 49 48
 etc.
2037 12 2117 2140 2141 4
2038 12 2117 2142 2143 50
coordinates
  3 2143
  1 2.8581-16 3.0392-16 0.000000+0
 2 9.18519-4 3.0406-16 0.000000+0
e tc.
2142 0.000000+0 0.000000+0 1.000000+0
2143 0.000000+0 0.000000+0 0.000000+0
define node set apply1_nodes
 46 92 etc.
define node set apply2_nodes
  1 47 etc.
define node set apply3_nodes
  1  2 etc.
define node set icond1_nodes
2117
define node set icond2_nodes
2117
orthotropic

1,1
3.606e+10 3.606e+10 9.183e+08 0.890e-02 0.526e+00 1.560e-01 9.545e+02
2.996e+08 2.797e+08 2.797e+08 0.000e+00 0.000e+00 0.000e+00 0.000e+00
0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00
damping

0.00000+0 0.00000+0 0.00000+0
1 2025

fail data

1 $\bullet$ 0
ufail composite

(geometry

0.50000-2 0.00000+0 0.00000+0 0.00000+0 0.00000+0 0.00000+0
1 to 2025

orientation

zx plane

1 to 2025

fixed disp

1 2 3 4 5 6
46 92 etc.

0.00000+0 0.00000+0 0.00000+0 0.00000+0 0.00000+0 0.00000+0

initial vel

0.00000+0 0.00000+0 7.00000+2 0.00000+0 0.00000+0 0.00000+0
2117

masses

3 2.75000-4
2117

gap data

1.00000-4 0.00000+0 6.00000+5 0.00000+0 0.00000+0 0.00000+0 0 0
2026 to 2038

no print

print node

1 $\bullet$ 36
tota vel

1 2117 4 5

print element

1 $\bullet$ 6

strain

1

post

26 16 17 0 0 19 20 0 51 0 6
91,1
91,2
91,3
58,1
58,2
58,3
Failure program

-1,1,Principal stress,layer 1
-1,2,Principal stress,layer 2
-1,3,Principal stress,layer 3
-4,2,Principal strain,layer 2
-8,1,Fibre Breakage layer 1
-8,2,Fibre Breakage layer 2
-8,3,Fibre Breakage layer 3
-9,1,Compression F,layer 1
-9,2,Compression F,layer 2
-9,3,Compression F,layer 3
-10,2,In-plane Shear F,layer 2
-11,1,total Shear F,layer 1
-11,2,total Shear F,layer 2
-11,3,total Shear F,layer 3
-12,1,thickn. Shear F,layer 1
-12,2,thickn. Shear F,layer 2
-12,3,thickn. Shear F,layer 3
-13,1,Delamination F,layer 1
-13,2,Delamination F,layer 2
-13,3,Delamination F,layer 3

initial state
1,4
0.
1 to 2025
1,2,3,4
1,2,3,
udump

end option

$...start of loadcase lcasel
control
99999 10 0 0 0 1 0 0 1 0
1.00000-1 1.00000-1 0.00000+0 0.00000+0 0.00000+0
dynamic change
5.0000000e-07 5.00000000e-05 102 0 0 0.00000+0 0.00000+0
continue
$...end of loadcase lcasel
$...
USER-SUBROUTINES:

subroutine ub�inc(inc,incsub)
   implicit real*8 (a-h,o-z)
C
   c...user subroutine that gets called at the beginning of each increment
   c...enables the user to define the maximum number of iterations (ncycme)
   c  for the power sweep eigenvalue extraction which precedes a dynamic
   c  transient analysis using the central difference method.
   c  The default value of ncycme is 40.
C
   common/nrgcom/ en,nrgaps,kdtdl(2100,4,3)
   common/failcom/ damo(2100,4,3,10),Dcr(2100,4,3,10),
&   dam(2100,4,3,10),sigma(2100,4,3,5),
&   ErE(2100,4,3,7),sigmao(2100,4,3,5),
&   ErEo(2100,4,3,7),alpha,beta,ErEm,ddam,
&   gErEm
C
   include '/usr/local/marc/marck6l/common/cdc'
   include '/usr/local/marc/marck6l/common/dimen'
   include '/usr/local/marc/marck6l/common/nzro1'
   include '/usr/local/marc/marck6l/common/array4'
   include '/usr/local/marc/marck6l/common/arrays'
   include '/usr/local/marc/marck6l/common/elmcom'
   include '/usr/local/marc/marck6l/common/heat'
   include '/usr/local/marc/marck6l/common/laas'
   include '/usr/local/marc/marck6l/common/college'
   include '/usr/local/marc/marck6l/common/dyns'
C
   include '/usr/local/marc/marck6l/common/concom'
C
   *********************
   c  CHOOSE FAILURE BEHAVIOUR:
   c...
   c  Reduce strength:
a=2.00
   c...
   c  Reduce stiffness:
b= 2.00
   ddam = 0.10
   c...
   c  Maximum reduction in-plane stiffnesses (ErEm < 1):
   ErEm=0.01
   c...
   c  Maximum reduction out-of-plane stiffnesses (ErEm < 1):
gErEm=0.10
   c...
   c Enter the number of gap-elements in model:
nrgaps=13
   c...
   c In case central difference time-integration is used:
   ncycme=400
   c...
   c Begin Do-loop
do 20, nelem=1,numel
do 20, nint=1,nstres
do 20, layer=1,neqst
c Damage parameter dtdl(1) to start subroutine anelas
   jdtdl=skddfl(nelem,nint,layer)
   vars(jdtdl)= 1.0d+0
c...
c... Initialize values of damo(),dam(),Dcr() en ErE()
   if (inc.gt.0) go to 10
   c...
      damo(nelem,nint,layer,1)=0.0d+0
      damo(nelem,nint,layer,2)=0.0d+0
      damo(nelem,nint,layer,3)=0.0d+0
      damo(nelem,nint,layer,4)=0.0d+0
      damo(nelem,nint,layer,5)=0.0d+0
      damo(nelem,nint,layer,6)=0.0d+0
      damo(nelem,nint,layer,7)=0.0d+0
      damo(nelem,nint,layer,8)=0.0d+0
      damo(nelem,nint,layer,9)=0.0d+0
      damo(nelem,nint,layer,10)=0.0d+0
   c...
      dam(nelem,nint,layer,1)=0.0d+0
      dam(nelem,nint,layer,2)=0.0d+0
      dam(nelem,nint,layer,3)=0.0d+0
      dam(nelem,nint,layer,4)=0.0d+0
      dam(nelem,nint,layer,5)=0.0d+0
      dam(nelem,nint,layer,6)=0.0d+0
      dam(nelem,nint,layer,7)=0.0d+0
      dam(nelem,nint,layer,8)=0.0d+0
      dam(nelem,nint,layer,9)=0.0d+0
      dam(nelem,nint,layer,10)=0.0d+0
   c...
      Dcr(nelem,nint,layer,1)=0.0d+0
      Dcr(nelem,nint,layer,2)=0.0d+0
      Dcr(nelem,nint,layer,3)=0.0d+0
      Dcr(nelem,nint,layer,4)=0.0d+0
      Dcr(nelem,nint,layer,5)=0.0d+0
      Dcr(nelem,nint,layer,6)=0.0d+0
      Dcr(nelem,nint,layer,7)=0.0d+0
      Dcr(nelem,nint,layer,8)=0.0d+0
      Dcr(nelem,nint,layer,9)=0.0d+0
      Dcr(nelem,nint,layer,10)=0.0d+0
   c...
      10 continue
   c...
      sigmao(nelem,nint,layer,1)=sigma(nelem,nint,layer,1)
      sigmao(nelem,nint,layer,2)=sigma(nelem,nint,layer,2)
      sigmao(nelem,nint,layer,3)=sigma(nelem,nint,layer,3)
      sigmao(nelem,nint,layer,4)=sigma(nelem,nint,layer,4)
      sigmao(nelem,nint,layer,5)=sigma(nelem,nint,layer,5)
   c...
      ErE0(nelem,nint,layer,1)=exp(-dmax1( &
      & damo(nelem,nint,layer,1),
      & damo(nelem,nint,layer,5)
      & )*beta))
      ErE0(nelem,nint,layer,2)=exp(- &
      & damo(nelem,nint,layer,2)
      & )*beta))
      ErE0(nelem,nint,layer,3)=exp(-dmax1( &
      & damo(nelem,nint,layer,3),
      & damo(nelem,nint,layer,6)
      & )*beta))
      ErE0(nelem,nint,layer,4)=exp(- &
      & damo(nelem,nint,layer,4)
      & )*beta))
      ErE0(nelem,nint,layer,5)=exp(- &
      & damo(nelem,nint,layer,5)
      & )*beta))
\[ \text{ErEo(nelem,nint,layer,5)} = \exp(-d_{\text{max}}(\text{damo(nelem,nint,layer,5)}, \text{damo(nelem,nint,layer,6)}, \text{damo(nelem,nint,layer,7)} \times \beta)) \]
\[ \text{ErEo(nelem,nint,layer,6)} = \exp(-d_{\text{max}}(\text{damo(nelem,nint,layer,6)}, \text{damo(nelem,nint,layer,8)}, \text{damo(nelem,nint,layer,10)} \times \beta)) \]
\[ \text{ErEo(nelem,nint,layer,7)} = \exp(-d_{\text{max}}(\text{damo(nelem,nint,layer,5)}, \text{damo(nelem,nint,layer,9)}, \text{damo(nelem,nint,layer,10)} \times \beta)) \]
\[ \text{damo(nelem,nint,layer,1)} = \text{damo(nelem,nint,layer,1)} \]
\[ \text{damo(nelem,nint,layer,2)} = \text{damo(nelem,nint,layer,2)} \]
\[ \text{damo(nelem,nint,layer,3)} = \text{damo(nelem,nint,layer,3)} \]
\[ \text{damo(nelem,nint,layer,4)} = \text{damo(nelem,nint,layer,4)} \]
\[ \text{damo(nelem,nint,layer,5)} = \text{damo(nelem,nint,layer,5)} \]
\[ \text{damo(nelem,nint,layer,6)} = \text{damo(nelem,nint,layer,6)} \]
\[ \text{damo(nelem,nint,layer,7)} = \text{damo(nelem,nint,layer,7)} \]
\[ \text{damo(nelem,nint,layer,8)} = \text{damo(nelem,nint,layer,8)} \]
\[ \text{damo(nelem,nint,layer,9)} = \text{damo(nelem,nint,layer,9)} \]
\[ \text{damo(nelem,nint,layer,10)} = \text{damo(nelem,nint,layer,10)} \]

