Numerical simulations of particle migration in a viscoelastic fluid subjected to Poiseuille flow

M.M. Villone

Dipartimento di Ingegneria Chimica, Università di Napoli Federico II,
Piazzale Tecchio 80, 80125 Napoli, Italy

G. D’Avino

Department of Chemical Engineering, K.U. Leuven, W. de Croylaan 46, B-3001 Leuven, Belgium

M.A. Hulsen

Department of Mechanical Engineering, Eindhoven University of Technology, PO Box 513, 5600 MB Eindhoven, The Netherlands

F. Greco

Istituto di Ricerche sulla Combustione, IRC-CNR, Piazzale Tecchio 80, 80125 Napoli, Italy

P.L. Maffettone*

Dipartimento di Ingegneria Chimica, Università di Napoli Federico II,
Piazzale Tecchio 80, 80125 Napoli, Italy

Abstract

In this work we present 2D numerical simulations on the migration of a particle suspended in a viscoelastic fluid under Poiseuille flow. A Giesekus model is chosen as constitutive equation of the suspending liquid. In order to study the sole effect of the fluid viscoelasticity, both fluid and particle inertia are neglected.

*Corresponding author. +39 0817682282; fax +39 0812391800
Email addresses: massimiliano_villone@hotmail.com (M.M. Villone),
gadavino@unina.it (G. D’Avino), m.a.hulsen@tue.nl (M.A. Hulsen),
greco@irc.na.cnr.it (F. Greco), pierluca.maffettone@unina.it (P.L. Maffettone)
The governing equations are solved through the finite element method with proper stabilization techniques to get convergent solutions at relatively large flow rates. An Arbitrary Eulerian-Lagrangian (ALE) formulation is adopted to manage the particle motion. The mesh grid is moved along the flow so as to limit particle motion only in the gradient direction to substantially reduce mesh distortion and remeshing.

Viscoelasticity of the suspending fluid induces particle cross-streamline migration. Both large Deborah number and shear thinning speed up the migration velocity. When the particle is small compared to the gap (small confinement), the particle migrates towards the channel centerline or the wall depending on its initial position. Above a critical confinement (large particles), the channel centerline is no longer attracting, and the particle is predicted to migrate towards the closest wall when its initial position is not on the channel centerline. As the particle approaches the wall, the translational velocity in the flow direction is found to become equal to the linear velocity corresponding to the rolling motion over the wall without slip.

**Key words:** Particle migration, Microfluidic, Viscoelasticity, Poiseuille flow, Numerical simulations, Arbitrary Lagrange Eulerian formulation

1. Introduction

In many practical processes, particles are suspended in fluids in order to give specific properties to the final composite material (for example filled polymers, paints, coatings). On the other hand, in several systems, particles are naturally transported in fluids (i.e. cells in blood, pollutants in gaseous flows etc.). Quite often, the channel dimensions where suspensions flow are comparable to the particles size (i.e. microfluidic devices), and the suspending liquid exhibits viscoelastic effects such as normal stresses and shear-rate dependence of the viscosity.

It is well known that particles suspended in liquids can show peculiar effects with a phenomenology strongly depending on the type of the flow as well as on the fluid rheology. One such effect, of interest here, is cross-streamline migration, i.e., the motion of the particle orthogonally to the direction of the main flow. This problem received great interest over the last fifty years.

The first experimental observations on the migration phenomenon were performed by Segré and Silberberg [1, 2]. The authors studied the macro-
scopic inertial migration of non-interacting, neutrally buoyant spheres in a Newtonian fluid in a tube flow. They found that the particles migrate away from both the wall and the channel centreline, and move towards an equilibrium radial position of about 0.6 times the tube radius.

Later on, the relevant phenomenology observed by Segré and Silberberg [1, 2] has been experimentally confirmed in several works [3, 4, 5]. Recently, the cross-streamline migration has been studied at moderately high Reynolds numbers (up to 2500) by Matas et al. [6]. They observed a motion towards the wall of the equilibrium radial position found by Segré and Silberberg as the Reynolds number is increased. The same observations were made by [7] where convective transport of neutrally buoyant spherical particles is studied in capillaries using an on-line particles detector.

