Development of laminar mixed convection in a horizontal square channel with heated side walls

J.J.M. Sillekens, C.C.M. Rindt *, A.A. van Steenhoven

Faculty of Mechanical Engineering, Eindhoven University of Technology, WH-3.129, P.O. Box 513, 5600 MB Eindhoven, The Netherlands

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Abstract

A finite element code is used for detailed analysis of mixed-convection flow in a horizontal channel heated from the side walls. The Reynolds number typically is \( \Re \approx 500 \), and the Grashof number varies around \( \Gr \approx 10^5 \). Firstly, the numerical code is validated by a quantitative comparison with results of particle-tracking experiments for the velocity and liquid crystal measurements for the temperature. It is concluded that the agreement between the numerical and experimental data is satisfactory and that the finite element approximation employed can be used in analyzing mixed-convection flow problems in complex geometries. Secondly, the velocity and temperature fields are further described by means of isovelocity and isotemperature lines. The results are quantified by the relative kinetic energy of secondary flow, the Fanning friction factor, and the Nusselt number as a function of the streamwise position. It seems that the resulting secondary flow induced by buoyancy forces causes a substantial increase in heat transfer, knowledge of which is of importance for the design of compact heat exchangers. © 1998 Elsevier Science Inc. All rights reserved.

Keywords: Mixed convection; Liquid crystal thermography; Particle-tracking velocimetry; Channel flow; Laminar flow

1. Introduction

The present study on mixed convection in a horizontal square channel is initiated by the research on the complicated flow patterns occurring in the coiled heat exchanger of a solar domestic hot water system (SDHWS). When such a system is operated according to the low-flow principle [the Reynolds number being \( \Re \approx \mathcal{O}(10^3) \) and the Grashof number being \( \Gr \approx \mathcal{O}(10^5) \)], the flow is laminar, and both natural and forced convection must be taken into account. Buoyancy and centrifugal forces give rise to a secondary flow perpendicular to the main axial flow. The relative magnitude of the secondary flow due to buoyancy forces can be estimated by

\[
\frac{U_{Gr}}{U} \approx \mathcal{O} \left( \sqrt{\frac{\Gr}{\Re}} \right)
\]

It is expected that the secondary flow due to buoyancy forces for high Prandtl number fluids is lower than for low Prandtl number fluids. This effect on secondary flow caused by buoyancy forces may be appreciated when the study of Le Fevre (1956) is considered, where buoyancy-induced flow at a vertical plate is described. From the similarity formulation for the boundary-layer equations governing this flow, it can be

\[\text{Notation} \]

- \( c_p \) specific heat capacity
- \( f \) Fanning friction factor
- \( g \) gravity acceleration
- \( \Gr \) Grashof number
- \( H \) height of channel or cavity
- \( K \) relative kinetic energy
- \( \Nu \) Nusselt number
- \( p \) pressure
- \( \Pr \) Prandtl number
- \( \Re \) Reynolds number
- \( T \) temperature
- \( U \) mean axial velocity
- \( u_{ax} \) axial velocity
- \( U_{Gr} \) buoyant secondary velocity
- \( u_{sec} \) secondary velocity
- \( \bar{u} \) velocity vector
- \( u, v, w \) velocity components
- \( \dot{x} \) position vector
- \( x, y, z \) Cartesian coordinates

- \( \mu \) dynamic viscosity
- \( \rho \) density
- \( \sigma_{xx}, \sigma_{xy}, \sigma_{xz} \) stress components
- \( \tau_w \) mean wall shear stress

- \( b \) cubic expansion coefficient
- \( \Delta t \) time interval
- \( \Delta T \) characteristic temperature difference
- \( \lambda \) thermal conductivity

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* Corresponding author. E-mail: c.c.m.rindt@wtb.tue.nl.

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deduced that the influence of the Prandtl number on secondary flow is as follows:

\[ \frac{U_{Gr}}{U} = \varepsilon \left( \frac{\sqrt{Gr}}{Re \sqrt{1 + Pr}} \right) \]  

(2)

The efficiency of the helically coiled heat exchangers is largely influenced by this secondary flow through which temperature boundary layers remain thin, and heat transfer increases.

In this investigation, attention is focussed on mixed-convection flow in a horizontal square channel heated from the side walls. In such a configuration, the physical phenomena occurring can be studied relatively easily using experimental methods, making it possible to validate the numerical methods developed. The literature on this subject up to 1989 has been reviewed by Hartnett and Kostic (1989) and Shah and Joshi (1987). The resulting flow field is determined by a relatively large set of parameters: the Grashof, Reynolds, and Prandtl numbers, the temperature boundary conditions on the walls, and the aspect ratio of the channel. Buoyancy-driven flows cause the heat transfer rate to increase, an effect more pronounced when the flow is as follows:

The conservation equations of mass, momentum, and energy are simplified using the Oberbeck–Boussinesq approximations (Oberbeck, 1888). In Gray and Giorgini (1976) the limitations to those approximations are discussed. For water, only minor errors are introduced when, as in the present study, the temperature differences are of \( O(10^8) \). The following dimensionless quantities are defined as

\[ \bar{u} = \frac{u}{U}; \quad T' = \frac{T - T_{wall}}{T_{wall} - T_{inlet}}; \quad \rho' = \frac{\rho - \rho_0}{\rho_0}; \quad \frac{g}{g'} = \bar{g}, \]

\[ \nabla' = H \nabla, \]

where the characteristic velocity \( U \) equals the mean axial velocity and the characteristic length scale the height \( H \) of the channel.