\[ \text{ErE(nelem,nint,layer,1)} = \exp(-d_{\text{max}}(\text{damo(nelem,nint,layer,1)}, \text{damo(nelem,nint,layer,5)} \times \beta)) \]
\[ \text{ErE(nelem,nint,layer,2)} = \exp(-d_{\text{max}}(\text{damo(nelem,nint,layer,2)} \times \beta)) \]
\[ \text{ErE(nelem,nint,layer,3)} = \exp(-d_{\text{max}}(\text{damo(nelem,nint,layer,3)}, \text{damo(nelem,nint,layer,6)} \times \beta)) \]
\[ \text{ErE(nelem,nint,layer,4)} = \exp(-d_{\text{max}}(\text{damo(nelem,nint,layer,4)} \times \beta)) \]
\[ \text{ErE(nelem,nint,layer,5)} = \exp(-d_{\text{max}}(\text{damo(nelem,nint,layer,5)}, \text{damo(nelem,nint,layer,6)}, \text{damo(nelem,nint,layer,7)} \times \beta)) \]
\[ \text{ErE(nelem,nint,layer,6)} = \exp(-d_{\text{max}}(\text{damo(nelem,nint,layer,6)}, \text{damo(nelem,nint,layer,8)}, \text{damo(nelem,nint,layer,10)} \times \beta)) \]
\[ \text{ErE(nelem,nint,layer,7)} = \exp(-d_{\text{max}}(\text{damo(nelem,nint,layer,5)}, \text{damo(nelem,nint,layer,9)}, \text{damo(nelem,nint,layer,10)} \times \beta)) \]

\[ \text{C... Make sure ErEo() and ErE() values are not smaller than ErEm} \]
\[ \text{if (ErEo(nelem,nint,layer,1).lt.ErEm) then} \]
\[ \text{ErEo(nelem,nint,layer,1)} = \text{ErEm} \]
\[ \text{endif} \]
\[ \text{if (ErEo(nelem,nint,layer,2).lt.ErEm) then} \]
\[ \text{ErEo(nelem,nint,layer,2)} = \text{ErEm} \]
\[ \text{endif} \]
if (ErE(nelem,nint,layer,3).lt.ErEm) then
 ErE(nelem,nint,layer,3)=ErEm
cendif
if (ErE(nelem,nint,layer,4).lt.ErEm) then
 ErE(nelem,nint,layer,4)=ErEm
cendif
if (ErE(nelem,nint,layer,5).lt.ErEm) then
 ErE(nelem,nint,layer,5)=ErEm
cendif
if (ErE(nelem,nint,layer,6).lt.gErEm) then
 ErE(nelem,nint,layer,6)=gErEm
cendif
if (ErE(nelem,nint,layer,7).lt.gErEm) then
 ErE(nelem,nint,layer,7)=gErEm
cendif

c... End Do-loop
20 continue

c...
return
cend
c

c...
subroutine anelas(m,nn,kc,drats,irdim,ndi,nshear,mats,dt,dtdl,b,
  * rprops,iprops)

** * * * * * * * * *

user subroutine for elastic constant ratios for
anisotropic options.

m element number
nn integration point number
kc layer number
drats elastic constant ratio matrix
irdim number of stress components
ndi number of direct components
nshear number of shear components
mats material id
dt state variable
dtdl incremental state variable
b original stress-strain law
rprops real parameters
iprops integer parameters