Perturbation analysis has been widely used to study the effect of the inertia on the particle migration in shear and Poiseuille flows giving satisfactory agreement with the experimental observations [8, 9, 10]. Direct numerical simulations have also been performed in order to examine the effect of a finite Reynolds number on the cross-streamline migration. An extensive literature has been produced, focusing on several aspects such as neutrally and non-neutrally buoyant particles [11, 12], lift-off correlations [13], existence of multiple equilibrium positions [14]. Recently, the analysis has been extended to 3D systems confirming the phenomenology experimentally observed [15, 16, 17]. For a comprehensive review of the inertial effects on the particle migration the reader can refer to [17] and the references therein.

Concerning the effect of the viscoelasticity of the suspending fluid, migration of solid spherical particles at low Reynolds number in non-Newtonian fluids has been studied experimentally by Mason and co-workers [18, 19, 20]. The results showed that the magnitude and direction of migration is strictly dependent on the rheological properties of the suspending medium. For an essentially non-elastic, shear-thinning fluid they observed that neutrally buoyant solid spheres migrate toward the wall in Poiseuille flow. On the contrary, in a viscoelastic medium the migration in the minimum shear-rate direction in Poiseuille flow (i.e. toward the center line) was observed. However, it should be mentioned that the rheological data were incomplete in these works.

Recently, the cross-streamline migration of bubbles in a viscoelastic channel flow has been studied [21]. When a surfactant is used, making the particle-fluid interface rigid, a transverse migration is observed with direction towards the channel walls. It must be remarked that in these experiments the particle
size is comparable with the channel dimension.

By using a perturbative method, Ho and Leal [22] developed an analytical theory by considering a Second Order Fluid as suspending medium. They found that, due to the normal stresses, a particle migrates in the direction of decreasing absolute shear rate, which is towards the axis channel in Poiseuille flow.

Joseph and co-workers [12, 13, 14] made 2D direct numerical simulations taking into account the viscoelasticity of the suspending liquid, modeled as an Oldroyd-B fluid with a Bird-Carreau shear rate viscosity dependence. They found that the migration direction depends on the competition of inertia, blockage ratio, elasticity and shear thinning of the fluid. Limiting to the Poiseuille flow case, they indicate that the elasticity of the fluid drives the particle towards the axis of the channel, whereas shear thinning makes the particle migrate towards the closest wall.

In this work we perform a systematic numerical study on the migration of a particle suspended in a viscoelastic fluid under Poiseuille flow at moderately large flow rates (as compared with [12]). The suspending medium is modeled as a Giesekus fluid [23], a model often capable of accurately describing experimental viscoelastic data. The study is carried out by neglecting fluid and particle inertia, and the analysis is performed through 2D Direct Numerical Simulations. The influence of the flow rate, particle dimension (as compared to the gap size) and the shear thinning on the migration velocity are investigated.

The momentum balance is discretized through the DEVSS (Discrete-Elastic-Viscous-Split-Stress) method that is one of the most robust formulations currently available. The viscoelastic constitutive equation is stabilized by implementing the SUPG (Streamline-Upwind-Petrov-Galerkin) technique. Furthermore, a log-conformation representation of the conformation tensor is used. Finally, an ALE particle mover [24] is adopted to handle the particle motion. To easily manage the particle motion, the mesh grid is translated along the flow direction with a velocity equal to the particle $x$-velocity. Consequently, the relative $x$-distance between the mesh nodes and the particle is kept unchanged and the particle only moves along the $y$-direction (i.e., the migration direction). In this way, remeshing due to ALE approach is only needed once-twice per simulation, always preserving the accuracy of the solution.
2. Governing equations

In figure 1a a schematic diagram of the problem is presented: a single, rigid, non-Brownian, inertialess, circular particle (2D problem) moves in a channel filled by a viscoelastic fluid in Poiseuille flow. The particle with diameter \( D_p = 2R_p \), denoted by \( P(t) \) and boundary \( \partial P(t) \), moves in a rectangular domain, \( \Omega \), with dimensions \( L \) and \( H \) along \( x \)- and \( y \)-axis respectively and external boundaries denoted by \( \Gamma_i \) \( (i = 1 \ldots 4) \). The Cartesian \( x \) and \( y \) coordinates are selected with the origin at the center of the domain. On the upper and lower boundaries, no-slip conditions are set whereas a flow rate \( Q \) is imposed on the left (inflow) boundary. Finally, periodicity is imposed on the left and right boundaries. For an unfilled Newtonian fluid, this would generate the well-known parabolic velocity profile depicted in figure 1b (dashed line). In the figure, the maximum velocity \( u_{\text{max}} \) and the average velocity \( \bar{u} \) are also reported. The solid line is the velocity profile for a viscoelastic fluid with a constitutive equation and model parameters chosen as discussed below and with the same flow rate as the Newtonian fluid.

