Virtual CMM using Monte Carlo Methods based on Frequency Content of the Error Signal

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ABSTRACT

In coordinate measurement metrology, assessment of the measurement uncertainty of a particular measurement is not a straightforward task. A feasible way for calculation of the measurement uncertainty seems to be the use of a Monte Carlo method. In recent years, a number of Monte Carlo methods have been developed for this purpose, we have developed a Monte Carlo method that can be used on CMM’s that takes into account, among other factors, the auto-correlation of the error signal. We have separated the errors in linearity errors, rotational errors, straightness errors and squareness errors. Special measurement tools have been developed and applied to measure the required parameters. The short-wave as well as the long-wave behavior of the errors of a specific machine have been calibrated. A machine model that takes these effects into account is presented here. The relevant errors of a Zeiss Prismo were measured, and these data were used to calculate the measurement uncertainty of a measurement of a ring gauge. These calculations were compared to real measurements.

1.UNCERTAINTY CALCULATION

Assessing measurement uncertainty implies assessing the distribution of the possible measurement results. Methods involving Monte Carlo simulation consist of a model of the measurement process, and knowledge of the most important influence quantities. Knowledge of influence quantities can be determined analytically or by calibration, and in both cases it should consist of a probability distribution. Such a model is called a virtual machine. In case of a coordinate measurement machine, it is called a virtual CMM [1]. This virtual CMM is used to numerically estimate the distribution of the possible results, which is a measure of the uncertainty. For every simulation, a sample of every influence quantity is taken using a random generator. These samples are evaluated by the model resulting in a distribution of virtual results.

In general, a measurement result \( M \) is a function of a number of measured values, the input quantities \( m_i \). \( M \) is calculated using a model function \( f \):

\[
M = f(m_1, m_2, \ldots, m_n)
\]

In coordinate metrology, this model function often is not a function, but an algorithm that is implemented in a computer. The measurement uncertainty of \( M \) is defined as:

\[
u_M^2 = \sum_i \left( \frac{\partial M}{\partial m_i} \right)^2 \cdot u(m_i)^2 + 2 \sum_{i \neq j} \left( \frac{\partial^2 M}{\partial m_i \partial m_j} \right) \cdot \left\{ u(m_i) \cdot u(m_j) \right\}
\]

Here, \( u(m_i)^2 \) is the uncertainty of input quantity \( m_i \) and \( \left\{ u(m_i) \cdot u(m_j) \right\} \) is the covariance of \( m_i \) and \( m_j \). Consider a simple linearity measurement with only two measurement points and one result: \( L = x_2 - x_1 \). The uncertainty is given by:

\[
u_L^2 = \left( \frac{\partial L}{\partial x_1} \right)^2 \cdot u_1^2 + \left( \frac{\partial L}{\partial x_2} \right)^2 \cdot u_2^2 + 2 \cdot \left( \frac{\partial L}{\partial x_1} \right) \cdot \left( \frac{\partial L}{\partial x_2} \right) \left\{ u(x_1) \cdot u(x_2) \right\}
\]

Here, \( u_1 = u(x_1) \) and \( u_2 = u(x_2) \) are the random errors on these positions. The term \( \left\{ u(x_1) \cdot u(x_2) \right\} \) represents the auto-correlation of the error signal, with lag \( \Delta x = x_2 - x_1 \). A satisfactory machine model should take into account the auto-correlation of the error signal.

A model that uses this approach using a virtual CMM, would require the errors of all simulated points within one simulation to be correlated with each other in the same way they are correlated in the actual machine. In the above
example, this would imply that the points \( x_1 \) and \( x_2 \) are to be drawn from a signal that has the same auto correlation as the original error signal of the machine, a so-called surrogate signal.

2. THE MACHINE MODEL

GEOMETRIC ERRORS

In this example, we only look at the errors a horizontal plane of the machine so the problem becomes two dimensional, the vertical axis of the machine is ignored. The model can be extended to three dimensions when desired. The CMM is modeled as a machine that consists of a number of stacked axes that each have a linearity error, straightness errors (these two are called translation errors), rotational errors, and squareness errors.

Linearity errors are errors that occur in the moving direction of a machine. For example, when the machine moves in \( x \)-direction, there’s an error in \( x \) direction. This error is called the \( f_x \) error. In a three dimensional measurement machine, there are three linearity errors, \( f_x, f_y, f_z \). In the two dimensional case, there are two. A straightness error can have a linear term, but this term is ignored at this point. A linear linearity error can be the result of either an actual linearity error or of incorrect temperature measurement during calibration. Therefore all linear terms of the linearity errors are accounted for in the temperature uncertainty of the model.

Straightness errors are the errors that occur perpendicular to the moving direction of the machine. For example, when the machine moves in \( x \)-direction and there’s an error in \( y \) direction, this error is called the \( f_{xy} \) error. In the three dimensional case there are six straightness errors, in the two dimensional case there are two. A linear straightness error can be the result of either misalignment during calibration or of the squareness error.