* * * * * * * * *

implicit real*8 (a-h,o-z) dp

dimension drats(irdim,irdim),dt(1),dtdl(1),b(irdim,irdim),n(2)
dimension rprops(1),iprops(1)

common/failcom/ damo(2100,4,3,10),Dcr(2100,4,3,10),
  & dam(2100,4,3,10),sigma(2100,4,3,5),
  & ErE(2100,4,3,7),sigmao(2100,4,3,5),
  & ErEo(2100,4,3,7),alpha,beta,ErEm,ddam,
  & gErEm

include '/usr/local/marc/marck6l/common/dimen'
include '/usr/local/marc/marck6l/common/nzro1'
include '/usr/local/marc/marck6l/common/concom'

include '/usr/local/marc/marck6l/common/matdat'

Make sure dimensions array icntr are large enough:
idimcntr=numel*nstres*neqst*10
if (idimcntr.gt.360000) call quit(9999)
3000*4*3*10=360000

if (dt(1).eq.0) then
  do 10 i=1,irdim
  do 10 j=1,irdim
  drats(i,j) = 1.0d0
10 continue
endif

if (dt(1).eq.0) go to 30
if (dt(1).eq.inc-1) go to 20
if (sigma(m,nn,kc,1).ge.0.0) then
  drats(1,1)=ErE(m,nn,kc,1)
else
  drats(1,1)=ErE(m,nn,kc,2)
endif

if (sigma(m,nn,kc,2).ge.0.0) then
  drats(2,2)=ErE(m,nn,kc,3)
else
  drats(2,2)=ErE(m,nn,kc,4)
endif
c...
  drats(1,2)=dmin1(ErE(m,nn,kc,1),ErE(m,nn,kc,3))
  drats(2,1)=dmin1(ErE(m,nn,kc,1),ErE(m,nn,kc,3))
  c...
  drats(3,3)=ErE(m,nn,kc,5)
  drats(4,4)=ErE(m,nn,kc,6)
  drats(5,5)=ErE(m,nn,kc,7)
  c...
  20 continue
  c...
  if (dt(l).eq.inc) go to 30
  c...
  if (sigmao(m,nn,kc,1).ge.0.0) then
    drats(1,1)=ErEo(m,nn,kc,1)
    else
    drats(1,1)=ErEo(m,nn,kc,2)
    endif
  c...
  if (sigmao(m,nn,kc,2).ge.0.0) then
    drats(2,2)=ErEo(m,nn,kc,3)
    else
    drats(2,2)=ErEo(m,nn,kc,4)
    endif
  c...
  drats(1,2)=dmin1(ErEo(m,nn,kc,1),ErEo(m,nn,kc,3))
  drats(2,1)=dmin1(ErEo(m,nn,kc,1),ErEo(m,nn,kc,3))
  c...
  drats(3,3)=ErEo(m,nn,kc,5)
  drats(4,4)=ErEo(m,nn,kc,6)
  drats(5,5)=ErEo(m,nn,kc,7)
  c...
  30 continue
  c...
  if (m.eq.1 and kc.eq.1 and nn.eq.1) then
    write(6,* 'anelas',inc,dt(1)
    write(6,* m,nn,kc,ncycle
    write(6,* drats(1,1),drats(2,2),drats(1,2)
    write(6,* drats(3,3),drats(4,4),drats(5,5)
    endif
    return
  end
subroutine ufail(nelem,nint,layer,matid,stress,strain,ndi,nshear, 
 1  failc)
C...
c... user subroutine to calculate a scalar function of the current 
c... stresses and strains for a composite material 
c...
c... nelem = user element number 
c... nint = integration point number 
c... layer = layer number 
c... matid = material id 
c... stress = current total stresses 
c... strain = current total strains 
c... ndi = number of direct stresses 
c... nshear = number of shear stresses 
c... failc = user defined failure criteria 
c...
  implicit real*8 (a-h,o-z)  
  dimension stress(5),strain(5),n(2)  
  common/failcom/  damo(2100,4,3,10),Dcr(2100,4,3,10),  
  & dam(2100,4,3,10),sigma(2100,4,3,5),  
  & ErE(2100,4,3,7),sigmao(2100,4,3,5),  
  & ErEo(2100,4,3,7),alpha,beta,ErEm,ddam,  
  & gErEm  
C...
  include '/usr/local/marc/marck6l/common/dimen'  
  include '/usr/local/marc/marck6l/common/nzro1'  
  include '/usr/local/marc/marck6l/common/concom'  
C...
  Make sure dimensions array's are large enough!  
idimcntr=numel*nstres*neqst*10  
if (idimcntr.gt.360000) call quit(9999)  
C...
*******************************************************************************  
*******************************************************************************  
C...
C... Enter maximum stresses:
C... Dynema [0/90]  
tll= 1.12d+09  
c1l= 1.90d+07  
t22= 1.12d+09  
c22= 1.90d+07  
t12= 3.50d+07  
t23= 1.05d+08  
t13= 1.05d+08  
c...
C... Select failure criteria (l=select,0=deselect):
C... Maximum tensile/compressive stress iactiv1=1  
iactiv1=1  
c... Fibre breakage iactiv2=1 | Tsai Wu iactiv2=2  
iactiv2=1  
c... In plane shear failure iactiv3=1  
iactiv3=1  
c... Through-thickness shear failure iactiv4=1  
c... USE ONLY FOR CROSS-PLY LAMINATES: t13=t23  
iactiv4=1  
c... Delamination  
iactiv5=1  
c... Choose quadratic failure criterium:  
c... tsai wu criterium: fxy=-0.5d0*sqr(1.0d0/(xt*xc*yt*yc))  
c... hoffman criterium: fxy=-0.5d0/(xt*xc)  
c... hill criterium : fxy=-0.5d0/(xt*xt)  
c... Therefor GOTO ' 40 continue' to select  
*******************************************************************************
c... The calculated stresses:
\[ s_1 = \text{stress}(1) \]
\[ s_2 = \text{stress}(2) \]
\[ s_{12} = \text{stress}(3) \]
\[ s_{23} = \text{stress}(4) \]
\[ s_{13} = \text{stress}(5) \]

\[ \sigma(n, \text{int}, \text{layer}, l) = \text{stress}(l) \]
\[ \sigma(n, \text{int}, \text{layer}, 2) = \text{stress}(2) \]
\[ \sigma(n, \text{int}, \text{layer}, 3) = \text{stress}(3) \]
\[ \sigma(n, \text{int}, \text{layer}, 4) = \text{stress}(4) \]
\[ \sigma(n, \text{int}, \text{layer}, 5) = \text{stress}(5) \]

c... Store stresses in common block:

\[ c... \]

\[ \text{... The residual stresses:} \]
\[ x_t = t_1^*(1+\alpha)/(1+\alpha/E_e(n, \text{int}, \text{layer}, 1)) \]
\[ x_c = c_1^*(1+\alpha)/(1+\alpha/E_e(n, \text{int}, \text{layer}, 2)) \]
\[ y_t = t_2^*(1+\alpha)/(1+\alpha/E_e(n, \text{int}, \text{layer}, 3)) \]
\[ y_c = c_2^*(1+\alpha)/(1+\alpha/E_e(n, \text{int}, \text{layer}, 4)) \]
\[ s_{xy} = t_{12}^*(1+\alpha)/(1+\alpha/E_e(n, \text{int}, \text{layer}, 5)) \]
\[ s_{yz} = t_{23}^*(1+\alpha)/(1+\alpha/E_e(n, \text{int}, \text{layer}, 6)) \]
\[ s_{xz} = t_{13}^*(1+\alpha)/(1+\alpha/E_e(n, \text{int}, \text{layer}, 7)) \]

