Parameter identification of a linear single track vehicle model

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D&C 2011.004

Traineeship report

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Eindhoven, January 2011
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1. Introduction

As vehicles are becoming more and more performing throughout the years, it turns out that their design process requires better knowledge of their behaviour. One of the ways to get to that knowledge is to use mathematical models, which describe the behaviour of the vehicle when given relevant parameters. Knowing these parameters is essential to run the models, and get the expected results.

Cornering behaviour of a road vehicle is mainly influenced by the tyre forces and moments [ref 5], [ref 8]. These forces depend on different parameters coming either from the conditions of the interaction between the tyre and the ground, such as slip conditions, and vertical load, or from the tyre itself, such as vertical stiffness or the so called cornering stiffness. Vehicle parameters, such as wheelbase length or yaw inertia, also greatly influence handling, and vehicle response to an input coming from both driver and road conditions.

This observation points out the importance of knowing all these parameters in order to better analyse the dynamic behaviour or to enhance vehicle control and stability.

This study will focus on a simple vehicle description, known as “single track model” also referred to as “bicycle model” [ref 7]. This model contains several parameters influencing its behaviour. Some of these parameters can be easily measured by equipping a car with some sensors while running standard tests on a special track (tests described in chapter 2), some others can be measured with a static vehicle (e.g. vehicle mass, steering ratio), but some parameters such as yaw inertia, or tyre parameters (cornering stiffness, relaxation length) are really difficult to measure properly, and need special and expensive equipment. This study will thus propose a method to estimate those parameters, by identifying them in functions derived from the equation of motion of the single track model, using numerical optimisation techniques.

Several studies have already been made on similar topics [ref 1], [ref 4], [ref 6]. These papers use a single track model and tyre force model to estimate the road-tyre interaction parameters such as side slip angle and tire force. All these papers use identification methods to evaluate the unknowns thanks to measurement signals, from actual road vehicles. Most of the papers focus on tyre properties, as they are considered to be the most influential in vehicle handling. Mainly two different types of estimations can be done: on-line (also known as real time) and off-line estimation. Real time estimation is mostly aimed at being used in control applications, such as ABS or ESP systems, and offline estimation is used with model to enhance knowledge of the vehicle, for instance to improve suspension geometry design. This Study will be focused on off-line estimation.
Our study will take into account the fact that some vehicle body parameters (e.g. yaw inertia) are also essential, when it comes to driving a car through a corner. The bicycle model, used in a cornering case, also involves yaw inertia effects when describing a dynamic motion, for instance random steering, or a lane change test.

This will make it possible to estimate the yaw inertia value, while keeping a simple bicycle model. For that, a first simple model will be derived from the equations of motion of the bicycle vehicle.

Another step of the study will be to build an enhanced model taking into account tyre relaxation effects. This is aimed to fit better to the actual behaviour, so as to obtain better results for the parameters values. This model will also enable us to estimate the tyre relaxation length, which is of major importance when one wants to estimate tyre side force in lateral dynamic behaviour.

The report will first introduce the single track representation used to model the vehicle, and the test which have been done in order to get the measured data. Then the different cases (steady state cornering, dynamic cornering) will be described by deriving the models including the unknown parameters from the equations of motion. After that the identification method will be presented, along with the way of organizing the MATLAB based identification program. Then the results will be shown and the relevance of the method and results it gives will be discussed in the final chapter.
2. Vehicle model and testing conditions

2.1. Description of the bicycle model

During this project, the vehicle will be modelled as a “single track” model (also known as “bicycle model”). This model assumes that the car can be described by only one front and one rear equivalent tyre, linked by the vehicle body. It thus doesn’t take into account effects such as body roll or lateral load transfer.

A representation of the single track representation is shown on Figure 2-1. The vehicle mass equals $m$. The distances from this centre of gravity to the front and rear axles are respectively $a$ and $b$. The yaw moment of inertia is noted $I_z$, and the wheelbase is noted $L$, steering angle $\delta$, and side slip angles $\alpha_1$ and $\alpha_2$, are assumed to be small.
2.2. Track tests

A car has been driven under different conditions on a test track. The purpose of these tests was to measure the vehicle dynamic performance of the car during cornering. Two different situations were run.

