MODELLING OF TWO-LAYER STRATIFIED STORES

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Abstract—A two-layer stratified storage process is studied analytically, numerically and experimentally. Detailed insight into the mixing phenomena is provided by the numerical and experimental model. It appears that the predominant mixing mechanism inside two-layer stores is shear-induced entrainment of thermocline fluid. An analytical model is developed, which is especially suited to quickly obtain indicative performance data under practical operation conditions. With the numerical model presented, more accurate performance data can be obtained. As a demonstration of how the numerical code can serve as an optimisation tool, the laboratory store as used in the experiments is optimised with respect to its inlet and outlet height. © 2000 Elsevier Science Ltd. All rights reserved.

1. INTRODUCTION

In 1973, thermal stratification was identified as a means to increase the efficiency of thermal energy storage (Fischer et al., 1975). Undoubtedly thermal stratification must have been present in stores before, however, its potential to increase the efficiency of, for example, a solar system was not recognised. Due to the separation of hot and cold water inside the store, the solar collector is supplied with relatively cold water. As a result the collector pump switch-on criterion (collector output temperature higher than collector input temperature) is more frequently satisfied which means that the solar system is in operation longer. In addition, hot water supplied by the collector is kept at a relatively high temperature which means that auxiliary heating is less frequently needed (which also results in a higher solar system yield). Many heat and mass transfer phenomena are known to have an adverse effect on thermal stratification. Noteworthy are natural convection flows caused by thermal conduction in the tank wall material and inserts (e.g. heat exchanger), see Hermansson (1993). The present paper is focused on distortion of thermal stratification by the inlet and outlet flows which, for short-term stores, is the dominating stratification detrimental mechanism. Though in practice the stratification in solar energy stores is continuously variable (due to the variable solar irradiation) here the attention is focused on two-layer hot and cold water stores because of their reduced complexity and higher relevancy with other heat stores (co-generation and chilled water).

Like in a previous paper (van Berkel et al., 1996), the two-layer store is studied in two modes; one in which the thermocline is preserved in the store by on-time flow reversal (multiple-sweep mode) and one in which the thermocline is formed, swept through the store and discharged through the outlet (once-through mode). The first mode is of more practical significance as in this mode strong variable outlet temperatures are avoided, which may otherwise lead to system malperformance. The second mode is of more theoretical significance. As this mode only comprises a single stroke, it takes less time to complete the analysis (both experimentally and numerically).

Depending on the store mode (multiple-sweep or once-through) the performance of a stratified store is expressed in two performance numbers. For the multiple-sweep mode the Cycle Thermal Efficiency (CTE) is used:

\[
CTE = \frac{\int T_{\text{per}} (T_{\text{hot, out}} - T_{\text{cold, in}}) \, dt}{1/T_{\text{per}} \int 0^{1/T_{\text{per}}} (T_{\text{hot, in}} - T_{\text{cold, in}}) \, dt}
\]

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in which in the first half period \((0-1/2 T_{\text{per}})\) the store is charged with hot water at a temperature \(T_{\text{hot,in}}\) and in the second half period \((1/2 T_{\text{per}} \rightarrow T_{\text{per}})\) the store is discharged. When discharging, the store is fed with cold water at a temperature \(T_{\text{cold,in}}\). If the volume of water which has been charged into a store is equal to the volume which can be extracted, this equation relates the heat contents charged to a store (denominator) to the heat contents which can be extracted from the store (numerator). For the once-through mode the Figure of Merit (FOM), see Wildin (1990), is applied:

\[
\Phi_{m} \int_{0}^{T_{\text{per}}} (T_{\text{hot,out}} - T_{\text{cold,in}}) \, dt
\]

\[
\text{FOM} = \frac{\Phi_{m} \int_{0}^{T_{\text{per}}} (T_{\text{hot,out}} - T_{\text{cold,in}}) \, dt}{M_{\text{store}} (T_{\text{hot,in}} - T_{\text{cold,in}})}
\]

with \(T_{\text{per}} = M_{\text{store}} / \Phi_{m}\). At the beginning of the charging process, the store is at a uniform temperature \(T_{\text{hot,in}}\). To obtain a practical measure, the cold water inlet temperature \((T_{\text{cold,in}})\) is taken as constant throughout the discharge process. In addition to these practical measures, the local entropy production rate is used to express the local mixing rate in a store:

\[
\dot{s} = \lambda \left( \frac{\nabla T}{T} \right)^{2}
\]

which expresses the production of entropy due to diffusion of thermal energy \((\lambda)\). For short-term storage, high temperature gradients are present in and near the thermocline. Temperature gradients associated with heat transfer through the tank wall are in general much smaller. As a result, in the present study, effects of heat losses to the ambient are neglected. In a water store, entropy production associated with the diffusion of impulse shows to be negligible, see van Berkel (1997).