\[ c... \]

\[ \text{if}(\text{activ1} = 0) \text{ go to 20} \]
\[ \text{if}(Dcr(n, \text{int}, \text{layer}, 1), \text{lt}, 1) \text{ then} \]
\[ \text{failed1} = s_1/x_t \]
\[ \text{if}(\text{failed1} > 1) \text{ then} \]
\[ \text{dam}(n, \text{int}, \text{layer}, 1) = \text{damo}(n, \text{int}, \text{layer}, 1) + \text{ddam} \]
\[ \text{else} \]
\[ \text{dam}(n, \text{int}, \text{layer}, 1) = \text{damo}(n, \text{int}, \text{layer}, 1) \]
\[ \text{endif} \]
\[ Dcr(n, \text{int}, \text{layer}, 1) = (1-\exp(-\text{dam}(n, \text{int}, \text{layer}, 1)*\beta))/(1-E_e) \]
\[ \text{endif} \]

\[ c... \]

\[ \text{if}(Dcr(n, \text{int}, \text{layer}, 2), \text{lt}, 1) \text{ then} \]
\[ \text{failed2} = s_1/x_c \]
\[ \text{if}(\text{failed2} > 1) \text{ then} \]
\[ \text{dam}(n, \text{int}, \text{layer}, 2) = \text{damo}(n, \text{int}, \text{layer}, 2) + \text{ddam} \]
\[ \text{else} \]
\[ \text{dam}(n, \text{int}, \text{layer}, 2) = \text{damo}(n, \text{int}, \text{layer}, 2) \]
\[ \text{endif} \]
\[ Dcr(n, \text{int}, \text{layer}, 2) = (1-\exp(-\text{dam}(n, \text{int}, \text{layer}, 2)*\beta))/(1-E_e) \]
\[ \text{endif} \]

\[ c... \]

\[ \text{if}(Dcr(n, \text{int}, \text{layer}, 3), \text{lt}, 1) \text{ then} \]
\[ \text{failed3} = s_2/y_t \]
\[ \text{if}(\text{failed3} > 1) \text{ then} \]
\[ \text{dam}(n, \text{int}, \text{layer}, 3) = \text{damo}(n, \text{int}, \text{layer}, 3) + \text{ddam} \]
\[ \text{else} \]
\[ \text{dam}(n, \text{int}, \text{layer}, 3) = \text{damo}(n, \text{int}, \text{layer}, 3) \]
\[ \text{endif} \]
\[ Dcr(n, \text{int}, \text{layer}, 3) = (1-\exp(-\text{dam}(n, \text{int}, \text{layer}, 3)*\beta))/(1-E_e) \]
\[ \text{endif} \]

\[ c... \]

\[ \text{if}(Dcr(n, \text{int}, \text{layer}, 4), \text{lt}, 1) \text{ then} \]
\[ \text{failed4} = s_2/y_c \]
\[ \text{if}(\text{failed4} > 1) \text{ then} \]
\[ \text{dam}(n, \text{int}, \text{layer}, 4) = \text{damo}(n, \text{int}, \text{layer}, 4) + \text{ddam} \]
\[ \text{else} \]
\[ \text{dam}(n, \text{int}, \text{layer}, 4) = \text{damo}(n, \text{int}, \text{layer}, 4) \]
\[ \text{endif} \]
Dcr(nelem,nint,layer,4)=(1-exp(-
& d-am(nelem,nint,layer,4)*beta))/(1-ErEm)
endif
20 continue

c...
if (iactiv2.eq.0) go to 40

c...
if (iactiv2.eq.2) go to 30

c...
if (s1.lt.0.06+0) then
  str1=0.06+0
else
  str1=s1
endif
if (s2.lt.0.06+0) then
  str2=0.06+0
else
  str2=s2
endif
failcr5=(str1**2)/(xt*xt)
& (str2**2)/(yt*yt)
& (s12**2)/(sxy**2)
if (failcr5.gt.l) then
  if (str1 .ge.str2.and.
& Dcr(nelem,nint,layer,5).lt.1) then
    dam(nelem,nint,layer,5)=damo(nelem,nint,layer,5)+ddam
  endif
  if (str2.ge.str1.and.
& Dcr(nelem,nint,layer,6).lt.1) then
    dam(nelem,nint,layer,6)=damo(nelem,nint,layer,6)+ddam
  endif
else
  dam(nelem,nint,layer,5)=damo(nelem,nint,layer,5)
  dam(nelem,nint,layer,6)=damo(nelem,nint,layer,6)
endif
if (Dcr(nelem,nint,layer,5).lt.l) then
  Dcr(nelem,nint,layer,5)=(1-exp(-
& d-am(nelem,nint,layer,5)*beta))/(1-ErEm)
endif
if (Dcr(nelem,nint,layer,6).lt.1) then
  Dcr(nelem,nint,layer,6)=(1-exp(-
& d-am(nelem,nint,layer,6)*beta))/(1-ErEm)
endif
30 continue

c...
if (iactiv2.eq.1) go to 40

c...
SELECT TSAI WU, HOFMANN OR HILL CRITERIUM

cWU  \ f_{xy}=0.5d0*sqrt(1.0d0/(xt*xt*yt*yt))

cHO \ f_{xy}=0.5d0/(xt*xt)

cHI \ f_{xy}=0.5d0/(xt*xt)

