DEVELOPMENT OF AN IMPROVED DUMMY HEAD
FOR USE IN HELMET CERTIFICATION TESTS

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ABSTRACT
First order improvements to the rigid headform, used in current helmet certification tests, are made by introducing a skull and brain structure. In developing the new headform certain requirements were taken into consideration. The new headform appears to meet all requirements but one. The 200 mm drop test with Hybrid-III skin padding on the anvil resulted in too low resultant linear head accelerations. Using a stiffer, more realistic padding on the anvil resulted in a resultant linear head acceleration which satisfies the requirements (100 - 150 g).

The padding plays an important role in the evaluation process. Because of the deformable skull, a fairly stiff padding has to be used in order to let the resultant linear head acceleration satisfy the requirements. In contrast to the 200 mm drop test experiments, the padding properties of the skin are of no importance when an EPS padding is placed between the skin and the anvil.

INTRODUCTION
In accidents, the human head is exposed to loads exceeding several times the loading capacities of its natural protection. Annually, approximately five thousand motorcyclists get killed in Europe as a result of traffic accidents. They account for 9% of all road fatalities. Wearing a helmet reduces the risk of fatality with about 50%. The exact manner in which helmets protect the head is still not understood. Furthermore, mechanisms causing head injury are also not clearly understood.
Current helmets are empirically designed to meet the shock absorption requirements of current test standards. These standards all require tests for impact energy absorption, but most of them do not require tests for the chin guard or resistance to penetration. Efficient energy absorption with a minimum tendency to induce rotational motion and a comprehensive evaluation of the whole helmet are features which require special attention.

Current crash helmet drop test results can only predict the protective capacity of a crash helmet in a limited way. The main limitations are the use of a rigid headform, because it does not model the dynamics of the real, flexible human head correctly and it does not model the deformations of the brain, which is likely to be correlated with brain injury. Furthermore, only translational accelerations of the headform are measured, which is also insufficient to predict injury.

The objective is to make some first order improvements to the rigid headform. Therefore, an anatomically more realistic headform is introduced, so that more, relevant parameters for predicting the protective capacity of crash helmets can be determined. High speed bi-plane X-ray equipment is implemented in a drop test setup to obtain these parameters. The drop test setup is built according to the European test standard ECE-22/R04 [1] for crash helmets. The headform contains a deformable skull, made of a composite material, which is filled with a gel representing the brain (Dow Corning Sylgard 527 A&B dielectric silicone gel). The geometry of the model is based on the headform used in the European test standard and a numerical model of the human head, developed by Bandak & Eppinger [2].

In this study, the construction of the more realistic headform is discussed and evaluated on the basis of experiments found in literature. Furthermore, a numerical Finite Element model of the physical headform is built and validated with results of experiments on the physical headform. The numerical model will be used in simulations of the crash helmet drop tests to acquire more, relevant field parameters like stresses and strains. Furthermore, the numerical model can be used to investigate the influence of the different anatomical structures (falx, tentorium cerebelli, etc.) on the head response in helmeted head impact.

METHODS

First of all a set of requirements for the physical headform is presented. After that, the way the headform is built up and experiments to validate the model are discussed. Finally, a numerical model of the developed headform is constructed.

Requirements

The human head is a complex structure with complex materials. In order to model the head correctly for use in crash helmet drop tests, it must be known which components are relevant in this particular impact situation. The most important component of the head, with respect to head injury, is the brain. This is because, in motorcycle accidents, brain injury is the most frequent cause of death.

The cranial skull should be modeled correctly, since deformations to the cranial skull can influence the mechanics and dynamics of the helmet [3]. Skull base fractures have a high occurrence in oblique chin impacts. However, very little effort is spent in helmet certification tests to establish the helmets protective capacities in chin impact. The falx and tentorium cerebelli, the two most important meninges in the head, inhibit the movements of the brain inside the head and thus influence the dynamics of the human head as a whole.

However, the falx and tentorium cerebelli are left out of the model, because there were
no quantitative data on the mechanical properties of these components found in literature. The injury mechanism for skull base fracture due to chin impact is not well understood and injury parameters are not available. Therefore, the chin is only modeled correctly by its shape. Furthermore, the model does not contain a scalp, since it is expected that the scalp is of little influence in helmeted head impact. This will be elaborated later on.