In dimensionless form, the equations for steady flow then read (with omission of the primes):

**Continuity:**

\[ \nabla' \cdot \bar{u} = 0 \]

**Momentum:**

\[ \bar{u} \cdot \nabla' \bar{u} + \nabla' \rho' - \frac{1}{Re} \nabla' \bar{u} + \frac{Gr}{Re} \frac{T_g}{T} = 0 \]

**Energy:**

\[ \bar{u} \cdot \nabla'T = -\frac{1}{Re Pr} \nabla'T = 0 \]

with \( Re = \rho U H/\mu \) denoting the Reynolds number, \( Pr = \mu c_p/\lambda \) denoting the Prandtl number, and \( Gr = g \beta_0 \Delta T H^3/\mu^2 \) denoting the Grashof number \( (\Delta T = T_{wall} - T_{inlet}) \). All fluid properties in the above mentioned equations are constant and taken at the reference temperature \( T_{ref} \). In the present study, channel flow is solved numerically by discretization of those equations by a finite element formulation. For details about the finite element technique one is referred to the standard text books (Cuvelier et al., 1986), for details about the implementation used in the present study one is referred to Sillekens et al. (1994a, b). Here only a brief description is given.

2.2. Solution procedure

The non-linear convective term is linearized using Picard's method. The penalty function method is used to decouple the momentum and continuity equations. In order to minimize the size of the matrices to be handled, the velocity and temperature fields are solved as separate problems in an iterative fashion. To avoid spatial oscillations in the temperature field the streamline upwind/Petrov–Galerkin (SUPG) method (Brooks and Hughes, 1982) is used for discretization of the energy equation. For \( C(h) \) accurate quadratic elements (which will be used throughout this study) the SUPG-method showed to be \( C(h) \) accurate for smooth solutions, even when the Peclet number is high (Segal, 1993).

The calculation domain is presented in Fig. 1. From the experimental results, it is seen that the flow in the channel is symmetric with respect to the plane \( y = 0 \). For the computations, therefore, only half of the channel is discretized. In the axial direction, the calculation domain equals \( 11 H (1 < x/H < 10) \). For the momentum equation \( 31 \times 5 \times 10 \) triquadratic hexahedral Crouzeix–Raviart elements are used in the \( x \)-, \( y \)- and \( z \)- direction, respectively. The element distribution in a cross section is shown in Fig. 1. The energy equation was solved on a \( 50 \times 8 \times 12 \) mesh of triquadratic hexahedra, which is refined about 1.5 times as compared to the mesh for the momentum equation because of the thinner boundary layers expected (\( Pr > 1 \)). To evaluate the temperature in the nodal points of the mesh for the momentum equation and to evaluate the velocity values in the nodal points for the energy equation, a triquadratic interpolation between the meshes is employed.

The problem is solved numerically on a Silicon Graphics Super Challenge (MIPS R8000 processor). When using a solution
at a lower Grashof number as a starting solution, typically 25 iterations were necessary to obtain a converged solution. Each iteration took 33 min CPU-time. Both computation times needed and memory limitations prevent significant mesh refinement studies on the computers available, making experiments indispensable.

2.3. Boundary conditions

The boundary conditions are given in Table 1 and in Fig. 1. The prescribed inlet profile $u(y, z)$ at $x = -H$ (plane IN in Fig. 1) is an approximation to within ±2% accuracy of the exact profile for a fully developed channel flow (Shah and Joshi, 1987). At the walls (SW-, SW+, TW, BW) the no-slip condition is applied; whereas, at the midplane $y = 0$ (SY), the symmetry condition holds. At $x = 10H$ (OUT) for the velocity field, the stresses are set equal to zero; i.e.,

$$
\sigma_{xx} = -p + \frac{2}{\text{Re}} \frac{\partial u}{\partial x} = 0, \quad \sigma_{xy} = \frac{1}{\text{Re}} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) = 0,
$$

$$
\sigma_{xz} = \frac{1}{\text{Re}} \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) = 0.
$$

For the temperature, all the boundaries act as adiabatic walls except the inflow boundary (IN: $T_{\text{inlet}}$) and the vertical side walls for $x \geq 0$ (SW+: $T_{\text{wall}} = 1$).

Outflow boundary conditions still receive a lot of attention in literature (Sani and Gresho, 1994) and are often based on intuition, experience, asymptotic behavior, and numerical experimentation (Papanastasiou et al., 1992). The outflow boundary conditions for the stresses used here are natural boundary conditions in the Galerkin weak formulation and result in a well-posed problem (Sani and Gresho, 1994). When a flow is fully developed the condition $\sigma_{xx} = 0$ sets the dynamic pressure $p$ at the outflow boundary to zero. A secondary flow, however, generally gives rise to a non-uniform pressure distribution over