First case: Steady state cornering

During this test, the car is driven on a flat surface, and follows a circular path with a constant radius. The yaw velocity is considered to stay constant for a given forward speed. The forward velocity of the car is varied through a wide range. This test shows how the car behaves during a steady state cornering.

Measurement such as lateral acceleration, yaw velocity, steering angle, and side slip angle are then made.

For this steady state case this study will mainly focus on the vehicle side slip angle (β), which is the slip angle measured at the centre of gravity, and the steering angle of the front wheel, both of these parameters being considered as functions of the lateral acceleration of the vehicle measured at its centre of gravity.

Second case: Random steering

The car is driven at constant velocity and a “pseudo” random steering input is applied either by a driver or a robot. This test shows the frequency response of the vehicle to a steering input.

For this case, the transfer function between steer angle input and the two output parameters which are lateral acceleration and yaw velocity will be studied.
Figure 2-2: linear behaviour up to lateral acceleration of 4m.s\(^{-2}\)

One can notice that for these conditions, the vehicle behaviour can be assumed to be linear below lateral acceleration of 4m.s\(^{-2}\). In order to keep the model simple, this project will consider only this linear range.

### 2.3. Modelling of the different cases

#### Steady state cornering

The first step of the project consists of modelling the steady state cornering behaviour, and determining both front and rear cornering stiﬁnesses. The following assumption are made:

- No body roll
- No slope/level surface
- Constant forward velocity \(u(\approx V)\)
- Constant steering input( which means \(\dot{r} = 0\))

\[
m(\dot{v} + ur) = F_{y1} + F_{y2}
\]

\[
I\dot{r} = aF_{y1} - bF_{y2}
\]

\[
F_{y1} = C_{1}\alpha_{1}
\]

\[
F_{y2} = C_{2}\alpha_{2}
\]

Definitions of the side slip angles:

\[
\alpha_{1} = \delta - \frac{1}{u}(v + ar), \quad \alpha_{2} = -\frac{1}{u}(v - br)
\]
Derivation of the expression of $\delta$, the steering angle:

By substitution of $F_1$ and $F_2$ in eq (1) and (2):

$$m(\dot{v} + ur) = -\frac{v}{u}(C_1 + C_2) + \frac{L}{u}(bC_2 - aC_1) + \delta C_1 \tag{6}$$

$$l\dot{r} = -\frac{r}{u}(a^2C_1 + b^2C_2) - \frac{v}{u}(aC_1 - bC_2) + a\delta C_1 \tag{7}$$

And after combination of (6) and (7) to eliminate $v$:

$$m l\dot{r} + [I(C_1 + C_2) + m(a^2C_1 + b^2C_2)]\dot{r} + \frac{1}{u}[C_1C_2l^2 - mu^2(aC_1 - bC_2)]r = muaC_1\delta + C_1C_2l\delta \tag{8}$$

The steady state cornering implies the following conditions: $\ddot{r} = 0, \dot{r} = 0, \dot{\delta} = 0$

And assuming that $\beta$ is small, one can write: $R = \frac{u}{r}$

This gives:

$$\frac{r}{u}[C_1C_2l^2 - mu^2(aC_1 - bC_2)] = C_1C_2l\delta \tag{9}$$

And from (9), the expression for $\delta$ can be derived:

$$\delta = \frac{L}{R} - \frac{mv^2}{RL} \left( \frac{a}{C_2} - \frac{b}{C_1} \right) \tag{10}$$

$$a_y = \frac{V^2}{R} \tag{11}$$

By replacing $a_y$ in (10), one can obtain:

$$\delta = \frac{L}{R} - \frac{ma_y}{l} \left( \frac{a}{C_2} - \frac{b}{C_1} \right) \tag{12}$$

The equation of $\beta$ can be derived from the expression of front slip angle:

$$\beta = -\frac{V}{u} \tag{13}$$

$$\alpha_2 = -\frac{1}{u}(v - br) = \beta + \frac{br}{u}$$

$$\beta = -\frac{b}{R} + \alpha_2 \tag{14}$$

And, from eq (3):

$$\beta = -\frac{b}{R} - \frac{a\alpha_y}{C_2l} \tag{15}$$

Equations (12) and (15) show that $\beta$ and $\delta$ are linearly dependent on the lateral acceleration $a_y$. 
Dynamic model without relaxation length

In order to describe the dynamic behaviour of the bicycle model, a state space model will be used, which can easily be derived from the equations of motion.