The axes of the machine are not perfectly perpendicular to each other. The expected value of the deviation of the right angle is defined the squareness error. In the three dimensional machine, there are three squareness errors, in the two dimensional case there is one squareness error, \( S \). In figure 1, the relevant errors for the two dimensional case are shown.

![Figure 1: Geometric errors in a two dimensional measurement machine](image)

A rotation error is the rotation of a carriage of the machine about any axis. Using the two instruments described below, it is not possible to measure the rotation errors of the machine directly. They can however, be derived from the translation errors, as shown in section 3.

The goal of calibrating these geometric errors, is to determine the autocorrelation function for all relevant values of \( \Delta x \) and \( \Delta y \) respectively. In this example, the smallest value of \( \Delta \) is chosen to be 1 micrometer, the largest value 300 is mm. It is clear that it is not possible to measure the complete axis with the smallest step, as this would require each axis to be calibrated on 300.000 positions. For this reason, the measurement of the error was split up in three levels of magnification. The first level consists of 41 points with a stepsize of micrometer, resulting in a total measurement length of 40 micrometer. The second level consists of 41 points with a stepsize of 40 micrometer, total length is 1.6
mm. The top level consists of 188 points of 1.6 mm, total length 299.2 mm. This scheme should provide us with sufficient data, only when we can assume that the short wave behavior of each axis on one position can assumed roughly the same as the short wave behavior of that axis on another position. Looking at the construction of a coordinate measurement machine with stacked axes, there is no reason to believe that this short wave behavior is much different on different positions. The guideways and scales that introduce the errors are produced using the same production process on every position. Measurements have been done to investigate the validity of this assumption, and the results of the investigated machine, a Zeiss Prismo, did not show this assumption was not valid.

TEMPERATURE ERROR

Both the temperature and expansion coefficients of both the workpiece and the machine scales are known with an uncertainty. This results in a uncertainty in linearity measurements that is linear dependent of the distance between two points. This error is added linearly to the geometric linearity error.

3. THE MEASUREMENT SETUP

STRAIGHTNESS ERRORS

Straightness errors of a CMM can be determined by probing a straightness gauge. A calibrated straightness gauge with a length of 320 mm and a straightness error smaller than 0.1 micrometer was used to determine both straightness errors. The straightness gauge is placed on the machine and probed on different positions using the data collection scheme mentioned in the previous section. Examples of measured straightness errors are shown in figure 3.

Figure 2 Measurement setup used to calibrate straightness errors
LINEARITY ERRORS

Linearity errors can be measured using a stepping gauge, but this instrument has a fixed step. An instrument that can measure linearity errors using a variable step has been developed, a laser stepping gauge. The setup is shown in figure 4. A steel plane is mounted on the carriage of a computer controlled linear positioning stage. The position of the carriage can be measured using a laser interferometer setup. The moving mirror is mounted on the carriage, the fixed mirror is mounted on the base of the instrument. The instrument is used as follows: the carriage is moved to the desired position, and stopped. It’s position is measured accurately using the laser interferometer. The CMM moves to the steel plane and determines it’s position by probing it. When this is done, the CMM moves back and waits for the carriage to move to the next position where the process repeats itself. The entire process is automated, which makes it possible to reliably collect large amounts of data. The positions given by the machine can now be compared to the positions given by the laser interferometer, which makes it possible to calculate the machine error.

Figure 3 measured straightness errors xTy, short wave and long wave behavior

Figure 4: Measurement setup used to calibrate linearity errors
ROTATIONAL ERRORS

The rotational errors of the y carriage about it’s z axis can be calculated by measuring the linearity error $y_T y$ on two different x positions. In figure 6, the measurement results of the short wave (stepsize 1 micrometer) error on two different x positions are shown. The distance between the two lines is chosen as large as possible. These errors show to be very similar. The long wave behavior for this error, as well as tests on other errors showed similar results. This implies that for this machine the rotational error is much smaller than the linearity and straightness errors. Because the effect of rotational errors seems to be a relatively small effect, it was chosen to neglect the effects of rotational errors completely.

Figure 6 $y_T y$ error on different x positions. Top picture is at $x=1$ mm, bottom picture is at $x=721$ mm.
SQUARENESS ERROR

The squareness error of a CMM can be measured using a calibrated squareness standard. This is a very straightforward measurement and is not explained in detail here. The squareness error of the machine was found to be 0.5 arcsec, about 2.5 µrad.

TEMPERATURE ERROR

The effect of the uncertainty in temperature can be determined by measuring a long steel gauge block a large number of times. The standard error of the measurement results is a measure of the standard uncertainty introduced by temperature effects. This standard error is found to be 2 µm m⁻¹ K⁻¹, or 2·10⁻⁶ K⁻¹.