<table>
<thead>
<tr>
<th>Plane</th>
<th>Boundary Condition</th>
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<tbody>
<tr>
<td>IN: $x = -H$</td>
<td>$u(y, z) = {1 - [y/(\frac{1}{2}H)]^2} \cdot {1 - [z/(\frac{1}{2}H)]^2}$, $v = w = 0$, $T_{\text{inlet}} = 0$</td>
</tr>
<tr>
<td>OUT: $x = 10H$</td>
<td>$2\text{Re}^{-1} \frac{\partial u}{\partial x} - p = 0$, $\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0$, $\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = 0$, $\frac{\partial T}{\partial x} = 0$</td>
</tr>
<tr>
<td>SY: $y = 0$</td>
<td>$v = 0$, $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial y} = 0$, $\frac{\partial T}{\partial y} = 0$</td>
</tr>
<tr>
<td>SW: $y = \frac{1}{2}H$; $x &lt; 0$</td>
<td>$u = v = w = 0$, $\frac{\partial T}{\partial y} = 0$</td>
</tr>
<tr>
<td>SW+: $y = \frac{1}{2}H$; $x \geq 0$</td>
<td>$u = v = w = 0$, $T_{\text{wall}} = 1$</td>
</tr>
<tr>
<td>TW: $z = \frac{1}{2}H$</td>
<td>$u = v = w = 0$</td>
</tr>
<tr>
<td>BW: $z = -\frac{1}{2}H$</td>
<td>$\frac{\partial T}{\partial z} = 0$</td>
</tr>
</tbody>
</table>
a cross-section. Therefore, the prescription of a zero normal stress generally leads to errors in the induced secondary flow field near the outflow boundary, referred to as artificial outflow boundary layers by Sani and Gresho (1994) and also observed by Evans and Greif (1993). By similar arguments, it can be deduced also that the other boundary conditions \( \sigma_{nx} = \sigma_{nx} = 0 \) are physically not correct. However, from numerical experience, it is known that these boundary conditions only affect the solution a few elements upstream of the outflow boundary (van de Vosse, 1987; Rindt et al., 1991). This will also be made plausible in the present paper when discussing the influence of the Grashof number on the temperature and velocity fields. The numerical results in the outflow region indicated as shaded areas in Figs. 8–10, should, because of the aforementioned, be handled with care.

3. Experimental procedure

3.1. Experimental setup

The considered configuration is sketched in Fig. 2 (top). Water flows through a horizontal square channel of 3.1 cm cross-section. The medium is symmetrically heated at the side walls by water flowing through the heat exchangers. The flow through the heat exchangers is separated from the flow through the channel by 0.25 mm thick polycarbonate (Lexan) sheets. Because of the high flow rate through the heat exchangers and the thin dividing walls, the temperature of the side walls can be kept at a relatively uniform temperature of \( T_{wall} \pm 0.15 \)\(^\circ\)C, as compared to a typical temperature difference of \( T_{wall} - T_{inlet} = 1.5 \)\(^\circ\)C. Fig. 2 (bottom) shows a scheme of the total experimental setup. The setup consists of two separate circuits. With the main circuit, the axial flow through the channel is controlled. Conditioned water flows from basin \( b_1 \) through an inflow section, which is sufficiently long (65 \( H \)) for the flow to become hydrodynamically fully developed. The developed flow with temperature \( T_{inlet} = 19.7 \)\(^\circ\)C enters the measurement section where it is heated by the heat exchangers (temperature \( T_{wall} \approx 21.2 \)\(^\circ\)C) of the secondary circuit. The flow through the main circuit is measured by flow meter \( V \) and is pumped by a peristaltic pump \( p_1 \) from basin \( b_2 \) to basin \( b_1 \). The flow rate can be adjusted using a valve at \( V \). The temperature of the water in the main circuit is controlled by cooler \( c \) and by a Julabo \( P_1 \) thermostatic heater \( h_1 \). The flow through the channel is heated by two heat exchangers in the measurement section. Water is pumped by centrifugal pump \( p_2 \) from the lower basin to the higher basin from which it flows through the heat exchangers. The temperature of this circuit is controlled by a Julabo \( P_1 \) thermostatic heater \( h_2 \).

Although the main circuit was very well insulated by 20 cm thick insulation plates, a heat flux of about 1 Wm\(^{-2}\) from the medium in the inflow section to the surrounding air could not be avoided. As a result of this small heat flux the medium in
the inflow section experienced a buoyancy effect with a Grashof number of $\text{Gr} = g \beta \rho^2 q H^4 / \nu^2 \approx 5 \times 10^3$. It will be shown that, as a result of this effect, the velocity profile at the entrance of the measurement section, $(x = 0)$ was somewhat shifted with respect to the fully developed velocity profile without this buoyancy effect.

The total accuracy of the setup in terms of Reynolds and Grashof numbers approximately equals 3% ($\text{Re} \approx 500$) and 15% ($\text{Gr} \approx 10^5$), respectively. The accuracy of the latter can be improved by using a channel of smaller cross-section (to the cost of worse optical accessibility) and thinner dividing walls between the main channel and the heat exchangers (to the cost of rigidity).

### 3.2. Particle-tracking velocimetry

Particle-tracking (PT) is a method for measuring the velocity field in a fluid by means of keeping track of individual particles suspended in the fluid (Adrian, 1991). The flow is hardly influenced by the particle seeding, because the concentration of particles can be kept very low. In this study, the PT facility of the computer package DigImage is used. Details of the method as implemented in DigImage can be found in Dalziel (1992a, b).