if (s1.lt.0.06+0) then
  str1=0.06+0
else
  str1=s1
endif
if (s2.lt.0.0d+0) then
  str2=0.0d+0
else
  str2=s2
endif
failcr5=(1/xt - 1/xc)*str1 + (1/yt - 1/yc)*str2 +
  & (str1**2)/(xt**2) + (str2**2)/(yt**2) +
  & (s12**2)/(sxy**2) + 2*sxy*str1*str2
c...
if (failcr5.gt.1) then
  if (str1.ge.str2.and.
  & (Dcr(nelem,nint,layer,5).lt.1) then
    dam(nelem,nint,layer,5)=damo(nelem,nint,layer,5)+ddam
  endif
if (str2.ge.str1.and.
  & (Dcr(nelem,nint,layer,6).lt.1) then
    dam(nelem,nint,layer,6)=damo(nelem,nint,layer,6)+ddam
  endif
c...
else
  dam(nelem,nint,layer,5)=damo(nelem,nint,layer,5)
  dam(nelem,nint,layer,6)=damo(nelem,nint,layer,6)
c...
endif
if (Dcr(nelem,nint,layer,5).lt.1) then
  Dcr(nelem,nint,layer,5)=(1-exp(-
  & dam(nelem,nint,layer,5)*beta))/(1-ErEm)
endif
if (Dcr(nelem,nint,layer,6).lt.1) then
  Dcr(nelem,nint,layer,6)=(1-exp(-
  & dam(nelem,nint,layer,6)*beta))/(1-ErEm)
endif
c...
40 continue
c...
if (iactiv3.eq.0) go to 50
c...
if (Dcr(nelem,nint,layer,7).lt.1) then
  failcr7=(dabs(s12)/sxy)
  if (failcr7.gt.1) then
    dam(nelem,nint,layer,7)=damo(nelem,nint,layer,7)+ddam
  else
    dam(nelem,nint,layer,7)=damo(nelem,nint,layer,7)
  endif
  Dcr(nelem,nint,layer,7)=(1-exp(-
  & dam(nelem,nint,layer,7)*beta))/(1-ErEm)
endif
c...
50 continue
c...
if (iactiv4.eq.0) go to 60
c...
if (s1.lt.0.0d+0) then
  str1=0.0d+0
else
  str1=s1
endif
if (s2.lt.0.0d+0) then
  str2=0.0d+0
else
  str2=s2
endif
failcr8=(str1**2)/(xt**2) + (str2**2)/(yt**2) +
  & (s13**2)/(sxx**2) + (s23**2)/(syx**2) +
  & (s12**2)/(sxy**2) + 2*sxy*str1*str2
if (failcr8.gt.l) then 
  if (dabs(s23).ge.dabs(s13) .and. 
    & Dcr(nelem,nint,layer,8),lt.l) then 
    dam(nelem,nint,layer,8)=damo(nelem,nint,layer,8)+ddam 
  endif 
  if (dabs(s13).ge.dabs(s23) .and. 
    & Dcr(nelem,nint,layer,9).lt.l) then 
    dam(nelem,nint,layer,9)=damo(nelem,nint,layer,9)+ddam 
endif
else
  dam(nelem,nint,layer,8)=damo(nelem,nint,layer,8) 
  dam(nelem,nint,layer,9)=damo(nelem,nint,layer,9) 
endif
if (Dcr(nelem,nint,layer,8).lt.l) then 
  Dcr(nelem,nint,layer,8)=(1-exp(-
    & dam(nelem,nint,layer,8)*beta))/((1-gErEm)) 
endif
if (Dcr(nelem,nint,layer,9).lt.l) then 
  Dcr(nelem,nint,layer,9)=(1-exp(-
    & dam(nelem,nint,layer,9)*beta))/((1-gErEm)) 
endif
60 continue
if (iactiv5.eq.0) go to 100
if (Dcr(nelem,nint,layer,10).lt.1) then 
  failc10=(s13**2)/(sxz**2) +(s23**2)/(syz**2) 
  if (failc10.gt.l) then 
    dam(nelem,nint,layer,10)=damo(nelem,nint,layer,10)+ddam 
  else 
    dam(nelem,nint,layer,10)=damo(nelem,nint,layer,10) 
  endif 
  Dcr(nelem,nint,layer,10)=(1-exp(-
    & dam(nelem,nint,layer,10)*beta))/((1-gErEm)) 
endif
100 continue
c
failm=dmax1(Dcr(nelem,nint,layer,1),Dcr(nelem,nint,layer,2) 
  & .Dcr(nelem,nint,layer,3),Dcr(nelem,nint,layer,4) 
  & .Dcr(nelem,nint,layer,5),Dcr(nelem,nint,layer,6) 
  & .Dcr(nelem,nint,layer,7),Dcr(nelem,nint,layer,8) 
  & .Dcr(nelem,nint,layer,9),Dcr(nelem,nint,layer,10))
if (failm.gt.0.0) then 
  failc=1.0d+00 
else 
  failc=0.0d+00 
endif
c
if (nelem.eq.1 .and. layer.eq.1 .and. nint.eq.1) then 
c  write(6,*), 'info ufail,ncycle',ncycle 
c  write(6,*), nelem,nint,layer,inc 
c  write(6,*), stress(1),xt,xc,failc 
c  write(6,*), stress(2),yt,yc 
c  write(6,*), stress(3),sxy 
c  write(6,*), stress(4),syz 
c  write(6,*), stress(5),sxz 
c  write(6,*), 'ErE 1,3,5,2,4,6,7'
Failure program

```c
c write(6,*) ExE(nelem,nint,layer,1),ExE(nelem,nint,layer,3)
c & ExE(nelem,nint,layer,5)
c write(6,*) ExE(nelem,nint,layer,2),ExE(nelem,nint,layer,4)
c write(6,*) ExE(nelem,nint,layer,6),ExE(nelem,nint,layer,7)
c write(6,*) 'damo 1,3,2:5,6,4:7:8,9,10'
c write(6,*) damo(nelem,nint,layer,1),damo(nelem,nint,layer,3),
c & damo(nelem,nint,layer,2)
c write(6,*) damo(nelem,nint,layer,5),damo(nelem,nint,layer,6),
c & damo(nelem,nint,layer,4)
c write(6,*) damo(nelem,nint,layer,7)
c write(6,*) damo(nelem,nint,layer,8),damo(nelem,nint,layer,9),
c & damo(nelem,nint,layer,10)
c endif
return
end
```
subroutine plotv(v~,sp,etot,eplas,ecreep,iel,iip,ila,ndirec, * nshr,jpltcd)
C
C ****************************************************
C
C v variable
C s (idss) stress array
C sp stresses in preferred direction
C etot total strain (generalized)
C eplas total plastic strain
C ecreep total creep strain
C t current temperature
C iel(1) user element number
C iel(2) internal element number
C iip integration point number
C fia layer number for beam or shell elements
C
C ****************************************************

implicit real*8 (a-h,o-z)
dimension s(l),etot(l),eplas(l),ecreep(l),sp(l),iel(2)
common/failcom, damo(2100,4,3,10), Dcr(2100,4,3,10),
& dam(2100,4,3,10), sigma(2100,4,3,5),
& ErE(2100,4,3,7), sigmano(2100,4,3,5),
& ErEo(2100,4,3,7), alpha,beta,ErEm,ddam,
& gErEm
include '/usr/local/marc/marck6llcommon/concom'

c calculation of principal stresses
if (jpltcd.eq.1) then
  all=(s(l)+s(2) + sqrt((s(l)+s(2))**2 -4*(s(l)*s(2) - s(3)**2)))
* /2
  a22=(s(l)+s(2) - sqrt((s(l)+s(2))**2 -4*(s(l)*s(2) - s(3)**2)))
* /2
  if (all.ge.a22) then
    v=all
  else
    v=a22
  endif
endif
c calculation of principal stress2
if (jpltcd.eq.2) then
  a11=(s(l)+s(2) + sqrt((s(l)+s(2))**2 -4*(s(l)*s(2) - s(3)**2)))
* /2
  a22=(s(l)+s(2) - sqrt((s(l)+s(2))**2 -4*(s(l)*s(2) - s(3)**2)))
* /2
  if (a11.ge.a22) then
    v=a11
  else
    v=a22
  endif
endif
c Calculation of largest absolute value of stress1 or stress2 with +/-
c if (jpltcd.eq.3) then
   a11=(s(1)+s(2) + SQRT((s(1)+s(2))**2 -4*(s(1)*s(2) - s(3)**2)))
   * 2
   a22=(s(1)+s(2) - SQRT((s(1)+s(2))**2 -4*(s(1)*s(2) - s(3)**2)))
   * 2
   if (abs(a11).ge.abs(a22)) then
      v=a11
   else
      v=a22
   endif
endif