As for short-term stores, mixing depends on a balance between the stratification promoting force (buoyancy) and the stratification detrimental force (inertia), the store performance is predominantly expressed as a function of the (overall) Richardson number:

\[
Ri_{H} = \frac{g \Delta \rho H}{\mu_{2}^{2}}.
\]

Alternatively the Richardson number can be based on the momentary position of the thermocline \(h\). Besides the Richardson number, the dynamics in the storage vessel is also a function of the inlet Reynolds number defined as:

\[
Re_{in} = \frac{u_{in} h_{in}}{\nu}
\]

In the present research, a two-layer store is studied analytically, numerically and experimentally. The goal of the research is to develop tools with which the design of two-layer stores can be optimised regarding the above-mentioned performance numbers. To this end, an analytical model for the multiple-sweep store and a numerical model for the once-through store is developed. The results obtained with these models are compared to experimental results for the laboratory store. Finally an optimisation is carried out.

### 2. RESEARCH TOOLS

#### 2.1. Analytical model for the multiple-sweep mode

As pointed out earlier, under practical (multiple-sweep) conditions the store always features a two-layer stratification. As a result the store can be considered one-dimensionally which enables development of a relatively simple analytical model. It is assumed that the thermocline oscillates between levels \(h_{\text{min}}\) and \(H - h_{\text{min}}\) (thereby not entering the thermocline exclusion zones with thickness \(h_{\text{min}}\)), see Fig. 1. An important assumption is that entrainment is a one-way process. Fluid is transported over the thermocline from the non-turbulent (outlet) layer (for the cold water charge cycle, the upper layer, see Fig. 1) to the turbulent (inlet) layer, not vice versa. This assumption, which is supported experimentally by means of fluid dye colouring tests, implies that during the upward stroke, the temperature of the upper layer does not change. The same applies for the bottom-layer temperature during the downward stroke.

The entrainment velocity can be coupled to the thermocline velocity by:

\[
\frac{dh}{dt} = \frac{\Phi_{e}}{A} + u_{e}
\]

which states that the true (Lagrangian) thermocline velocity \((dh/dt)\) is the combined effect of fluid displacement \(\Phi_{e}/A\) and entrainment of fluid over the thermocline \(u_{e}\).

The entrainment velocity can be assessed by evaluating the equation of mechanical energy, which provides the link between the cause of entrainment (kinetic energy) and its effect (change of potential energy). In integral form it states that the change rate of kinetic and potential
energy within a system is equal to the kinetic and potential energy fluxes over its boundary, the mechanical work acting on the boundary plus the dissipation of mechanical energy within the system (Kundu, 1990). To simplify this equation, the following assumptions are made:

1. Inlet and outlet areas are equally sized and small compared with the tank height.
2. Inlet and outlet velocities are block-profiled, hence \( u_{in} = u_{out} = u \). Besides it is assumed that no shear stresses are present at the inlet and outlet. Hence, fluid stress is caused by pressure alone.
3. Density variations are accounted for only in the potential energy and pressure terms.
4. Flow separation (from the vertical tank wall) occurs at the inlet. As a result the fluid pressure at the inlet equals the static pressure at that height in the store.
5. Flow separation does not occur in the converging outlet flow. As a result the fluid pressure at the outlet equals the local static pressure inside the store at that height, minus the dynamic pressure head.
6. In first approximation, it is assumed that only a fraction \( \eta \) of the jet kinetic energy influx is converted into the change of potential energy. The remaining part \( (1 - \eta) \) is dissipated into heat and neglected.