The vector \( x_p = (x_p, y_p) \) gives the position of the center of the particle \( P \) whereas the particle angular rotation is denoted by \( \Theta = \Theta \hat{k} \) where \( \hat{k} \) is the unit vector in the direction normal to the \( x - y \) plane. The particle moves according to the imposed flow and its rigid-body motion is completely defined by the translational velocity, denoted by \( U_p = dx_p/dt = (U_p, V_p) \) and angular velocity, \( \omega = d\Theta/dt = \omega \hat{k} \).

The governing equations for the fluid domain, \( \Omega - P(t) \), neglecting inertia, read as follows:

\[ \nabla \cdot \sigma = 0 \]  
(1)

\[ \nabla \cdot u = 0 \]  
(2)

\[ \sigma = -pI + 2\eta_s D + \tau \]  
(3)

Equations (1)-(3) are the momentum balance, the mass balance (continuity) and the expression for the total stress, respectively. In these equations \( \sigma \), \( u \), \( p \), \( I \), \( \eta_s \), \( D \), are the stress tensor, the velocity vector, the pressure, the \( 2 \times 2 \) unity tensor, the viscosity of a Newtonian ‘solvent’, and the rate-of-deformation tensor, respectively. The viscoelastic stress tensor, \( \tau \), is written as (for the constitutive model chosen, see below):

\[ \tau = \frac{\eta}{\lambda}(c - I) \]  
(4)
where \( c \) is the ‘conformation tensor’, \( \eta \) is the polymer viscosity, and \( \lambda \) is the relaxation time.

We will model the viscoelastic fluid with the Giesekus constitutive equation (for \( c \)):

\[
\lambda \frac{\nabla}{\partial t} c + c - I + \alpha (c - I)^2 = 0 \tag{5}
\]

where \( \alpha \) is the so-called mobility parameter that modulates the shear thinning behavior. The symbol \( \nabla \) denotes the upper-convected time derivative, defined as:

\[
\frac{\nabla}{\partial t} c = \frac{\partial c}{\partial t} + u \cdot \nabla c - (\nabla u)^T \cdot c - c \cdot \nabla u \tag{6}
\]

Notice that the zero-shear-rate viscosity is given by \( \eta_0 = \eta_s + \eta \).

The boundary and initial conditions are:

\[
u = U_p + \omega \times (x - x_p) \quad \text{on } \partial \Omega(t) \tag{7}
\]

\[
u = 0 \quad \text{on } \Gamma_1 \text{ and } \Gamma_3 \tag{8}
\]

\[
c|_{t=0} = c_0 \tag{9}
\]

Equations (7)-(8) express the rigid-body motion on the particle boundary and the no-slip conditions on the upper and lower fluid boundaries. Since inertia is neglected, no initial condition of the velocity is required whereas the initial conformation tensor condition is necessary (Eq. (9)). In our simulations, we use a stress-free state, i.e. \( c|_{t=0} = I \), as initial condition over the whole fluid domain.

Furthermore, we impose periodic boundary conditions between the inflow \( \Gamma_4 \) and the outflow \( \Gamma_2 \) together a flow rate in inflow:

\[
u(-L/2, y) = \nu(L/2, y) \quad \forall \ y \in [-H/2, H/2] \tag{10}
\]

\[-t(-L/2, y) = t(L/2, y) - \Delta p \quad \forall \ y \in [-H/2, H/2] \tag{11}
\]

\[
\int_{\Gamma_4} u \ dy = Q \tag{12}
\]

In Eq. (11), \( t = \sigma \cdot n \) is the traction on the inflow/outflow curves, \( \Delta p = \frac{dp}{dx} L \) where \( \frac{dp}{dx} \) is the pressure gradient and \( n \) is the outwardly directed unit normal vector. The flow rate in Eq. (12) is imposed through a constraint where the associated Lagrange multiplier is identified as the pressure difference which is an unknown quantity determined by the Eq. (12) [25].
The equation governing the rigid-body motion (Eq. (7)) adds (for the 2D case) three additional unknowns, namely, the translational and angular velocities of the particle. So, to obtain the particle motion, it is necessary to consider the balance equations for drag force and torque acting on the particle boundary. Under the assumptions of absence of particle inertia, and of no ‘external’ forces and torques (force- and torque-free particle), such balance equations are given by:

\[ F = \int_{\partial P(t)} \sigma \cdot n ds = 0 \]  \hspace{1cm} (13)

\[ T = \int_{\partial P(t)} (x - x_p) \times (\sigma \cdot n) ds = 0 \]  \hspace{1cm} (14)

where \( F = (F_x, F_y) \) and \( T = Tk \) are the total force and torque on the particle boundary, respectively, and \( n \) is the outwardly directed unit normal vector on \( \partial P \).

