LINEAR PROPAGATION OF PULSATILE WAVES IN VISCOELASTIC TUBES

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Abstract—An experimental and theoretical analysis is made of pulsatile wave propagation in deformable latex tubes as a model of the propagation of pressure pulses in arteries. A quasi one-dimensional linear model is used in which, in particular, attention is paid to the viscous phenomena in fluid and tube wall. The agreement between experimental and theoretical results is satisfactory. It appeared that the viscoelastic behaviour of the tube wall dominates the damping of the pressure pulse. Several linear models are used to describe the wall behaviour. No significant differences between the results of these models were found.

INTRODUCTION

The study of wave propagation in liquid-filled deformable tubes is often motivated by its application to arterial blood flow. The full axi-symmetrical linear approach is based on the work of Morgan and Kiely (1954) and Womersley (1957) which is later extended by several authors with regard to material properties: viscoelasticity, prestressing, tethering and anisotropy of the tube wall. Examples of a detailed linear study are Klip et al. (1967) and Kuiken (1984). An overview is given by Pedley (1980) and Milnor (1982). Several linear models have included nonlinearities in their two-dimensional model. The work of Gerrard (1985) is mentionable as an attempt to verify experimentally a full axi-symmetrical linear theory.

The inclusion of non-linearities and the extension to more complex geometries in two-dimensional models is difficult. Furthermore, typical two-dimensional phenomena such as higher order modes, are hardly observed in an in vivo situation (Milnor, 1982). Many authors therefore choose a quasi one-dimensional approach in these cases. A description of the basic linear, elastic, nonviscous theory is given by Lighthill (1978). Similar models are developed in which in some way fluid viscosity, wall viscoelasticity, reflections or non-linearities are included (e.g. Anliker et al., 1971; Gally et al., 1979; Holenstein et al., 1984; Atabek, 1972) have included nonlinearities in their two-dimensional model. The work of Gerrard (1985) is mentionable as an attempt to verify experimentally a full axi-symmetrical linear theory.

The experimental set-up used in the present research is shown in Fig. 1. The most important elements are the pressure vessel (PV), the magnetic valve (V) and the deformable tube (T). An important consideration in designing the set-up was to avoid non-linearities as much as possible by using small velocities and pressure disturbances. The tube (silicon rubber, Penrose Drain, length 90 cm, diameter 18 mm, wall thickness 0.2 mm) is fixed on both ends (slightly pre-stressed) and is supported by a rigid flat plate. Collapse of the tube is prevented by a transmural pressure of 2.4 kPa, adjusted by the fluid height of the
reservoir (R). The tube can be connected to the pressure vessel or the damping vessel (DV) by a 3-way magnetic valve. The overpressure in the pressure vessel is 50 kPa.

Before the start of the experiment both tube and fluid (water) are at rest, the tube is in connection with the damping vessel. The experiment is started by switching the magnetic valve, so that the deformable tube is connected to the pressure vessel; the damping vessel has been shut off. After 12–14 ms the magnetic valve is reswitched. Then the pressure pulse (initial amplitude 250 Pa) propagates downstream with a velocity of about 3.2 m/s (see Fig. 2). The maximum fluid velocity is 0.08 m/s. By adjusting the flow resistance between the damping vessel and the deformable tube, the resonance of the magnetic valve can be critically damped to suppress troublesome pressure oscillations. Pressure and diameter disturbances can be measured locally with a catheter-tip manometer (Millar) and a photonic sensor. The manometer is positioned through a metal tube (outer diameter 4 mm, open on the upper side) on the axis of the deformable tube. Its influence on the wave phenomena is ignored.