The flow configuration is intersected by a thin light sheet. The perpendicular reflection of the particles is recorded on S-VHS tape. The video system is capable of grabbing each field of the recording separately. Suppose that two subsequent fields are grabbed, say field $P$ and field $Q$ at time $t_P$ and $t_Q$, $t_Q = t_P + \Delta t$, where $\Delta t = 0.04$ s, the time interval between two complete video fields. When the distance between the individual particles in field $P$ is larger than the displacement of the particle during $\Delta t$, then every particle in field $P$ can be matched exactly to a particle in field $Q$. The average velocity in the light sheet of a particle at position $\vec{x}_P$ in $P$ then can be computed using the Lagrangian definition of velocity: $\vec{u} = (\vec{x}_P - \vec{x}_Q) / \Delta t$, $\vec{x}_Q$ being the position of the particle in $Q$. For details about the matching algorithm one is referred to Dalziel (1992b).

In the experiments Optimage particles were used with a typical diameter of 100 $\mu$m and in a concentration of about 0.001 vol%. A cross-section of the channel equal to 6 cm in the axial direction and 3.1 cm in the transverse direction was visualized by a 3 mm thick light sheet. The acquired resolution was 85 pixels/cm in the axial direction and 120 pixels/cm in the transverse direction, respectively. To avoid interlace effects, only the odd video fields were used, and the effective transverse resolution is reduced to 60 pixels/cm. Because the volume centroid of a particle was used to determine its position, the particles could be located with subpixel accuracy (Dalziel, 1992a). The resolution of particle location approximately equals $\pm 2 \mu$m and the velocity resolution then is equal to $\pm 0.01$ cm/s.

The density $\rho_p$ of the particles used is only slightly higher than that of water, i.e. $\rho_p \approx 1000$ kg/m$^3$. From the experiments performed, it was estimated that the vertical velocity of the particles attributable to this density difference is less than $10^{-3}$ cm/s.

The timescale on which suspended particles are able to follow rapid changes in velocity can be estimated by equating inertia forces and viscous forces acting on a particle. For the channel flow considered, this timescale is about $5 \times 10^{-4}$ s. A velocity increase of 1 cm/s will be caught up with within 5 $\mu$m.

Furthermore it is known from the experiments of Segrée and Silberberg (1962) that small particles suspended in a Hagen–Poiseuille flow have a tendency to move to a specific radial position. From these experiments and from the analysis of Ho and Leal (1974), it can be concluded that this inertia-induced migration (also referred to as the Magnus effect) results in a transverse velocity of less than 0.01 cm/s.

### 3.3. Liquid crystal thermography

Liquid crystal thermography (LCT) is used for quantitative visualization of the temperature fields. The method is based on the optical properties of a cholesteric liquid crystal. The cholesteric liquid crystal is characterized by its typical layered structure. Because of this layered structure, they show constructive interference as a result of optical path differences of incoming light. There are two parameters influencing the resulting reflection of cholesteric liquid crystal: the angle of incidence $\theta$ and the pitch $p_{LC}$. When the temperature of cholesteric liquid crystal is changed, the orientation of the molecules is changed so that the pitch decreases with increasing temperature. This phenomenon makes it possible to use the cholesteric liquid crystal phase for temperature visualization purposes: at a higher temperature shorter wavelengths are reflected. In order to avoid mechanical stress influences and chemical influences, the cholesteric material can be encapsulated in gum arabic. Encapsulation further prevents liquids crystals from contamination, which is important for use in water for instance. A major disadvantage of encapsulation is that the reflection of encapsulated liquid crystal is less intense than the reflection of unencapsulated liquid crystal. Cholesteric liquid crystal is commercially available with red start temperatures ranging from $-30$ to $115$ °C and with bandwidths from 1 to $50$ °C (Parsley, 1991).

The time response of encapsulated cholesteric liquid crystal is dependent on the intrinsic time response of the liquid crystal and on the time response of the capsule material. According to Fergason (1968), the intrinsic time response (the time the molecules need to rearrange as a result of a sudden increase in temperature) is of the order of 0.1 s. The response time of the capsule material is estimated to be less than 0.1 s.

For calibration of a certain type of liquid crystals, a suspension in demineralized water of approximately 0.04 vol% is made. To calibrate the reflected colours of liquid crystals against temperature, a small transparent container is used in which the suspension is cooled slowly, starting from the higher temperature in the temperature bandwidth. The 90° reflection of a 3 mm halogen light sheet crossing the container, is recorded on an S-VHS tape, and the temperature of the suspension is registered simultaneously using a thermocouple. During calibration, the suspension is stirred by a mixer.

### 4. Comparison between experimental and numerical results

For a channel flow with heated side walls, the density of the medium in the temperature boundary layers near those walls decreases as a result of the heat transferred from the side walls to the medium in the channel. Because this gives rise to a body force, the medium flows upwards near the side walls and returns in the core of the channel. The relative magnitude of the secondary flow compared to the primary flow can be estimated to be $\sqrt{\text{Gr} / [\text{Re} \sqrt{(1 + \text{Pr})}]} = 0.24$ for the case of $\text{Re} = 500$, $\text{Gr} = 10^5$, and $\text{Pr} = 6$ as described in this section. Because the onset of these secondary vortices, the dynamic pressure in the upper region of the channel increases, and the axial velocity profile will change along the channel. For validation of the code, in this section a comparison is made between the experimentally measured and numerically calculated velocities and temperatures. Before doing so, the method of measuring temperatures was first tested in a differentially heated cavity.
4.1. An initial test: Differentially heated cavity