C c Calculation of principal strain1
C if (jpltcd.eq.4) then
   a11=(etot(1)+etot(2) + SQRT((etot(1)+etot(2))**2 -4*(etot(1)*etot(2) - (etot(3)/2)**2)))
   * 2
   a22=(etot(1)+etot(2) - SQRT((etot(1)+etot(2))**2 -4*(etot(1)*etot(2) - (etot(3)/2)**2)))
   * 2
   if (a11.ge.a22) then
      v=a11
   else
      v=a22
   endif
endif

C c Calculation of principal strain2
C if (jpltcd.eq.5) then
   a11=(etot(1)+etot(2) + SQRT((etot(1)+etot(2))**2 -4*(etot(1)*etot(2) - (etot(3)/2)**2)))
   * 2
   a22=(etot(1)+etot(2) - SQRT((etot(1)+etot(2))**2 -4*(etot(1)*etot(2) - (etot(3)/2)**2)))
   * 2
   if (a11.ge.a22) then
      v=a11
   else
      v=a22
   endif
endif

C c Calculation of largest absolute value of strain1 or strain2 with +/-
c if (jpltcd.eq.6) then
   a11=(etot(1)+etot(2) + SQRT((etot(1)+etot(2))**2 -4*(etot(1)*etot(2) - (etot(3)/2)**2)))
   * 2
   a22=(etot(1)+etot(2) - SQRT((etot(1)+etot(2))**2 -4*(etot(1)*etot(2) - (etot(3)/2)**2)))
   * 2
   if (abs(a11).ge.abs(a22)) then
      v=a11
   else
      v=a22
   endif
endif

C if (jpltcd.eq.7) return

C c Damage variables; make sure that jpltcd 8 is called for all layers(-8,1 etc.)
C The temperature t activates progressive failure in subroutine anelas
C if (jpltcd.eq.8) then
   v=dmax1(Dcr(iel(l),iip,ila,l),Dcr(iel(l),iip,ila,3),
    & Dcr(iel(l),iip,ila,5),Dcr(iel(l),iip,ila,6))
endif
if (jpltcd.eq.9) then
  v=dmax1(Dcr(iel(l),iip,ila,8),Dcr(iel(l),iip,ila,9),
  & Dcr(iel(l),iip,ila,5),Dcr(iel(l),iip,ila,6),
  & Dcr(iel(l),iip,ila,10))
endif
if (jpltcd.eq.10) then
  v=Dcr(iel(l),iip,ila,7)
endif
if (jpltcd.eq.11) then
  v=dmax1(Dcr(iel(l),iip,ila,8),Dcr(iel(l),iip,ila,9),
  & Dcr(iel(l),iip,ila,5),Dcr(iel(l),iip,ila,6),
  & Dcr(iel(l),iip,ila,10))
endif
if (jpltcd.eq.12) then
  v=dmax1(Dcr(iel(l),iip,ila,8),Dcr(iel(l),iip,ila,9))
endif
if (jpltcd.eq.13) then
  v=Dcr(iel(l),iip,ila,10)
endif
if (ipltcd.lt.8) go to 10
   v=0.0d+0.or.v.gt.1.0/(1.0-ErEm))
   write(6,*,'plotv',iel(l),iip,ila)
   write(6,*' v
    call quit(9999)
  endif
  continue
  return
  end
  c...
Failure program

```
subroutine e1evar(iel,ilp,ila,gstran,gstres,stress,pstran,
  1xstran,ystran,cauchy,eplas,equivc,swell,kttyp,prang,dt,
  2gsv,ngenx,ngen1,nstass,nstax,thstr)
  c
  c Variable ngenx also used in common block el1com, therefore changed to ngens
  c
  c user routine to obtain element quantities
  c**********************************************************************
  c
  c... compatible with marc k61
  c... please note that the arguments n, nn, layer.
  c have been renamed to iel, ilp, ila respectively
  c
  c iel(1) user element number
  c iel(2) internal element number
  c ilp integration point number
  c ila layer number for beam or shell elements
  c
  c******************************************************************************
  c implicit real*8 (a-h,o-z) dp
  dimension gstran(ngenx),gstres(ngenx),
  1stress(ngen1),pstran(ngen1),cstran(ngen1),vstran(ngen1),
  2cauchy(ngen1),dt(nstats),gsv(1)
  2cauchy(ngen1),dt(*),gsv(1)
  3,thstr(1)
  dimension iel(2)
  dimension vector(72000)

  include '/usr/local/marc/marck61/common/cdc'
  include '/usr/local/marc/marck61/common/array4'
  include '/usr/local/marc/marck61/common/arrays'
  include '/usr/local/marc/marck61/common/dimen'
  include '/usr/local/marc/marck61/common/elmcom'
  include '/usr/local/marc/marck61/common/heat'
  include '/usr/local/marc/marck61/common/lass'
  include '/usr/local/marc/marck61/common/nzro1'
  include '/usr/local/marc/marck61/common/space'
  include '/usr/local/marc/marck61/common/stmen'
  include '/usr/local/marc/marck61/common/array2'
  include '/usr/local/marc/marck61/common/dyns'
  include '/usr/local/marc/marck61/common/concom'

  c common/nrgcom/ en,nrgaps,kdtdk(2100,4,3)
  c common/failcom/ damo(2100,4,3,10),Dct(2100,4,3,10),
  & dam(2100,4,3,10),sigma(2100,4,3,5),
  & ErE(2100,4,3,7),sigmuo(2100,4,3,5),
  & ErEs(2100,4,3,7),alpha,beta,ErEm,ddam,
  & gErEm

  c logical first, last
  c
  data en / 0.0 /

  c
  c Damage parameter dt and dt1 to start subroutine anelas
  dt(1) = inc
  c Il=1+ncpt*nstres*nstax
  c dt(I)=1.0d+0
  lms = ilp+1
  lms2 = lms*ncpt
  lms3 = lms2*nstax
  lms4 = (ila - 1)*nstax+lms3
  lofr = (n-1)*nelstr
  jdt(d)= lms3 + lofr + jdt(d)
  kdtdk(n,iil,ila)=jdt(d)
```
Calculation of energy

dof = numnp*ndeg

c...check if array vector is big enough

c if (ncheck.gt.72000) call quit(9999)

ielbuf = n
n8wel = nelstr
n4wel = nelsto
mla = neqpt
nel = numel
nip = intel

c...get pointers to MARC database

ljmpel = (ielbuf - 1) * n8wel
ljmpip = ljmpel + iip - 1
ljmpla = ljmpel + (iip - 1) * mla + ila - 1
ked = itoten
kvoip = ijacoc
kthip = ithick
kgeom = igeomr
kcoip = icrpt

led = ked + ljmpla
lvoip = kvoip + ljmpip
lthip = kthip + ljmpla
lgeom = kgeom + ljmpel
lthip0 = kthip + ljmpla
lcoip = kcoip + ljmpla + (iip - 1) * ncrd