The particle position and rotation are updated by integrating the following kinematic equations:

\[ \frac{dx_p}{dt} = U_p, \quad x_p|_{t=0} = x_{p,0} \] \hspace{1cm} (15)

\[ \frac{d\Theta}{dt} = \omega, \quad \Theta|_{t=0} = \Theta_0 \] \hspace{1cm} (16)

Finally, we define the Deborah number as \( De = \lambda Q/H^2 \). We prefer to choose \( (Q/H^2)^{-1} \) rather than \( (u_{\text{max}}/H)^{-1} \) as characteristic time of the process because of its independence of the fluid. Furthermore, according to such a definition, the Deborah number is directly linked to a macroscopic quantity.

3. Numerical procedure and code validation

The complete set of equations has been solved numerically through the Finite Element method. The particle motion is handled by using an ALE moving mesh method [24]: every time step the mesh nodes are moved in order to follow the particle motion. The mesh displacement (in terms of velocity) is calculated by solving a Laplace equation assuring smooth variation [24]. To take into account the relative motion of the mesh nodes with respect to the fluid velocity, the node velocities \( \dot{u} \) must be subtracted in the convective
terms of the constitutive equation (since inertia is neglected, no convective term appears in the momentum balance). The main advantage of the ALE formulation with respect to other methods is the use of a boundary-fitted mesh during the whole simulation leading to high accuracies around the particle where the largest gradients are expected. However, during the particle motion the mesh becomes more and more distorted reducing the solution accuracy. Therefore, a new mesh is generated and the fields on the old mesh need to be projected on the new one. Since the projection step involves interpolation, it is time-consuming and the accuracy is also affected so it should be reduced as much as possible.

In our formulation, due to the simple geometry involved, we use an ALE grid that rigidly moves along the flow direction with the particle $x-$velocity $U_p$. In such a way, the relative $x-$distance between the particle and each mesh node is kept constant and the only motion allowed is along the gradient direction (because of the particle migration). Such a technique drastically reduces the mesh distortion and the remeshing/projection/interpolation steps are limited to once-twice per simulation. Therefore the node velocity is given by $\mathbf{u} = (U_p, \hat{u}_y)$ where $\hat{u}_y$ is the node velocity along the $y-$axis, i.e. the solution of the Laplace equation as discussed above.

A DEVSS-G/SUPG formulation [26, 27, 28] with a log-representation for the conformation tensor [29, 30] is implemented in order to stabilize the system and achieve convergence up to high Deborah numbers. Finally, a second-order Crank-Nicolson/Adams-Bashforth scheme is used for time integration. The weak form and implementation details can be found in [31].

Mesh and time convergence are checked in all the calculations presented in this work. The length $L$ of the domain is checked as well to assure that the particle does not feel its image across the periodic boundaries. In what follows, we report some convergence results in order to show the sensibility of the mesh and time step size to the fluid parameters.

In figure 2 the migration velocity profiles as function of time are reported for two initial particle positions, $y_{p,0} = 0.20$ (left) and $y_{p,0} = 0.38$ (right). The parameters chosen are: $\beta = 0.1$, $De = 1.0$, $\alpha = 0.2$, $\eta_s/\eta_p = 0.1$ where $\beta = D_p/H$ is the blockage ratio. The details of the meshes labeled as M1 and M2 are reported in Table I for the two initial positions. In both cases, a mesh with 60 elements on the particle boundary is sufficient to achieve convergence although an extra refinement between the particle and the upper boundary is needed when the particle starts quite close to the wall. Regarding the time convergence, we found that a smaller step size should be chosen as the particle
is close to the wall due to fast dynamics involved. To better appreciate the mesh and time convergence, we report in Table II the values of the migration velocity at two times (t=2 and t=4) for the meshes and time steps used in figure 2. The largest discrepancies appear when the particle is close to the wall. However, the deviations are within 1%.