Liquid crystal thermography was tested in a differentially heated cavity with an aspect ratio equal to $H/B = 2.33$. The installed Grashof number (based on the height of the cavity) was equal to $Gr = 1.3 \times 10^6$ based on the height of the cavity. The temperature measurements were compared to finite element calculations. Fig. 3 shows a satisfactory agreement between the experimental and numerical results, although it seems that the measured temperature field is somewhat shifted towards higher temperatures (more red and less blue), which may well be due to experimental uncertainties in the boundary conditions. The temperature boundary layers are resolved quite well by LCT, showing the good spatial resolution of the method. In the lower temperature range, the agreement between the experimental and the numerical results is fair. In the higher range, the agreement is somewhat worse. In global terms, one can say that the mean relative error is about 5% but that the maximal relative error can go up to 10%. For more detailed information about this experiment, the processing of the liquid crystal images, and the system calibration, one is referred to Sillekens (1995).

4.2. Channel flow: Midplane velocities

As mentioned above, because of the onset of secondary vortices in a channel flow with heated side walls, the dynamic pressure in the upper region of the channel increases and the axial velocity profile will change along the channel. This clearly is observed in Fig. 4 where the measured velocity field at the symmetry plane of the channel for $Re = 500$, $Gr = 10^6$, and $Pr = 6$ is presented.

In Fig. 4 (top), it is seen that the velocity field (indicated by the arrows) is still rather symmetric with respect to the plane $z = 0$. From the gray background field, indicating the magnitude of the shear $\partial u/\partial z$, it can be observed that the maximum in the axial velocity (indicated by the white band) remains at about $z = 0$. Fig. 4 (bottom) shows that the maximal axial velocity has shifted towards lower values of $z$, and a remarkable change in the velocity profile is observed, which is caused by the convective transport of the medium by the secondary motion.

The results in Fig. 4 show that particle tracking is a very powerful tool for quantitative visualization of this flow field. It would, however, be interesting to measure the secondary flow field in a cross-section of the channel at a particular axial position. Unfortunately, this is not feasible: as will be shown, the axial velocity is an order of magnitude larger than the secondary velocity, and, therefore, particles would pass through the light sheet too fast to be tracked.

In order to compare the numerical results to the experimental results, the axial and vertical velocities at the symmetry plane $y = 0$ will be considered at three axial positions: $x = -H$, where a fully developed velocity profile is expected; $x = 4.1H$, where the magnitude of the secondary velocity has reached its maximum; and further downstream at $x = 8H$. From Fig. 5 (top), it is seen that the measured inflow velocity profile differs somewhat from the analytical profile for fully developed channel flow. As has been mentioned before, this is attributable to buoyancy effects in the inflow section, where the flow is cooled slightly by the environment. The resulting secondary velocity causes the inflow profile to be shifted upwards slightly. The right graph of Fig. 5 (top), however, shows that the magnitude of the secondary flow is very low and is of the order of the uncertainty of the measurement.

Fig. 5 (middle) shows the axial and vertical velocity components at axial position $x = 4.1H$. The computed vertical velocity is seen to be in a good agreement with the experimental result, the measured axial velocity, however, still seems to suffer from the shifted inflow effect. A fair comparison between the numerical and experimental results is, therefore, not possible. It can be observed, however, that the downward movement of the maximum in the axial velocity profile from the inflow profile (Fig. 5, top) is about 0.07$H$, both for the measured and for the computed profile.

At $x = 8H$, the maximum in the axial velocity profile is seen to be shifted by about 0.16$H$. Although the measured profile still seems to suffer from the shifted inflow, the agreement between the measured and the computed profile is good. Again, there is good agreement between the measured and computed vertical velocity, although the measured vertical velocity suffers from the fact that in the upper region of the channel ($z > 0.3$), hardly any particles were observed. The reason

Fig. 3. Temperatures in a differentially heated cavity with aspect ratio 2.33; left: using LCT; right: computed temperatures.
4.3. Channel flow: Temperatures

To visualize the temperature field in the channel, LCT is employed. As explained before, the medium in the channel flows upwards near the side walls of the channel and flows downward in the core region. In the symmetry plane \( y = 0 \) the medium thus becomes stratified: the warm medium, coming from the heated side walls, pushes away the cold medium. In Fig. 6 (left) the build-up of the warmer layer in the symmetry plane is clearly visualized. The experimentally visualized temperature field is indicated by the color field, whereas, the numerical result is given by means of contours of equal temperature. Both from the experimental and the numerical results, it is seen that the size of the hot layer grows slightly in the axial direction, indicating the downward transport in the symmetry plane. Fig. 6 (right) shows that this downward transport is the largest near the center of the channel. Both the experimental and numerical results exhibit a typical M-shaped temperature boundary layer, showing that, although the magnitude of the secondary flow is small, its influence on the temperature field (and thus on the heat transfer) is large. From this figure, it is also seen that the physical situation, indeed, shows symmetry in the plane \( y = 0 \). The blue spots at the left and right upper corners in Fig. 6 (right) are the result of reflections at the wall of the channel.