After amplification and filtering (200 Hz, 24 dB/octave) the pressure and diameter signals are sent to a computer. The repeated triggering of valve and computer (interval 1–4 s) and resetting of amplifiers and other equipment, is done automatically. To improve the accuracy of the experiments and to diminish stochastic deviations, on every location the mean is taken over 10 experiments. From these mean values the decay of the pulse amplitude, the broadening of the pulse and the wave velocity are determined. The pulse width is defined as the full width at the half maximum of the local pulse amplitude. The pulse propagation velocity is defined as the velocity of the pulse top. The inaccuracies based on a 95% reliability interval are: pulse amplitude ±2%, pulse width ±4%, wave velocity ±1%. The dynamic pressure–area relation is determined by measuring the transfer function between pressure and diameter signals with a Fourier analyser (HP 5423A). Because the diameter disturbances are small (<2%) a linear relationship between diameter and area was assumed. To improve the accuracy, five independent experiments are performed. The pressure–area relation is also determined by adjusting the height of the fluid reservoir and measuring the tube diameter with a micrometer. The value found in this way will be denoted as the static one.

With this experimental set-up the following experiments are performed: propagation experiments (measuring of pressure pulses on different locations and determination of wave velocity and changes of pulse amplitude and pulse width), analysis of tube wall behaviour (static and dynamic pressure–area relation). Furthermore, the validity of the linear approach is tested and the influence of the rigid support of the tube is checked.

Figure 2 gives a characteristic result of a propagation experiment. The pressure pulse propagates downstream and is spread out and damped by dispersion and dissipation. Figure 3 gives an impression of the initial frequency spectrum of the pressure pulse. The Womersley parameter $\alpha = R \sqrt{\omega p / \eta}$ ranges from 10 to 120.

**THEORETICAL DESCRIPTION**

Based on previous experiences (Van Steenhoven and Van Dongen, 1986) and the described exper-
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iments, we make the following assumptions in formu-
lating a theoretical model: non-linearities in fluid and
tube wall are ignored; the fluid dynamical phenomena
are quasi one-dimensional; the tube is uniform; the
fluid is incompressible and Newtonian; no influence of
gravity.

Then the system is described by the following set
equations:
\[ A_t + A_0 U_x = 0 \]  conservation of mass (1)
\[ \rho U_t + p_x = \frac{2\tau}{R} \]  conservation of momentum. (2)

Equations (1) and (2) follow from the integration of the
differential forms of continuity and impulse equations
over the radial coordinate \( r \). \( A \) is the cross-sectional
area of the tube, and \( U \) and \( p \) are area-averaged
velocity and pressure of the fluid. \( \tau = \eta \partial u/\partial r \) is the
wall shear stress, \( R \) the radius of the tube and \( \rho \) the
fluid density. The subscripts \( x \) and \( t \) denote the spatial
and time derivatives. As the equations are linear,
homogeneous solutions for \( A, U, p \) and \( \tau \) are substituted:
\[ A = \text{Re} \{ A(\omega) \exp(j(\omega t - kx)) \} \quad \text{etc.} \quad (3) \]
\( \omega \) is the angular frequency and \( k \) the wave number.
The constitutive relation of the tube wall can be given
in the general form
\[ \hat{\tau} = F(\omega) \hat{A}. \quad (4) \]
The real part of \( E \) describes the elastic behaviour, the
imaginary part the viscous one. Several expressions
for \( E \) can be used. For our calculations we use
\[ E = 1/C \] (5a)
\[ E = 1/C + ja \] (5b)
\[ E = 1/C + j\omega b \] (5c)
\[ E = 1/C + j(c + \omega d) \] (5d)
\[ E = E_{\text{exp}}(\omega). \] (5e)

\( C \) is the compliance of the tube, and \( a \) through \( d \) are
real constants. When using expression (5a) the tube is
treated as purely elastic. Expression (5b) approximates
the behaviour of arteries (Milnor, 1980) and is the
limiting case of the models used by e.g. Gerrard (1985).
Expression (5c) is used by Van Steenhoven and Van
Dongen (1986).