All the simulations presented in the next Section are performed by choosing a constant time step size $\Delta t = 0.01$ when the particle is far from the wall and $\Delta t = 0.005$ when it is close to it (with a distance less than one particle diameter).

Finally, we monitored the particle angular velocity as well (not reported). As in [31], we found that $\omega$ is less influenced by the mesh refinement and time step size, likely due to the larger values it assumes.

4. Results

As it is well known, an inertialess particle immersed in a Newtonian and inertialess fluid in Poiseuille flow indefinitely keeps its initial height, i.e. the particle goes along with the flow and no motion in the gradient direction occurs. This has also been verified for a viscous model, i.e., a generalized Newtonian fluid with a viscosity (nonlinearly) dependent on the shear rate. No migration is, in fact, found by varying the shear thinning content as well as the flow rate, suggesting that the transversal motion is a purely viscoelastic effect.

In the following, the effect of the viscoelasticity of the suspending fluid on the cross-streamline particle migration is, then, investigated. All the quantities to be discussed are dimensionless. The gap $H$ is chosen as the characteristic length, and $(H^2/Q)$ as the characteristic time scale.

In figure 3, the trajectories $y_p(t)$ for different initial values $y_{p,0}$ are reported. The parameters chosen are: $\beta = 0.1$, $De = 1.0$, $\alpha = 0.2$, $\eta_s/\eta_p = 0.1$. Such parameters are selected as they provide substantial viscoelastic effects ($De$) and shear thinning ($\alpha$). The shaded area in the figure represents the part of the channel that cannot be accessed by the particle. Because of symmetry, we only report (in figure 3 as well as in all the following figures) the curves which refer to the upper half of the channel (from $y = 0$ to $y = 0.5$). It is observed that the direction of the migration depends on the initial position of the particle: there is a ‘critical’ position $y_N$ along the gap above which the particle migrates towards the wall, and below which
it migrates towards the axis of the channel. In words, we are observing a
dynamical situation with a multiplicity of stable states \((y = 0, \ y = H/2 - R)\) separated by an unstable one \(y_N\).

It should be noticed in figure 3 that the dynamics is much faster when the
particle tends to the wall, whereas the migration velocity slowly changes in
time when the particle goes to the channel center. In figure 4, the temporal
trends of \(V_p\) corresponding to the trajectories \(y_p\) presented in figure 3 are
reported. The same notation as in figure 3 is adopted. For the sake of clarity,
we split the figure in two parts: in the upper part, the curves corresponding
to the particle motion towards the wall are reported whereas the curves in
the lower part refer to the migration towards the channel centerline. For each
initial particle position below \(y_N\), \(V_p\) follows a damped oscillatory transient
eventually approaching 0. The long term dynamics is monotonic, and the
stable equilibrium position \((V_p = 0)\) is reached from negative velocity values.
The initial velocity oscillations are due to the stress build-up around the
particle in the start-up of the process. On the other hand, when the particle
is initially located above the critical height, after the initial oscillation it
achieves a positive \(y-\)velocity, hence migrates towards the wall. When the
particle is close to the wall, \(V_p\) achieves a maximum before steeply decreasing
to 0 when approaching the solid boundary.

By combining the information supplied by figures 3 and 4, we report in
figure 5 \(V_p\) as a function of \(y_p\). The same notation of the two previous figures
is used. We notice that, excluding the fast initial stages of the dynamics at
various \(y_{p,0}\), the curves overlap, i.e., there exists a single master curve which
relates the migration velocity of the particle to its vertical position. A similar
slow attracting manifold was already identified in the case of simple shear
flow [31, 32]. In the Poiseuille flow case, however, the master curve is run in
two opposite directions from the critical value \(y_N\).

The initial fast transients when starting close to \(y_N\) are better appreci-
ated in the zoomed inset in figure 5. In all cases, starting from \(V_p = 0\), the
migration velocity first becomes negative, which corresponds to the early under-
shoots in figure 4. For the supercritical \((y_{p,0} > y_N)\) initial condition, the
migration velocity then becomes and remains positive, while for subcritical
conditions (and very close to \(y_N\)) the migration velocity, after passing through
a positive maximum, attains negative values when entering the master curve.
Those latter features translate into the circulation of the trajectory around
the initial condition (see inset).