The flow separation assumptions 4 and 5 are supported by experimental observation. Adopting these assumptions and taking \( z = 0 \) as the reference level for potential energy, the equation of mechanical energy for a system as shown in Fig. 1 (cold water charge cycle) becomes, see van Berkel (1997):

\[
\Phi_{v}(\rho_{in} - \rho) + u_{c}A(\rho - \rho_{top}) = \eta \frac{\rho u^{2}}{gh} \Phi_{v} \tag{7}
\]

This equation states that the kinetic energy available for mixing (right hand side) is used for thermocline entrainment (second term left hand side) and mixing of the inlet flow with the lower layer contents (first term left hand side).

The additional equation is the conservation of mass equation. During the upward stroke, the temperature of the bottom layer may change due to the inflow of cold fluid and entrainment of hot fluid. Assuming a uniform bottom layer temperature, conservation of mass during the propagation of the thermocline from \( h \) to \( h + dh \) yields (see van Berkel, 1997):

\[
d(h, \rho) = \left( \frac{\Phi_{v}}{A} \rho_{in} + u_{c} \rho_{top} \right) dt \tag{8}
\]

In summary, the evolution of the bottom-layer density \( \rho \), the interface level \( h \) and the entrainment velocity \( u_{c} \) for the cold water charge cycle is given by the system of Eqs. (6)–(8). In addition to these equations, the periodic condition holds, which means that at the end of a cycle, the temperatures are the same as in the beginning. Besides, assuming that in the upward stroke the temperature of the upper layer does not change, and vice versa, means that the upper and lower layer temperatures, when the thermocline is at position \( h = h_{min} \), are equal to the corresponding values when the thermocline is at \( h = H - h_{min} \).

For solving the coupled set of Eqs. (6)–(8), a numerical procedure is applied. At each time step, the relevant parameters (\( \rho, h \) and \( u_{c} \)) are calculated after which (Euler forward) time stepping proceeds, until the final thermocline level is reached. The accuracy of the solution is checked by back substitution of the solution into the system of equations.

Quantitative analytical model results will be given later in comparison with experimental results. Here the results are discussed in a qualitative sense. A typical result is presented in Fig. 2.
where the time evolution of the bottom-layer temperature $T$, the thermocline level $h$ and the entrainment velocity $u_e$ are given for the cold water charge cycle. The dimensions of the store and relevant process parameters are given in the caption of the Fig. 2. Using the analytical model, it is found that in the initial stage of the cold water charge cycle, when the thermocline is close to the inlet, the entrainment rate is maximal, causing a sharp increase of bottom-layer temperature. Soon after, however, the entrainment velocity levels-off because a larger fraction of the jet kinetic energy available for mixing is required to mix the cold inlet flow with the bottom layer. When the thermocline moves away from the inlet, the entrainment velocity decreases further. In this stage, the constant supply of cold water starts to dominate over thermocline entrainment, causing a decrease in bottom-layer temperature. When the thermocline reaches the upper extreme level, the temperature is back at its initial value (thereby satisfying the periodic conditions). During the downward hot water charge stroke, the temperature of the bottom layer does not change and the system returns to its initial conditions from which point the charge process may start again. As a result of the mixing process, the temperature of the lower layer during downward motion is higher than the inlet temperature. Water can not be discharged with the same temperature at which it has been charged. The $CTE$ is therefore lower than unity.

2.2. GFD model for the once-through mode

To gain more insight in the store thermohydrodynamics, two store geometries have been studied using a Computational Fluid Dynamics (CFD) model. The most advanced simulations have been done in 3D on the interaction process of the jet, formed by the inlet flow, and the thermocline. The results of these simulations will be presented elsewhere. More practical simulations comprise 2D-simulations of a real store (like the one shown in Fig. 1), the results of which will be presented in this paper. With regard to the simulations, common assumptions are made for the physics involved: Boussinesq approximation, no viscous dissipation of mechanical energy, no radiative heat transfer. The equations to be solved then read:

\begin{align}
\nabla \cdot \mathbf{u} &= 0 \\
\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u} \mathbf{u}) &= -\frac{1}{\rho} \nabla p + g \beta (T - T_{\text{av}}) + \nu \nabla^2 \mathbf{u} \\
\frac{\partial T}{\partial t} + \nabla \cdot (\mathbf{u} T) &= \alpha \nabla^2 T
\end{align}