...initialize energy en

first = iel(1) .eq. 1 .and. iip eq. 1 .and. ila .eq. 1
if(first) then
end if

...get energy density ed, integration point volume voip and thickness

thip

ed = vars(lled)

ed = 0.5*(stress(1)*gstran(1) + stress(2)*gstran(2) +
& stress(3)*gstran(3) + stress(4)*gstran(4) +
& stress(5)*gstran(5))

voip = vars(lvoip)

thip = vars(lthip)

c...get number of layers nla and thickness of layer thla

call ssect1(iel(1),ielbuf,nla,vars(lgeom),iip ,
& rdum1 ,vars(lthip0),rdum2 ,vars(kcoip),1)

do 10 kc=1,nla
call ssect2(kc,nla,vars(lgeom),thip,fc,kkc2,thla,ar,kcold)
10 continue

c...add contribution den to energy and store den in v

c
den = voip * thla * ed
en = en + den
if(first) then
write(6,*) 'elevar', stress(1),gstran(1),stress(1)*gstran(1)
write(6,*) 'elevar', stress(2),gstran(2),stress(2)*gstran(2)
write(6,*) 'elevar', stress(3),gstran(3),stress(3)*gstran(3)
write(6,*) 'elevar', stress(4),gstran(4),stress(4)*gstran(4)
write(6,*) 'elevar', stress(5),gstran(5),stress(5)*gstran(5)
else
write(6,*) 'elevar', ed ,inc
write(6,*) 'elevar'
end if
write(6,*) 'information',iel(1),nel-nrgaps,iip,nip,ila,nla
if(last) then
write(36,*)
write(36,1000) en
end if
write(0,*); information is written to log-file
compute kinetic energy and total

call scla(vector,0.0d0,ndeg,numnp,0)
call nptx(vars(isxxrh),ints(inprh),vector,vars(idynv),
$ ndeg,numnp,maxnpr,nums,0,0)
ekin=0.5d0*ddot(ndof,vector,1,vars(idynv),1)
write(36,1001) ekin
etotal =ekin+en
write(36,1002) etotal

PERFORATION CRITERIUM:
perf= (Dcr(4,1,1,5)+Dcr(4,1,2,5)+Dcr(4,1,3,5)+
$ Dcr(4,2,1,5)+Dcr(4,2,2,5)+Dcr(4,2,3,5)+
$ Dcr(4,3,1,5)+Dcr(4,3,2,5)+Dcr(4,3,3,5)+
$ Dcr(4,4,1,5)+Dcr(4,4,2,5)+Dcr(4,4,3,5)+
$ Dcr(49,1,1,5)+Dcr(49,1,2,5)+Dcr(49,1,3,5)+
$ Dcr(49,2,1,5)+Dcr(49,2,2,5)+Dcr(49,2,3,5)+
$ Dcr(49,3,1,5)+Dcr(49,3,2,5)+Dcr(49,3,3,5)+
$ Dcr(49,4,1,5)+Dcr(49,4,2,5)+Dcr(49,4,3,5))/24
write(37,*) perf
write(36,1003) perf
write(36,1004) sigma(4,1,1,1)
end if

1000 format(10x,'total strain energy summed over whole model',e13.5)
1001 format(10x,'total kinetic energy summed over whole model',e13.5)
1002 format(10x,'energy summed over whole model',e13.5)
1003 format(10x,'perforation near the projectile',e13.5)
1004 format(10x,'stress 1',e13.5)
return
end
subroutine thru(b,bplas,e,eelas,
  * s,sinc,gf,
  * nstas,mats,ndi,nshear,ngens,m,a,nx,nnx,kc,iprops,jprops,
  * iprops,propiort,jtype,jhour,e,
  * dt,dtdl,
  * eis,gnu,kit,
  * ifailp,icompss
  * yd,yd1,
  * jfailn)
  implicit real*8 (a-h,o-z)
  dimension (ngens,ngens),bplas(ngens,ngens),e(1),
  * eelas(1),s(1),sinc(1),
  * gf(1),
  * dt(1),dtdl(1),
  * iprops(1),jprops(1),mdnm(2),propa(1)
  b
  bplas
  e
  eelas
  s
  sinc
  gf
  nstas
  mats
  ndi
  nshear
  ngens
  m
  n
  nn
  kc
  iprops
  jort
  jtype
  jhour
  dt
  dtdl
  eis
  gnu
  kit
  ifailp
  icompss
  yd
  yd1
  jfailn
  c
  c
  c
  stress strain relation
  temp stress-strain relation
  incremental strain
  elastic strain
  stress
  stress increment
  stress change due to temp effects
  number of state variables
  material i.d.
  number of direct components
  number of shear components
  size of stress-strain (ndi+nshear) +1 (if Herrmann)
  element number
  element/elasto number
  integration point number
  kc number
  material properties
  constitutive identifiers
  flag for curvilinear coordinates
  element type
  increment number
  subincrement number
  cycle number
  storage flag
  new nonlinearity flag
  state variables
  incremental state variables
  creep strain
  called by thru
  c
  c
  c
  c
  dimension young(3,2),poison(3,2),shear(3,2),snuno(6)
  include '/usr/local/marc/marck6l/common/space'
  include '/usr/local/marc/marck6l/common/table'
  include '/usr/local/marc/marck6l/common/hrgls1'
  include '/usr/local/marc/marck6l/common/pstelm'
  include '/usr/local/marc/marck6l/common/strtl1'
  include '/usr/local/marc/marck6l/common/concom'
  include '/usr/local/marc/marck6l/common/arrays'
  include '/usr/local/marc/marck6l/common/vstcl1'
  include '/usr/local/marc/marck6l/common/harm1on'
  include '/usr/local/marc/marck6l/common/ltsubs'
  include '/usr/local/marc/marck6l/common/mers'
  include '/usr/local/marc/marck6l/common/makers'
  include '/usr/local/marc/marck6l/common/prepro'
  include '/usr/local/marc/marck6l/common/maters'
  c
  c
  c
  c
  call mtrace(6i,thrue ,1.0)
Failure program

```fortran
if(m.eq.2.and.nn.eq.3.and.kc.eq.2) write(6,9876) dt(l),dtdl(l)
if(m.eq.2) then
  write(6,*) 'm,kc,nn',m,kc,nn
  write(6,*), 'dt(i),dtdl(i)',dt(l),dtdl(l)
endif
9876 format('thme-dt,dtdl',2e13.5)
mdum(1)=m
mdum(2)=nn
noniso=iprops(knoniso)
isotrp=iprops(kisotrp)
ianels=iprops(kianels)
ianiso=iprops(kianiso)
kinnrd=iprops(kknhrd)
mohrc=iprops(kmohrc)
joakr=iprops(kjoakr)
joakrm=iprops(kjoakrm)
moooney=iprops(kmoooney)
ipela=iprops(kpela)
iiisel=iprops(kvisel)
irheol=iprops(kirheol)
ingenpl=iprops(kgenpl)
mroz=iprops(kmroz)
jjviscp=iprops(kjjviscp)
jjcrack=iprops(kjcrack)
jjsoil=iprops(kjsoil)
jcmarcl=iprops(kjcmarcl)
jogden=iprops(kjogden)
jhip=iprops(kjhip)
jdamage=iprops(kjdamage)
ipgrcr=iprops(kipgrcr)
tcomp=iprops(ktcomp)
tvisc=iprops(kttvisc)
ianmat=iprops(kianmat)
if(kit.ge.1) goto 120
if(linear.ne.2) go to 1
if(idyn.ne.0.and.inc.lt.2) go to 1
if(jel.eq.l.and.lovl.eq.4) go to 1
go to 20
1 continue
lofr=(n-l)*nelstr
loff=(n-l)*nelsto
jfailn=0
if(ipgrcr.eq.i) jfailp=ifailp
jfachk=ipgrcr
do 301 i=l,ngens
  do 301 j=l,ngens
    bplas(ij)=O.Od0
    b(i,j)=O.Od0
  301 continue
  if((yd.le.O.or.ydl .le.O).and.noypt.gt.O) then
    call scla(s,0.d0,ngens,l,l)
go to 10
  endif
  il=2*noslps*(mats-l)
  islx=il+iesl
  igslx=i+igsl
  if(dtdl(l).eq.O.d0.and.jfachk.eq.O) go to 10
  if(icomps.eq.1 .or. noniso.eq.1) go to 65
301 continue
```