In order to better identify the master curve from simulation results, fig-
Figure 6 reports data points taken upon complete extinction of the fast transients. Such data, with exclusion of those very close to the wall, are well interpolated by the odd polynomial function $V_p = -ay_p + by_p^3 + cy_p^{11}$, odd-ity being required by the symmetry with respect to the axis of the channel. (An odd polynomial was also found to properly describe migration data in Couette flow [31, 32].) The polynomial expression fits data quite well, but has no physical meaning. The intersection of the fitting curve and the horizontal $V_p = 0$ identifies the critical value $y_N$, which is 0.36 for the chosen parameters. The master curve for $V_p$ is equal to 0 in correspondence of three different vertical positions: The axis of the channel, the neutral height, and the close-to-wall height. By inspection, it is apparent that the neutral height represents an unstable equilibrium position for the particle, i.e., any external disturbance would make it move away from $y_N$.

The existence of a master curve for $V_p(y_p)$ implies that all the trajectories superimpose by shifting them in time. Therefore, two and only two trajectories $y_p(t)$ exist, one leading to the channel axis, and the other to the wall. Time shifts of data of figure 3 leads to figure 7, where the solid lines result from the integration of the evolution equation $dy_p/dt = V_p(y_p) = -ay_p + by_p^3 + cy_p^{11}$. Indeed, the different curves reported in figure 3 are parts of two master trajectories, below and above the separatrix, travelled by the particle from different initial conditions.

A master curve is found for the particle angular velocity, as well. This is shown in figure 8 where $\omega$ is reported versus $y_p$ for different initial values $y_{p,0}$. In the upper part of the channel $\omega$ is always negative regardless the direction of the migration, i.e., the particle rotates counterclockwise; in the lower part of the channel (not shown) rotation is clockwise. Needless to say, at the channel axis the particle does not rotate. It is worth remarking here that the master curve for $\omega$ is an odd function, while it was an even function in the case of Couette flow [31, 32], which is of course explained by the different symmetries of the imposed flows.

Notice that $\omega$ does not tend to zero when the particle is approaching the wall. The particle, indeed, rotates even when very near to the wall. Simulation accuracy allows us to study in detail the situation when the particle-wall distance is very small. In figure 9, we report the dimensionless $x-$translational velocity and the dimensionless tangential velocity at the particle contour as a function of the vertical coordinate. It is observed that those quantities converge to each other at the wall: the equality $U_p = |R_p \omega|$ is indeed the condition of rolling motion without slip.
The effect of confinement on migration is investigated by varying the parameter $\beta$. The master curves $V_p$ vs $y_p$ for different $\beta$—values are reported in figure 10. The particle position is here scaled by $H/2 - R_p$, to account for the variation of the accessible region of the channel with $\beta$: In this way, the curves in the figure vary between 0 and 1. It can be noted that the nonmonotonic character of the master curve at low $\beta$—value (solid line) is lost at high $\beta$ (dashed line). In other words, when the blockage ratio increases, the migration always occurs towards the wall.

Such feature of the migration dynamics is clearly illustrated in figure 11 where the critical height $y_N$ is reported as a function of the blockage ratio. The line to guide the eye in the figure divides the semi-channel into two parts: above the line the particle goes to the wall, whereas below the line it goes to axis. It is apparent that at very large blockage ratios the wall becomes the only attractor. This observation compares well (if qualitatively) with the experimental data of [21]. The authors, indeed, always found migration towards the wall at their very strong confinement. The migration velocity profile reported in figure 4 of [21] is qualitatively similar to the one reported in figure 10 for $\beta = 0.8$ including the terminal region towards the wall, where $V_p$ achieves a maximum and decreases to zero. A remark on small blockage ratios is in order. Due to computational limitations, it is not possible to run simulations with $\beta < 0.025$, so we can not assess whether or not there are some very low values of $\beta$ where the particle migrates towards axis for strictly every value $y_{p,0}$.

For what matters the effect of the fluid parameters on the particle migration, the same general conclusions as in [31] are found: by increasing both the Deborah number and the fluid shear thinning ($\alpha$), migration (towards the wall or the channel axis) becomes faster, still showing the phenomenology reported above, i.e. with a migration direction depending on the particle initial position. It is also interesting to examine the influence of these two parameters on the position of the neutral height. Regarding the Deborah number, we in fact do not find any significant variation in $y_N$ when varying $De$ up to 3. On the contrary, shear thinning has a more relevant influence as reported in figure 12 for $De = 1.0$. A smaller $\alpha$—value leads to a larger extension of the region where the particle migrates towards the channel centerline, i.e., the neutral height moves up. In particular, even for strong confinements, the axis attraction still exists. At larger $\alpha$—values, the separatrix moves instead towards the axis of the channel, thus reducing the center-line attracting region until it disappears at large $\beta$. 