The compliance \( C \) is determined from the static
pressure–area relation. The parameters in (5c) and (5d)
are found by a least-square approximation of the
imaginary part of the dynamic relation. Averaging
over the frequency range of interest gives the param-
eter in (5b). Furthermore, the experimental dynamic
relation is also substituted directly into the calcu-
lations, without approximation by some parameter
model. This is stated by expression (5c).

Also the wall shear stress \( \tau \) can be given in a general
form
\[ \frac{2\tau}{\rho R} = - F(\omega) \hat{U}. \] (6)

For the friction function \( F \) the following expressions
are used:
\[ F = 0 \] (7a)
\[ F = 8\eta/\rho R^2 \] (7b)
\[ F = 8\eta/\rho R^2 \]
\[ = \frac{2\eta}{\rho R^2} \frac{J_1(z)}{2J_1(z)-zJ_0(z)} \quad \omega > 0 \] (7c)
\[ z = \frac{3/2}{\sqrt{\frac{\omega^2}{\eta}}.} \]

\( J_0 \) and \( J_1 \) are the Bessel functions of the first kind and
of zeroth and first order, respectively. \( \eta \) is the dynamic
viscosity of the fluid and \( R \) the radius of the tube. In
expression (7a) the fluid is regarded as an ideal fluid.
(7b) corresponds to the fully developed stationary
laminar flow situation. In the derivation of (7c) the
two-dimensional theory of an unsteady (harmonic)
fully developed flow in a longitudinally tethered tube
according to Womersley (1957) has been used.

For the solution of the set equations (1) and (2)
equations (4) and (6) are substituted there. This yields
homogeneous set equations in \( A \) and \( U \) and the
following dispersion relation:
\[ k^2 = \frac{\rho}{\omega E} \omega(\omega - jF). \] (8)

When \( E \) is real and \( F \) equals zero the propagation is
dispersionless; the propagation velocity then equals
the Moens–Korteweg velocity \( c = A/\rho C \) (Lighthill,
1978).

The propagation of a pressure disturbance with
frequency \( \omega/2\pi \) is calculated by defining a transfer
function \( H \) as
\[ H(\omega, x) = p(\omega, x)/p(\omega, 0) \]
\[ = \exp[-jk(\omega)x]. \] (9b)

The calculation procedure of the pressure as a func-
tion of time on an arbitrary location is then as follows:
(1) computation of transfer function \( H \) from equa-
tions (8) and (9b) with expressions for \( E \) and \( F \)
equations (5) and (7));
(2) Fourier transform of the pressure signal on
\( x = 0; \)
(3) computation of the propagation of the Fourier
components with equation (9a);
(4) computation of the pressure signal at arbitrary \( x \)
by inverse Fourier transform.

The calculations are performed with frequency steps
of 0.65 Hz in the range from 0 to 30 Hz. Results of this
calculation procedure have been compared with ana-
lytical results. For that purpose the experimental pulse
was modelled as a Gaussian shape. Two simplified
cases were analysed: a viscoelastic tube with an ideal
fluid and an elastic tube with a frequency-dependent
wall shear stress. The analytical results were obtained
in a similar way as described by Van Steenhoven and
Van Dongen (1986). The deviations were less than 2%.

To evaluate the influence of errors in $E$ on the inaccuracy of the calculated pulse amplitude and width, the calculations with the viscoelastic tube models are executed five times each, with five different sets of material parameters obtained from independent experiments. The final results are obtained by averaging of pulse amplitude and width over these five numerical results on every location of interest and for every tube model.