The state space model can be written in the following way:

\[
\begin{align*}
\dot{x} &= Ax + Bu \quad (16) \\
y &=Cx + Du \quad (17)
\end{align*}
\]

Where:

\[
x = \begin{bmatrix} v \\ r \end{bmatrix}, \quad y = \begin{bmatrix} a_y \\ r \end{bmatrix}, \quad u = \delta
\]

And:

\[
A = -\begin{bmatrix}
\frac{c_1 + c_2}{l_x u} & \frac{ac_1 - bc_2}{l_x u} \\
\frac{ac_1 - bc_2}{l_x u} & \frac{a^2 c_1 + b^2 c_2}{l_x u}
\end{bmatrix} \quad B = \begin{bmatrix}
c_1 \\
\frac{c_1 + c_2}{mu} \\
\frac{ac_1 - bc_2}{mu}
\end{bmatrix} \quad C = \begin{bmatrix}
c_1 + c_2 \\
\frac{ac_1 - bc_2}{mu} \\
0 \\
\frac{ac_1 - bc_2}{mu} - 1
\end{bmatrix} \quad D = \begin{bmatrix}
c_1 \\
\frac{c_1 + c_2}{mu} \\
0 \\
\frac{ac_1 - bc_2}{mu}
\end{bmatrix}
\]

From this model, the transfer function of the system can be derived as follows:

Laplace transform of the state space equations ((16) and (17))

\[
sx(s) = Ax(s) + Bu(s) \quad (19)
\]

\[
y(s) = Cx(s) + Du(s) \quad (20)
\]

From (19):

\[
x(s) = \frac{B}{s-A} u(s) \quad (21)
\]

Replacing \(x(s)\) in (20) gives:

\[
y(s) = \left( \frac{c_B}{s-A} + D \right) u(s) \quad (22)
\]

\[
H(s) = c_B + \frac{D}{s-A} \quad (23)
\]
Dynamic model including tyre relaxation effect

The first dynamic model does not fit really well to the actual behaviour. A second model which includes the tyre relaxation effect has been built to describe better the vehicle dynamics.

The equations of motion are:

\[
m(v + ur) = C_1 \alpha'_1 + C_2 \alpha'_2
\]

\[
I_z \dot{r} = aC_1 \alpha'_1 - bC_2 \alpha'_2
\]

\[
\frac{\alpha_1}{u} \alpha'_1 + \alpha'_1 = \delta - \frac{1}{u} (v + ar)
\]

\[
\frac{\alpha_2}{u} \alpha'_2 + \alpha'_2 = -\frac{1}{u} (v - br)
\]

From these equations, a state space model is built as follows:

\[
\dot{x} = Ax + Bu
\]

\[
y = Cx + Du
\]

Where:

\[
x = \begin{bmatrix} u \\ r \\ \alpha'_1 \\ \alpha'_2 \end{bmatrix} \quad y = \begin{bmatrix} \alpha_y \\ r \end{bmatrix} \quad u = \delta
\]

And:

\[
A = \begin{bmatrix} 0 & -u & c_1 & c_2 \\ 0 & 0 & \frac{a c_1}{l} & \frac{b c_2}{l} \\ -\frac{1}{\sigma_1} & -\frac{1}{\sigma_1} & -\frac{u}{\sigma_1} & 0 \\ -\frac{1}{\sigma_2} & \frac{b}{\sigma_2} & 0 & -\frac{u}{\sigma_2} \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ \frac{1}{u} \\ \frac{a c_1}{l} \\ \frac{b c_2}{l} \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 & \frac{c_1}{m} & \frac{c_2}{m} \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\]

The transfer function of the system can be easily found by using the same method as used previously (cq (23)).
3. Identification method

3.1. Problem formulation

The purpose of the parameter identification is to minimize an objective function built as the
with the least mean square method. This function is the squared difference between the
values given by the model and the actual measured data. By doing that, the minimizers will
be found for each variable, which are the values of the wanted parameters.