The mass and size of the human head varies strongly from person to person. For this reason, the total mass of the head model is chosen to be 5.6±0.15 kg according to the ECE-22/R04 regulations [1]. The outer geometry and size of the head model is also chosen according to the ECE regulations. This results in a head circumference of 600 mm.

The ECE regulations have no requirements as to the moments of inertia of the head model. Values for the inertial properties of the human head were found in literature [4]. However, the headform used in the ECE regulations includes the neck and has a head circumference of 600 mm. Therefore, the values for the moments of inertia need to be scaled up and the influence of the neck on these values has to be accounted for. The scaling method and the adding of the neck is elaborated in the appendix at the end of the paper. The results are shown in Table 6. The deviation from these moments of inertia should be no greater than 50 kg·m², which is about the standard deviation of the principal moments of inertia of the human head.

The cranial bone model material should absorb as little X-radiation as possible, in order to still be able to look into the head model during impact. The thickness of the skull is 6.91±1.19 mm and the Young’s modulus of cranial bone is 5.6±3 GPa [5].

The brain model material should mimic the dynamic material properties of the human brain [6]. Since the human brain has a complex structure as well as material properties, it is difficult to find such a material. Therefore, it is also hard to give tolerance criteria on the material properties of the brain model material.

The circumference of the head model used in the drop tests is 600 mm. However, data on brain volume in relation to head size is not found in literature. In order to find indications on brain volumes, several Finite Element models were scaled up to a head circumference of 600 mm. The model of the National Highway Traffic Safety Administration (NHTSA) [2] and the Eindhoven University of Technology model [7] gave unrealistically high brain volumes of almost 2000 ml. They were probably scaled too much, because these models did not include the scalp. Another reason could be that scaling of the human head is not so trivial. In order to introduce as little error as possible in the scaling, the HUMOS database is used. The HUMOS model is a Finite Element model of a large human head, built from MRI data. This model also includes the scalp, which, in combination with the already large dimensions, introduces little scaling errors. After scaling to a head circumference of 600 mm, a brain volume of 1700 ml was found, which seems realistic. Even though the physical head model does not contain a scalp, it was chosen to implement a realistic brain volume rather than an overscaled brain.

It is very difficult, if not impossible, to model the brain anatomically correct with respect to its geometry as well as with respect to its boundary conditions. With respect to its geometry,
the model is chosen to be hemispherical. This means it is flat on the side of the skull base. This approach has the advantage that it does not bring too many constructional problems. This geometry simplification is also used in the NHTSA Finite Element model [2].

The head model

As a brain model material, the Dow Corning Sylgard 527 A&B dielectric silicone gel will be used. This material models the dynamic properties of brain tissue in a sufficient manner [6]. It resembles brain tissue at low frequencies, but becomes stiffer and more viscous at higher frequencies. Thus only a qualitative impression of the behaviour of the brain deformations during impact will result when this material is used. Despite this disadvantage, this material still is one of the best and most widely applied brain model materials [8; 9]. Furthermore, the gel is very adhesive. This means that the boundary conditions between the brain and the skull are well defined (no-slip condition). The advantage is that it can easily be modelled numerically. A disadvantage is that the cerebrospinal fluid (CSF) can not be modeled without using a lubricant.

Because the Sylgard gel is expensive, in the pilot experiments a gelatin mixture is used. The edible bone gelatin is used by the North Atlantic Treaty Organisation (NATO) as a tissue mimicking material [10]. The gelatin was prepared by dissolving one part of gelatin powder in nine parts of water (10% solution) at 50°C. This liquid mixture is partly poured into the composite skull and left to cure at 4°C. When this part of the gelatin is cured, plastic X-ray absorbing markers are placed onto the gelatin. The markers have a density of 1.24 kg/m^3, which is about the same density as the gelatine (1.0 kg/m^3) and the silicone gel (0.97 kg/m^3). Finally, the remaining part of the liquid mixture is poured into the skull and left to cure.

To model the dynamic properties of cranial bone correctly, a composite material is chosen. Composites are fiber reinforced polymers or resins. These materials are famous for their strength in combination with low weight, just like bone material. The stiffness of the material can be adjusted by varying the fiber volume fraction. More fibers result in a stiffer material.