in which the subscript av denotes the average over...
As it was anticipated that all fluid motion length scales could be resolved, no turbulence model was incorporated in the numerical program. In essence the solution method is a pressure-correction method. It projects an intermediate velocity field $u^*$ obtained from the Navier–Stokes equation (excluding pressure), to a divergence-free velocity field $u^{n+1}$ by evaluating the pressure gradient term in the separate correction/projection step. In this step, a resulting Poisson equation for pressure is solved directly using a finite difference approximation on an equidistant staggered grid (Schumann and Sweet, 1976). Discretized in time the equations read:

$$
\frac{u^n - u^{n-1}}{2\Delta t} = -\nabla \cdot (uu^n) + g\beta(T - T_n)n
+ \nu \nabla^2 u^{n-1}
$$

(12)

$$
\frac{u^{n+1} - u^n}{2\Delta t} = -\nabla p^{n+1}
$$

(13)

$$
-\nabla^2 p^{n+1} = \nabla \left( \frac{u^{n+1} - u^n}{2\Delta t} \right)
= -\nabla \left( \frac{u^n}{2\Delta t} \right)
$$

(14)

$$
\frac{T^{n+1} - T^n}{\Delta t} = -\nabla \cdot (u T^n) + \alpha \nabla^2 T^n
$$

(15)

For the advection and buoyancy components of the momentum equation, time stepping is performed according to the time central, second-order accurate, neutrally stable leap-frog scheme. As a time central treatment is unconditionally unstable for diffusion, this term is treated with an Euler forward scheme over $2\Delta t$. To attain second-order accuracy for the treatment of buoyancy, this term is evaluated at time level midpoint $n$. Splitting between odd and even time levels is avoided by means of a time filter. More information on the discretisation of advection and the implementation of the boundary conditions can be found in van Berkel et al. (1996).

All simulations are started with a zero velocity field and a uniform temperature field and stepwise increase of the inlet velocity. The simulations presented in this paper are performed on a uniform $80 \times 160$ grid, for 10,000 time steps of 0.04 s. This simulation requires about 1 h to run on a Pentium 300 MHz computer, which roughly corresponds with $3 \cdot 10^{-5}$ s per cell, per time step.

### 2.3. Experiments

To provide overall insight in the storage tank flow patterns for representative operating conditions, visualisation experiments are performed in a laboratory size transparent storage tank. Besides, the experimental set-up is also used for validation of the numerical 2D-model for the once-through system and the analytical 1D-model for the multiple-sweep store.

The two-layer store set-up is configured such as to enable multiple-sweep and once-through thermocline operation, see Fig. 3. The size of the actual store volume is (width × height × depth = $400 \times 800 \times 400$ mm). Inlet and outlet chambers are applied for proper introduction and withdrawal. Water enters the inlet chamber from one side

![Fig. 3. Two-layer stratified store set-up with dimensions width × height × depth = $400 \times 800 \times 400$ mm.](image-url)
via a tubular inlet distribution manifold to assure an even distribution of the flow. Hot water is prepared and stored in a separate large (150 litre) head tank. During testing, hot water is pumped to and from the test tank by means of a centrifugal pump in the hot water line. Without the storage function of the head tank, hot water should be prepared instantaneously, which would require a high power heating rate (50 kW maximum). By occasionally heating the head tank, the hot water contents could be kept at a constant temperature.

To avoid usage of a cooling device in the cold water circuit, cold water is drawn directly from the tap. It is fed to the test tank, via the small (50 litre) cold water head tank, which serves as an overflow. The overflow imposes a constant pressure in the test tank and thus protects the test tank from high static pressure heads. The flow-rate is measured with a calibrated water capacity meter. The measurement accuracy of the water volume is approximated to be $0.5 \cdot 10^{-3}$ m$^3$ over the total measurement time.

In addition to the inlet and outlet temperature sensors, the tank contents are monitored by nine sensors, equidistantly placed at the centerline of the tank. The temperatures are recorded with 0.5- and 1-mm Thermocoax TK105/25/D-type thermocouples, having estimated response times less than 0.1 and 1 s, respectively. Tests showed typical values for the random error of 0.05°C. The temperature data were recorded with an 80386 CPU Personal Computer/data-logger, typically at a sample rate of 10 Hz and either first grabbed into RAM-memory or directly recorded to hard-disk.