Objective function: Steady state cornering

\[\varepsilon_\beta = \sum_{i=1}^{n} (\beta_{ms}(i) - \beta_{th}(i))^2 \quad (29)\]

\[\varepsilon_\delta = \sum_{i=1}^{n} (\delta_{ms}(i) - \delta_{th}(i))^2 \quad (30)\]

\[\varepsilon = \varepsilon_\beta + \varepsilon_\delta \quad (31)\]

With:

\[\varepsilon\] error function

\[\beta_{ms}, \delta_{ms}\] measured values

\[\beta_{th}, \delta_{th}\] values obtained from the model

\[i\] Index of the corresponding time step, for \(n\) steps

In the circular steady state steering case, the values of steering angle and vehicle side
slip angle are real numbers. Then for each design variable \(\beta\) and \(\delta\), a squared difference can
be derived. Then the objective function is built as a sum of the two error function on \(\beta\)
and \(\delta\), as shown by (31). The two terms of the sum are of different nature: the first is a slip
angle, and the second is a steering angle. Then one can wonder if there is a need to weight each term of the sum, to make them compatible, and obtain homogeneous orders of magnitude. It turns out that the values of \( \beta \) and \( \delta \) are really close, and adding a weighting coefficient does not change the result, up to a coefficient of ten for one or the other term. It will be then considered that weighting this function is not necessary.

Objective function: Dynamic cornering

When it comes to studying the dynamic case, the two variables are lateral acceleration \( a_y \) and yaw velocity \( r \). These variables can be computed by using the transfer function of the model. The derivation of the error function is given below.

Using the transfer function given by eq (23):

\[
H_{th}(w) = a_{th} + i b_{th} \tag{32}
\]

\[
H_{ms}(w) = a_{ms} + i b_{ms} \tag{33}
\]

Then, the distance between the two numbers is given by:

\[
|H_{th} - H_{ms}| = \sqrt{(a_{th} - a_{ms})^2 + (b_{th} - b_{ms})^2} \tag{34}
\]

The squared error is:

\[
|H_{th} - H_{ms}|^2 = (a_{th} - a_{ms})^2 + (b_{th} - b_{ms})^2 \tag{35}
\]

The objective function is based on the distance in complex plane (34) between the two complex numbers which are measured value and value from the model, which will be squared (35).

This error will be weighed by the coherence of the signal, to take into account that low coherence points are less trustworthy.
As we want to identify two series of values, which are lateral acceleration and yaw velocity frequency responses, we make a sum of both squared errors (38), where each one is normalized by its own mean value to avoid “over-optimization” on one or the other variable.

\[
\varepsilon_{ay}(w) = \left| H_{ay/\delta,th} - H_{ay/\delta,ms} \right|^2 \frac{c_{ay}}{|H_{ay/\delta,ms}|}
\]

\[
\varepsilon_r(w) = \left| H_{r/\delta,th} - H_{r/\delta,ms} \right|^2 \frac{c_r}{|H_{r/\delta,ms}|}
\]

\[
\varepsilon = \varepsilon_{ay} + \varepsilon_r
\]

Where:

- \( H_{ay/\delta,ms} \): transfer function between \( a_y \) and \( \delta \) (measurements)
- \( H_{ay/\delta,th} \): transfer function between \( a_y \) and \( \delta \) (theoretical value from the model)
- \( H_{ay/\delta,ms} \): transfer function between \( r \) and \( \delta \) (measurements)
- \( H_{ay/\delta,th} \): transfer function between \( r \) and \( \delta \) (theoretical value from the model)
- \( \varepsilon \): error function
- \( \bar{H}_{ay/\delta,ms} \): average value of \( H_{ay/\delta,ms} \)
- \( \bar{H}_{r/\delta,ms} \): average value of \( H_{r/\delta,ms} \)
4. Results

4.1. Steady state cornering

The unknown variables are front and rear cornering stiffnesses \( C_1 \) and \( C_2 \).