Cranial bone has a Young’s modulus of 5.6 GPa ± 3 GPa [5]. The Young’s modulus $E_c$ of a composite material can be computed according to:

$$\eta_g E_g f_g + \eta_r E_r = E_c$$  \hspace{1cm} (1)

where $\eta_g$ is the glass fiber volume fraction to be determined, $E_g$ is the Young’s modulus of the glass fibers (70 GPa), $f_g$ is a correction factor for the glass fiber distribution (3/8 for randomly distributed fibers), $\eta_r$ is the resin volume fraction, $E_r$ is the Young’s modulus of the resin (2.0 GPa), and $E_c$ is the desired Young’s modulus of the composite (5.6 GPa). Since the total volume fraction equals one ($\eta_g + \eta_r = 1$), the glass fiber volume fraction $\eta_g$ can be determined from equation (1) as follows:

$$\eta_g = \frac{E_c - E_r}{E_g f_g - E_r} = \frac{5.6 - 2.0}{70 \times \frac{3}{8} - 2.0} = 0.15$$  \hspace{1cm} (2)

For the lower part of the skull, the requirements are less strict, because in helmet crash tests this part of the head is not impacted. This part is made of PMA. The outer geometry is in confirmation with the ECE regulations. The internal structure can be milled to adjust the inertial properties of the total head model and to provide space for the placement of the sensors.
A numerical model of the physical headform is used for tuning the inertial properties of the headform (Section: Numerical model).

To attach the composite skull model to the PMA part of the head, the goal was to make a liquid tight connection between the two components to prevent the gel from squeezing out of the cranial vault during impact. With gluing, there is no need to drill holes in the connected parts. Furthermore, stresses are equally distributed over the glued surfaces, which reduces the chance of failure due to fatigue. However, the strength of a glued combination is hard to test non-destructively. To test the glued combination of the composite skull with the PMA lower head, a series of tests was conducted. It turned out that the PMA material must be sandpapered to make a lasting connection with the composite material.

In case of skull fracture or gel replacement, the skull must be removed from the lower head. This can only be done by soaking the connected parts in acetone. It takes a few days before the glued connection lets loose, but then the acetone has soaked in the composite material, making it useless for further use.

Another way to connect the skull to the lower head is by means of screws. Screws have the advantage to connect and disconnect the two components time and time again. Since metal screws absorb the X-radiation, plastic screws are preferable.

In pilot experiments, with the skull clamped to the lower part of the head model, the skull did not come off. So, it might not be necessary to use an additional fixation of the skull.

Figure 1. Anatomically improved headform, composed of a PMA lower part and a gelatin filled cranial skull, made of a fiber reinforced polymer.
Evaluation experiments

When the headmodel meets all the requirements, it does not mean that it represents the human head sufficiently well. To investigate whether the physical head model is really biofidelic, it has to be evaluated on the basis of cadaver experiments found in literature. Several researchers investigated the mechanical response of cadaver heads. The most relevant of them will be discussed in the following.

Nahum et al. [11] performed two series of blunt head impacts on stationary unembalmed human cadavers. They used a rigid impactor covered with a variety of padding materials to provide a means of altering the duration of load application. Unfortunately, they did not mention padding thickness and density, which plays a crucial role when these experiments are to be used for evaluation purposes.

Hodgson et al. [12] developed a physical head model for use in evaluation of impact attenuation properties of football helmets. They evaluated their head model in helmeted cadaver head experiments. They reproduced these experiments with their head model, so that evaluation experiments exactly match the cadaver experiments. However, the type of helmet used in their experiments is no longer available. The influence of a different type of helmet on the outcome of the experiments is not known. Due to these uncertainties, these experiments can not be used to properly evaluate the physical head model.

The resonance and antiresonance frequencies of human cadaver heads are reported in literature [13; 14]. When the head is excited on the frontal bone, the head has an anti-resonant frequency at about 350 Hz and a resonant frequency of about 950 Hz. In antiresonance the forehead is relatively stationary and in resonance it is vibrating at a high amplitude. The resonance frequency of the head model was determined from the frequency response function (FRF). The FRF was measured with a standard impulse test method and computed with DIFA data acquisition software (DSA200). This software has a built-in anti-aliasing filter. A PCB piezoelectric impact hammer (model 208A05) was used in combination with a Kistler piezoelectric force transducer (model 8704B50).