During the multiple-sweep mode, the thermocline is kept in the tank by on-time flow reversing. The tank mode (cold water charge/hot water charge) is controlled on the basis of temperature signals provided by two thermocouples, placed 114 mm from the tank top and the tank bottom. As soon as the top thermocouples record a temperature lower than the top switch temperature, the flow is reversed manually from cold water charge to hot water charge. Similarly, the hot water charge flow is reversed when the bottom switch thermocouple records a temperature higher than the bottom switch temperature. The accuracy of the charge/discharge times is estimated to be 1 s. All tests are started with a uniform initial temperature distribution in the store. It is found that after six cycles, the main store parameters (charge/discharge times, thermocline thickness, inlet and outlet temperatures) do not change notably for the subsequent cycles.

Flow visualisation is performed mainly by fluorescent dye colouring. A 10–15 mm thick vertical light sheet is created by illuminating the domain from both sides through slotted masks, attached to the side walls, thereby creating a 2D-cut of the flow field. The domain contents are prepared by adding premixed fluorescent dye in a mass ratio of 1:2000. To preserve the dye in the test set-up, it is added to the hot water contents. Visualisation results will be given for cold water charge tests. The colour pattern then visualises the entrainment process in the bottom, transparent turbulent layer.

All tests are performed for a flow-rate of $0.32 \cdot 10^{-3}$ m$^3$ s$^{-1}$, corresponding with an inlet Reynolds number of $Re_{in} = 615$. The Richardson number is set via the top-layer temperature. It may be noted that in case of the multiple-sweep mode for $Ri_H = 3.3$ and 9.8, the switch temperatures are equal to the mean layer temperature. For higher $Ri_H$-numbers, however, this strategy causes partial withdrawal of the thermocline fluid as the thermocline grows thicker than the 114-mm distance between the thermocouples and the tank bottoms. To overcome this problem, for these cases the switch temperatures are set closer to the warm and cold water temperatures.

### 3. RESULTS

#### 3.1. Flow phenomena for the once-through mode

A global impression of the flow pattern inside the laboratory two-layer storage tank (for $Ri_H = 9.8$ and $Re_{in} = 615$) is provided by two fluorescent dye snap shots, taken during cold water charging, see Figs. 4 and 5. In Fig. 4 the inlet jet enters the store bottom right. After it has crossed the tank bottom, it collides with the vertical tank wall opposite to the inlet. Then, the jet is deflected upwards and finally collides with the thermocline, at which stage entrainment of top-layer fluid occurs. Marked by the arrows, are overturning motions near the thermocline zone, which are likely to be attributed to the Kelvin–Helmholtz shear-layer instability. A rough estimate obtained from Fig. 4 yields, for the first Kelvin–Helmholtz eddy diameter, a value of 30–40 mm and for the initial distance between two subsequent eddies a value of 90 mm.

Thermocline vortices are also found in the side view plane, see Fig. 5. Fluid filaments (cusps) are visible which are drawn into the bottom layer, thereby contributing to the entrainment process.
Furthermore, it becomes clear that entrainment mainly takes place by withdrawal of a thermocline fluid filament into the bottom layer. Occasionally overturning motions occur (marked by the arrow), quite similarly as observed during the experiments. The vorticity plots (Fig. 6b) reveal the presence of many whirls in the bottom layer, originating from the inlet wall jet. It furthermore appears that the jet detaches from the wall approximately half way to the tank bottom. As this effect is not observed during the experiments, it is hypothesised that jet detachment in the numerical results is caused by large-scale sub-thermocline circulation connected to the standing thermocline wave (which is found in the numerical results only). The cause for the numerical artificial standing wave motion is most likely the 2D inverse energy cascade for kinetic energy.

In the development of the analytical model it is assumed that entrainment is a one-way process, i.e. fluid is transported over the thermocline from the non-turbulent (outlet) layer to the turbulent (inlet) layer and not vice versa. From both the visualisation experiments and the calculations it may be concluded that this assumption holds, at least for the Richardson number and the inlet Reynolds number under consideration.