As it has been seen before, the problem is well bounded and unconstrained. Thus, in order to solve it, the Matlab command “fminsearch” is used. This function calls the objective function “error_func”, and stores the results in a vector “c”. To start the identification process, a guessed value has to be given for the design variables: this value is chosen at 100,000 N/rad for each cornering stiffness value, which is a good starting point.

<table>
<thead>
<tr>
<th>Mass(kg)</th>
<th>( C_1 ) (N/rad)</th>
<th>( C_2 ) (N/rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>front</td>
<td>rear</td>
<td></td>
</tr>
<tr>
<td>unloaded</td>
<td>784</td>
<td>522.5</td>
</tr>
<tr>
<td>loaded</td>
<td>809.5</td>
<td>840</td>
</tr>
</tbody>
</table>

Table 1: cornering stiffness values for different loads

The optimization problem is then solved for each load case. The results are shown in table 1. Although these results show that cornering stiffness increases with the load, one can notice that the relation is not proportional.
The results are shown below:

**Figure 4-1**: steering angle and Vehicle slip Angle vs. lateral acceleration (unloaded case)

**Figure 4-2**: steering angle and Vehicle slip Angle vs. lateral acceleration (loaded case)

Figures 4-1 and 4-2 show measured points and tuned models as functions of lateral acceleration, for both loaded and unloaded case. The good fitting of the model confirms that the linear solution is a good approximation.
4.2. Dynamic cornering model without relaxation effects

Once the cornering stiffnesses have been determined using the steady state measurements, it is necessary to model the dynamic cornering case, including non-zero yaw acceleration, in order to determine the vehicle yaw inertia. The measured data file contains the response function of $a_y$ and $r$ to a dynamic steering input. The aim of the identification process will fit the previously built model to this data. Two different methods will be used for this:

First case: Front and rear cornering stiffnesses values are taken from the steady state case identification, and the yaw Inertia is the only parameter identified.

Second case: All parameters are identified at the same time, only with the dynamic model.

The results of the identification can be seen in tables 2 and 3.

<table>
<thead>
<tr>
<th>Mass (kg)</th>
<th>C_1 (N/rad)</th>
<th>C_2 (N/rad)</th>
<th>I_z (kg.m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>front</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>unloaded</td>
<td>784</td>
<td>522.5</td>
<td>67,473</td>
</tr>
<tr>
<td>loaded</td>
<td>809.5</td>
<td>840</td>
<td>73,263</td>
</tr>
</tbody>
</table>

Table2: identification only with dynamic model: corn. stiffness and inertia for different loads

<table>
<thead>
<tr>
<th>Mass (kg)</th>
<th>C_1 (N/rad)</th>
<th>C_2 (N/rad)</th>
<th>I_z (kg.m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>front</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>unloaded</td>
<td>784</td>
<td>522.5</td>
<td>44,184</td>
</tr>
<tr>
<td>loaded</td>
<td>809.5</td>
<td>840</td>
<td>63,896</td>
</tr>
</tbody>
</table>

Table3: identification with corn. stiffness from steady state model: corn. stiffness and inertia for different loads

The results of the identification including steady state value seem to be quite good, but the ones from the identification with only dynamic models seem really unrealistic.
The results are given by figures IV-3 to IV-6:

Figure 4-3: lateral acceleration frequency response: unloaded case

Figure 4-4: yaw velocity frequency response: unloaded case
Figures 4-3 to 4-6 show that, although for loaded case both identification processes give similar results, for the unloaded case, it is clearly better to take the cornering stiffnesses from the steady state case, for both lateral acceleration response and yaw velocity response.
4.3. Dynamic cornering model including relaxation effects

As the dynamic model used previously does not give really accurate results, identification is now done with the model including tyre relaxation length, described in chapter 2.3. As in the previous case, a comparison is made between identification of all the parameters at the same time, and identification with cornering stiffnesses taken from the steady state case.

The results can be found in table 4 and 5, for both identification cases.