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5. Conclusions

In this work, the cross-streamline migration of a single, inertialess, solid particle in a viscoelastic fluid under Poiseuille flow is investigated through 2D numerical simulations. The balance equations are solved by the finite element method. An ALE particle mover is combined with a DEVSS-G/SUPG formulation and with a log-representation of the conformation tensor to easily manage the particle motion and guarantee numerical convergence at relatively large Deborah numbers. In order to reduce the mesh elements deformation, the grid is translated along the flow direction with velocity equal to the $x -$translational velocity of the particle.

Viscoelasticity induces particle crossflow migration. Particle trajectories for different initial positions are shown to collapse on master curves, and the existence of two (symmetric) separatrices in the gap is found. In each half gap, a particle starting between the separatrix and the channel centerline moves towards the channel axis, whereas a particle starting in the region between the separatrix and the wall moves to the solid boundary. In other words, three equilibrium positions in each half gap are identified along the gradient direction: the centerline (stable), the separatrix (unstable) and the wall (stable). This multiplicity is at variance with the simple shear flow case examined in [31, 32], where migration was always towards the closest wall. On the other hand, a similar multiplicity was found in the case of a sphere in a Newtonian liquid at non-zero Reynolds numbers [10], though migration in the latter case is reversed with respect to our findings, i.e., the sphere migrates towards two symmetric vertical positions (the counterparts of our separatrices).

Degree of confinement as well as fluid properties do influence the vertical position of the separatrices. In particular, stronger confinements move the neutral position towards the channel axis, increasing, in fact, the channel portion where the particle migrates to the wall. When the particle size is comparable to the channel gap, our simulations show that the separatrix coincides with the centerline: the particle migrates towards the wall regardless of its initial position. The latter result qualitatively resembles the experimental observations reported in [21]. Conversely, when the sphere is very small, the attraction towards channel axis extends to the whole channel, in qualitative agreement with calculations in [22]. Regarding the fluid rheology, it is found that large Deborah numbers lead to a faster migration, and shear thinning promotes the displacement of the neutral positions towards
the centerline.

In concluding, we wish to emphasize that all the results presented in this paper rest on the 2D nature of the flow cell. On the other hand, our previous findings for the shear flow case [31, 32], where the qualitative features of the 2D and 3D solutions were found to be essentially the same, make us confident that the present calculations, in fact, catch the main phenomenology of the 3D case. To proceed to compare with the available experimental data, extension of our calculations to 3D situations seems mandatory, and of course requires heavier computational load. Work is in progress in this direction.

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References


Captions

Figure 1. Sketch of the flow cell and of the adopted coordinate system. The undisturbed velocity profile for a Newtonian (dashed line) and a viscoelastic (solid line) fluid is also shown for the same flow rate.

Figure 2. Migration velocity of the particle $V_p$ as function of time $t$ for different mesh resolutions and time step sizes. Two initial particle positions are reported ($y_{p,0} = 0.20$ on the left and $y_{p,0} = 0.38$ on the right). The other parameters are: $\beta = 0.1$, $De = 1.0$, $\alpha = 0.2$, $\eta_s/\eta_p = 0.1$.

Figure 3. Vertical position of the particle $y_p$ as function of time $t$ for different values of its initial position $y_{p,0}$. The other parameters are: $\beta = 0.1$, $De = 1.0$, $\alpha = 0.2$, $\eta_s/\eta_p = 0.1$. The shaded area represents the region where the center of the particle cannot access.

Figure 4. Migration velocity of the particle $V_p$ as function of time $t$ for different values of its initial position $y_{p,0}$. The upper plot refers to the cases when the particle migrates towards the wall whereas the curves in the lower part correspond to the migration towards the channel centerline. The other parameters are: $\beta = 0.1$, $De = 1.0$, $\alpha = 0.2$, $\eta_s/\eta_p = 0.1$.