The European Enhanced Vehicle-safety Committee [15] has reviewed several drop and impactor tests in relation to appropriateness for defining side impact dummy biomechanical targets. They found that Hodgson and Thomas [16] performed tests with sample size and energy level appropriate to be used for definition of a side impact head performance requirement. These tests involve a 200 mm head only drop test on a rigid horizontal surface. The head is positioned so that its mid-sagittal plane makes an angle of 35° with the impact surface and its a-p axis is horizontal. The head is dropped using a quick-release mechanism. During impact, head peak resultant acceleration of the head model must be between 100 g and 150 g, with g the gravitational acceleration (9.81 m/s²).

The head model does not contain a scalp, because it is expected to be of little influence in helmeted head impact. In the evaluation experiments, the scalp most probably is of influence. In the experiments, the influence of the scalp was accounted for by using a padding on the anvil. As a padding the scalp of a Hybrid-III dummy was used. The thickness of this scalp is about 12 mm.

The evaluation experiments were carried out on a drop test setup which is built in accordance to the ECE regulations for crash helmet testing. The test setup is depicted in Figure 2. The head is positioned on a guided cart. The cart can only move in vertical direction in order to drop the head in a controlled way. The head is equipped with a triaxial accelerometer (Entran Devices, model EGCS3-A) and three uniaxial angular rate sensors (ATA Sensors, model ARS-01). The axes of the three angular rate sensors coincide with the three axes of the triaxial accelerometer.
Brain injury is found to correlate with strain and strain rate [17; 18]. To determine the strain and strain rate within the brain during helmeted head impact, markers are placed within the brain. The markers are tracked using high speed bi-plane X-ray equipment, which is implemented in the drop test. The Philips image intensifiers of this equipment have a maximum field of view of 38 cm. The high voltage for the Philips SRO2550 ROT350 X-ray tubes is generated by two CREOS 330 ESU generators. The bi-plane X-ray images are recorded at a rate of 1000 frames per second with a resolution of 256×240 pixels. For mono-plane recording of the images, a frame rate of 4000 fps with a resolution of 256×256 pixels is possible. This difference is due to the use of two different high speed cameras (Kodak Motion Corder Analyzer Monochrome, model SR-1000 and Kodak Ektapro HS Motion Analyser).

Al-Bsharat et al. [19] performed cadaver experiments where not only head acceleration was measured, but also brain deformations were determined. These deformations were determined from tracks of markers within the brain. These experiments can be reproduced with the physical headform on the drop test setup as described above. The X-ray experiments are planned in the near future. As pilot experiments, the X-ray setup was implemented in the FRF experiments.

**Numerical model**

Stress and strain distributions in the physical head model can not be measured directly. Therefore, a numerical model of the physical head model was developed (Figure 3).

The numerical model has to be validated against experiments on the physical head model. This validation is planned in the near future. The numerical model can already be used as a quick and reliable method for determining the inertial properties of the physical head model.
For numerical simulations, the Finite Element package MADYMO is used. The model consists of 20976 elements (22998 nodes). In Table 2 an overview of the number of elements and of the material models is given for the numerical model. With $E$ the Young’s modulus in MPa, $\rho$ the density in kg/m$^3$, $G_\infty$ the elastic shear modulus in Pa and $K$ the bulk modulus in MPa. For the visco-elastic material model of the silicone gel, a 4-mode Maxwell model was fitted on the data presented by Brands et al. [6]. As seen in Figure 4 this model fits very well. The additional material parameters for the silicone gel are given in Table 3.

### RESULTS

The mass of the physical model was measured on an electronic balance, and turned out to be 5.61 kg, which is well within the required range of 5.6 ±0.15 kg. The inertial properties as computed with MADYMO are compared to the requirements in Table 4. MADYMO computed a total head mass of 5.61 kg, which is exactly the same as the measurement. The results are within the range of the requirements.

The results of the impulse test are depicted in Figure 5. There is an antiresonance at a

<table>
<thead>
<tr>
<th>Component</th>
<th>#elements</th>
<th>material model</th>
<th>material parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cranial skull</td>
<td>628</td>
<td>Linear elastic</td>
<td>$E=5.3$, $\rho=1300$</td>
</tr>
<tr>
<td>Brain</td>
<td>7040</td>
<td>Visco-elastic</td>
<td>$G_\infty=216.4$, $K=1.096$</td>
</tr>
<tr>
<td>Lower head</td>
<td>8220</td>
<td>Linear elastic</td>
<td>$E=3.0$, $\rho=1410$</td>
</tr>
<tr>
<td>Sensor unit</td>
<td>3632</td>
<td>Linear elastic</td>
<td>$E=1.0$, $\rho=1890$</td>
</tr>
<tr>
<td>Sensor holder</td>
<td>1456</td>
<td>Linear elastic</td>
<td>$E=70.0$, $\rho=2710$</td>
</tr>
</tbody>
</table>
frequency of about 440 Hz, where in literature [16; 14] an antiresonance frequency of about 360 Hz was measured. The FRF was determined for forehead impact, where the accelerometer was also placed on the forehead. Other impact directions were also measured and compared with the literature data. The results are summarised in Table 5.

In Figure 6, the resultant linear head acceleration from the 200 mm drop test is depicted. The maximum resultant linear head acceleration is 63 g, which is lower than the minimum required acceleration of 100 g.

The X-ray images recorded during the test are shown in Figure 7. The markers are not clearly visible in these images. The images are depicted less bright in order to make the headform visible. In the sequence of images, it is seen that the headform starts to rotate as a result of the eccentric impact. The maximum rotational head acceleration can be derived from the measurement of the resultant angular head velocity (Figure 8). The maximum rotational head acceleration equals the maximum slope of the curve. This slope is also depicted in the figure.
and equals:

\[ \alpha = \frac{\Delta \omega}{\Delta t} = 3.9 \times 10^3 \text{rad/s}^2 \quad (3) \]

where \( \Delta \omega \) is the change in resultant angular head velocity during time interval \( \Delta t \) and \( \alpha \) is the resultant angular acceleration.

**DISCUSSION**

The moments of inertia of the head model are well within the range of the requirements. This is important for the 200 mm drop tests, since an eccentric loading condition is used in these tests. When the moments of inertia are too low, too much rotation is induced. This results in a lower resultant linear head acceleration. When the moments of inertia are too high, the resultant linear head acceleration will be overestimated by the model.

The dynamics of the headform is determined by means of impulse testing. For top parietal impact the measurements match the literature data very well (330 Hz vs. 300 Hz). For side
parietal, frontal and especially occipital impact, the differences are larger. These regions are close to the lower part of the head. That part is modelled less accurate, which is probably the cause of these differences.

In the drop tests discussed in this paper the maximum resultant linear head acceleration of the headform does not meet the requirement of the 200 mm drop tests. This is probably due to the use of the Hybrid-III skin as padding on the anvil. This skin is developed to make the Hybrid-III dummy head biofidelic. In this head, the skull is modelled too stiff (metal). For this reason the Hybrid-III skin is much softer than the real human skin. The combination of a softer skin and a stiffer skull makes the Hybrid-III dummy biofidelic according to the 200 mm drop test.

A second 200 mm test was performed with stiffer rubber padding of about the same thickness as the Hybrid-III skin. In Figure 9 the resultant linear head acceleration of the 200 mm drop test with Hybrid-III skin and with rubber padding are compared. With the stiffer rubber padding the maximum resultant head acceleration is 120 g. This is within the range of the requirements (100 - 150 g). Thus, the padding plays an important role in the drop test experiments. This confirms the choice not to use the experiments carried out by Nahum et al. [11], where the mechanical properties of the used padding were not given.

One of the reasons to build an anatomically improved headform was to account for the dynamics of the head. Since both helmet and head are dynamic systems the interaction between them will have a significant effect on the dynamic behaviour of both of them. With the stiff rubber padding, the oscillations in the signal are larger than with the Hybrid-III skin padding (Figure 9). These oscillations are probably caused by the dynamics of the head.

The maximum resultant angular velocity during the 200 mm drop test with the stiffer rubber padding is about 10,000 rad/s² (Figure 10). This is about 2.5 times higher than with the Hybrid-III skin padding.