<table>
<thead>
<tr>
<th></th>
<th>Mass(kg)</th>
<th>$C_1$ (N/rad)</th>
<th>$C_2$ (N/rad)</th>
<th>$I_z$ (kg.m²)</th>
<th>$\sigma_1$ (m)</th>
<th>$\sigma_2$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>front</td>
<td>rear</td>
<td>front</td>
<td>rear</td>
<td></td>
<td></td>
</tr>
<tr>
<td>unloaded</td>
<td>784</td>
<td>522.5</td>
<td>66,243</td>
<td>91,920</td>
<td>1356.4</td>
<td>0.881</td>
</tr>
<tr>
<td>loaded</td>
<td>809.5</td>
<td>840</td>
<td>58,255</td>
<td>97,913</td>
<td>1705</td>
<td>0.670</td>
</tr>
</tbody>
</table>

Table 4: identification only with dynamic model cornering stiffness values, inertia and tyre relaxation length for different loads

<table>
<thead>
<tr>
<th></th>
<th>Mass(kg)</th>
<th>$C_1$ (N/rad)</th>
<th>$C_2$ (N/rad)</th>
<th>$I_z$ (kg.m²)</th>
<th>$\sigma_1$ (m)</th>
<th>$\sigma_2$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>front</td>
<td>rear</td>
<td>front</td>
<td>rear</td>
<td></td>
<td></td>
</tr>
<tr>
<td>unloaded</td>
<td>784</td>
<td>522.5</td>
<td>67,473</td>
<td>91,611</td>
<td>1353.7</td>
<td>0.902</td>
</tr>
<tr>
<td>loaded</td>
<td>809.5</td>
<td>840</td>
<td>73,263</td>
<td>130,450</td>
<td>2211.9</td>
<td>0.600</td>
</tr>
</tbody>
</table>

Table 5: identification with corn. stiff. from steady state model: cornering stiffness values, inertia and tyre relaxation length for different loads

Tables 4 and 5 show that for both cases values are close for unloaded conditions, but there is a great change when it comes to loaded case.
The results are given in figure IV-7 to IV-10:

**Figure 4-7:** lateral acceleration frequency response: unloaded case

**Figure 4-8:** yaw velocity frequency response: unloaded case
Figures 4-7 to 4-10 shows that for the case including tyre relaxation length, the identification results quite close, when the cornering stiffnesses come the from steady state identification or when all parameters are identified only with the dynamic model.
5. Conclusions and recommendations

5.1. Limits of the bicycle model

When considering the results of this project, one has to keep in mind that the model which has been used gives an approximation of the actual car behaviour. The single track model doesn’t take into account neither body roll effects nor load transfer. The wheel inclination angle is also neglected.

And finally, only the linear part of the behaviour has been studied, which means that the results are close to reality only for small slip and steering angles, and for small lateral acceleration (under $4\text{m.s}^{-2}$). Beyond this point, the tyre characteristic becomes non-linear. To describe this behaviour it would be necessary to make a new non-linear model.

5.2. Accuracy of the fitting

As one can notice; even with the second method, the frequency response does not follow exactly the curve of measured data.

The important lack of accuracy for lateral acceleration frequency responses could be assumed to come from the fact that the single track model does not take into account the effect of body roll when cornering. Adding this parameter could improve the fitting of the model, but this would mean building a four-wheel model [ref 7], which is much more complicated. The bicycle model has been chosen in this study for its simplicity, so if one wants to keep using this simple model, he may only trust the acceleration values up to a frequency of 1Hz. It can be seen that after this limit, even with optimized parameters, the model cannot describe well the actual vehicle. Nevertheless, for the yaw velocity parameters, the results are still quite relevant even in higher frequencies, and the results can be trusted at least up to 3Hz.
5.3. Importance of weighting in the error function

The objective function of the identification problem is made of a sum of several error functions, coming from lateral acceleration value, yaw velocity, side slip angle, or steering angle. The value of these parameters are not in the same order of magnitude, and if nothing is done to change this, some part of the error function will be given more importance than the others by the least mean square method. Such as to minimize this effect, each error term has been divided by the norm of the correspondent parameter, so that all differences of order disappear. When it comes to building the dynamic case error, the error function has been multiplied, at each point, by the coherence of the signal, to reflect the fact that the low-coherence measurement points are less trustworthy. Moreover, it turns out that giving more emphasis to one or the other parameter in the objective function could lead to a better fitting of the model.