RESULTS

Pressure–area relations

An example of an experimental dynamic pressure–area relation is given in Fig. 4. The elastic component is fairly constant up to 20 Hz and decreases slightly up to 30 Hz. From the static experiments a value for the compliance is found of $3.1 \times 10^{-9}$ m$^2$/Pa (±4%). This is in good agreement with the dynamic values for low frequencies. The viscous contribution increases slightly with increasing frequency up to 30 Hz. The elastic as well as the viscous component show rather large fluctuations. Nevertheless, they can be described by a linear least-

![Diagram](image-url)
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Table 2. Material parameters

<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.52</td>
<td>0.47</td>
<td>0.25</td>
<td>0.29</td>
</tr>
<tr>
<td>2</td>
<td>0.47</td>
<td>0.44</td>
<td>0.17</td>
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<tr>
<td>3</td>
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<td>0.50</td>
<td>0.24</td>
<td>0.32</td>
</tr>
<tr>
<td>4</td>
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<td>0.34</td>
<td>0.21</td>
<td>0.19</td>
</tr>
<tr>
<td>5</td>
<td>0.32</td>
<td>0.31</td>
<td>0.15</td>
<td>0.23</td>
</tr>
<tr>
<td>Average</td>
<td>0.45</td>
<td>0.41</td>
<td>0.20</td>
<td>0.27</td>
</tr>
<tr>
<td>±</td>
<td>0.10</td>
<td>0.10</td>
<td>0.06</td>
<td>0.07</td>
</tr>
</tbody>
</table>

The square approximation fairly well. Estimations for the material parameters are given in Table 2. The results beyond 30 Hz are regarded as unreliable, because of the sudden change in behaviour at 30 Hz, the shape of the coherence spectrum and the decreasing amplitude of the Fourier components for higher frequencies.

Wave propagation

As visible in Fig. 2 the propagating pressure pulse is subjected to considerable damping. So fluid viscosity and/or wall viscoelasticity have to be included in the theoretical model. The damping appears to be highly underestimated in a purely elastic model with a steady wall shear stress (7b). When a more realistic frequency-dependent wall shear stress (7c) is used, the pulse amplitude shows a slight decay (Fig. 5). The pulse width however is unaffected. Furthermore, the pulse shows an asymmetrical deformation, which is not observed experimentally to such an extent. A model which includes the viscoelastic behaviour of the tube wall gives a much better resemblance (Fig. 6).

Figure 7 gives a more quantitative comparison of experimental and different theoretical results. Experimentally, a decay of the pulse amplitude of about 60% and a broadening of about a factor 2 over a distance of 0.8 m are observed. The models in which viscoelasticity is accounted for predict a damping in the same order of magnitude, whereas a model with a purely elastic behaviour of the tube does not. So it is confirmed that the viscoelastic behaviour is the dominating factor in the observed damping. The agreement between experimental and theoretical data is fairly good (if viscoelasticity is included), although the damping is somewhat overestimated. Results obtained from different viscoelastic models show little variation. The choice of a particular model is therefore not essential. A possible explanation of the overestimation of the damping is described in the next section. The discrepancy in the decay of the pulse amplitude is smaller than in the pulse broadening.

In Table 1 the propagation velocities are compared. The propagation velocity is defined as the velocity of the pulse top. Because there is only little dispersion in the dominant, lower range of the frequency spectrum the difference between the phase and group velocity is negligible. The relative differences between the theoretical values are small (2.5%). The difference with the experimental result is, however, 13–16%. It is unlikely that this is caused by numerical inaccuracies, since the numerical and analytical values agree quite well. More attention is paid to this subject in the section on the influence of rigid support.

Non-linearities

The linear approach assumes that the observed phenomena are independent of the amplitude of the
pressure pulse. This is verified by using three different initial pulse amplitudes. The results are shown in Fig. 8. Small but significant differences are observed. The decay of the pulse amplitude is smaller for higher initial amplitudes. The difference is in the same direction and of the same order of magnitude as the discrepancy between experimental and theoretical results. Therefore, non-linearities may be an explanation for this discrepancy. Of course a non-linear approach is necessary for a definite conclusion. The wave velocity is not affected by non-linearities (Table 1).

**Influence of rigid support**

Because of the rigid support the deformable tube does not expand axi-symmetrically. The static difference between horizontal and vertical diameter is small (1.3%), but it is not a priori clear how this support affects the pulse propagation. Investigation of this influence by removing the support raises many experimental problems (Gerrard, 1985). Therefore, the experiment is repeated while supporting the tube both on the top and bottom side. It is expected that the
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Fig. 8. Experimental decay of pulse amplitude (ph) and pulse broadening (pb) for different initial pulse amplitudes: \( \Delta, p(0) = 100 \) Pa; \( \bigcirc, p(0) = 250 \) Pa; \( \square, p(0) = 450 \) Pa.

Fig. 9. Experimental decay of pulse amplitude (ph) and pulse broadening (pb) for one-sided and two-sided support: \( \bigcirc \), one-sided; \( \bigotimes \), two-sided.

The effect of a one-sided support will be enhanced with a two-sided support. The rigid top support was mounted parallel to the bottom support at a distance of \( \approx 18 \) mm. The results are given in Fig. 9. The decay of the pulse amplitude is hardly affected. The broadening is somewhat diminished. The wave velocity is significantly increased, as indicated in Table 1. This is plausible, because in a way the tube is made less compliant. However, no definite conclusions about the influence of the support can be inferred from these experiments. The phenomenon is not well understood. Flaud et al. (1985) find a much better agreement between theoretical and experimental results. Greenwald and Newman (1982) find deviations of 3–13%.

Possibly, dynamic asymmetrical expansion of the thin-walled tube is (partly) responsible. Determination of the asymmetrical dynamic tube displacements, with one- and two-sided supports, and a theoretical analysis of the influence of this asymmetrical behaviour on the wave propagation, can possibly give a more definite conclusion about the deviation of the wave velocity and the influence of the rigid support.

**CONCLUDING DISCUSSION**

With the experimental set-up pulsatile pressure and diameter variations can be generated reproducibly and measured accurately. The dynamic pressure-area relation can be determined fairly accurately up to 30 Hz. The observed phenomena in the pulse propagation experiments are fairly well described by a linear, quasi one-dimensional incompressible model, although the damping is somewhat overestimated and the wave velocity somewhat underestimated. The viscoelastic behaviour of the tube wall appears to be the dominating damping factor. The unsteady wall shear stress makes a small contribution to the decay of the pulse amplitude. The discrepancies between experimental and theoretical results are partially explained from non-linearities. The initial amplitude affects the decay of the pulse amplitude and slightly the pulse broadening. The rigid support of the tube could be another explanation for the discrepancies. A more severe restriction of the tube displacement by the support diminishes the pulse broadening as well, and raises the wave velocity by about 10%. However, more research on the role of both phenomena is necessary.

The deviations in the theoretical results are mainly caused by the inaccuracies in the determination of the material parameters. Especially, the positioning of the photonic sensor relative to the position of the pressure transducer is very critical. As described earlier, the theoretical results presented in Fig. 7 are the mean results of five calculations with different sets of parameters to determine the deviations of the mean results. When the averaged material parameters
(Table 2) are used the difference is small (<2%).

In the present investigation it was possible to perform a coupled analysis of viscous behaviour of both fluid and tube wall, with various descriptions, in a rather simple way. This is an advantage of the chosen linear approach, but it limits the application to cases with small displacements. Experimental non-linearities were in our case intentionally as small as possible. It was verified that their influence was small indeed.

An interesting result of this investigation is that, although the wall viscoelasticity is the dominating damping factor, the actual model to describe this behaviour is not very critical, provided that the ratio of imaginary to real parts of $E$ is of the right order of magnitude in the frequency range of interest. For aortic walls this ratio is of the order of 0.1–0.2 (Pedley, 1980), which is essentially in the same range as the one found for the latex tube in our experiments. Therefore, in principle, the method outlined above yields an accurate description of propagation, damping and dispersion of small pulsed shaped waves in arteries, which possibly can be used for the detection of atherosclerosis.

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