One of the assumptions was that the skin was not important in helmeted head impact. To verify this assumption, the 200 mm drop tests were repeated with Expanded Polystyrene (EPS) foam padding on the anvil. EPS foam is the material used in the protective padding liner in crash helmets. This material has excellent shock absorbing capacities. The resultant linear
head accelerations of the drop tests with and without rubber padding on the EPS foam padding are compared in Figure 11. It is clearly seen that there is hardly any difference between the drop tests with and without the rubber padding. The EPS foam (the helmet) is much softer in compression than the rubber padding (the skin). The use of the EPS foam padding on the anvil lower the peak acceleration from 120 g to 23 g.

According to the 200 mm drop tests the head model with rubber padding satisfies the requirements as well as the Hybrid-III head with the Hybrid-III skin. Since the Hybrid-III skin is much softer than the rubber padding, it is not obvious that the skin is also of no importance in helmeted Hybrid-III head impact. Experiments to validate this were not performed, because the Hybrid-III head was not available.

Even though the head model with gelatine as a brain simulant is biofidelic, it can not be used to predict brain deformation during impact, since gelatine is too stiff and nearly perfectly elastic [6]. It can be expected that, with the right padding, the head model with the silicone gel also is biofidelic. This head model would be capable of predicting brain deformations, but only qualitatively. Furthermore, this head model probably will model the dynamics of the human head even better.
CONCLUSIONS

An improved physical head model for use in helmet certification tests is developed according to proposed requirements. The head model model meets all the requirements but one. The 200 mm drop test with Hybrid-III skin padding on the anvil resulted in too low resultant linear head accelerations. Using a stiffer, more realistic padding on the anvil resulted in a resultant linear head acceleration which satisfies the requirement (100 - 150 g).

The padding plays an important role in the evaluation process. Because of the deformable skull, a fairly stiff padding has to be used in order to let the resultant linear head acceleration satisfy the requirements. In contrast to the 200 mm drop test experiments, the padding properties of the skin are of no importance when the anvil is padded with EPS foam. This can partly be
Figure 10. Resultant angular head velocity during 200 mm drop test for Hybrid-III skin padding and rubber padding on the anvil.

Figure 11. Comparison of the 200 mm drop tests with and without rubber padding on an EPS foam padded anvil.

attributed to the use of a deformable skull.

ACKNOWLEDGMENTS

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REFERENCES


The mean principal moments of inertia of the human head are shown in Table 6. These values differ from the values found in [4], because some orientations were switched to be closer to the orientations from Figure 12.

The cubic root of the mean head volume is chosen as a reference characteristic length scale for the scaling. With the mean head volume $\bar{V}$ is $3785.2$ ml, the reference characteristic length
Table 6. Mean principal moments of inertia of the human head.

<table>
<thead>
<tr>
<th>( \bar{I}_{xx} ) [kg ( \cdot ) cm(^2)]</th>
<th>( \bar{I}_{yy} ) [kg ( \cdot ) cm(^2)]</th>
<th>( \bar{I}_{zz} ) [kg ( \cdot ) cm(^2)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>206.3</td>
<td>177.8</td>
<td>151.3</td>
</tr>
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</table>

The moments of inertia of the neck around the head COG are computed by multiplying the neck mass \( m_{\text{neck}} \) with the square of the distance \( d \) from the neck COG to the appropriate axis through the head COG. The shifting of the position of the total COG as a result of the adding of the neck will have effect on the moment of inertia. This effect is assumed negligible.

The total principal moments of inertia are computed as follows:

\[
\begin{align*}
I_{xx} &= \bar{I}_{xx} \lambda^2 + m_{\text{neck}} [d \cos(30^\circ)]^2 = 206.3 \cdot 1.05^2 + 1.2 \cdot (10 \cdot \frac{\sqrt{3}}{2})^2 = 317 \\
I_{yy} &= \bar{I}_{yy} \lambda^2 + m_{\text{neck}} d^2 = 177.8 \cdot 1.05^2 + 1.2 \cdot 10^2 = 316 \\
I_{zz} &= \bar{I}_{zz} \lambda^2 + m_{\text{neck}} [d \sin(30^\circ)]^2 = 151.3 \cdot 1.05^2 + 1.2 \cdot (10 \cdot \frac{1}{2})^2 = 197
\end{align*}
\]