4.4. Conclusion

From the above, it can be concluded that agreement between the experimental and numerical velocity data is quite satisfactory. The differences that occur in the axial component of the velocity field are probably mainly due to differences in the inflow conditions. Even the quite small secondary component of the velocity field detected in the experiments agrees quite well with the numerically calculated one. Differences are smaller than 1% of the mainstream velocity component. The agreement between the experimental and numerical temperature data is less satisfactory and is of the order of 10% of the installed temperature difference. This is mainly due to the inaccuracies in the temperatures determined with the liquid crystal technique. Also in the cavity problem, differences were found in the order of 5% to 10%. On the basis of these findings, it is concluded that the numerical model in the range of Grashof numbers under consideration in the present study, produces reliable results and can be used for further investigation.

5. Influence of Grashof number

5.1. Temperature and velocity fields

In Fig. 7 both the temperature (left half of each picture) and the velocity fields (right half) are displayed at two axial positions \( x = 4 \, H \) (left-hand side of the page) and \( x = 8 \, H \) (right-hand side of the page) for the three different values of the Grashof number: top: \( \text{Gr} = 5 \times 10^4 \), middle: \( \text{Gr} = 10^5 \), and bottom: \( \text{Gr} = 2 \times 10^5 \). The temperature field is given by contours of constant temperature, the difference between the contours is \( \Delta T/10 \). The velocity field is given by contours of constant axial velocity (with a difference of \( U/2.5 \)) and by vectors indicating the direction and the magnitude of the secondary flow field. All pictures show that the temperature boundary layer at the side wall remains small. The heated medium flows upwards near the side wall, along which the temperature boundary layer grows. This is quite similar to the flow along a heated vertical plate. At the top wall, the medium has to bend and initially a flow develops, similar to a density current (flow that occurs when a fluid with a density different from the bulk fluid penetrates along a wall) as can be observed in Fig. 7 (top), left side. Farther downstream, the warmer medium starts to fill up the upper part of the channel, and in the core, a stably stratified flow develops. At higher Grashof numbers, the warm fluid layer in the core is thicker than at a lower Grashof number, indicating that the secondary flow is more intense at a higher Grashof number.

Comparing the velocity fields at different Grashof numbers, it is indeed seen that the secondary flow field intensifies for higher Grashof numbers. Near the side wall, the secondary velocities clearly are larger than in the core region. The secondary flow, however, remains an order of magnitude lower than the primary flow. The secondary flow consists of one vortex in either side of the channel. The centre of this vortex is observed to shift downward along the channel, an effect that is more pronounced when then Grashof number is higher.

Further it is interesting to observe that for the case of \( \text{Gr} = 2 \times 10^5 \), at \( x = 8 \, H \) (Fig. 7) the medium “bounces” against the upper wall, and a corner structure similar to that observed by Lankhorst (1991) forms: first the medium reflects from the upper wall, then flows slightly upwards, but flows downward again near the symmetry plane.
Finally, as discussed before, the axial velocity profile is seen to be shifted downward. The upper left picture resembles the prescribed (fully developed) inlet velocity profile the most, the lower right picture shows how this profile has changed because of buoyancy.

5.2. Flow and heat transfer parameters

The analysis of the mixed convective flow field in the channel will be completed with a description of the heat transfer from the side walls to the medium, in relation to the magnitude of the secondary flow, followed by a description of the friction experienced by the flow in the channel.

Heat transfer will be expressed as function of the axial position \( x/H \) in terms of the local Nusselt number. The local Nusselt number is based on the height of the channel and the heat transfer coefficient averaged over the perimeter at a particular cross-section. Heat transfer is thought to be in a close relation with the magnitude of the secondary flow: when the heat transfer is high, the buoyancy forces will be relatively large, resulting in a higher magnitude of the secondary flow; when a strong secondary flow is present, the temperature boundary layer at the side wall will be small, and thus the heat transfer will be high. The magnitude of the secondary flow is expressed as a ratio of the kinetic energy of the secondary flow to the kinetic energy of the primary flow at a cross-section and will be referred to as the relative kinetic energy \( K \).

\[
K = \frac{\int u_{sec}^2 \, dy \, dz}{\int u_{ax}^2 \, dy \, dz},
\]

Fig. 5. Measured and computed axial (\( u \)) and vertical (\( w \)) velocity components at various axial distances for \( Re = 500, Gr = 10^5 \) and \( Pr = 6 \); top: \( x = -H \); middle: \( x = 4.1 \, H \); bottom: \( x = 8H \). (–) Analytical (top) or numerical (middle and bottom) data; (––) experimental data.
where \( u_{av} = \sqrt{v^2 + w^2} \), \( u_{av} \) is the axial velocity, and \( A \) is the area of the cross-section.

Fig. 8 gives the computed value of the Nusselt number as a function of the axial position in the channel for five different values of the Grashof number and for \( \text{Re} = 500 \) and \( \text{Pr} = 6 \). Near the entrance of the channel, the Nusselt number is the highest and approximately equal for all Grashof numbers considered. Because the magnitude of the secondary flow here is small (see Fig. 9), conduction of heat from the side wall to the medium is equal at all Grashof numbers. The temperature boundary layer is still very thin, and therefore the relative heat transfer is high. From position \( x = 0.5H \), effects of the secondary flow become important. For \( x > 8H \), oscillations occur in the solution that are caused by the incorrect boundary conditions prescribed. For all cases, in the fully developed situation, a Nusselt number of \( \text{Nu}_{x = 5.72} \) will be reached (Shah and Joshi, 1987). Comparing the Nusselt number for the different Grashof numbers considered, it is clear that the heat transfer is much better at high Grashof numbers than at low Grashof numbers. The Nusselt number for \( \text{Gr} = 2 \times 10^5 \) is up to three times as large as the Nusselt number for \( \text{Gr} = 0 \). These differences in the heat transfer coefficient lead to differences in the bulk temperature. Because the Nusselt numbers for the Grashof numbers under consideration are more or less constant for \( x > 1H \), the bulk temperatures show almost an exponential decrease. The values of the bulk temperatures at the outlet \( (x = 10H) \) are presented in Table 2.