### 3.2. Multiple-sweep analysis: experimental and analytical model results

The evolution of temperatures inside the store and the inlet and outlet temperatures during a multiple-sweep test are shown in Fig. 7 for $Ri_H = 9.8$. Shown is the eleventh cycle which, as analysis of previous cycles revealed, represents stabilised conditions. During the cycle, the thermocline first moves downwards (hot water charging) and then upwards (cold water charging). In the narrow bands between the vertical lines, the pump is switched off and the store throughput is zero.
A first thing to notice is that (within the 1-s inaccuracy due to imprecise timing of valve operation) the time interval for charging is almost equal to the time interval for discharging (277 and 275 s, respectively). Under the assumption of constant flow-rate, this indicates that the water volume charged is equal to the volume discharged, a finding which was predicted by the analytical model. Just visible in Fig. 7 is that, due to entrainment, warm water which has been charged into the store, is discharged with a slightly lower temperature. Similarly, cold water is discharged with a slightly higher temperature.

The difference in charge and discharge temperatures is used to determine the store Cycle Thermal Efficiency for several operation conditions, see Fig. 8a.

The measured values for the CTE can be used to estimate the main parameter in the analytical model which is the efficiency $\eta$ for the conversion of jet kinetic energy flux into the change in potential energy (entrainment). When the store geometry and the operation conditions are incorporated in the analytical model, the conversion factor $\eta$ can be calculated for which the analytical model predicts the actually measured CTE, see Fig. 8b.
Fig. 7. Oscillating thermocline test for $Ri_H=9.8$. The numbered solid lines represent the internal thermocouples, positioned 114, 171, 228, 342, 399, 456, 570, 627, 684 mm from the bottom. The dark solid lines denote the inlet and outlet temperatures. Hot water charging starts at time 7120 and ends at 7400. Cold water charging subsequently starts at 7450 and ends at 7730. The positions of the thermocouples are indicated in Fig. 3.

Fig. 8b. A tentative explanation for the observed increase of $\eta$ with $Ri_H$ is that the stronger the effect of buoyancy (higher $Ri_H$), the more turbulent motions in and near the thermocline are suppressed, the less jet kinetic energy dissipates and the larger the fraction which remains for entrainment. This explanation is speculative and a better understanding of the functional behaviour of the efficiency remains for further research.

On the basis of Fig. 8, it is concluded that $Ri_H=15$ is an apt design condition. The CTE sharply drops for lower $Ri_H$-values whereas it increases just slightly for higher $Ri_H$-values.

3.3. Once-through analysis: experimental and numerical model results

With respect to numerical simulation, a choice has to be made with regard to the temporal and spatial resolutions. In Fig. 9a the convergence of the FOM-error is shown, defined as the difference
Fig. 9. The error in FOM (a) and the outlet temperatures (b), as a function of grid spacing for the laboratory store base case. Initial temperature 23°C, cold water inlet temperature 11°C and flow-rate 0.032 m³ s⁻¹ ($R_{\text{ni}} = 10$, $Re_{\text{ni}} = 615$).

between the FOM computed for a specific grid size and the FOM computed with the finest grid, see Fig. 9a. Despite the anomalous result for the 2.5-mm grid spacing, a general convergence with grid spacing can be recognised. However, despite the apparent convergence in the FOM, no convergence is found in the store dynamics, represented by the store outlet temperature, see Fig. 9b. Most remarkable is that a fluctuating outlet temperature is found for the numerical simulation (which intensifies for finer grids), while it is absent for the experiment. Closer analysis reveals that the fluctuating outlet temperature signal is caused by the standing wave motion of the thermocline (with a wavelength equal to half the tank width). As the outlet temperature fluctuation is not observed in the corresponding experiments (nor is the thermocline standing wave), the thermocline oscillation is possibly a result of the 2D inverse energy cascade.

Further insight in the accuracy can be gained by comparison of the store simulation results with
store experimental results. For a fixed geometry ($h_{in} = h_{out} = 20$ mm), constant (11°C) inlet temperature and various initial storage temperatures (19°C, 23°C, 27°C and 43°C), the computed and the experimental $FOM$ values are presented in Fig. 10. In addition the measured results are given, corrected for mixing in the inlet and outlet chambers (see van Berkel, 1997). A first thing to notice is that the magnitude of the $FOM$ is about 0.02–0.03 lower than the measured $CTE$ (see Fig. 8). This effect may be explained by the large mixing rate encountered during the density current stage (which is only present in the $FOM$ tests).