4. GENERATING THE SURROGATE ERROR SIGNAL

GEOMETRIC ERRORS

There are several ways of generating a signal with the same auto correlation as the measured signal. A known method is to calculate one of them is to determine the amplitude spectrum of the signal using a Fourier transform, generate a set of random phases, and transform this back to the position domain using an inverse Fourier transform. This method is known as the method of surrogate data [2]. However, if this method is used in combination with the above mentioned measurement scheme the results are not always satisfactory. Especially in cases where the begin and the end of the signal doesn’t match, higher harmonics are introduced in the spectrum. This spectrum will generate surrogate signals that look totally different from the original signal. This situation does not seem ideal.

An other way of generating a surrogate signal with the same auto correlation is to take the original signal and perform a transformation on it that does not affect the auto correlation. There are four simple transformations which randomize the signal but do not affect the auto correlation:

- Keep the original signal and don’t change it at all;
- Reverse the sign;
- Flip the signal from left to right;
- Reverse the signal and flip it from left to right.

This would allow four simulations to be done with one signal. Because the signal is measured in three levels with different stepsize, there are three signals on every axis. This gives a total of 4³=64 possible simulations for one axis. In this example with four axes in a two dimensional machine, there are 64⁴ possible simulations. In figure 7, an example of four simulated signals is given. One signal is not changed, the other ones have a reversed sign, are flipped from left to right or both.
The three levels of magnification are added linearly. In figure 8, an example of a simulation showing the first four points of the second level of magnification and the first three signals of the smallest level of magnification (smallest stepsize) is given. The points of the second level are notional, for the purpose of explanation of the method. The stars are points of a simulation of the second level. The straight lines represent a linear interpolation between these points. The simulations of the smallest level of magnification are added to these lines as shown in figure 8. The first and last points of the simulated signals are placed exactly on the simulated points of the second level of magnification (the stars). In this way, a continuous signal will be created.
TEMPERATURE ERROR

For one measurement, an error in temperature measurement would result in a systematic error. To express the uncertainty of this error, it is simulated as if it were a random error. The length dependent error is simulated as if every simulated measurement were performed with a different random error in the temperature measurement. Like in a real measurement, the entire measurement volume is scaled linearly. In figure 9, an example of four simulated temperatures is given. The standard deviation of the temperatures is chosen in such a way that the standard error is of the length dependent uncertainty is 2 µm m^{-1} K^{-1}.

![Figure 9 simulation of different temperatures, to introduce linear length dependent error in x-direction](image)

SQUARENESS ERROR

The squareness error of a CMM results in a systematic error that is the same in all measurements. Analogous to the length dependent error, to express the uncertainty of the squareness error, it is simulated as if it were a random error. The squareness error is simulated as if every simulated measurement were performed with a different CMM with a different squareness error. In figure 10, an example of four simulated squareness errors is given. The standard deviation of the simulated squareness errors is 2.5 µrad.

![Figure 10 simulation of different squareness errors in the xy-plane](image)

5. SIMULATING MEASUREMENTS

The entire model is implemented in a computer program using Matlab. In addition to the simulation model, some geometric elements that can be measured on the machine are implemented. A measurement uncertainty of a given geometric element is done as follows. The positions of the measurement points of the element are the nominal points that feed the simulation software. Every simulation consists of a set simulated measurement points. Through every set of simulated points, the desired geometric element is fitted. The desired parameters of the geometric element, in this example the diameter of the circle, are calculated. The distribution of the simulated diameters can be used to estimate the measurement uncertainty. In figure 11, this is illustrated for the element circle. For the figure, All geometric errors are multiplied by a factor 2000 to make it possible to distinguish the different simulations.
6. RESULTS

The validity of the model is tested comparing the simulated uncertainties with real measurements of the diameter of a calibrated ring gauge. Two ring gauges were measured, one of 50 mm diameter, and one of 110 mm diameter. In figure 12, the measurement setup is shown. Both ring gauges were put on 9 different measurement positions in the measurement volume of the CMM. On every position, for both gauges, the diameter was measured five times, using 4, 6, 10, 16 and 32 measurement points. These measurements were simulated using the described computer model. The results are shown in figure 13. The o-marks represent dispersion of the measured diameters of the ring, the x-marks represent the calculated uncertainty.

Figure 11 graphical representation of simulation of two circles

Figure 12 measurement of calibrated ring gauge, diameter 110 mm
Figure 13 comparison of modeled and measured uncertainty

7. CONCLUSIONS

Figure 13 obviously shows that the deviations are simulated with the correct order of magnitude. The uncertainties predicted by the model could be somewhat too high as the repeatability enters in all calibration components, and because the short-range errors are somewhat present in the larger range too. Both effects make the simulations too pessimistic. In future work, together with making the 3D model complete, this will be rectified.

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