In Fig. 9 it is observed that for all situations considered (except for \( \text{Gr} = 0 \)), the relative kinetic energy initially grows, reaches a maximum, and then decreases. The sudden increase at the end of the channel \( (x > 9H) \) is a result of the physically incorrect outflow boundary conditions and will be discussed below. In the first part of the channel, the secondary flow gains momentum because of buoyancy forces. Because of the rise in temperature of the medium in the downstream direction, the buoyancy forces in the channel gradually decrease. Besides, fluid is heated at the side walls, flows upwards, and results in a thermal stratification (see Fig. 7). This stratification leads to damping of the secondary flow field. The influence of the decreasing forcing rate and the stratification on the relative kinetic energy is clearly visible in Fig. 9. It can be seen that the relative kinetic energy shows a more or less exponential decrease for the Grashof numbers under consideration. Note that because the buoyancy forces become zero when the flow is in thermal equilibrium with the side walls, the relative kinetic energy finally will stabilize at a value of zero. The behavior of the secondary flow field is also reflected in the Nusselt number. The Nusselt number reaches a local maximum (at \( x = 5.5H \) for \( \text{Gr} = 10^5 \)) and then decreases slightly. Note that the occurrence of the maximum is somewhat lagging behind the occurrence of the maximum in the kinetic energy ratio, indicating that at this stage the heat transfer is increased by the secondary flow.

In Fig. 10 the Fanning friction factor is shown as a function of the axial position for the different values of the Grashof numbers. The Fanning friction factor \( f \) is defined as:

\[
f(x) = \frac{\bar{\tau}_w}{\frac{1}{2} \rho u^2},
\]

where \( \bar{\tau}_w \) is the average wall shear stress at a certain axial position:

\[
\bar{\tau}_w(x) = \frac{1}{4 \text{Re} \text{Fr}} \oint_{\partial} - \frac{\partial \bar{u}}{\partial n} \, dx.
\]

Here, \( \partial \) denotes the perimeter of the channel, and \( n \) denotes the local normal to the perimeter. The Fanning friction factor is a measure of the local skin friction experienced by the flow. For fully developed flow in a square channel a value of \( f_{ax} \cdot \text{Re} = 14.2271 \) will be reached (Shah and Joshi, 1987).

Comparing the different curves, it is seen that friction is increased by buoyancy forces. As has been shown before, the axial velocity profile changes because of buoyancy, and especially in the lower half of the channel, the normal axial velocity gradient substantially increases (see Fig. 7). Surprisingly, also the friction for \( \text{Gr} = 0 \) changes with axial position. Although the inlet velocity profile prescribed only differs from the analytical profile by about 2% (Shah and Joshi, 1987), this difference is enough to observe a development towards the analytical ex-

Fig. 6. Measured temperatures using LCT and computed temperature contours for \( \text{Re} = 500 \), \( \text{Gr} = 10^5 \), and \( \text{Pr} = 6 \); left: in the symmetry plane \( 8H < x < 9H \), \( y = 0 \); right: in cross-section at \( x = 8H \), \( \Delta T = T_{wall} - T_{air} = 1.5^\circ C \) and difference between the contours is \( \Delta T/10 \).
The expected value for the friction factor. The Fanning friction factor does not show a local maximum like the relative kinetic energy or the Nusselt number. It is interesting, however, to observe that the rate of increase of the friction factor in axial direction is closely related to the magnitude of the relative kinetic energy:

\[ \text{Axial position } x/H \]

\[ \text{N}_\text{u}-\text{number} \]

\[ \text{Axial position } x/H \]

\[ \text{K}_\text{e}-\text{number} \]

Fig. 7. Calculated temperature and velocity fields for Re = 500, Pr = 6 and various Grashof numbers; left: \( x = 4H \); right: \( x = 8H \); isotherms (left side of each figure) and isotachs and secondary velocity vectors (right side of each figure); a vector length equal to the height of the channel corresponds to the mean axial velocity; top: \( Gr = 5 \times 10^4 \); middle: \( Gr = 10^5 \); bottom: \( Gr = 2 \times 10^5 \).

Fig. 8. Calculated Nusselt number for Re = 500, Pr = 6; \( I: Gr = 0; II: Gr = 5 \times 10^4; III: Gr = 10^5; IV: Gr = 1.5 \times 10^5; V: Gr = 2 \times 10^5 \).

Fig. 9. Calculated relative kinetic energy for Re = 500, Pr = 6; \( I: Gr = 0; II: Gr = 5 \times 10^4; III: Gr = 10^5; IV: Gr = 1.5 \times 10^5; V: Gr = 2 \times 10^5 \).