c temperature dependent elastic modulus and poisson’s ratio
temp=dt(1)
  if(jprops(1).ne.0) then
    call tabva2(rips(1),ets1,jprops(1))
  else
    call value(ets1,frac,vars(ieslx),rprops(1),temp1,noesl)
  endif
  if(jprops(4).ne.0) then
    call tabva2(rprops(4),gnu1,jprops(4))
  else
    call value(gnu1,frac,vars(igslx),rprops(4),temp1,nognsl)
  endif
c elastic stress-strain matrix at start of increment
call belas(bplas,ets1,gnu1,ndi,nshear,ngens,mdum,m,k,c,mats,
  * dt,dtdl,jfailp,young(1,1),poison(1,1),shear(1,1),
  * ianiso,iort,rprops,iprops)
go to 68
65 continue
call compos(m,nn,k,c,ngens,ndi,nshear,ngens,mdum,m,k,c,mats,bplas,
  * dt,dtdl,jfailp,young(1,1),poison(1,1),shear(1,1),
  * ianiso,iort,rprops,iprops)
go to 12
10 continue
c ******
temp=dtsum
  if(jprops(1).ne.0) then
    call tabva2(rips(1),ets,jprops(1))
  else
    call value(ets,frac,vars(ieslx),rprops(1),temp1,noes1)
  endif
  if(jprops(4).ne.0) then
    call tabva2(rprops(4),gnu,jprops(4))
  else
    call value(gnu,frac,vars(igslx),rprops(4),temp1,nognsl)
  endif
c elastic stress-strain matrix at end of increment
c ******
call belas(b,ets,gnu,ndi,nshear,ngens,mdum,m,k,c,mats,dt,dtdl,
  * jfailp,young(1,2),poison(1,2),shear(1,2),ianiso,iort,
  * rprops,iprops)
go to 13
12 continue
call compos(m,nn,k,c,ngens,ndi,nshear,ngens,mdum,m,k,c,mats,b ,
  * dtsum,
  * vars(ieslx),vars(igslx),vars(ishrx),noesl,nognsl,nshear,
  1 iort,jtype,jfailp,young(1,2),poison(1,2),shear(1,2),
  2 noniso,ianels,ianmat,rprops,iprops,propa,iprops)
  ets=young(1,2)
  gnu=poison(1,2)
13 continue
call mcpy(b,elasml,ngens,ngens,1)
c do 1002 i=1,ngens
  sinc(i)=0.d0
  do 1001 k=1,ngens
    sinc(i)=sinc(i)+b(i,k)*c(k)
  1001 continue
  snuno(i)=s(i)+sinc(i)
1002 continue
c
if(jfachk.eq.0) go to 200
kfail=kfailp
    call progcr(snuno,eelas,e,ndi,nshear,kfail,m,nn,kc,
        mats,ngens)
if(kfail.eq.jfailp) go to 200
loaduq=1
jfain=1
if(ifstore.eq.1) ifailpskfail
if(ifcomp.eq.1.or.noniso.eq.1) go to 205
    call belas(b,ets,gnu,ndi,nshear,ngens,mdum,m,nn,kc,mats,
        mats,ngens)
if(kfail.eq.jfailp) go to 200
loaduq=l
jfailn=l
if(istore.eq.1) ifailp=kfail
if(icomps.eq.1.or.noniso.eq.1)
go to 205
    call belas(b,ets,gnu,ndi,nshear,ngens,mdum,m,nn,kc,mats,
        mats,ngens)
go to 202
205 continue
    call compos(m,nn,kc,ngens,ndi,nshear,mats,b,dtsum,
        mats,ngens)
    do 200 i=1,ngens
        bplas(i,k)=b(i,k)-bplas(i,k)
        gf(i)=gf(i)+bplas(i,k)*eelas(k)
        if(linear.ne.2) go to 25
        la1=iblprm+lofr+(nnx-1)*ngens*ngens
        call mcpy(b,vars(la1),ngens,ngens,0)
go to 25
    20 continue
    la1=iblprm+lofr+(nnx-1)*ngens*ngens
    call mcpy(vars(la1),b,ngens,ngens,0)
    if(jfailn.eq.0) go to 123
    temporary update of total elastic strain components
    do 1006 i=1,ngens
        eelas(i)=eelas(i)+e(i)
go to 121
    120 continue
        if(linear.ne.2) go to 25
        la1=iblprm+lofr+(nnx-1)*ngens*ngens
        call mcpy(b,vars(la1),ngens,ngens,0)
go to 25
    20 continue
    la1=iblprm+lofr+(nnx-1)*ngens*ngens
    call mcpy(vars(la1),b,ngens,ngens,0)
    if(jfailn.eq.0) go to 123
    temporary update of total elastic strain components
    do 1006 i=1,ngens
        eelas(i)=eelas(i)+e(i)
go to 121
    120 continue
        if(linear.ne.2) go to 25
        la1=iblprm+lofr+(nnx-1)*ngens*ngens
        call mcpy(b,vars(la1),ngens,ngens,0)
go to 25
    20 continue
    la1=iblprm+lofr+(nnx-1)*ngens*ngens
    call mcpy(vars(la1),b,ngens,ngens,0)
    if(jfailn.eq.0) go to 123
    temporary update of total elastic strain components
    do 1006 i=1,ngens
        eelas(i)=eelas(i)+e(i)
go to 121
    120 continue
        if(linear.ne.2) go to 25
        la1=iblprm+lofr+(nnx-1)*ngens*ngens
        call mcpy(b,vars(la1),ngens,ngens,0)
go to 25
    20 continue
    la1=iblprm+lofr+(nnx-1)*ngens*ngens
    call mcpy(vars(la1),b,ngens,ngens,0)
306 \text{sinc}(i) = \text{sinc}(i) + g(f(i)) \\
4 \text{continue} \\
\text{if(houre.eq.1.and.tdil(1).eq.0.0.and.jfachk.eq.0)} \\
* \text{call mcpy(elasm1,elasm0,ngen,ngen,1)} \\
\text{call mtrace(66htrue,2,0) tr} \\
\text{return} \\
\text{end}