Though the levels of the measured and simulated $FOM$ agree reasonably well, the measurements result in a flatter profile. The discrepancy is ascribed to the artificial thermocline standing wave motion, due to which cold fluid is discharged too early (when the wave crest has passed the outlet). As the wave-like thermocline motion is most intense at small temperature differences, the $FOM$-discrepancy is largest at low $Ri_H$-numbers. Like the $CTE$-test, the present $FOM$-tests indicate optimal store conditions around an overall Richardson number of 10.

### 3.4. Store optimisation

It must be stressed here that both the $CTE$-tests and the $FOM$-tests outlined in the previous paragraphs are performed for a fixed geometry and variable store temperatures. In practice however, the store temperatures are predetermined and a suitable store geometry (in particular the relative size of the inlet and outlet) must be chosen. As a demonstration of how the numerical code can serve as an optimisation tool, the laboratory store is optimised with respect to its inlet and outlet size (which are assumed to be identical). This optimisation serves as a demonstration. Though in practice inlets and outlets are composed of pipes and manifolds (having circular cross-sections) general validity is obtained by expressing the store performance as a function of the overall Richardson number. As this number depends on the inlet and outlet velocity, it is insensitive of the actual shape of the inlet and outlet.

The optimisation routine involves computation of the $FOM$-value for eight different inlet and outlet sizes while holding the initial store temperature, the inlet temperature and flow-rate constant at 23°C, 11°C and 0.32·$10^{-3}$ m$^3$ s$^{-1}$, respectively. All computations are carried out with a 80×160 grid and 0.04 s time step. The result is shown in Fig. 11. The $FOM$ simulation results indicate that a maximum for the $FOM$ is attained at an overall Richardson number of 65. A comparison between Fig. 10 (obtained for variable temperature) and Fig. 11 (obtained for variable inlet velocity) shows an agreement for small Richardson numbers (when the outlet size is so small that withdrawal of cold fluid does not occur long before the thermocline reaches the tank top). This indicates that under these conditions the $Ri_H$...
is a consistent parameter (only its value matters, not whether it has been changed by varying the temperature difference or inlet velocity). For larger outlet openings (larger $Ri_H$ in Fig. 11) early withdrawal occurs, which causes the discrepancy with Fig. 10.

For interpretation of the functional behaviour of the $FOM$ with the relative size of the inlet and outlet, it must be reminded that for the once-through mode, mixing takes place during the initial density current intrusion stage, the thermocline phase and the withdrawal phase. Optimal store conditions are found when the sum of the mixing contributions of the three phases are minimal. For a small inlet and outlet, a high velocity jet is formed which induces a high mixing rate during the thermocline phase. On the other hand, when the inlet and outlet openings are large, a high mixing rate is encountered especially during the withdrawal phase when thermocline fluid is withdrawn long before the thermocline reached the tank top.

4. CONCLUSIONS

It must be noted that in the present study the optimal store conditions are determined for the laboratory store under specific conditions. As a consequence no general design rules (for other stores) can be given. Apart from the question whether or not simple but accurate design rules truly exist, it can be argued that they loose importance as analytical and numerical models may offer a higher accuracy, at acceptable costs. After further validation and development, these models can be applied to predict the performance of new stores. The analytical and numerical models may thereby serve different purposes. The analytical model is especially suited to quickly obtain indicative CTE-performance data. As the model is tentative (its prime parameter, the jet kinetic energy conversion efficiency in generally unknown) the model results are indicative too. A major advantage of the model is that it does not strongly depend upon the store geometry. The numerical model provides more insight and more accurate $FOM$-performance data. However, due to the type of spatial discretisation, the present numerical model is restricted to rectangular stores. With respect to the accuracy, the present study has shown that though the dynamics of the store is not fully represented, for optimal laboratory store conditions 2D-simulation on an $80 \times 160$ grid is sufficiently accurate. With respect to computational speed, it can be stated that the simulations can be performed on an ordinary personal computer (a single run typically then takes in the order of 1 h). This makes it possible...
to use the numerical code as a future engineering tool for optimisation of a stratified thermal store. To summarise the final conclusions of this study are:

- Store mixing is found to be a two-stage process. Firstly, thermocline entrainment takes place, mainly as a result of shear exerted by the deflected inlet jet. Shear layers form, which, after detachment from the thermocline, may become unstable due to Kelvin–Helmholtz-like waves. In the span-wise direction, cusp entrainment takes place due to vortices which are a result of inhomogeneous jet penetration and back flow. Secondly, once drawn into the bottom layer, stretching and folding takes place. Due to the increased interface area and decreased normal width, diffusion of heat effectively mixes thermal energy. Entropy is produced mainly in the thermocline, which, as a result of entrainment, is kept thin.

- The analytical model results (for $\eta = 1$), overestimate the mixing rate and underestimate the CTE. It is concluded that $\eta$ must increase (roughly proportionally) with the Richardson number. A tentative explanation for the behaviour is that turbulent motions are more suppressed for stronger stratification (higher $R_i$), due to which jet kinetic energy is more efficiently converted into change of potential energy (entrainment).

- The experimental visualisation results correspond well with the numerical simulation results, though the simulated flow pattern is more dynamical. The experimental and numerical simulation results show a deviation (in terms of $FOM$) of less than 0.04. The mismatch in store dynamics is ascribed to the inverse cascade for kinetic energy which forms a fundamental drawback of 2D simulations.

- The experimental and numerical analyses show to be complementary. Where the experiment is a true realisation of the thermocline entrainment process, the numerical simulation provides the temporal and spatial resolution for detailed analysis. Moreover the experiments facilitated validation of the numerical as well as the analytical model.

The store model results leave some ambiguity with respect to the behaviour of the jet conversion efficiency as a function of $R_i$. To come to a physical understanding of this behaviour, the conversion efficiency should be determined experimentally with a more accurate model, for a wide range of operation conditions. To eliminate the 2D artificial thermocline wave excitation, short-term thermally stratified store simulations should be performed in 3D. The computational work can be kept within limits by considering only a part of the tank. For the laboratory store, this would typically require a grid of width $3 \times height \times depth = 80 \times 160 \times 20$ cells, which can be solved within hours on a Gigaflop (super) computer. In addition, the numerical code should be made fit to cope with complex geometries, possibly by the use of boundary fitted co-ordinates. With the numerical code, the store performance can be evaluated for many parameter settings (tank aspect ratio, inlet and outlet size). From the results, more insight can be gained of the functional behaviour of the store Figure of Merit on parameters like the overall Richardson number and the inlet Reynolds number.

**NOMENCLATURE**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$A$</td>
<td>cross-sectional area</td>
</tr>
<tr>
<td>$CTE$</td>
<td>Cycle Thermal Efficiency</td>
</tr>
<tr>
<td>$FOM$</td>
<td>Figure of Merit</td>
</tr>
<tr>
<td>$g$</td>
<td>acceleration due to gravity</td>
</tr>
<tr>
<td>$h$</td>
<td>position of thermocline</td>
</tr>
<tr>
<td>$H$</td>
<td>height of the tank</td>
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<tr>
<td>$M$</td>
<td>mass</td>
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<tr>
<td>$Re$</td>
<td>Reynolds number</td>
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<tr>
<td>$Ri$</td>
<td>Richardson number</td>
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<tr>
<td>$s$</td>
<td>entropy</td>
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<tr>
<td>$t$</td>
<td>time</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature</td>
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<tr>
<td>$u_e$</td>
<td>entrainment velocity</td>
</tr>
<tr>
<td>$u$</td>
<td>velocity vector</td>
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</table>

**Greek symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
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<tbody>
<tr>
<td>$\alpha$</td>
<td>thermal diffusivity</td>
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<tr>
<td>$\beta$</td>
<td>cubic expansion coefficient</td>
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<tr>
<td>$\eta$</td>
<td>dynamic viscosity</td>
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<tr>
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<td>$\nu$</td>
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<tr>
<td>$\Phi_a$</td>
<td>mass flux</td>
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<tr>
<td>$\Phi_v$</td>
<td>volume flux</td>
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<tr>
<td>$\rho$</td>
<td>density</td>
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**REFERENCES**