5.4. Conclusion

This study has achieved its purpose, by proposing an identification method which estimates the parameters of a linear single track model for several cornering conditions, and vehicle loads. The differences between different have been shown, particularly by separating the identification into two different steps, in order to obtain first the cornering stiffness parameter by identification in the steady state model, and then using these parameters in the dynamic model. This method gives better results mainly with the dynamic model without tyre relaxation effect, as we can see that when including this tyre relaxation length parameter, the differences between the two methods tend to be highly reduced.
6. References

1. Baffet G., Charara A., Lechner D., “Estimation of Vehicle Sideslip, Tire Force and Wheel Cornering Stiffness”, HEUDIASIAC Laboratory, UMR CNRS 6599, Université de Technologie de Compèigne, Compiègne, France; INRETS-MA Laboratory (Department of Accident Mechanism Analysis) Chemin de la croix Blanche, Salon de Provence, France.


8. Smith N. D., “Understanding Parameters Influencing Tire Modeling”, Colorado State University, Department of Mechanical Engineering, Colorado State University, 2004
7. Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>Front cornering stiffness (N.rad$^{-1}$)</td>
</tr>
<tr>
<td>$C_2$</td>
<td>Rear cornering stiffness(N.rad$^{-1}$)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Vehicle side slip angle (at the centre of gravity)(rad)</td>
</tr>
<tr>
<td>$a_y$</td>
<td>Lateral acceleration (m.s$^{-2}$)</td>
</tr>
<tr>
<td>$\alpha'_1$</td>
<td>Front dynamic side slip angle (rad)</td>
</tr>
<tr>
<td>$\alpha'_2$</td>
<td>Rear dynamic side slip angle (rad)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Steering angle (rad)</td>
</tr>
<tr>
<td>$a$</td>
<td>Centre of gravity to front axle distance (m)</td>
</tr>
<tr>
<td>$b$</td>
<td>Centre of gravity to rear axle distance (m)</td>
</tr>
<tr>
<td>$I_z$</td>
<td>Yaw inertia (kg.m$^2$)</td>
</tr>
<tr>
<td>$L$</td>
<td>Wheelbase length (m)</td>
</tr>
<tr>
<td>$F_{y1}$</td>
<td>Front lateral tyre force (N)</td>
</tr>
<tr>
<td>$F_{y2}$</td>
<td>Rear lateral tyre force (N)</td>
</tr>
<tr>
<td>$u$</td>
<td>Forward velocity (m.s$^{-1}$)</td>
</tr>
<tr>
<td>$r$</td>
<td>Yaw velocity (rad.s$^{-1}$)</td>
</tr>
<tr>
<td>$m$</td>
<td>Vehicle mass (kg)</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>Front tyre relaxation length (m)</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>Rear tyre relaxation length (m)</td>
</tr>
<tr>
<td>$Hay/\delta^{ms}$</td>
<td>transfer function between $a_y$ and $\delta$ (measurements)</td>
</tr>
<tr>
<td>$Hay/\delta^{th}$</td>
<td>transfer function between $a_y$ and $\delta$ (theoretical value from the model)</td>
</tr>
<tr>
<td>$Hay/\delta^{ms}$</td>
<td>transfer function between $r$ and $\delta$ (measurements)</td>
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<td>$Hay/\delta^{th}$</td>
<td>transfer function between $r$ and $\delta$ (theoretical value from the model)</td>
</tr>
</tbody>
</table>
8. Acknowledgements

I would like to thank my supervisors, prof.dr. H. Nijmeijer and dr. Ir. I.J.M. Besselink, for having helped me to achieve this study, and advised me during all the time of the project.

On the personal side, this topic helped me to understand better vehicle dynamics, particularly what happens during cornering, and how a road vehicle reacts to an input. It also enabled me to increase my knowledge about how to use Matlab tools and programming commands.