...the higher the magnitude of the secondary flow, the more the axial velocity profile changes, and thus the higher the increase in shear. It is expected that a maximum in shear will be reached when the axial velocity profile is maximally shifted.
In the present study, the finite element method (FEM) is employed for detailed analysis of mixed-convection flow in a horizontal straight channel with heated side walls. The Reynolds and Grashof numbers are such that the flow is laminar and steady. Because of the simple geometry, a detailed comparison could be made with velocity measurements using particle tracking and temperature measurements using liquid crystals. The agreement is quite satisfactory: differences can be explained by the differences in inlet conditions and the inaccuracies of the experimental methods used. It is, therefore, concluded that the numerical method used in the present study is a very powerful tool for analyzing mixed-convection flows in complex geometries, such as helically coiled heat exchangers or corrugated tubes. Besides, using the method employed, detailed insight into the physics can be obtained, such as the influence of the buoyancy-induced secondary flows on the main flow characteristics and the heat transfer, knowledge of which is of great importance for the design of new heat exchanger concepts.

Particle-tracking velocimetry seems to be a powerful tool for measuring velocity fields. In the present study, only velocities are detected in the midplane (symmetry plane) of the channel where the flow is quasi two-dimensional (2-D). At all other positions, the flow is essentially 3-D, leading to velocity components perpendicular to the light sheet. This makes proper matching of particles in successive images a difficult job, certainly when axial and secondary velocities are of the same order. Therefore, 3-D particle-tracking velocimetry seems to be indispensable in analyzing complex flow problems.

In the present study, the steady momentum and energy equations are solved, supposing that no temporal instabilities occur. In most studies on the temporal stability of a channel flow, the case of air is considered (Hosokawa et al., 1993; Evans and Greif, 1993; Huang and Lin, 1995), and other boundary conditions are applied (mostly heated from below). As far as known, no data are available of the situation under consideration in the present study (water and heated side walls). However, from the liquid crystal and particle-tracking experiments, it may be deduced that no instabilities were present. Also, for higher Grashof numbers, the aforementioned techniques may be extremely useful in detecting the occurrence of unstable flow phenomena.

Acknowledgements

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References


Table 2

<table>
<thead>
<tr>
<th>Number</th>
<th>Dimensionless bulk temperature at the outlet for various Grashof numbers</th>
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</thead>
<tbody>
<tr>
<td>$5 \times 10^4$</td>
<td>0.82</td>
</tr>
<tr>
<td>$10^5$</td>
<td>0.79</td>
</tr>
<tr>
<td>$1.5 \times 10^5$</td>
<td>0.77</td>
</tr>
<tr>
<td>$2 \times 10^5$</td>
<td>0.75</td>
</tr>
</tbody>
</table>

Fig. 10. Calculated Fanning friction factor for $Re = 500$, $Pr = 6$; I: $Gr = 0$; II: $Gr = 5 \times 10^4$; III: $Gr = 10^5$; IV: $Gr = 1.5 \times 10^5$; V: $Gr = 2 \times 10^5$.

5.3. Influence outflow boundary conditions

As mentioned before the results beyond position $x = 8 H$ should be handled with care. In Figs. 8–10, the numerical results in the outflow region are, therefore, indicated as shaded areas. The physically incorrect outflow boundary conditions lead to wiggles in the Nusselt number and Fanning friction factor and to a sudden increase of the relative kinetic energy in the outflow region. For the case where no buoyancy forces are present ($Gr = 0$; line I in Figs. 8–10), the flow should be fully developed over the full length of the calculation domain, implying no changes in the axial flow field and zero secondary velocities. For this case, setting the normal stress equal to zero ($\sigma_n = 0$) is a proper boundary condition. As already mentioned, the change in the Fanning friction factor for this case (see Fig. 10) is attributed to the small differences between the inlet profile prescribed and the exact profile for fully developed flow in a square channel. The secondary velocities for this case (see Fig. 9) are almost zero over the entire domain. In the outflow region, a sudden increase is observed which is attributed to the physically incorrect boundary conditions for the tangential stresses ($\sigma_r = \sigma_u = 0$). For the cases where buoyancy forces are present (lines II to IV in Figs. 8–10) also, the zero normal stress boundary condition is physically incorrect because of pressure differences over a cross-section. This results in an ‘extra’ secondary flow in the outflow region, the effect of which on the relative kinetic energy is clearly seen in Fig. 9. For higher Grashof numbers, larger errors are introduced because then larger pressure differences are present over a cross-section. However, from the results presented it can be concluded that the upstream influence of the outflow boundary conditions is restricted to an axial distance of about 2 $H$.

6. Conclusions

In the present study, the finite element method (FEM) is employed for detailed analysis of mixed-convection flow in a horizontal straight channel with heated side walls. The Reynolds and Grashof numbers are such that the flow is laminar and steady. Because of the simple geometry, a detailed comparison could be made with velocity measurements using particle tracking and temperature measurements using liquid crystals. The agreement is quite satisfactory: differences can be explained by the differences in inlet conditions and the inaccuracies of the experimental methods used. It is, therefore, concluded that the numerical method used in the present study is a very powerful tool for analyzing mixed-convection flows in complex geometries, such as helically coiled heat exchangers or corrugated tubes. Besides, using the method employed, detailed insight into the physics can be obtained, such as the influence of the buoyancy-induced secondary flows on the main flow characteristics and the heat transfer, knowledge of which is of great importance for the design of new heat exchanger concepts.

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