Figure 5. Migration velocity of the particle $V_p$ as function of its position $y_p$ for different values of its initial position $y_{p,0}$. In the inset a zoom around the position of the separatrix is reported. The other parameters are: $\beta = 0.1$, $De = 1.0$, $\alpha = 0.2$, $\eta_s/\eta_p = 0.1$. The shaded area represents the region where the center of the particle cannot access.

Figure 6. Migration velocity of the particle $V_p$ as function of its position $y_p$ for different values of its initial position $y_{p,0}$ after the transient extinguished (symbols). The solid line is the interpolation of the simulation data (see text). The other parameters are: $\beta = 0.1$, $De = 1.0$, $\alpha = 0.2$, $\eta_s/\eta_p = 0.1$. The shaded area represents the region where the center of the particle cannot access.

Figure 7. Vertical position of the particle $y_p$ as function of the shifted time for different values of its initial position $y_{p,0}$. The solid line is the integration of the evolution equation $dy_p/dt = V_p(y_p)$ (see text). The other parameters
are: $\beta = 0.1$, $De = 1.0$, $\alpha = 0.2$, $\eta_s/\eta_p = 0.1$. The shaded area represents the region where the center of the particle cannot access.

**Figure 8.** Angular velocity of the particle $\omega$ as function of particle position $y_p$ for different values of its initial position $y_{p,0}$. The other parameters are: $\beta = 0.1$, $De = 1.0$, $\alpha = 0.2$, $\eta_s/\eta_p = 0.1$. The shaded area represents the region where the center of the particle cannot access.

**Figure 9.** $x$–translational velocity $U_p$ (dashed line) and the absolute value of angular velocity times the particle radius $|\omega R_p|$ as a function of particle position $y_p$. The black circle is the intersection point between the two curves.

**Figure 10.** Migration velocity of the particle $V_p$ as function of its position $y_p$ (scaled by $H/2 - R_p$) for different values of the blockage ratio $\beta$. The other parameters are: $De = 1.0$, $\alpha = 0.2$, $\eta_s/\eta_p = 0.1$.

**Figure 11.** Position of the neutral height $y_N$ as function of blockage ratio $\beta$. The other parameters are: $De = 1.0$, $\alpha = 0.2$, $\eta_s/\eta_p = 0.1$. The shaded area represents the region where the center of the particle cannot access (dependent on $\beta$).

**Figure 12.** Position of the neutral height $y_N$ as function of blockage ratio $\beta$ for different values of $\alpha$. The other parameters are: $De = 1.0$, $\eta_s/\eta_p = 0.1$. The shaded area represents the region where the center of the particle cannot access (dependent on $\beta$).
<table>
<thead>
<tr>
<th>Mesh label</th>
<th>M1</th>
<th>M2</th>
<th>M1</th>
<th>M2</th>
</tr>
</thead>
<tbody>
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<td>0.38</td>
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<td>70</td>
<td>60</td>
<td>70</td>
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<tr>
<td>#el. in the mesh</td>
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<td>12616</td>
<td>7334</td>
<td>9602</td>
</tr>
</tbody>
</table>

Table 1: Mesh parameters (for $\beta = 0.1$)
\[ y_{p,0} = 0.20 \begin{array}{ccc} \text{t = 2} & \text{t = 4} \\ \\ y_{p,0} = 0.20 \end{array} \]

\[
\begin{array}{l}
\text{M1, } \Delta t = 0.01 -2.7212 \cdot 10^{-3} -3.5420 \cdot 10^{-3} \\
\text{M2, } \Delta t = 0.01 -2.7144 \cdot 10^{-3} -3.5388 \cdot 10^{-3} \\
\text{M1, } \Delta t = 0.0075 -2.7220 \cdot 10^{-3} -3.5419 \cdot 10^{-3} \\
\text{M1, } \Delta t = 0.005 4.0106 \cdot 10^{-3} 4.8724 \cdot 10^{-3} \\
\text{M2, } \Delta t = 0.005 3.9943 \cdot 10^{-3} 4.8140 \cdot 10^{-3} \\
\text{M1, } \Delta t = 0.0025 4.0145 \cdot 10^{-3} 4.8687 \cdot 10^{-3} \\
\end{array}
\]

Table 2: Migration velocity at \( t = 2 \) and \( t = 4 \) for the meshes and time steps as in figure 2.
Figure 1:
Figure 2:
Figure 4:
Figure 5:
Figure 6:
Figure 8:
Figure 9:
Figure 10:
Figure 11:
Figure 12: